

**ADITYA INSTITUTE OF TECHNOLOGY AND MANAGEMENT, TEKKALI
(AUTONOMOUS)**

**I B. Tech II Semester Supplementary Examinations, August-2015
ENGINEERING MATHEMATICS-III
(Common to CE, ME, CSE, IT, ECE &EEE)**

Time: 3 hours

Max Marks: 70

PART- A

Answer all questions

[10 x 1=10M]

1. a) Determine the rank of unit matrix of order 5(I_5).
- b) Are there any linear systems without solution? with one solution? with more than one solution? Justify?
- c) The eigen values of a matrix $A = \begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$ are 1, 2, 3. Determine the eigen values of the matrix $\text{adj}A$.
- d) Write the quadratic form corresponding to the symmetric matrix $\begin{bmatrix} 1 & 2 \\ 2 & -4 \end{bmatrix}$.
- e) Is Fourier series exist for the periodic function $f(x) = \sqrt{x}$ in $(0, 5)$? If not give reasons.
- f) If the Fourier transform of e^{-x^2} is $\sqrt{f} e^{-\frac{s^2}{4}}$, then find inverse Fourier transform of $e^{-\frac{s^2}{4}}$.
- g) Find the Z-transform of $\sin(3n)$.
- h) Find the inverse Z –transform of $\frac{z}{(z-1)^2}$.
- i) Find $S(1,2)$
- j) Compute $\Gamma(\frac{1}{4})\Gamma(\frac{3}{4})$.

PART-B

Answer one question from each unit

[5 X 12 = 60 M]

UNIT - I

2. a) Reduce the matrix $\begin{bmatrix} 8 & 1 & 3 & 6 \\ 0 & 3 & 2 & 2 \\ -8 & -1 & -3 & 4 \end{bmatrix}$ in to its normal form and hence find its rank. [6M]
- b) Find the values of a and b for which the equations $x + ay + z = 3$; $x + 2y + 2z = b$; $x + 5y + 3z = 9$ are consistent. When will these equations have a unique solution? [6M]
(OR)
3. a) Reduce to echelon form and hence find the rank of the matrix $A = \begin{bmatrix} 1 & 2 & -4 & 5 \\ 2 & -1 & 3 & 6 \\ 8 & 1 & 9 & 7 \end{bmatrix}$. [6M]
- b) Solve the system of equations $8y + 2z = -7$, $3x + 5y + 2z = 8$, $6x + 2y + 8z = 26$ by Gauss Elimination method. [6M]

UNIT-II

4. a) Two eigen values of the matrix $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$ are 1, 1. Find the eigen values of A^{-1} . [4M]

- b) Find the characteristic equation of the matrix, $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$ and hence compute by [8M]

using Cayley's Hamilton Theorem. Also find the matrix represented by $A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I$.

(OR)

5. Reduce the quadratic form $3x^2 + 5y^2 + 3z^2 - 2xy - 2xz + 2yz$ into canonical form by orthogonal transformation. Also identify its nature, rank, signature and index. [12M]

UNIT-III

6. a) Expand the function $f(x)$ as a Fourier series [8M]
 $f(x) = \begin{cases} - & ; - < x < 0 \\ x & ; 0 < x < \end{cases}$ and hence deduce that $1/1^2 + 1/3^2 + 1/5^2 + \dots = \pi^2/8$.

- b) Find the Fourier transform of $f(x) = \begin{cases} 1 - x^2; & |x| \leq 1 \\ 0; & |x| > 1 \end{cases}$ [4M]

(OR)

7. a) Expand $f(x) = x$ as a half range cosine series in the interval $0 < x < \pi$ and hence deduce $1/1^2 + 1/3^2 + 1/5^2 + \dots = \pi^2/8$. [6M]

- b) Solve the integral equation [6M]

$$F_c(S) = \int_0^\infty f(t) \cos st dt = \begin{cases} 1-s, & 0 \leq s \leq 1 \\ 0, & s > 1, \end{cases} \quad \text{Hence evaluate } \int_0^\infty \frac{\sin^2 t}{t^2} dt$$

UNIT-IV

8. a) Evaluate u_3 from $U(z) = \frac{2z^2 + 5z + 14}{(z-1)^4}$, by using initial value theorem. [6M]

- b) Using Z-transform, solve the difference equation $u_{k+2} - 2u_{k+1} + 3u_k = 2^k$, with $u_0 = 2$, $u_1 = 1$. [6M]

(OR)

9. a) Using damping rule, show that (i) $Z(na^n) = \frac{az}{(z-a)^2}$ (ii) $Z(n^2a^n) = \frac{az^2 + a^2z}{(z-a)^3}$ [6M]

- b) Evaluate $Z^{-1}\left(\frac{z^2}{(z-4)(z-9)}\right)$, by using Convolution theorem. [6M]

UNIT-V

10. a) Define gamma function and evaluate $\Gamma\left(\frac{1}{2}\right)$. [6M]

- b) Show that $\int_0^\infty x^m e^{-a^2x^2} dx = \frac{1}{2a^{m+1}} \Gamma\left(\frac{m+1}{2}\right)$. [6M]

(OR)

11. a) Show that $\int_0^1 \frac{x^4}{\sqrt{1-x^2}} dx = \frac{3f}{16}$. [6M]

- b) Define beta function and show that $S(m, n) = S(n, m)$. [6M]