

**DIFFERENTIAL EQUATIONS
(Common to all Branches)****Time: 3 Hours****Max Marks: 60**

Answer ONE Question from each Unit

All Questions Carry Equal Marks

All parts of the Question must be answered at one place

UNIT-I

1. a) Solve $y(\log y)dx + (x - \log y)dy = 0$ 5M
 b) A body originally at 80°C cools down to 60°C in 20 minutes, the temperature of the air being 40°C . What will be the temperature of the body after 24 minutes from the original? 5M

(OR)

2. a) Solve $(x^3 + y^3)dx - (xy^2)dy = 0$. 5M
 b) Find the orthogonal trajectories of $x^2 + cy^2 = 1$. 5M

UNIT-II

3. Solve $(D^2 - 4D + 3)y = e^x \cos 2x + 1$. 10M

(OR)

4. Apply method of variation of parameters to solve $(D^2 + 1)y = (\operatorname{cosec} x) \cot x$. 10M

UNIT-III

5. Find the Fourier series of $f(x) = (\pi - x)/2$ in the interval $(0, 2\pi)$. 10M
 Hence, show that $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots$

(OR)

6. Find the Fourier cosine series expansion for 10M

$$f(x) = \begin{cases} \frac{1}{4} - x, & \text{if } 0 < x < \frac{1}{2} \\ x - \frac{3}{4}, & \text{if } \frac{1}{2} < x < 1 \end{cases}$$

UNIT-IV

7. Find the extreme value of 10M

$$f(x, y) = x^4 + y^4 - 2x^2 + 4xy - 2y^2$$

(OR)

8. a) Find Taylor's series expansion for $x^2y + 3y - 2$ about $(1, -2)$ 5M
 b) A rectangular box open at the top is to have volume of 32 cubic ft. Find the dimensions of the box requiring least material for its construction. 5M

UNIT-V

9. a) Form the partial differential equation by eliminating the arbitrary constants a and b from $x^2 + y^2 + (z - a)^2 = b^2$ 5M
b) Solve $p^2 z^2 + q^2 = p^2 q$. 5M

(OR)

10. a) Form the partial differential equation by eliminating the arbitrary function from $xyz = f(x + y + z)$ 5M
b) Solve $(y^2 + z^2)p - xyq + zx = 0$ 5M

UNIT-VI

11. A bar 100cm long, with insulated sides, has its ends kept at 0°C and 100°C until steady state conditions prevail. The two ends are then suddenly insulated and kept so. Find the temperature distribution. 10M
(OR)
12. A tightly stretched string with fixed end points $x=0$ and $x=l$ is initially in a position given by $y = y_0 \sin^3(\pi x/l)$. If it is released from rest from this position, find the displacement $y(x,t)$ 10M