NATIONAL TAIWAN NORMAL UNIVERSITY
Department of Computer Science and Information Engineering

Numerical Methods

Final Examination
Wednesday 01/08/2020

Instructions:

- When the exam begins, write your name on every page of this exam booklet.
- This exam contains 5 programming problems and 3 written problems, some with multiple parts. You have 90 minutes.
- This exam is closed book. No internet or mobile phones are permitted.
- Show your work, as partial credit will be given. You will be graded not only on the correctness of your answer, but also on the clarity with which you express it.
- **Good luck!**

Part 1: Computing problems (50). Round off to the 4th decimal place if the answer is not an integer.

(a) Let $P(x)$ be the degree 10 polynomial through the 11 points: (-5, 5), (-4, 5), (-3, 5), (-2, 5), (-1, 5), (0, 5), (1, 5), (2, 5), (3, 5), (4, 5), (5, 42). Calculate $P(6)$. **(10 pts)**

(b) In Newton's divided difference method a polynomial for interpolating n points $\{x_1, x_2, \ldots, x_n\}$ can be written in this form:

$$P(x) = c_0 + c_1(x - x_1) + c_2(x - x_1)(x - x_2) + \cdots + c_{n-1}(x - x_1)\cdots(x - x_{n-1})$$

Find the coefficients c_0, c_1, c_2, and c_3 of the interpolating polynomial of the four points: (-1, 1), (0, 1), (1, 2) and (2, 0). **(10 pts)**

(c) Find the best exponential fit $y = c_1 e^{c_2 t}$ of the data points. Report c_1 and c_2. **(10 pts)**

t	y
0	10
1	5
2	2
3	1

(d) Apply Gram-Schmidt orthogonalization to find the reduced QR factorization of the matrix:

$$\begin{bmatrix} 1 & -2 & -1 \\ 2 & 0 & 1 \\ 2 & -4 & 2 \\ 4 & 0 & 0 \end{bmatrix}$$

Report the matrix Q. **(10 pts)**

(e) Find the point (x, y) that minimizes the sum of squares distance to the following three circles. **(10 pts)**
C1: center (2, 1), radius 1
C2: center (5, 3), radius 2
C3: center (5, -3), radius 1

Part 2: Written problems (50)

Problem #1 (20). Please answer the following questions regarding *polynomial interpolation*, along with a brief explanation of your answer.

(a) Suppose we have n distinct points. How many polynomials of degree $n - 1$ or less that interpolate these points? **(5)**

(b) Following the previous question, how many degree n polynomials pass through these points? **(5)**

(c) Let $P_3(x)$ be the interpolating polynomial for the data (0, 0), (0.5, y), (1, 3), and (2, 2). The coefficient of x^3 in $P_3(x)$ is 6. Find y. **(10)**

Problem #2 (10). Please find the least squares solution of k of the following equations. Show how you derive your answer.

$$4k = 3$$
$$7k = 5$$
$$11k = 8$$

Problem #3 (20). The three-part problem is stated as follows: Divide 10 into three parts such that they shall be in continued proportion to each other and the product of the first and the last two shall be 6. Taking x, y, and z as three parts, this problem can be represented as a system as follows:

$$x + y + z = 10$$
$$x/y = y/z$$
$$xy = 6$$
$$yz = 6$$

It can be solved by the Gauss-Newton method discussed in the class. Please write down the vector $r()$ and the Jacobian matrix $D_r()$ for applying the Gauss-Newton method to find the least squares solution of the three-part problem.