Matlab Project Assignment

Task #01

The data $x[n] = \theta n + w[n]$for $n = 0, 1, 2, \ldots N-1$ are observed, where $w[n]$ is zero-mean WGN with variance σ^2.

The objective is to estimate θ.

Showing that the estimator $\theta = \frac{\sum_{n=0}^{N-1} x[n]n}{\sum_{n=0}^{N-1} n^2}$ is efficient by Monte Carlo (MC) simulation.

```
close all ; clear all ; clc ;


MC = 10000 ; % Number of Monte Carlo loops
N = 10 ; % Number of samples in the data
n = [0:N-1] ; % row vector to represent the index of each sample
noise_var = 2 ; % Variance of the AWGN
theta = 10 ; % Mean


noise = 0 + randn(MC,N)*sqrt(noise_var) ; % w[n] ==> AWGN with zero mean and variance = no
signal = theta*n ; % theta*n ==> signal of interset
x = signal + noise ; % captured samples with MC rows and N columns

estimate = sum(x.*n,2)/sum(n.^2) ; % (sum of wiegthed samples / sum of squared indecies)

mean_estimate = mean(estimate) % mean of the estimate along MC loops
```

```
mean_estimate = 9.9998
```

```
var_estimate = var(estimate) % variance of the estimate along MC loops
```

```
var_estimate = 0.0071
```

```
CRLB = (noise_var)/(sum(n.^2)) % (variance / sum of squared indecies )
```

```
CRLB = 0.0070
```

```
% The estimate is efficient ("good enough") if the error in its statistics is less than so
if abs((mean_estimate-theta)/theta)<.03 && abs((var_estimate-CRLB)/CRLB)<.03
    disp('The estimator is an efficeint estimator ')
else
    disp('The estimator is not an efficeint estimator ')
end
```

```
The estimator is an efficeint estimator
```

Generating figure that shows that simulated and theoretical probability density functions agree for the previous estimator.

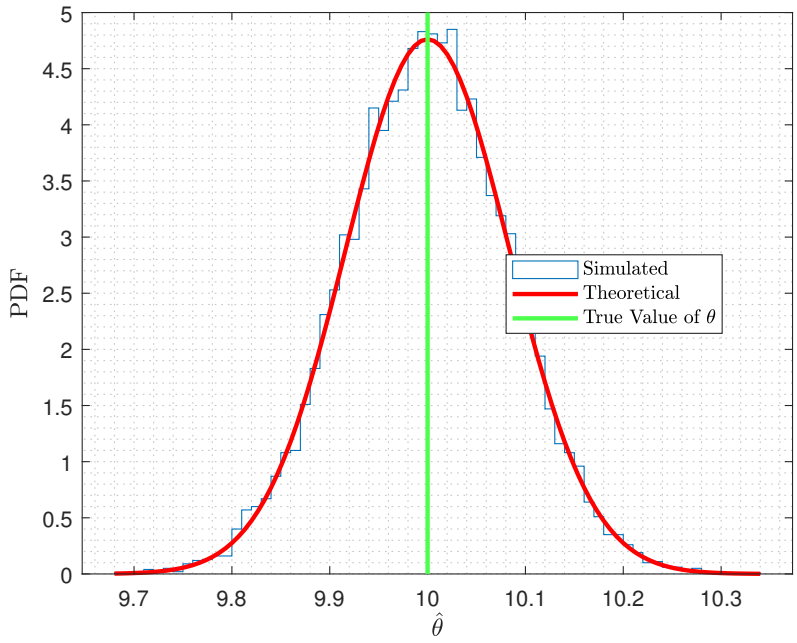
```
figure

H = histogram(estimate,'Normalization','pdf','DisplayStyle',"stairs") ;
hold on
x2 = linspace(H.BinLimits(1),H.BinLimits(2),100) ; % range of x for theoretical results

theor_PDF = normpdf(x2,theta,sqrt(CRLB)) ;
plot(x2, theor_PDF,'r-','Linewidth',2) ;

xline(theta,'-g','Linewidth',2) ;

grid minor
legend('Simulated','Theoretical','True Value of $\theta$','interpreter','latex','Location
xlabel('$\hat{\theta}$','interpreter','latex') ;
ylabel('PDF','interpreter','latex') ;
```




```
%%% saving plot
%% saveas(gca,'two_PDFs.pdf');
% export_fig two_PDFs.pdf
% system('pdfcrop two_PDFs.pdf two_PDFs.pdf');
```

What happens when $N = 1$ and $N \to \infty$?

```
% close all ; clear all ; clc ;
MC = 10 ; % Number of Monte Carlo loops (reduced to speed up the iterations)
N = 10.^[0:7] ; % Number of samples in the data (swept)
noise_var = 2 ; % Variance of the AWGN
theta = 10 ; % Mean

for ii = 1:length(N) % used as an index to sweep N
    n = [0:N(ii)-1] ; % row vector to represent the index of each sample

    x = theta*n + randn(MC,N(ii))*sqrt(noise_var)  ; % captured samples with MC rows and N

    estimate = sum(x.*n,2)/sum(n.^2) ; % (sum of wiegthed samples / sum of squared indecie
```

```
    mean_estimate(ii) = mean(estimate) ; % mean of the estimate along MC loops
    var_estimate(ii) = var(estimate) ; % variance of the estimate along MC loops

    CRLB(ii) = (noise_var)/(sum(n.^2)) ; % (variance / sum of squared indecies )
end
format long
mean_estimate
```

```
mean_estimate = 1x8
                NaN  10.025149688833483   9.998238217515659   9.999998553326240   9.9999
                   10.000000018113194   9.999999999526327  10.000000000026077
```

```
var_estimate
```

```
var_estimate = 1x8
                NaN   0.006266279213217   0.000012346504144   0.000000006148877   0.0000
                   0.000000000000008   0.000000000000000   0.000000000000000
```

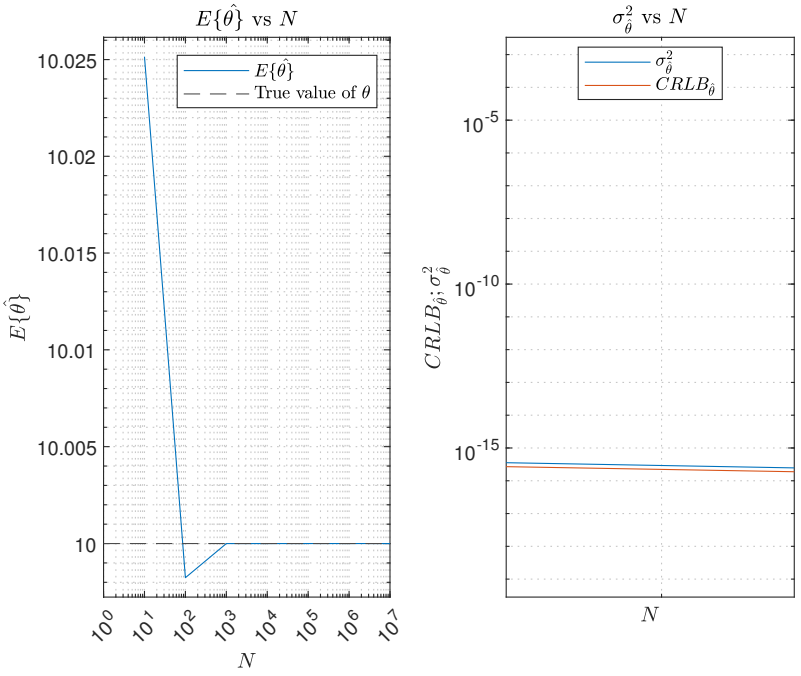
```
CRLB
```

```
CRLB = 1x8
                Inf   0.007017543859649   0.000006091061367   0.000000006009011   0.0000
                   0.000000000000006   0.000000000000000   0.000000000000000
```

```
% plotting spagetti
subplot(1,2,1)
loglog(N,mean_estimate)
hold on
yline(theta,'k--')
hold off
ylim([0.9999*min(mean_estimate) 1.0001*max(mean_estimate)])
grid minor
title('$E\{\hat{\theta\}}$ vs $N$','interpreter','latex')
xlabel('$N$','interpreter','latex')
xticks(N)
ylabel('$E\{\hat{\theta\}}$','interpreter','latex')
legend('$E\{\hat{\theta\}}$','True value of $\theta$','interpreter','latex')

subplot(1,2,2)
loglog(N,var_estimate)
hold on
loglog(N,CRLB)
hold off
```

```
grid minor
title('$\sigma^2_{\hat{\theta}}$ vs $N$','interpreter','latex')
xlabel('$N$','interpreter','latex')
xticks(N)
ylabel('$CRLB_{\hat{\theta}} ; \sigma^2_{\hat{\theta}}$','interpreter','latex')
legend('$\sigma^2_{\hat{\theta}}$','$CRLB_{\hat{\theta}}$','interpreter','latex','Locatio
```




```
%%% saving plot
%% saveas(gca,'N_difference.pdf');
% export_fig N_difference.pdf
% system('pdfcrop N_difference.pdf N_difference.pdf');
```