

AR16

CODE: 16BS1002

SET-1

**ADITYA INSTITUTE OF TECHNOLOGY AND MANAGEMENT, TEKKALI
(AUTONOMOUS)**

I B.Tech II Semester Supplementary Examinations, Oct./Nov,2020

**ENGINEERING MATHEMATICS – II
(Common to all branches)**

Time: 3 Hours

Max Marks: 70

Answer ONE Question from each Unit

All Questions Carry Equal Marks

All parts of the Question must be answered at one place

UNIT-I

1. a Compute a real root of $x^3 - 2x - 5 = 0$ correct to 3 decimal places by Regula Falsi Method **7M**
b Compute a real root of the equation $x^4 - x - 10 = 0$ by Newton Raphson method **7M**

(OR)

2. a Compute $y(17)$ using Newton's backward difference formula, from the following data **7M**

x	8	10	12	14	16	18
y	10	19	32.5	54	89.5	15.4

- b Find the polynomial such that $f(1) = 9, f(4) = 18, f(12) = 130$ using Lagrange's interpolation formula. **7M**

UNIT-II

3. a Using Taylor's method, solve $\frac{dy}{dx} = 2y + 3e^x$ with $y(0) = 0$ at $x = 0.2$ **7M**
b Using Runge-Kutta method of fourth order, solve $\frac{dy}{dx} = \frac{2xy + e^x}{x^2 + xe^x}$ with $y(1) = 0$ at $x = 1.2$ **7M**

(OR)

4. a Evaluate $\int_0^6 \frac{dx}{1+x^2}$ by using Trapezoidal rule **7M**
b Determine $f'(50)$ using Newton's forward difference formula, from the following data **7M**

x	50	55	60	65
$f(x)$	1.6990	1.7404	1.7782	1.8129

UNIT-III

5. a Find the Laplace transforms of the function $f(t) = e^{-t} \sin^2 t$ **7M**
b Find $L^{-1} \left\{ \frac{3s+2}{s^2-s-2} \right\}$ **7M**

(OR)

6. a Find the Laplace transform of the full wave rectifier function **7M**
defined by
 $f(t) = E \sin \omega t$ if $0 < t < \frac{\pi}{\omega}$ having period $\frac{\pi}{\omega}$
b Solve $\frac{d^4 y}{dt^4} - k^4 y = 0$, where $y(0) = 1, y'(0) = y''(0) = y'''(0) = 0$ by using Laplace transform method. **7M**

UNIT-IV

7. a Obtain the Fourier series for $f(x) = 1 - x^2$ in $(-1, 1)$ **7M**
b Obtain the half range cosine series $f(x) = (x-1)^2$ in $(0, 1)$ **7M**
(OR)
8. a Determine the Fourier series for $f(x) = x - x^2$ in $(-\pi, \pi)$ **7M**
b Obtain the half range sine series $f(x) = x$ in $(0, 2)$ **7M**

UNIT-V

9. a Solve the partial differential equation **7M**
 $x(y^2 - z^2)p + y(z^2 - x^2)q = z(x^2 - y^2)$
b Solve $y \frac{\partial u}{\partial x} + x \frac{\partial u}{\partial y} = 3u$ and $u(x, 0) = e^{x^2}$ by the method of **7M**
separation of variables
(OR)
10. a Find a partial differential equation from $z = f(x+at) + g(x-at)$ **7M**
b The ends A and B of a rod of 20 cm long, have the **7M**
temperature at 30°C and 80°C until steady state prevails. The
temperature of the ends are changed to 40°C and 60°C
respectively. Find the temperature distribution in the rod at
time t

Time: 3 Hours

Max Marks: 70

PART-A

ANSWER ALL QUESTIONS

[1 x 10 = 10 M]

1. a) If $A = \begin{bmatrix} 1-i & i+2 & -i \\ 3 & 5 & i+6 \end{bmatrix}$, then $\bar{A} = \dots$
- b) The eigen values of $A = \begin{bmatrix} i/2 & \sqrt{3}/2 \\ \sqrt{3}/2 & i/2 \end{bmatrix}$
- c) If $\lambda^2 - 5\lambda - 2 = 0$ is the characteristic equation of A then $A^{-1} = \dots$
- d) The matrix of the quadratic form $x^2 + y^2 + z^2$ is
- e) What are the conditions required to express a function $f(x)$ as a Fourier series
- f) Write Fourier sine integral formulas for $f(x)$
- g) Find the Z-Transform of na^n
- h) Write the initial value theorem of Z-Transform
- i) Using Gamma function find the value of $\int_0^\infty e^{-x^2} dx$
- j) Find the value of $\beta\left(\frac{1}{4}, \frac{3}{4}\right)$

PART-B

Answer one question from each unit

[5x12=60M]

UNIT-I

2. a) Reduce the matrix $\begin{bmatrix} 2 & 1 & 3 & 5 \\ 4 & 2 & 1 & 3 \\ 8 & 4 & 7 & 13 \\ 8 & 4 & -3 & -1 \end{bmatrix}$ to Echelon form and find its rank 6M
- b) Find the values of a and b for which the equations $x + y + z = 3$,
 $x + 2y + 2z = 6$, $x + ay + 3z = b$ have (i) no solution (ii) a unique solution
(iii) an infinite number of solutions. 6M
- (OR)
3. a) Find what value of k the matrix $\begin{bmatrix} 4 & 4 & -3 & 1 \\ 1 & 1 & -1 & 0 \\ k & 2 & 2 & 2 \\ 9 & 9 & k & 3 \end{bmatrix}$ has rank 3. 6M
- b) Solve completely the following system of equations 6M
 $x + y - 3z + 2w = 0$, $2x - y + 2z - 3w = 0$, $3x - 2y + z - 4w = 0$, $-4x + y - 3z + w = 0$

UNIT-II

4. a) Find the Eigen values and Eigen vectors of the matrix $\begin{bmatrix} 1 & -6 & -4 \\ 0 & 4 & 2 \\ 0 & -6 & -3 \end{bmatrix}$ 8M
- b) Write down the quadratic form corresponding to the matrix $\begin{bmatrix} 0 & 5 & -1 \\ 5 & 1 & 6 \\ -1 & 6 & 2 \end{bmatrix}$ 4M

(OR)

5. a) Find the Eigen values and Eigen vectors of the matrix $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ 8M
- b) Represent the quadratic form $x^2 + 4y^2 + z^2 - 4xy + 2xz - 4yz$ in matrix notation. 4M

UNIT-III

6. a) Find a Fourier series to represent $x - x^2$ in the interval $(-\pi, \pi)$ 6M
- b) Find the Fourier transform of $f(x) = \begin{cases} 1 - x^2, & \text{if } |x| \leq 1 \\ 0, & \text{if } |x| > 1 \end{cases}$. Hence evaluate 6M

$$\int_0^{\infty} \frac{x \cos x - \sin x}{x^3} \cos \frac{x}{2} dx$$

(OR)

7. a) Expand $f(x) = \begin{cases} \frac{1}{4} - x, & \text{if } 0 < x < \frac{1}{2} \\ x - \frac{3}{4}, & \text{if } \frac{1}{2} < x < 1 \end{cases}$ as the Fourier series of sine terms. 6M
- b) Find the Fourier cosine transform of e^{-x^2} 6M

UNIT-IV

8. a) If $\bar{u}(z) = \frac{2z^2 + 5z + 14}{(z-1)^4}$, evaluate u_2 and u_3 6M
- b) Find the inverse Z-Transform of (i) $\frac{2z^2 + 3z}{(z+2)(z-4)}$ (ii) $\frac{2z^3 - 20z}{(z-2)^3(z-4)}$ 6M
9. a) Show that $Z\left[\frac{1}{n!}\right] = e^{1/z}$. Hence evaluate $Z\left[\frac{1}{(n+1)!}\right]$ and $Z\left[\frac{1}{(n+2)!}\right]$ 6M
- b) Solve the difference equation $u_{n+2} - 2u_{n+1} + u_n = 3n + 5$ 6M

UNIT-V

10. a) Prove that $\beta(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$ 6M
- b) Prove that $\beta(n, n+1) = \frac{1}{2} \frac{[\Gamma(n)]^2}{\Gamma(2n)}$ hence find the value of $\beta\left(\frac{1}{4}, \frac{5}{4}\right)$ 6M
- (OR)
11. a) Express the following integrals in terms of gamma functions 6M
- (i) $\int_0^1 \frac{dx}{\sqrt{1-x^4}}$ (ii) $\int_0^{\pi/2} \sqrt{\tan \theta} d\theta$
- b) Show that (i) $\Gamma(n) = \int_0^1 \left\{ \log \left(\frac{1}{x} \right) \right\}^{n-1} dx$ (ii) $\beta(m, n+1) + \beta(m+1, n) = \beta(m, n)$ 6M