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13BS1003 **SET-1**

ADITYA INSTITUTE OF TECHNOLOGY AND MANAGEMENT, TEKKALI
(AUTONOMOUS)
I B.Tech II Semester Supplementary Examinations, July-2016
ENGINEERING MATHEMATICS-III
(Common to CE, EEE, ME, ECE, CSE & IT)

Time: 3 hours **Max Marks:70**

PART-A

Answer all questions **[10 x 1 = 10M]**

1 (a) Define echelon form of a matrix.

(b) Find the sum and product of the eigen values of the matrix $\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$.

(c) Find the matrix of the quadratic form $x^2 - 6xy + 3y^2$.

(d) State Cayley-Hamilton theorem.

(e) If $f(x) = x$ in $(-\pi, \pi)$, then find the Fourier coefficient a_2.

(f) State complex form of Fourier integral of a function..

(g) Find the finite Fourier sine transform of $f(x) = \sin ax$ in $(0, \pi)$.

(h) Find the inverse Z – transform of $\frac{4z}{z-a}$.

(i) Write the relation between beta and gamma functions.

(j) Evaluate $\int_0^{\pi/2} \sqrt{\cot\theta}\, d\theta$.

PART-B

Answer one question from each unit. **[5×12=60M]**

UNIT-I

2 (a) Find the rank of $A = \begin{bmatrix} 1 & -1 & -1 & 2 \\ 4 & 2 & 2 & -1 \\ 2 & 2 & 0 & -1 \end{bmatrix}$ **[6M+6M]**

(b) For what values of k the equations $x + y + z = 1,\ 2x + y + 4z = k, 4x + y + 10z = k^2$ have a solution and solve them completely in each case.

(OR)

3 (a) Test the following system for consistency and if consistent solve it **[6M+6M]**
$x + 2y + z = 3,\ 2x + 3y + 2z = 5,\ 3x - 5y + 5z = 2,\ 3x + 9y - z = 4.$

(b) Express the following system in matrix form and solve by Gauss elimination method: $2x_1 + x_2 + 2x_3 + x_4 = 6,\ 6x_1 - 6x_2 + 6x_3 + 12x_4 = 36,$
$4x_1 + 3x_2 + 3x_3 - 3x_4 = -1,\ 2x_1 + 2x_2 - x_3 + x_4 = 10.$

UNIT-II

4 (a) Determine the eigen values and eigen vectors of $\begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$. **[6M+6M]**

(b)Verify Caley-Hamilton theorem for the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ and find A^{-1}.

(OR)

5. Diagonalize the matrix $A=\begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$ and hence find A^4. **[12M]**

UNIT-III

6 (a) Obtain the Fourier series to represent $f(x)=\frac{1}{4}(\pi-x)^2$ in $0<x<2\pi$.

Hence show that $\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+\cdots\cdots\cdots=\frac{\pi^2}{12}$.

(b) Find the Fourier transform of $e^{-x^2/2}, -\infty<x<\infty$. **[6M+6M]**

(OR)

7 (a) Find the Fourier series to represent $f(x)=x^2-2$, when $-2\le x\le 2$.

(b) Find the Fourier transform of $f(x)$ defined by $f(x)=\begin{cases} 1-x^2, \ if \ |x|\le 1 \\ 0, \ if \ |x|>1 \end{cases}$.

Hence evaluate $\int_0^\infty \frac{x\cos x-\sin x}{x^3}\cos\frac{x}{2}\,dx.$ **[6M+6M]**

UNIT-IV

8 (a) Find $Z[\cosh at \sin bt]$.

(b) Solve the difference equation using Z – transform: $u_{n+2}-3u_{n+1}+2u_n=0$ given that $u_0=0, u_1=1$. **[4M+8M]**

(OR)

9 (a) If $f(z)=\frac{2z^2+3z+4}{(z-3)^3}, |z|>3,$ then find the values of $f(1)$, $f(2)$ and $f(3)$.

(b) Using Convolution theorem, find $Z^{-1}\left[\frac{z^2}{(z-4)(z-5)}\right]$. **[6M+6M]**

UNIT-V

10 (a)Show that $\int_0^1 y^{q-1}\left(\log\frac{1}{y}\right)^{p-1}dy=\frac{\Gamma(p)}{q^p}$, where p>0, q>0.

(b) Prove that $\Gamma\left(n+\frac{1}{2}\right)=\frac{\Gamma(2n+1)\sqrt{\pi}}{2^{2n}\,\Gamma(n+1)}$. **[6M+6M]**

(OR)

11 (a) Show that $\beta(m,n)=\int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}}dx=\int_0^\infty \frac{x^{n-1}}{(1+x)^{m+n}}dx.$

(b) Evaluate $\int_0^1 x^4\left(\log\frac{1}{x}\right)^3 dx.$ **[6M+6M]**