

AR13

SET 01

Code: 13BS1001

**ADITYA INSTITUTE OF TECHNOLOGY AND MANAGEMENT, TEKKALI
(AUTONOMOUS)**

I B.Tech I Semester Regular Examinations, February-2014

**Engineering Mathematics - I
(Common to all Branches)**

Time: 3 hours

Max Marks: 70

PART-A

Answer all Questions

[10X1=10M]

- 1)
 - a) Form a differential equation of $x = a \sin(\omega t + b)$
 - b) When do you say that a differential equation is exact?
 - c) Solve $\frac{d^4 y}{dx^4} + 13 \frac{d^2 y}{dx^2} + 36y = 0$
 - d) Show that the function $y_1 = \sin 2x$ and $y_2 = \cos 2x$ are linearly independent solutions of $y'' + 4y = 0$
 - e) Define Jacobian of u, v with respect to x, y
 - f) Write Taylor's expansion of $f(x, y)$ in powers of $(x - a)$ and $(y - b)$.
 - g) Evaluate $\int_0^5 \int_0^{x^2} x(x^2 + y^2) dx dy$
 - h) Write the formula to find area in Polar coordinates.
 - i) If $\mathbf{R} = x\mathbf{I} + y\mathbf{J} + z\mathbf{K}$, show that $\nabla \cdot \mathbf{R} = 3$
 - j) When do you say that a vector \mathbf{F} is solenoidal and irrotational vectors?

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PART-B

Answer one question from each unit

[5X12=60M]

Unit-I

2. a) Solve $(1 + y^2)dx = (\tan^{-1}y - x)dy$

b) A body originally at $80^{\circ}C$ cools down to $60^{\circ}C$ in 20 minutes. The temperature of the air be $40^{\circ}C$. What will be the temperature of the body after 45 minutes from the original.

[6M+6M]

(OR)

3. a) Solve $\frac{dy}{dx} - \frac{tany}{1+x} = (1+x)e^x$ Secy

b) Find the Orthogonal trajectories of the Cardioids $r = a(1 - \cos\theta)$

[6M+6M]

Unit-II

4. a) Solve $(D^2 - 1)y = x \sin 3x + \cos x$

b) Solve $(D^2 - 4D + 4)y = 8x^2 e^{2x} \sin 2x$

[6M+6M]

(OR)

5. Using Method of Variation of Parameters, Solve

$$y'' - 2y' + 2y = e^x \tan x$$

[12M]

Unit-III

6. Expand $e^x \log(1+y)$ in powers of x and y up to terms of third degree.

(OR)

[12M]

7. a) In Spherical Polar Coordinates

$x = r \sin\theta \cos\phi, y = r \sin\theta \sin\phi, z = r \cos\theta$ then Show that

$$\frac{\partial(x,y,z)}{\partial(r,\theta,\phi)} = r^2 \sin\theta$$

b) Expand $f(x, y) = x^2y + 3y$ in powers of $(x - 1)$ and $(y + 2)$ using Taylors theorem. [6M+6M]

Unit-IV

8. Change the order of integration in $I = \int_0^1 \int_{x^2}^{2-x} xy \, dx \, dy$ and hence evaluate it. [12M]

(OR)

9. Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{dx \, dy \, dz}{\sqrt{1-x^2-y^2-z^2}}$ [12M]

Unit-V

10. a) Find the directional derivative of $\phi = 5x^2y - 5y^2z + 2.5z^2x$ at the point P(1,1,1) in the direction of the line $\frac{x-1}{2} = \frac{y-z}{-2} = z$

b) Show that $\nabla^2(r^n) = n(n+1)r^{n-2}$ [6M+6M]

(OR)

11. Verify Greens theorem for $\int_C [(xy + y^2)dx + x^2dy]$ where C is bounded by $y = x$ and $y = x^2$. [12M]