NATIONAL TAIWAN NORMAL UNIVERSITY
Department of Computer Science and Information Engineering

Numerical Methods

Midterm Examination
Wednesday 11/06/2019

Instructions:

- This exam contains two parts: computing problems and written problems. You have 90 minutes.
- This exam is closed book. No hand-held devices are permitted.
- Show your work, as partial credit will be given. You will be graded not only on the correctness of your answer, but also on the clarity with which you express it.
- **Good luck!**

Part 1: Computing problems (50)

Please use your codes to solve the following mathematical problems. If the answer is not an integer, please round off to the **4th** decimal place.

(a) $x^3 + x^2 - 1 = 0$ **(5)**

(b) $e^{-x} - x = 0$ **(5)**

(c) Solve the following linear system. **(10)**

$$\begin{bmatrix} 2 & 1 & 0 & 0 & 0 & 0 \\ 1 & 2 & 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 & 1 & 0 \\ 0 & 0 & 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ -1 \end{bmatrix}$$

(d) Return *u*, *v* and *w* of the following nonlinear system. **(15)**

$$2u^2 - 4u + v^2 + 3w^2 + 6w + 2 = 0$$

$$u^2 + v^2 - 2v + 2w^2 - 5 = 0$$

$$3u^2 - 12u + v^2 + 3w^2 + 8 = 0$$

(e) Return the *P*, *L* and *U* of the *PA* = *LU* factorization of the matric: **(15)**

$$A = \begin{bmatrix} 1 & 0 & 0 & 1 \\ -1 & 1 & 0 & 1 \\ -1 & -1 & 1 & 1 \\ -1 & -1 & -1 & 1 \end{bmatrix}$$

Part2: Written problems (50)

Problem #1 (10). The fixed points of $g(x) = 0.39 - x^2$ are 0.3 and -1.3. Please answer the following questions.

(a) To which of the fixed points is the FPI method locally convergent? Why?

(b) Does the FPI method converge to this fixed point faster or slower than the Bisection method? Why?

Problem #2 (20). Newton's method can be used to solve equations. However, this method does not always work. Here we discuss scenarios where it may fail.

(a) Newton's method can be seen as a fixed-point iteration $x = g(x)$. Please write down Newton's $g(x)$ that solves $\sin x = 0$. **(5)**

(b) Continuing from (a), we take $\pi/2$ as the starting point ($x_0 = \pi/2$). What is the result of the next step? Please explain why Newton's method fails to find a solution for this example? **(5)**

(c) Now let's consider another example. Please write down Newton's $g(x)$ that solves $-x^3 + 4x^2 - 2x + 2 = 0$. **(5)**

(d) Continuing from (c), we take 0 as the starting point ($x_0 = 0$). What are the results of the next two steps (x_1 and x_2)? Please explain why Newton's method fails to find a solution for this example? **(5)**

Problem #3 (8).

Swamping is a significant source of error in naïve Gaussian elimination. Please briefly describe what it is and how it can be solved.

Problem #4 (12).

Please compute the first two steps of the Jacobi **and** the Gauss-Seidel methods. Let $\underline{x} = [u, v, w]$ and $\underline{x}_0$ be the starting vector [0, 0, 0]. Report $\underline{x}_2$.

$$\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}$$