#3 Т11

$H_0 \quad \xi \sim \rho_0(x) = 1\{(0,1)\}$

$H_1 \quad \xi \sim \rho_1(x) = \dfrac{e}{e-1} e^{-x} \{(0,1)\}$

а) $n = 1 \qquad \alpha$

ТНП

$$\rho = \frac{L_1}{L_0} = \frac{\frac{e}{e-1} e^{-x}}{1} \geq c$$

$$e^{-x} \geq B \qquad x \leq A$$

$$P(x \leq A \mid H_0) = \alpha$$

$$\int_0^A 1\, dx = A = \alpha$$

$G \quad x \leq A$

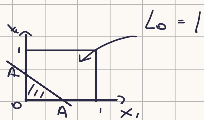
$\alpha_1 = \alpha$

$\alpha_2 = 1 - w \qquad w = P(x \leq A \mid H_1) = \int_0^\alpha \frac{e}{e-1} e^{-x} dx = \frac{e}{e-1}(1 - e^{-\alpha})$

б) $n = 2$

$$\rho = \frac{L_1}{L_0} = \frac{\left(\frac{e}{e-1}\right)^2 e^{-x_1} e^{-x_2}}{1 \cdot 1} \geq c$$

$$e^{-(x_1 + x_2)} \geq B \qquad x_1 + x_2 \leq A$$



$$P(x_1 + x_2 \leq A \mid H_0) = \alpha$$

$$\iint\limits_{x_1 + x_2 \leq A} 1\, dx_1 dx_2 = \frac{A^2}{2} = \alpha \qquad A = \sqrt{2\alpha} \qquad 2\alpha < 1$$

$G \quad x_1 + x_2 \leq \sqrt{2\alpha}$

$\alpha_1 = \alpha \qquad w = P(x_1 + x_2 \leq A \mid H_1) = \iint\limits_{x_1 + x_2 \leq A} \left(\frac{e}{e-1}\right)^2 e^{-x_1 - x_2} dx_1 dx_2$

$= \left(\frac{e}{e-1}\right)^2 \int_0^A dx_1 \int_0^{A - x_1} e^{-x_1} e^{-x_2} dx_2 = \left(\frac{e}{e-1}\right)^2 \int_0^A e^{-x_1}(1 - e^{-(A - x_1)})\, dx_1 =$

$= \left(\frac{e}{e-1}\right)^2 \int_0^A e^{-x_1} - e^{-A}\, dx_1 = \left(\frac{e}{e-1}\right)^2 (1 - e^{-A} - A e^{-A}) \qquad \alpha_2 = 1 - w$

$A = \sqrt{2\alpha}$

c) $\rho = \frac{L_1}{L_0} = \prod_{i=1}^{n} \frac{p_1(x_i)}{p_0(x_i)} \geq c$

$$\ln \rho = \sum \underbrace{\ln \frac{p_1(x_i)}{p_0(x_i)}}_{\eta_i} \geq \ln c$$

$$\frac{\sum \eta_i - n M\eta}{\sqrt{n D\eta_i}} \sim N(0,1)$$

$$P(\ln \rho \geq \ln c \mid H_0) = \alpha$$

$$\eta = \ln \left(\frac{e}{e-1} e^{-x}\right) = \ln \frac{e}{e-1} - x$$

$$\ln \rho = \sum \ln \frac{e}{e-1} - \sum x_i \geq \ln c \qquad G \quad \sum x_i \leq A$$

"избавились от логарифмов"

$$P(\sum x_i \leq A \mid H_0) = \alpha$$

$$P\left(\frac{\sum x_i - n M x}{\sqrt{n D x}} \leq \frac{A - n M x}{\sqrt{n D x}} \Big|_{H_0}\right) = \alpha$$

$$Mx = \frac{1}{2}$$
$$Dx = \frac{1}{12}(b-a)^2$$

$$\frac{A - n\frac{1}{2}}{\sqrt{n\frac{1}{12}}} = u$$

$$A = n\frac{1}{2} + u_\alpha \sqrt{\frac{n}{12}}$$

$$G \quad \sum x_i \leq n\frac{1}{2} + u_\alpha \sqrt{\frac{n}{12}}$$

$\alpha_1 \quad \alpha$

$$W = P(\sum x_i \leq A \mid H_1) = P\left(\frac{\sum x_i - n M x}{\sqrt{n D x}} \leq \frac{A - n M x}{\sqrt{n D x}} \Big|_{H_1}\right)$$

$$Mx = \int_0^1 x \frac{e}{e-1} e^{-x} dx \overset{!}{=} \frac{e-2}{e-1} \qquad Mx^2 = \frac{2e-5}{e-1}$$

$$Dx = \frac{e^2-3e+1}{(e-1)^2}$$

$$W = \int_{-\infty}^{B} \frac{1}{\sqrt{2\pi}} e^{-x^2/2} dx \underset{n\to\infty}{\to 1} \qquad B = \frac{\frac{n}{2} + u_\alpha\sqrt{\frac{n}{12}} - n\frac{e-2}{e-1}}{\sqrt{n \frac{e^2-3e+1}{(e-1)^2}}} \ominus \qquad \alpha_2 = 1 - W$$

$$\ominus \frac{\sqrt{n}\overbrace{\left(\frac{1}{2} - \frac{e-2}{e-1}\right)}^{0{,}082} + u_\alpha\sqrt{\frac{1}{12}}}{\frac{e^2-3e+1}{(e-1)^2}} \underset{n\to+\infty}{\longrightarrow} +\infty$$

$u_\alpha < 0$

крит состоятельный

9 $\quad G: x_{min} \leq c \qquad H_0: \xi \sim R(0,1)$

$P(\bar{x}_n \in G \mid H_0) = \alpha$

$\alpha = P(x_{min} \in G \mid H_0)$

$$P(x_{min} \leq c) = 1 - (1 - F(c))^n = \alpha$$
$$\alpha - 1 = -(1-c)^n$$
$$c = 1 - \sqrt[n]{1-\alpha}$$

$\alpha = \alpha$

$$W = P(\bar{x}_n \in G \mid H_1) = P(x_{min} \leq c \mid H_1) = 1 - (1 - F_1(c))^n \circleddash$$

$$F_1 = \int_0^x \frac{e}{e-1} e^{-t}\, dt = \frac{e}{e-1}\left(1 - e^{-x}\right)$$

$$\circleddash\ 1 - \left(1 - \frac{e}{e-1}\left(1 - \exp(-1 + \sqrt[n]{1-\alpha}\right)\right)^n = 1 - \left(1 - \frac{e}{e-1} + \frac{\exp(\sqrt[n]{1-\alpha})}{e}\frac{e}{e-1}\right)^n \underset{n\to\infty}{\not\to} 1$$

сост

$$\exp(\sqrt[n]{1-\alpha}) = \exp\left(\exp\left(\frac{1}{n} \ln(1-\alpha)\right)\right) \overset{\text{Тейл}}{=}$$

$$= \exp\left(1 + \frac{1}{n}\ln(1-\alpha) + o\left(\frac{1}{n}\right)\right) = e\left(1 + \frac{1}{n}\cdot \ln(1-\alpha) + o\left(\frac{1}{n}\right)\right)$$

$$\Rightarrow W = 1 - \left(1 + \frac{e + \frac{e}{n}\ln(1-\alpha) + o\left(\frac{1}{n}\right)}{e-1} - \frac{e}{e-1}\right)^n =$$

$$= 1 - \left(1 + \frac{e\ln(1-\alpha)}{e-1}\frac{1}{n} + \underbrace{o\left(\frac{1}{n}\right)}_{\to 0}\right)^n \xrightarrow[n\to\infty]{} \exp\left(\frac{e}{e-1}\ln(1-\alpha)\right) =$$

$$= 1 - (1-\alpha)^{\frac{e}{e-1}} \neq 1$$

$$(1-\alpha)^{\frac{e}{e-1}} \overset{\Rightarrow}{\neq} 0$$

не явл сост

ТП2

$P(x)$ - вер выпад x

H_0: $P(1) = P(2) = \frac{12/2}{24} = \frac{1}{4} = \theta_1 = \theta_2$

$P(3) = \frac{1}{6} = \theta_3$

$P(4) = = \frac{8}{24} = \frac{1}{3} = \theta_4$

H_1 кость сделана „честно"

$P(1) = P(2) = P(3) = P(4) = \frac{1}{4} = \theta_e$

$n = 2$ $\quad \alpha = 0{,}2$

H_0 P	1	2	3	4
1	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{24}$	$\frac{1}{12}$
2	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{24}$	$\frac{1}{12}$
3	$\frac{1}{24}$	$\frac{1}{24}$	$\frac{1}{36}$	$\frac{1}{18}$
4	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{18}$	$\frac{1}{9}$

H_1 P	1	2	3	4
1	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$
2	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$
3	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$
4	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$

$G \quad \ell \geq c$

$P(\ell \geq c \mid H_0)$

$$\ell = \frac{g_1(x_1)\, g_1(x_2)}{g_0(x_1)\, g_0(x_2)}$$

ℓ	1	2	3	4
1	1	1	$\frac{3}{2}$	$\frac{3}{4}$
2	1	1	$\frac{3}{2}$	$\frac{3}{4}$
3	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{9}{4}$	$\frac{9}{8}$
4	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{9}{8}$	$\frac{9}{16}$

Самое большое $\frac{9}{4}$

$P(\ell \geq \frac{9}{4} \mid H_0) = \frac{1}{36} > 0{,}2$

нет $\Rightarrow$ дальше возьмем $\frac{3}{2}$

$P(\ell \geq \frac{3}{2} \mid H_0) = \frac{7}{36} < 0{,}2 \Rightarrow c = \frac{3}{2}$

$\alpha_1 = \frac{7}{36}$

$\alpha_2 = P(\ell < \frac{3}{2} \mid H_0) = 11 \quad \frac{1}{16} = \frac{11}{16}$

$w = 1 - \alpha_2 = \frac{5}{16}$

T13

$\xi_1 \sim (a_1, \varphi_1^2)$ $\eta_1 \sim (b_1, \psi_1^2)$

$\xi_2 \sim (a_2, \varphi_2^2)$ $\eta_2 \sim (b_2, \psi_2^2)$

$H_0 : \varphi_1^2 = \psi_1^2 \quad \varphi_2^2 = \psi_2^2$

$H_1 \quad \varphi_1^2 \neq \psi_1^2 \quad \varphi_2^2 \neq \psi_2^2$

$H_0 \quad \frac{S_{x_1}^2}{S_{y_1}^2} \sim F(n-1, m-1)$

$\frac{S_{x_2}^2}{S_{y_2}^2} \sim F(n-1, m-1)$

$\frac{\alpha}{2} \le$ p-value (length) $\le 1 - \frac{\alpha}{2}$

$\frac{\alpha}{2} \le$ p-value (width) $\le 1 - \frac{\alpha}{2}$

$W = 1 - P(u_{\frac{\alpha}{2}} \le x \le u_{1-\frac{\alpha}{2}})$

Cell ipynb.

ПЧ

$H_0 \quad a = b \qquad H_1 \quad a > b$

$$H_0 \quad \frac{\bar{x} - \bar{y}}{\sqrt{\frac{S_x^2}{n} + \frac{S_y^2}{m}}} \sim N(0, 1) =$$

$$p \text{ value} = \int_{-}^{+\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx$$

сш 'руль'