\documentclass[12pt]{article}

\usepackage{amsmath}

\usepackage{graphicx}

\usepackage{hyperref}

\usepackage[latin1]{inputenc}

\title{Columbia Unversity\\

\ COMSW4771\\

\ Dr. James McInerney\\

\ Homework 0}

\author{Submitted By: Alexander Stein, as5281}

\date{September 10th, 2017}

\begin{document}

\maketitle

\item

Problem 1.1 \\ \\

(i) What is the marginal distribution of X?

%Usage: \btable{table specs}{caption}{reference label}

\newcommand{\btable}[3]{

\begin{table}[htbp]

\begin{center}

\caption{#2\label{#3}}

\begin{tabular}{#1}}

\newcommand{\etable}{\end{tabular}

\end{center}

\end{table}}

%Usage: \mc{number of columns spanned}{major column heading}

\newcommand{\mc}[3]{\multicolumn{#1}{c}{#2}}

\btable{l|ccc|cc}{Marginal Probabilities}{results} \hline\hline

%6 fields, justified left, center x 5

%double horizontal line at top, 1 vertical bar

& Y1 & Y2 & Y3 & P\textsubscript x(i) \\ \hline

X1 & 0.1 & 0.2 & 0.3 & 0.6 & \\

X2 & 0.2 & 0.1 & 0.1 & 0.4 & \\ \hline\hline

P\textsubscript y(j) & 0.3 & 0.3 & 0.4 & 1.00 & \\ \hline

\etable \\

(ii) What is Pr[Y=1 \text{\textbar} X=2]? \\ \\

P(Y=1\text{\textbar}X=2) = P(Y=1, X=2) / P(X=2) \\

P(Y=1\text{\textbar}X=2) = (0.2)/(0.4) \\

P(Y=1\text{\textbar}X=2) = 1/2 \\ \\

(iii) Let f : x \rightarrow x^2.\text{ What is E[f(x)\text{\textbar}Y=1]?} \\ \\

$$E(x|Y=y) \\

= \sum\_{x \in X}xP(X=x | Y=y) \\

= \sum\_{x \in X}xP(x=x, Y=y) / P(Y=y)$$ \\ \\

$$E(f(x)|Y=1)\\

= \sum\_{x \in X}x^2 P(X=x, Y=1) / P(Y=1) \\

= (1^2)\*(0.1)/(0.3) + (2^2)\*(0.2)/(0.3) \\

= 1/3 + 8/3 \\

= 3 $$\\ \\

Problem 1.2 \\ \\

(i) Verify that $\frac{1}{\theta}e^{-x/\theta}$ is a probability distribution.\\ \\

To verify, we will show that the integral from 0 to $\infty$ is 1 \\

$$\int\_{0}^{\infty} \frac{1}{\theta}e^{x/\theta} dx $$

$$u = -\frac{x}{\theta} \rightarrow dx = -\theta du $$

$$= -\frac{1}{\theta}\int\_{0}^{\infty}\theta e^u du$$

$$= -e^u = -e^{-x/\theta}\mid\_{0}^{\infty}$$

$$= (-e^{-\infty/\theta}) - (-e^{-0/\theta})$$

$$= 1$\\

(ii) What is E[X] ? \\

$$\mu = E[X] = \int\_{-\infty}^{\infty}xf(x)dx = \frac{1}{\theta}\int\_{0}^{\infty}e^{-x/\theta}$$

$$u=x/\theta dx = \theta du$$

$$=\theta\int ue^{-u}du$$

$$f=u \text{ and } g'=e^{-u}$$

$$f'=1 \text{ and } g=-e^{-u}$$

$$=fg - \int f'g \rightarrow -u\theta e^{-u} + \theta\int e^{-u}du$$

$$=-ue^{-u} -e^{-u} = -xe^{-x/\theta}-\theta e^{-x/\theta} \mid\_{0}^{\infty}$$

$$=(0)-(-\theta) = \theta$$ \\

(iii) What is Var(X) ? \\

$$ Var(X) = E((X-\mu)^2) = E(X^2) - E(X)^2 = E(X^2) - \mu $$

$$ \text{so all we need to find is } E(X^2) $$

$$E(X^2) = \int\_{0}^{\infty}\frac{x^2}{\theta}e^{-x/\theta}dx$$

$$u=x/\theta \text{ and } dx = \theta du $$

$$=\theta^2\int u^2e^{-u}du $$

$$f=u^2 \text{ and } g'=e^{-u} $$

$$f'=2u \text{ and } g=-e^{-u} $$

$$=fg - \int f'g \rightarrow \theta^2[ -u^2e^{-u} - \int\_{0}^{\infty} -2ue^{-u}du] $$

$$f\_{1}=u \text{ and } g\_{1}'=e^{-u}$$

$$f\_{1}'=1 \text{ and } g\_{1}=-e^{-u}$$

$$=\theta^2[ -u^2e^{-u} +2[-ue^{-u} + \int\_{0}^{\infty} - e^{-u}du]]$$

$$=\theta^2[-u^2e^{-u}-2ue^{-u}-2e^{-u}]$$

$$=-(x^2e^{-x/\theta}+2\theta xe^{-x/\theta}+2\theta^2e^{-x/\theta})\mid\_{0}^{\infty}$$

$$=(0)-(-2\theta^2) = 2\theta^2$$\\

$$Var(X) = E(X^2) - E(X)^2 = 2\theta^2 - \theta^2 = \theta^2$$ \\ \\ \\ \\

Problem 1.3 \\

\newcommand{\btable}[3]{

\begin{table}[htbp]

\begin{center}

\caption{#2\label{#3}}

\begin{tabular}{#1}}

\newcommand{\etable}{\end{tabular}

\end{center}

\end{table}}

%Usage: \mc{number of columns spanned}{major column heading}

\newcommand{\mc}[3]{\multicolumn{#1}{c}{#2}}

\btable{l|ccc|cc}{Burglary Probabilities}{results} \hline\hline

%6 fields, justified left, center x 5

%double horizontal line at top, 1 vertical bar

& Robbery(5\%) & No Robbery(95\%) \\ \hline

Alarm & 99\% & 10\% \\

No Alarm & 1\% & 90\% \\ \hline\hline

\etable \\

We are effectively using Bayes Theroem to solve this problem:

$$P("true\\_alarm")=P(Robbery)\*P(Alarm\mid Robbery)$$

$$=(0.05) \* (0.99) = 0.0495$$

$$P("false\\_alarm")=P(No\\_Robbery)\*P(Alarm|No\\_Robbery)$$

$$=(0.95)\*(0.10) = 0.095$$

$$P("any\\_alarm")=P("true\\_alarm") + P("false\\_alarm")$$

$$=0.1445$$ \\ \\

$$P(Robbery\mid "any\\_alarm") = \frac{P("true\\_alarm")}{P("any\\_alarm")} = \frac{0.0495}{0.1445} = 0.34256$$ \\ \\

Problem 2.1 \\ \\

Subspace S is spanned by

$a\_1 = \left|\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} \right|$,

$a\_2 = \left| \begin{array}{c} 2 \\ 8 \\ 3 \\ 2

\end{array} \right|$,

$a\_3 = \left| \begin{array}{c} 3 \\ 10 \\ 6 \\ 6

\end{array} \right|$. \\ \\

(i) what isthe dimension of the subspace S? \\ \\

we can see that $a\_1 + a\_2 - a\_3 = 0$, But there is no scalar multiple k such that $ka\_1=a\_2$. Thus, $a\_1$ and $a\_2$ are linearly independent, and the dimension of S is 2. \\ \\

(ii) Project the vector $\left| \begin{array}{c} 6 \\ 5 \\ 9 \\ 2 \end{array} \right|$ onto the subspace S\\

First we need an orthogonal basis, we will use Graham-Schmitt Process\\ \\

$$v\_1 = a\_1$$

$$v\_2 = a\_2 - \frac{(a\_2,v\_1)}{\left| \left| v\_1 \right| \right|^2}v\_1$$

$$v\_2 = \left| \begin{array}{c} 5/6 \\ 17/3 \\ -1/2 \\ -8/3 \end{array} \right|$$ \\ \\

Now we can project... \\ \\

$$proj\_s u=\frac{(u,v\_1)}{\left| \left| v\_1 \right| \right|^2}v\_1 + \frac{(u,v\_2)}{\left| \left| v\_2 \right| \right|^2}v\_2 = 1.7v\_1 + 0.58v\_2$$ \\ \\

Problem 2.2 - SKIPPED, don't know how \\ \\

Problem 3.1 - SKIPPED, don't know how \\ \\ \\ \\ \\

Problem 4.1 \\ \\

CODE:

\begin{verbatim}

import numpy as np

import matplotlib.pyplot as plt

from scipy.io import loadmat

from scipy import linalg as LA

mat\_dict = {}

mat\_dict.update(loadmat('hw0data.mat'))

M = mat\_dict['M']

print "The dimensions of matrix M: ", M.shape

print "The 4th row of M: ", M[3,:]

print "The 5th column of M: ", M[:,4]

print "Mean value of the 5th column of M: ", np.mean(M[:,4])

print "histogramming the 4th row..."

plt.hist(M[3,:], rwidth=0.5)

plt.title("Histogram of M[3,:]")

plt.xlabel("Value")

plt.ylabel("Frequency")

plt.show()

M\_square = np.dot(np.transpose(M), M)

e\_vals, e\_vecs = LA.eig(M\_square)

print "Top three eigenvalues of M\_transpose \* M: ", e\_vals[0:3]

\end{verbatim}

\end{document}