

**LINEAR ALGEBRA AND CALCULUS
(Common to All Branches)****Time: 3 Hours****Max Marks: 60**

Answer ONE Question from each Unit

All Questions Carry Equal Marks

All parts of the Question must be answered at one place

UNIT-I

1. Test for consistency and solve $2x - 3y + 7z = 5, 3x + y - 3z, 2x + 19y - 47z = 32$. 12M

(OR)

2. Find Eigen values and corresponding Eigen vectors for the matrix 12M

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

UNIT-II

3. Prove that $\frac{b-a}{1+b^2} < \tan^{-1} b - \tan^{-1} a < \frac{b-a}{1+a^2}$, if $0 < a < b < 1$. 12M

(OR)

4. Determine the maximum and minimum distances of the point $(3, 4, 12)$ from the sphere $x^2 + y^2 + z^2 = 4$. 12M

UNIT-III

5. a) Prove that the area of a loop of the curve $x^3 + y^3 = 3axy$ is $\frac{3a^2}{2}$ 6M

- b) Find the perimeter of the loop of the curve $3ay^2 = x(x-a)^2$ 6M

(OR)

6. Evaluate $\int_0^1 \frac{\sin^{-1} x}{x} dx$. 12M

UNIT-IV

7. Evaluate $\int_0^1 \int_{e^x}^e \frac{dy dx}{\log y}$ by changing the order of integration. 12M

(OR)

8. Evaluate $\int_1^e \int_1^{\log y} \int_1^{e^x} \log z dz dy dx$ 12M

UNIT-V

9. Show that $\text{div}(\text{grad } r^n) = n(n+1)r^{n-2}$ 12M

(OR)

10. Verify Greens theorem for $\int_C [(3x - 8y^2) dx + (4y - 6xy) dy]$ where C is the boundary of the region bounded by $x = 0, y = 0$ and $x + y = 1$ 12M

AR16

CODE: 16BS1001

SET-1

**ADITYA INSTITUTE OF TECHNOLOGY AND MANAGEMENT, TEKKALI
(AUTONOMOUS)**

I B.Tech I Semester Supplementary Examinations, April-2021

**ENGINEERING MATHEMATICS – I
(Common to All Branches)**

Time: 3 Hours

Max Marks: 70M

Answer ONE Question from each Unit

All Questions Carry Equal Marks

All parts of the question must be answered in one place only

UNIT-I

1. a) Solve $(x^3y^2+x) dy + (x^2y^3-y)dx = 0$ 7M

b) Find the orthogonal trajectories of family of curves $ay^2 = x^3$. 7M

(OR)

2. a) Solve $2ydx + x(2\log x - y)dy = 0$ 7M

b) If the air is maintained at 30°C and the temperature of the body cools from 80°C to 60°C in 12 minutes, find the temperature of the body after 24 minutes 7M

UNIT-II

3. Solve $(D-2)^2y = 8(e^{2x} + \sin 2x + x^2)$ 14M

(OR)

4. Solve $(D^4 + 2D^2 + 1)y = x^2 \cos x$ 14M

UNIT-III

5. a) If $F = xu + v - y$, $G = u^2 + vy + w$, $H = zu - v + vw$, compute $\frac{\partial(F,G,H)}{\partial(u,v,w)}$ 7M

b) Expand $f(x,y) = xy^2 + \cos(xy)$ about the point $(1, \frac{\pi}{2})$ upto 3rd degree terms. 7M

(OR)

- 6 Examine the following function for extreme values. 14M
 $f(x, y) = x^4 + y^4 - 2x^2 + 4xy - 2y^2$

UNIT-IV

- 7 Evaluate the integral $\int_0^1 \int_x^{\sqrt{2-x^2}} \frac{x \, dy \, dx}{\sqrt{x^2 + y^2}}$ by changing of order of integration . 14M

(OR)

- 8 Evaluate the integral $\int_0^{\log 2} \int_0^x \int_0^{x+\log y} e^{x+y+z} \, dx \, dy \, dz$. 14M

UNIT-V

- 9 a) Find the total work done in moving a particle in a force field given by $\vec{F} = 3xy\vec{i} - 5z\vec{j} + 10x\vec{k}$ along the curve $x = t^2 + 1, y = 2t^2, z = t^3$ from $t = 1$ to $t = 2$ 7M
- b) Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point $(2, -1, 2)$. 7M

(OR)

10. Verify Stoke's theorem for the vector field $\vec{F} = (2x - y)\vec{i} - yz^2\vec{j} - y^2z\vec{k}$ over the upper half surface of $x^2 + y^2 + z^2 = 1$, bounded by its projection on the xy - plane . 14M

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ENGINEERING MATHEMATICS - I
(Common to All Branches)

Time: 3 Hours

Max Marks: 70

PART-A

ANSWER ALL QUESTIONS

[1 x 10 = 10 M]

1. a) Is $(y^2 - 2xy)dx = (x^2 - 2xy)dy$ exact? 1M
- b) Solve $xdy + ydx = 0$ 1M
- c) Solve $y' + y = e^{e^x}$. 1M
- d) If $f(D) = (D^2 + 4D + 4)$, then find $\frac{1}{f(D)} \sin 2x$. 1M
- e) If $x = r \cos \theta, y = r \sin \theta$ then find $\frac{\partial(x,y)}{\partial(r,\theta)}$. 1M
- f) What is a Stationary value of a two variable function? 1M
- g) Solve $\int_{\theta=0}^{\pi} \int_0^{\theta} r \, dr d\theta$. 1M
- h) Evaluate $\int_0^2 \int_0^x e^{x+y} \, dy dx$. 1M
- i) Show that $\text{grad } \vec{r} = \vec{r}/|\vec{r}|$. 1M
- j) Prove that $\nabla f \times \nabla g$ is solenoidal. 1M

PART-B

Answer one question from each unit

[5x12=60M]

UNIT-I

2. a) A body kept in air with temperature 25°C cools from 140°C to 80°C in 20 minutes. Find when the body cools down to 35°C . 6M
 - b) Solve $\frac{dy}{dx} + y \tan x = y^2 \sec x$ 6M
- (OR)
3. a) Solve $x dx + y dy = \frac{xy - y dx}{x^2 + y^2}$. 6M
 - b) The number N of bacteria in a culture grew at a rate proportional to N. The value of N was initially 100 and increased to 332 in one hour. What was the value of N after $1\frac{1}{2}$ hours. 6M

UNIT-II

4. Solve $\frac{d^2y}{dx^2} + \frac{3dy}{dx} + 2y = x e^x \sin x$ 12M
- (OR)

5. Solve $(D^3 - 7D^2 + 14D - 8)y = e^x \cos 2x$ 12M

UNIT-III

6. a) Expand $\tan^{-1}x$ about the origin using Taylor's theorem. 6M
 b) If $u = x^2 - y^2, v = 2xy$ where $x = r \cos \theta, y = r \sin \theta$, show that $\frac{\partial(u,v)}{\partial(r,\theta)} = 4r^3$ 6M

(OR)

7. Examine the function for extreme values 12M
 $f(x, y) = x^4 + y^4 - 2x^2 + 4xy - 2y^2$ ($x > 0, y > 0$).

UNIT-IV

8. By change of order of integration evaluate $\int_0^a \int_{\frac{x^2}{a}}^{2a-x} xy^2 dy dx$ 12M

(OR)

9. a) Evaluate $\int \int r dr d\theta$ over the cardioids $r = a(1 - \cos \theta)$ about the initial line. 6M
 b) Find the area enclosed by curves $y^2 = ax$ and $x^2 = ay$. 6M

UNIT-V

10. a) If $\vec{f} = xy^2\vec{i} + 2x^2yz\vec{j} - 3yz^2\vec{k}$, find $\text{div } \vec{f}$ at the point (1, -1, 1). 6M
 b) Prove that $\nabla^2(r^n) = n(n+1)r^{n-2}$ 6M

(OR)

11. Verify Stoke's theorem for $\vec{F} = (y - z + 2)\vec{i} + (yz + 4)\vec{j} - xz\vec{k}$ where S is the surface of the cube $x = y = z = 0, x = y = z = 2$ above the xy-plane. 12M