

Answer ONE Question from each Unit

All Questions Carry Equal Marks

All parts of the Question must be answered at one place

**UNIT-I**

1. a) Define rank of the matrix? Apply Echelon form to find the rank of the matrix 5M

$$\begin{bmatrix} 2 & 1 & 3 & 5 \\ 4 & 2 & 1 & 3 \\ 8 & 4 & 7 & 3 \\ 8 & 4 & -3 & -1 \end{bmatrix}$$

- b) Reduce the given matrix into Normal form and hence find the rank 5M

$$\begin{bmatrix} 6 & 1 & 3 & 8 \\ 4 & 2 & 6 & -1 \\ 10 & 3 & 9 & 7 \\ 16 & 4 & 12 & 15 \end{bmatrix}$$

**(OR)**

2. a) Discuss for what values of  $\lambda, \mu$  the simultaneous equations have a Unique solution. 5M

$$x + y + z = 6, \quad x + 2y + 3z = 10, \quad x + 2y + \lambda z = \mu$$

- b) Test for consistency and solve the following simultaneous equations 5M

$$x + 2y - z = 0, \quad 2x + y + z = 0, \quad x - 4y + 5z = 0.$$

**UNIT-II**

3. Calculate the Eigen values of the matrix  $\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$  and the corresponding Eigen vectors. 10M

**(OR)**

4. Calculate Eigen values of the matrix  $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$  and the corresponding eigen vectors. 10M

### UNIT-III

5. a) Calculate the area enclosed by the Parabolas  $y^2 = x$  and  $x^2 = y$ . 5M
- b) Evaluate  $\iint_R xy(x+y) dx dy$ , over the region R bounded by  $y = x^2$  and  $y = x$  5M

(OR)

6. Find the integral  $\int_0^{4a} \int_{x^2/4a}^{2\sqrt{ax}} dy dx$  by changing the order of integration 10M

### UNIT-IV

7. Show that  $\int_0^{\pi/2} \sin^m \theta \cos^n \theta d\theta = (1/2)B(\frac{m+1}{2}, \frac{n+1}{2})$  10M

(OR)

8. a) Show that  $\int_{-1}^1 (1+x)^{m-1} (1-x)^{n-1} dx = 2^{m+n-1} B(m, n)$  5M
- b) Show that  $\Gamma(1/2) = \sqrt{\pi}$  5M

### UNIT-V

9. Find the angle between the surfaces  $x^2 + y^2 + z^2 = 9$  and  $z = x^2 + y^2 - 3$  at the point  $(2, -1, 2)$  10M

(OR)

10. Show that the vector field  $\vec{A} = 2xyz^2 \vec{i} + (x^2z^2 + z \cos yz) \vec{j} + (2x^2yz + y \cos yz) \vec{k}$  is irrotational. Find the scalar potential function. 10M

### UNIT-VI

11. Verify Green's theorem in plane for  $\oint_C (3x^2 - 8y^2)dx + (4y - 6xy)dy$ , where C is the region bonded by  $y = \sqrt{x}$  and  $y = x^2$ . 10M

(OR)

12. Verify Gauss Divergence theorem for  $\vec{F} = x^2 \vec{i} + y^2 \vec{j} + z^2 \vec{k}$  over the cube formed by the planes  $x = 0, x = a, y = 0, y = b, z = 0, z = c$  10M

## LINEAR ALGEBRA AND CALCULUS

(Common to CE, EEE, ME, ECE, CSE, IT Branches)

Time: 3 Hours

Max Marks: 60

Answer ONE Question from each Unit

All Questions Carry Equal Marks

All parts of the Question must be answered at one place

UNIT-I

1. a) Discuss for what values of  $\lambda, \mu$ , the simultaneous equations  $x + y + z = 6$ ,  $x + 2y + 3z = 10$ ,  $x + 2y + \lambda z = \mu$  have a Unique solution 6M
- b) Solve the equations  $3x + y + 2z = 3$ ,  $2x - 3y - z = -3$ ,  $x + 2y + z = 4$ , Using Gauss elimination method 6M

(OR)

2. Find the eigen values and eigen vectors of the matrix 12M

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

UNIT-II

3. a) If  $u = \frac{yz}{x}$ ,  $v = \frac{zx}{y}$ ,  $w = \frac{xy}{z}$  then show that  $\frac{\partial(u,v,w)}{\partial(x,y,z)} = 4$  6M
- b) If  $x = u(1+v)$ ,  $y = v(1+u)$  then prove that  $\frac{\partial(xy)}{\partial(u,v)} = 1 + u + v$  6M

(OR)

4. A rectangular box open at the top is to have volume of 32 cubic ft. Find the dimensions of the box requiring least material for its construction 12 M

UNIT-III

5. Calculate the surface area of the sphere generated by the circle  $x^2 + y^2 = 16$  about its diameter. 12M

(OR)

6. Find the volume of the solid generated by the revolution of the cardioid  $r = a(1 + \cos \theta)$  about the initial line  $\theta = 0$ . 12M

UNIT-IV

7. Evaluate  $\int_0^1 \int_{x^2}^{2-x} xy dy dx$  by changing the order of integration 12 M

(OR)

8. Evaluate the double integral  $\int_0^1 \int_x^{\sqrt{x}} (x^2 + y^2) dx dy$  12M

UNIT-V

9. a) Find the unit normal vector to the given surface  $x^2y + 2zx = 4$  at  $(2, -2, 3)$  6M
- b) Find the curl  $\vec{f}$  where  $\vec{f} = \text{grad}(x^3 + y^3 + z^3 - 3xyz)$  6M

(OR)

10. Verify Green's theorem in a plane for  $\oint (3x^2 - 8y^2) dx + (4y - 6xy) dy$ , where C is the region bounded by  $y = \sqrt{x}$  and  $y = x^2$  12M

**ENGINEERING MATHEMATICS – I****(Common to CE, EEE, ME, ECE, CSE & IT Branches)****Time: 3 Hours****Max Marks: 70**

Answer ONE Question from each Unit

All Questions Carry Equal Marks

All parts of the Question must be answered at one place

**UNIT-I**

1. a) Solve  $y(x^2y^2 - 1)dx + x(x^2y^2 + 1)dy = 0$ . 7M  
 b) Show that the family of parabolas  $y^2 = 4cx + 4c^2$  is "self-orthogonal". 7M  
**(OR)**

2. a) Solve  $2(y - 4x^2)dx + xdy = 0$ . 7M  
 b) A body is heated to  $110^\circ\text{C}$  and placed in air at  $10^\circ\text{C}$ . After 1 hour its temperature is  $60^\circ$ . How much additional time is required for it to cool to  $30^\circ\text{C}$ ? 7M

**UNIT-II**

3. Solve  $(D^2 + 4D + 4)y = 2e^{4x} + \cos 2x$ . 14M  
**(OR)**  
 4. Solve  $(D^2 + 1)y = \log \cos x$  by the method of variation of parameters. 14M

**UNIT-III**

5. If  $x = \sqrt{wv}$ ,  $y = \sqrt{uw}$ ,  $z = \sqrt{uv}$  and  $u = r \sin \theta$ ,  $\cos \phi$ ,  $v = r \sin \theta \sin \phi$ ,  $w = r \cos \theta$ , Calculate  $\frac{\partial(x,y,z)}{\partial(r,\theta,\phi)}$  14M  
**(OR)**  
 6. Find the maximum and minimum values of  $f(x, y) = x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$ . 14M

**UNIT-IV**

7. Evaluate  $\iiint x^2yz \, dx \, dy \, dz$  taken over the volume bounded by the surface  $x^2 + y^2 = 9$ ,  $z = 0$ ,  $z = 2$ . 14M  
**(OR)**  
 8. Change the order of integration and evaluate  $\int_0^a \int_{\frac{y^2}{a}}^{2a-y} xy \, dx \, dy$ . 14M

**UNIT-V**

9. a) Determine the directional derivative of  $f = xy + yz + zx$  in the direction of the vector  $i + 2j + 2k$  at the point  $(1, 2, 0)$  7M  
 b) Prove that  $\vec{A} = (6xy + z^3)\vec{i} + (3x^2 - z)\vec{j} + (3xz^2 - y)\vec{k}$  is irrotational. Find a scalar function  $f(x, y, z)$  such that  $\vec{A} = \nabla f$ . 7M  
**(OR)**  
 10. Verify Stokes' theorem for  $\vec{A} = xz\vec{i} - y\vec{j} + x^2y\vec{k}$  where  $S$  is the surface of the region bounded by  $x = 0$ ,  $y = 0$ ,  $z = 0$ ,  $2x + y + 2z = 8$  which is not included in the  $xz$ -plane 14M

# AR13

CODE: 13BS1001

SET-2

ADITYA INSTITUTE OF TECHNOLOGY AND MANAGEMENT, TEKKALI  
(AUTONOMOUS)

I B.TECH I SEM SUPPLEMENTARY EXAMINATIONS, JUNE, 2022

## ENGINEERING MATHEMATICS - I (Common to All Branches)

Time: 3 Hours

Max Marks: 70

### PART-A

ANSWER ALL QUESTIONS

[1 x 10 = 10 M]

1. a) Find the orthogonal trajectories of the family  $y^2 = 4ax$ .
- b) Find the integrating factor of the linear differential equation  $\frac{dy}{dx} + \frac{y}{x} = \frac{\log x}{x}$ .
- c) Solve  $(D^2 - 1)y = 0$ .
- d) If  $f(D) = D^2 + 4$ , then find  $\frac{1}{f(D)} \cos 3x$ .
- e) If  $U = x^2 + y^2$ ,  $x = t^2$  and  $y = 2t$ , find  $du/dt$ .
- f) Find the stationary points of  $f(x, y) = x^2 + y^2 - 6x + 12$ .
- g) Evaluate  $\int_0^2 \int_0^3 xy dx dy$
- h) Transform to Cartesian form  $\int_0^{\pi/2} \int_0^a r^3 \sin \theta \cos \theta d\theta dr$ .
- i) If  $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ , then find  $\text{curl } \vec{r}$
- j) State Stoke's theorem.

### PART-B

Answer one question from each unit

[5x12=60M]

#### UNIT-I

2. a) Form the differential equation of the family of circles having centre on x-axis and passing through the origin. [6M]
  - b) Solve  $\frac{dy}{dx} + y = x^3 y^6$ . [6M]
- (OR)
3. a) Solve  $(1 - x^2) \frac{dy}{dx} + 2xy = x\sqrt{1 - x^2}$ . [6M]
  - b) Solve  $x^2 y dx - (x^3 + y^3) dy = 0$  [6M]

#### UNIT-II

4. Solve  $(D^2 + 1)y = e^{-x} + x^2 + e^x \sin x$ . [12M]
- (OR)
5. Solve  $(D^3 - 3D^2 + 3D - 1)y = x e^x$  [12M]

#### UNIT-III

6. a) Find the Taylor series of  $f(x, y) = e^{xy}$  in powers of  $x-1$  and  $y-1$  [6M]
  - b) If  $x + y + z = u$ ,  $y + z = uv$  and  $z = uvw$  find  $\frac{\partial(x, y, z)}{\partial(u, v, w)}$ . [6M]
- (OR)
7. Find the maximum and minimum values of  $f(x, y) = x^3 - 3xy^2 - 15x^2 - 15y^2 + 72x$  [12M]

**UNIT-IV**

8. Change the order of integration and evaluate  $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y} dy dx$  [12M]

**(OR)**

9. Find the volume common to the cylinders  $x^2+y^2=a^2$  and  $x^2+z^2=a^2$ . [12M]

**UNIT-V**

10. a) Find  $\text{div } \vec{f}$  where  $\vec{f} = (x^2-yz)\vec{i} + (y^2-xz)\vec{j} + (z^2-xy)\vec{k}$  at (1,2,1) [6M]  
b) Find a unit normal to the surface  $x^2+y^2+2z^2=26$  at (2,2,6) [6M]

**(OR)**

11. Evaluate  $\int \vec{F} \cdot \vec{n} dS$  where  $\vec{F} = z\vec{i} + x\vec{j} - 3y^2z\vec{k}$  and S is the surface  $x^2+y^2=16$  included in the first octant between  $z=0$  and  $z=5$ . [12M]