UE18MA251 - LINEAR ALGEBRA AND ITS APPLICATIONS

QUESTION BANK

UNIT 1 - MATRICES AND GAUSSIAN ELIMINATION

1	Use elimination and back substitution to solve the system of equations 2x + y + 5z + u = 5, x + y -3 z - 4 u = -1 , 3x + 6 y -2z + u = 8 and 2x + 2y + 2z - 3u = 2. Answer : x = 2, y = 1/5, z = 0 and u = 4/5
2	Do the three planes x + 2y + z = 4 , y – z = 1 and x + 3y = 0 have at least one common point of intersection ? Explain. Is the system consistent if the last equation is changed to x + 3y = 5 ? If so, solve the system completely. Answer: The planes do not have a common point. If the last equation is changed as x + 3y = 5 then the system is consistent with infinity of solutions. Answer : x = 2 – 3k , y = 1 + k and z = k where k is real.
3	Explain the row approach to solve the system 2x – 3y = 1 ; x + 3y = -2 with a neat diagram. What happens to the solution when the second equation is replaced by x + 3y = ½ ? Answer: Solution x = -1/3, y = -5/9 ; New solution is x = ½ , y = 0
4	Write down all the permutation matrices of order 3 and identify their inverses. If P is any such permutation matrix, find a nonzero vector x such that (I – P) x = 0. What is the inverse of I – P and what can you say about its determinant value? Answer : $P_{12}P_{13}$ and $P_{12}P_{23}$ are inverses of each other and the other matrices are self inverses. The vector x = (1, 1, 1) is one solution. The matrix I –P is not invertible and its determinant value is 0.
5	When is a system of equations called singular? Explain the different types of singular systems of two equations in two unknowns with a suitable example for each case. Also provide a neat sketch of the row picture of these cases.
6	Use Gaussian elimination to test for consistency or otherwise of the system of linear equations : x – 2 y – 3 z = 0 , y + z = - 8 , - x + y + 2z = 3. Find the solution if the system is consistent. In case of inconsistency, change the coefficient of z suitably in the third equation (viz 2) so that the system yields a unique solution with z = 1. Solve also for x and y. Answer : Inconsistent. Replace 2 by – 3 to get x = - 15 , y = -9 , z = 1.
7	Given A = $\begin{bmatrix} 0 & a & b \\ 0 & 0 & c \\ d & e & f \end{bmatrix}$, what conditions apply on the entries of A to get a full set of pivots? Write down the permutation matrices that need to be used to reduce A to an upper triangular form.
8	Explain why the system x + y + z = 2, x + 2y + 3z = 1 and y + 2z = 0 is singular by finding a combination of the three equations that adds up to 0 = 1. What value should replace the zero on the right side to allow the equations to have solution? What is one of the solutions?
9	Which number q makes this system singular and which right-hand side t gives it infinitely many solutions? Find the solution that has z = 1 x + 4y – 2z = 1 x + 7y- 6z = 6 3y + qz = t .

10	Factor the matrix A into LDU where A = $\begin{bmatrix} 1 & 3 & 5 \\ 3 & 12 & 18 \\ 5 & 18 & 30 \end{bmatrix}$. How are L and U related and why? Answer: **L =** $\begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 5 & 1 & 1 \end{bmatrix}$, D = $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \end{bmatrix}$, U = $\begin{bmatrix} 1 & 3 & 5 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ L and U are transposes of each other since A is symmetric.
11	Find L and U for A = $\begin{bmatrix} 2 & 4 & -1 & 5 & -2 \\ -4 & -5 & 3 & -8 & 1 \\ 2 & -5 & -4 & 1 & 8 \\ -6 & 0 & 7 & -3 & 1 \end{bmatrix}$. What is the rank of A ? Answer : L = $\begin{bmatrix} 1 & 0 & 0 & 0 \\ -2 & 1 & 0 & 0 \\ 1 & -3 & 1 & 0 \\ -3 & 4 & 2 & 1 \end{bmatrix}$, U = $\begin{bmatrix} 2 & 4 & -1 & 5 & -2 \\ 0 & 3 & 1 & 2 & -3 \\ 0 & 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 & 5 \end{bmatrix}$, The rank of A is 4.
12	Find the matrices L , D and U for A = $\begin{bmatrix} 2 & 1 & 1 & 0 \\ 4 & 3 & 3 & 1 \\ 8 & 7 & 9 & 5 \\ 6 & 7 & 9 & 8 \end{bmatrix}$
13	If the inverse of A = $\begin{bmatrix} 1 & a & b \\ 1 & a & 2 \\ 1 & 0 & b \end{bmatrix}$ is known to be $\begin{bmatrix} 1 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{bmatrix}$, use Gauss – Jordan elimination on [A I] to find the values of a and b. Answer: a= 1 and b = 1
14	Solve for the columns of $A^{-1} = \begin{bmatrix} x & u \\ y & v \end{bmatrix}$ given the systems of equations $\begin{bmatrix} 10 & 20 \\ 20 & 50 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 10 & 20 \\ 20 & 50 \end{bmatrix}\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$. Also find the matrix B such that A^{-1} B = I Answer: $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1/2 \\ -1/5 \end{bmatrix}$, $\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} -1/5 \\ 1/10 \end{bmatrix}$, B = 100 $\begin{bmatrix} 1/10 & 1/5 \\ 1/5 & 1/2 \end{bmatrix}$
15	Use the Gauss- Jordan method to invert A = $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1/4 & 1 & 0 & 0 \\ 1/3 & 1/3 & 1 & 0 \\ 1/2 & 1/2 & 1/2 & 1 \end{bmatrix}$

	Answer: $\begin{bmatrix} 1 & 0 & 0 & 0 \\ -1/4 & 1 & 0 & 0 \\ -1/4 & -1/3 & 1 & 0 \\ -1/4 & -1/3 & -1/2 & 1 \end{bmatrix}$
16	Determine the equation of the polynomial $y = f(x)$ of degree 2 whose graph passes through the points (1, 6) , (2, 3) and (3, 2). Answer : $y = 11 - 6x + x^2$.
17	Propane is a common gas used for cooking and home heating. Each molecule of propane is comprised of 3 atoms of carbon, and 8 atoms of hydrogen written as C_3H_8. When propane burns, it combines with oxygen gas O_2 to form carbon dioxide CO_2 and water H_2O. Balance the chemical equation $C_3H_8 + O_2 \rightarrow CO_2 + H_2O$ that describes this process. Answer : $2\,C_3H_8 + 10\,O_2 \rightarrow 6\,CO_2 + 8\,H_2O$

UNIT 2 - VECTOR SPACES

1	If the vectors (1, 1, 2) , (1. 2. 4) , (2, 4, 8) span the column space of A, determine whether or not the vector b = (2, 3 , 5) is in C (A). What value should replace the third component " 5 " in the vector b so that the system Ax = b has infinitely many solutions ? Express this new vector b as a linear combination of columns of A.
2	Find the condition on a, b, c so that the vector (a, b, c) belongs to the space spanned by u = (2, 1, 0) , v = (1, -1, 2) and w = (0, 3, -4). Do the vectors u, v, w span R^3 ? Answer: The vector (a, b, c) belongs to the space spanned by u, v, w if and only if 2a -4b -3c = 0. The vectors u, v, w do not span R^3
3	Define the null space of an m by n matrix. Construct a matrix with (1, 0, 1) and (1, 2, 0) as a basis for its row space and column space. Why can't this be a basis for the row space and null space? Answer: $A = \begin{bmatrix} 1 & 1 \\ 0 & 2 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 1 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 2 & 1 \\ 2 & 4 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ Dim $C(A^T) = 2 =$ dim C(A) and n = 3. Therefore dim N(A) = 1 The given vectors can not be a basis for row space and null space since, in that case dim $C(A^T)$ + dim N(A) =2 + 2 = 4 ≠ 3
4	Find a basis for the column space and the null space of $A = \begin{bmatrix} 2 & 4 & 6 & 4 \\ 2 & 5 & 7 & 6 \\ 2 & 3 & 5 & 2 \end{bmatrix}$ Answer : A basis for C(A) is (2, 2, 2) , (4, 5, 3) and a basis for N(A) is (-1, -1, 1, 0) and (2, -2, 0, 1).
5	If the column space of A is spanned by the vectors (1, 4, 2) , (2, 5, 1) and (3, 6, 0) find all those vectors that span the left null space of A. Determine whether or not the vector b = (4, -2, 2) is in that subspace. What are the dimensions of C (A^T) and N (A^T) ? Answer: Solutions of $A^T x = 0$ are (2, -1, 1). The vector b is in $N(A^T)$. Dim $C(A^T) = 2$ and dim $N(A^T) = 1$

6	For what value of λ will the vectors (1, 3, -5) , (0, 5, λ) and (-2, -1, 0) span a two dimensional subspace ? For this value of λ, (i) express (-2, -1, 0) as a linear combination of the other two vectors and (ii) find a vector in R^3 that is not in the span of these vectors Answer: For λ = -10, the vectors span a 2-d subspace. (i) (-2, -1, 0) = -2 (1, 3, -5) + 1 (0, 5, -10) (ii) when λ = -10, the last entry is -a + 2b + c. Any vector (a, b, c) that satisfies -a+2b+c ≠ 0 will not be in the span of the given vectors.
7	If the row space of a matrix A is spanned by the vectors (2, 4, 6, 4) , (2, 5, 7, 6) and (2, 3, 5, c) find the value of c for which C (A) is (i) a plane in R^3 (ii) the whole of R^3. For this value of c in case (i), identify the free variable(s) and write down the special solution(s) of Ax = 0
8	Find the dimension and a basis for the null space and the left null space of the matrix whose columns are (1, 2, - 1), (3, 6, - 3) , (3, 9, 3) and (2, 7, 4).
9	Decide whether or not the vectors (1, 1, 0, 0), (1, 0, 1, 0), (0, 0, 1, 1) and (0, 1, 0, 1) are linearly independent. Do they span R^4 ? Explain. What is the dimension of the space spanned by these vectors? Answer : There are only 3 independent columns, they do not span R^4. Dim = 3
10	If A = $\begin{bmatrix} 2 & -6 & -8 \\ -4 & 12 & a \\ 1 & b & 2 \end{bmatrix}$ find the values of a and b so that the column space of A is (i) the whole of R^3 (ii) a 2-dimensional subspace of R^3 (iii) a 1-dimensional subspace of R^3. Find a basis for N (A) in the second case choosing a = 22 Answer : (i) C(A) is the whole of R^3 if a ≠ 16 and b ≠ -3 , (ii) C(A) is a 2 dim plane if b = - 3 , (iii) a and b do not exist. Basis for N(A) = { (3, 1, 0) }.
11	Obtain the special solution of Ax = 0 if A = $\begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$. What is the basis and dimension of the null space of this matrix? Answer: Special solution is (1, 0, 1, 0) , Dim N(A) = 1 and a basis is (1,0,1,0)
12	Find the four fundamental subspaces , their dimensions and a basis given A = $\begin{bmatrix} 0 & 3 & -1 & -2 & 6 \\ -2 & 1 & 2 & 1 & -3 \\ 1 & -1 & 2 & -2 & 3 \end{bmatrix}$

UNIT 3 – LINEAR TRANSFORMATIONS AND ORTHOGONALITY

1	Define $L: R^3 \to R^2$ *by* $L(x_1, x_2, x_3) = (x_3 - x_1, x_1 + x_2)$ (a) Find $L(e_1), L(e_2)$ *and* $L(e_3)$ (b) Show that L is a linear transformation (c) Show that $L(x_1, x_2, x_3) = x_1 L(e_1) + x_2 L(e_2) + x_3 L(e_3)$ Answer: L(1,0,0)=(-1,1) , L(0,1,0)=(0,1), L(0,0,1)=(1,0)

2	Let $L: R^3 \to R^4$ be a linear transformation. Suppose we know that L(1,0,1)= (-1,1,0,2), L(0,1,1)=(0,6,-2,0) and L(-1,1,1)=(4,-2,1,0). Determine L(1,2,-1). Answer: (-13, 41,-16,-6)
3	Let $F=\left\{\begin{bmatrix}1&0\\0&0\end{bmatrix},\begin{bmatrix}0&1\\0&0\end{bmatrix},\begin{bmatrix}0&0\\1&0\end{bmatrix},\begin{bmatrix}0&0\\0&1\end{bmatrix}\right\}$. Thus F is the standard basis of $M_{22}. \, Let \; B=\begin{bmatrix}-2&1\\3&4\end{bmatrix}. \, Define \; L: M_{22}\to M_{22} \; by \, L(x)=Bx.$. Find the matrix representation of L with respect to standard basis F of M_{22}. Answer : $L=\begin{bmatrix}-2&0&1&0\\0&-2&0&1\\3&0&4&0\\0&3&0&4\end{bmatrix}$
4	The three vectors $v_1=[1,2,1]^T, v_2=[2,1,-4]^T, v_1=[3,-2,1]^T$ are mutually orthogonal. Express the vectors $v=[7,1,9]^T$ as a linear combination of v_1, v_2, v_3. Answer : $v=3v_1-v_2+2v_3$
5	Find the matrix of the orthogonal projection $proj_W = R^3 \to R^3 \; where \; W = span\left\{\begin{bmatrix}1\\1\\1\end{bmatrix},\begin{bmatrix}1\\-1\\0\end{bmatrix}\right\}$ Answer : $\begin{bmatrix}5/6&-1/6&1/3\\-1/6&5/6&1/3\\1/3&1/3&1/3\end{bmatrix}$
6	Find the least squares solution to the system Ax = y where $A=\begin{pmatrix}1&1\\2&1\\-1&1\\0&1\end{pmatrix}, \; y=\begin{pmatrix}2\\3\\1\\1\end{pmatrix} and \; x=\begin{pmatrix}m\\b\end{pmatrix}$ Answer : $\begin{pmatrix}7/10\\7/5\end{pmatrix}$
7	Find the least squares solution to x+2y=3, 3x+2y=5, x+y=2.09. Answer: x=1, y=1.01.
8	Let T(x,y,z)=(5x-3y+z, 2z+4y, 5x+3y), what is the standard matrix of T? Answer: $A=\begin{bmatrix}5&-3&1\\0&4&2\\5&3&0\end{bmatrix}$
9	Let T(x, y, z)=(2x+3y, 3y-z). Write down the standard matrix of T and use it to find T(0,1,-1). Answer : T(0,1,-1)=(1,4)

10	Let T be the reflection in the line y=x in R^2. So T(x,y)=(y,x). (a) Write down the standard matrix of T. (b) Use the standard matrix to compute T(3,4) Answer : $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, T(3,4)=(4,3)$
11	What is the projection of $\vec{x}=\begin{bmatrix} 6 \\ 5 \\ 4 \end{bmatrix}$ onto the line spanned by $\vec{a}=\begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix}$ Answer : $\begin{bmatrix} 12/11 \\ -12/11 \\ 36/11 \end{bmatrix}$
12	Find the orthogonal projection $proj_w = R^3 \to R^3$, where W is the plane $x_1 + x_2 + x_3 = 0$ $Ans: proj_w(y) = \frac{1}{3}\begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$
13	Find a basis for the orthogonal complement of the space spanned by (1,0,1,0,2), (0,1,1,1,0) and (1,1,1,1,1).
14	Project b=(2,4,1) onto a=(3,2,5) and c=(3,-1,-2).
15	Determine the matrix induced by the composition of reflection about y-axis followed by another reflection about x-axis
16	Find the equation of the line that runs through (1,-1), (4,11),(-1,-9) and (-2,-13) by the least square method
17	Find the constant function that is the least square fit to the following data <table><tr><td>x</td><td>0</td><td>1</td><td>2</td><td>3</td></tr><tr><td>f(x)</td><td>1</td><td>0</td><td>1</td><td>2</td></tr></table> Answer: c=1
18	Let V and W be subspaces of R^2 spanned by (1,1) and (1,2) respectively. Find vectors $v \in V,\ w \in W\ so\, v + w = (2,-1)$ $Ans: v = (5,5),\ and\ w = (-3,-6)$

UNIT 4 – ORTHOGONALIZATION, EIGEN VALUES AND EIGEN VECTORS

1	Perform the Gram-Schmidt process on the following sets and also give the associated QR factorization. i) $a=(1, 2, 2)$, $b=(1, 3, 1)$ ii) $a=(2, 2, 1)$, $b=(1, 1, 5)$ iii) $a=(2, 2, 1)$, $b=(-2, 1, 2)$, $c=(18, 0, 0)$

	Answer : (i) $q1=\frac{1}{3}\begin{bmatrix}1\\2\\2\end{bmatrix}$,q2 = (1/sqrt(2))*(0,1,-1) and $\begin{bmatrix}1/3 & 0\\2/3 & 1/\sqrt{2}\\2/3 & -1/\sqrt{2}\end{bmatrix}\begin{bmatrix}3 & 3\\0 & \sqrt{2}\end{bmatrix}$ (ii) $q1=\frac{1}{3}\begin{bmatrix}2\\2\\1\end{bmatrix}, q2=\frac{1}{\sqrt{18}}\begin{bmatrix}-1\\-1\\-4\end{bmatrix}$ and $\begin{bmatrix}2/3 & -1/\sqrt{18}\\2/3 & -1/\sqrt{18}\\1/3 & 4/\sqrt{18}\end{bmatrix}\begin{bmatrix}3 & 3\\0 & \sqrt{18}\end{bmatrix}$ (iii) $q1=\frac{1}{3}\begin{bmatrix}2\\2\\1\end{bmatrix}, q2=\frac{1}{3}\begin{bmatrix}-2\\1\\2\end{bmatrix}, q3=\frac{1}{3}\begin{bmatrix}1\\-2\\2\end{bmatrix}$ and $\frac{1}{3}\begin{bmatrix}2 & -2 & 1\\2 & 1 & -2\\1 & 2 & 2\end{bmatrix}\begin{bmatrix}3 & 0 & 12\\0 & 3 & -12\\0 & 0 & 6\end{bmatrix}$
2	Find the distance from the point z = (0, 0, 1, 0) to the plane P that passes through the point x_0 = (1, 0, 0, 0) and is parallel to the vectors v_1 = (1, -1, 1, -1) and v_2 = (0, 2, 2, 0). Answer: Sqrt (3)/2
3	What multiple of a_1= (1, 1) should be subtracted from a_2 = (4, 0) to make the result orthogonal to a1? Factor the matrix in to QR with Orthonormal vectors in Q. Answer : multiple 2, A = QR = $\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\1 & -1\end{bmatrix}\begin{bmatrix}\sqrt{2} & 2\sqrt{2}\\0 & 2\sqrt{2}\end{bmatrix}$
4	Let P be the plane in R^3 spanned by vectors x_1= (1, 2, 2) and x_2 = (-1, 0, 2). i) Find an Orthogonal basis for P ii) Extent it to an Orthonormal basis for R^3 Answer: i) $\frac{1}{3}(1, 2, 2), \frac{1}{3}(-2, -1, 2)$ ii) $\frac{1}{3}(1, 2, 2), \frac{1}{3}(-2, -1, 2), \frac{1}{3}(2, -2, 1)$
5	Determine the Eigen values and corresponding Eigen vectors for the following matrices: (i) $\begin{bmatrix}-5 & 3 & 0\\-6 & 4 & 2\\2 & -1 & 1\end{bmatrix}$ (ii) $\begin{bmatrix}1 & -3 & 3\\3 & -5 & 3\\6 & -6 & 4\end{bmatrix}$ Answer: (i) -1,0,1 and k1(-3,-4,1), k2(-3,-5,1) , k3(0.5,1,0) where k1,k2,k3 are real (ii) 2,-2,-2 and x3 (0.5,0.5,1) , x3 (1,0,1)+x2(-1,1,0), where x1, x2,x3 are real
6	Find the numerically largest Eigen vector of (i) $\begin{bmatrix}1 & -3 & 2\\4 & 4 & -1\\6 & 3 & 5\end{bmatrix}$ (ii) $\begin{bmatrix}1 & 6 & 1\\1 & 2 & 0\\0 & 0 & 3\end{bmatrix}$ starting with an initial approximation of (i) (1 , 0, 0) (ii) (1, 1, 1) Answer : (i) λ = 7 , x = (0.3 , 0.07, 1) (ii) λ = 4 , x = (1, 0.5 , 0)
7	If possible diagonalize the matrix A = $\begin{bmatrix}1 & 1\\-1 & 4\end{bmatrix}$

	Answer: A= $\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}\begin{bmatrix}2 & 0\\ 0 & 3\end{bmatrix}\begin{bmatrix}2 & -1\\ -1 & 1\end{bmatrix}$
8	Diagonalize the following matrices $\begin{bmatrix}3 & 0 & 0\\ -3 & 4 & 9\\ 0 & 0 & 3\end{bmatrix}$, $\begin{bmatrix}-4 & -4 & -8\\ 4 & 6 & 4\\ 6 & 4 & 10\end{bmatrix}$ Answer: $\begin{bmatrix}3 & 1 & 0\\ 0 & 3 & 1\\ 1 & 0 & 0\end{bmatrix}\begin{bmatrix}3 & 0 & 0\\ 0 & 3 & 0\\ 0 & 0 & 3\end{bmatrix}\begin{bmatrix}0 & 0 & 1\\ 1 & 0 & -3\\ -3 & 1 & 9\end{bmatrix}$ $\begin{bmatrix}-2 & -1 & -2\\ 1 & 0 & 1\\ 1 & 1 & 2\end{bmatrix}\begin{bmatrix}2 & 0 & 0\\ 0 & 4 & 0\\ 0 & 0 & 6\end{bmatrix}\begin{bmatrix}-1 & 0 & -1\\ -1 & -2 & 0\\ 1 & 1 & 1\end{bmatrix}$
9	Find a formula for $\begin{bmatrix}1 & -6\\ 2 & -6\end{bmatrix}^k$ by diagonalzing the matrix. Answer: $A^k = \begin{bmatrix}3 & 2\\ 2 & 1\end{bmatrix}\begin{bmatrix}(-3)^k & 0\\ 0 & (-2)^k\end{bmatrix}\begin{bmatrix}-1 & 2\\ 2 & -3\end{bmatrix}$
10	Calculate A^6, where $A = \begin{bmatrix}1 & 1\\ -2 & 4\end{bmatrix}$, Answer: $A^6 = \begin{bmatrix}-600 & 665\\ -1330 & 1394\end{bmatrix}$ $=PD^6P^{-1}$
11	Show that $A = \begin{bmatrix}1 & 7 & 6\\ 0 & -1 & 3\\ 0 & 0 & 2\end{bmatrix}$ is diagonalizable.
12	If A has $\lambda_1 = 1$ with Eigen vector $X_1 = \begin{bmatrix}-1\\ 1\end{bmatrix}$ and $\lambda_2 = 5$ with Eigen vector $X_2 = \begin{bmatrix}3\\ 1\end{bmatrix}$, find A.
13	Let $A = \begin{bmatrix}1 & 6\\ 5 & 2\end{bmatrix}$ u = $\begin{bmatrix}6\\ -5\end{bmatrix}$, , $v = \begin{bmatrix}3\\ -2\end{bmatrix}$. Are u and v eigenvalues of A ? Answer : u but not v is an eigenvalue of A.
14	Find the eigen values and the corresponding eigenvectors of $A = \begin{bmatrix}4 & -1 & 6\\ 2 & 1 & 6\\ 2 & -1 & 8\end{bmatrix}$ Answer : The eigenvectors for $\lambda = 2$ are a(1, 2, 0) and b(-3 , 0, 1) The eigenvector for $\lambda = 9$ are c (1, 1, 1) where a, b, c are nonzero scalars

UNIT 5 – SINGULAR VALUE DECOMPOSITION

1	For what range of numbers b is the matrix A positive definite? $A=\begin{bmatrix}1 & 2 & 4\\ 2 & b & 8\\ 4 & 8 & 7\end{bmatrix}$
2	Decide whether the following matrices are positive definite, negative definite, semidefinite or indefinite: $A=\begin{bmatrix}1 & 2 & 3\\ 2 & 5 & 4\\ 3 & 4 & 9\end{bmatrix}$ $B=A^{-1}$.
3	For x in R^3, write $5x_1^2 + 3x_2^2 + 2x_3^2 - x_1 x_2 + 8 x_2x_3$ as x^TAx.
4	For the semidefinite matrices $A=\begin{bmatrix}2 & -1 & -1\\ -1 & 2 & -1\\ -1 & -1 & 2\end{bmatrix}$ $B=\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}$ write x^TAx as a sum of two squares and x^TBx as one square.
5	Is $Q(x) = 3x_1^2 + 2x_2^2 + x_3^2 + 4x_1 x_2 + 4 x_2x_3$ positive definite ? Answer : The eigen values are 5, 2, -1 and hence Q is not positive definite.
6	Compute SVD of the singular matrix $A=\begin{bmatrix}1 & 4\\ 2 & 8\end{bmatrix}$
7	Find the SVD from the eigenvectors v_1, v_2 of A^TA and $Av_i=\sigma_i u_i$. $A=\begin{bmatrix}1 & 1\\ 1 & 0\end{bmatrix}$
8	Find the SVD of the following matrices : (i) A = $\begin{bmatrix}4 & 11 & 14\\ 8 & 7 & -2\end{bmatrix}$ (ii) $A=\begin{bmatrix}1 & -1\\ -2 & 2\\ 2 & -2\end{bmatrix}$