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13BS1003 **SET-2**

ADITYA INSTITUTE OF TECHNOLOGY AND MANAGEMENT, TEKKALI
(AUTONOMOUS)
I B.Tech II Semester Regular/Supplementary Examinations, May-2016

ENGINEERING MATHEMATICS-III
(Common to CE, ME, CSE, IT, ECE & EEE)

Time: 3 hours **Max Marks: 70**

PART-A

Answer all questions **[10 x 1 = 10M]**

1 (a) Define the rank of a matrix.
(b) When does a non homogeneous system consistent?
(c) Define the latent root and latent vector.
(d) Write the nature of $-3y_1^2 - 2y_2^2 - y_3^2$.
(e) State Euler's formulae in Fourier series of a function f(x) with period 2π.
(f) State Fourier integral theorem.
(g) Find the finite Fourier sine transform of $f(x) = x^3$ in $(0, \pi)$.
(h) Find the Z – transform of $\cos n\theta$.
(i) Define gamma function.
(j) Compute $\Gamma\left(\frac{3}{2}\right)$.

PART-B

Answer one question from each unit. **[5 x 12=60M]**

UNIT-I

2 (a) Determine the rank of a matrix $A = \begin{bmatrix} -2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$ by reducing it to normal form.
(b) Investigate for what values of λ and μ the equations
$x + y + z = 6,\ \ x + 2y + 3z = 10,\ \ x + 2y + \lambda z = \mu$ have
(i) no solution (ii) a unique solution (iii) an infinite number of solutions. **[6M+6M]**

(OR)

3 (a) Solve by Gauss-Seidel method, the equations:
$2x + y + 6z = 9,\ \ 8x + 3y + 2z = 13,\ \ x + 5y + z = 7.$
(b) Test for consistency and solve the equations
$2x - y + 3z - 9 = 0;\ x + y + z = 6;\ x - y + z - 2 = 0.$ **[6M+6M]**

UNIT-II

4 Verify Cayley-Hamilton theorem for $A = \begin{bmatrix} 2 & -1 & 0 \\ 3 & 1 & -1 \\ 2 & 0 & 3 \end{bmatrix}$ and hence find A^{-1} and A^4.

[12M]

(OR)

5. Reduce the quadratic form $3x_1^2+3x_2^2+3x_3^2+2x_1x_2+2x_1x_3-2x_2x_3$ into sum of squares form by an orthogonal transformation and give the matrix of transformation. Also find nature, rank, index and signature of the quadratic form. **[12M]**

UNIT-III

6 (a) Find the Fourier series to represent the function $f(x)=x\sin x, -\pi<x<\pi$.

(b) Solve the integral equation $\int_0^\infty f(x)\sin tx\,dx=\begin{cases}1, 0\le t<1\\ 2, 1\le t<2\\ 0, t\ge 2\end{cases}$

[6M+6M]

(OR)

7 (a) Find the half range cosine series for $f(x)=\begin{cases} kx, 0\le x\le \frac{l}{2}\\ k(l-x), \frac{l}{2}\le x\le l\end{cases}$.

Deduce the sum of the series $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\frac{1}{7^2}+\cdots\cdots\cdots$.

(b) Find the Fourier cosine transform of $f(x)$ defined by $f(x)=\frac{1}{1+x^2}$ and hence find the Fourier sine transform of $f(x)=\frac{x}{1+x^2}$. **[6M+6M]**

UNIT-IV

8 (a) Find the Z – transform of $e^{-an}\sin n\theta$.

(b) Find $Z^{-1}\left[\frac{z}{z^2+11z+24}\right]$. **[6M+6M]**

(OR)

9 (a) Find $Z[2\cdot 3^n+5\cdot n]$ and deduce $Z[2\cdot 3^{n+3}+5\cdot(n+3)]$ using shifting theorem.

(b) Using Z – transform, solve $u_{n+2}-6u_{n+1}+9u_n=0$. **[6M+6M]**

UNIT-V

10 (a) Prove that $\int_0^1 \frac{x}{\sqrt{1-x^5}}dx=\frac{1}{5}\beta\left(\frac{2}{5},\frac{1}{2}\right)$

(b) State and prove the relation between beta and gamma functions.

[6M+6M]

(OR)

11 (a) Evaluate $\int_0^\infty \frac{x^8(1-x^6)}{(1+x)^{24}}dx$.

(b) Show that $\int_0^\infty \frac{x^{m-1}}{(a+bx)^{m+n}}dx=\frac{\beta(m,n)}{a^n b^m}$. **[6M+6M]**