AR20

CODE: 20BST101 **SET-1**

ADITYA INSTITUTE OF TECHNOLOGY AND MANAGEMENT, TEKKALI

(AUTONOMOUS)

I B.Tech I Semester Regular/Supplementary Examinations, April, 2022

LINEAR ALGEBRA AND CALCULUS

(Common to All Branches)

Time: 3 Hours **Max Marks: 60**

Answer ONE Question from each Unit
All Questions Carry Equal Marks
All parts of the Question must be answered in one place

UNIT-I

1. a) Determine the rank of $\begin{bmatrix} 4 & 2 & 3 \\ 8 & 4 & 6 \\ -2 & -1 & -1.5 \end{bmatrix}$. (5M)

   b) Solve the system by Gauss elimination method 2x+3y – z=5; 4x+4y-3z=3; 2x-3y+2z=2. (5M)

(OR)

2. a) Determine the values of a and b for which the system x+ 2y+ 3z=6; x+3y+5z=9; 2x+5y+az=b. (5M)

   b) Solve the following system of equations, if consistent: x+y+z=3; x+y – z=1; 3(x+y)-5z=1. (5M)

UNIT-II

3. Given that $A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$, verify that the eigenvalues of A^2 are the squares of those of A. (10M)

(OR)

4. Reduce the quadratic form $2x_1^2 + x_2^2 + x_3^2 + 2x_1x_2 - 2x_1x_3 - 4x_2x_3$ to canonical form by orthogonal transformation. Also find the rank, index, signature an nature of the quadratic form. (10M)

UNIT-III

5. Change the order of integration in $\int_0^a \int_0^{\frac{b}{a}\sqrt{a^2-x^2}} x^2 dydx$ and then integrate it. (10M)

(OR)

6. Evaluate $\int_0^1 \int_0^x \int_0^{x+y} (x+y+z)dzdydx$. (10M)

UNIT-IV

7. a) Prove that $\int_0^\infty e^{-ax}x^{n-1}\,dx = \frac{\Gamma(n)}{a^n}$, where a and n are positive. (5M)

   b) Evaluate $\int_0^1 x^m(1-x^n)^p\,dx$ in terms of Gamma functions. (5M)

(OR)

8. Show that $\beta(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}, m > 0 \ \& \ n > 0$ (10M)

UNIT-V

9. a) Show that $\bar{u} = (2x^2 + 8xy^2z)\bar{\imath} + (3x^3y - 3xy)\bar{\jmath} - (4y^2z^2 + 2x^3z)\bar{k}$ is not solenoidal (5M)

b) When $\phi = x^3 + y^3 + z^3 - 3xyz$, find $\nabla\phi$ and $\nabla.\nabla\phi$ at the point (1, 2, 3) (5M)

(OR)

10. a) Find the directional derivative $\phi = x^2yz + 4xz^2$ at the point P (1, -2, -1), (i) that is maximum, (ii) in the direction of PQ, where Q is (3, -3, -2). (5M)

b) Find the angle between the surfaces $x^2 - y^2 - z^2 = 11$ and $xy + yz - zx = 18$ at the point (6, 4, 3). (5M)

UNIT-VI

11. Verify Green's theorem in a plane with respect to $\int_C [(x^2 - y^2)dx + 2xy\, dy]$, where C is the boundary of the rectangle in the xy-planebounded by the lines $x = 0, x = a, y = 0$ and $y = b$. (10M)

(OR)

12. Verify Gauss divergence theorem for $\bar{F} = x^2\bar{\imath} + y^2\bar{\jmath} + z^2\bar{k}$, where S is the surface of the cuboid formed by planes $x = 0, x = a, y = 0, y = b, z = 0$ and $z = c$. (10M)

AR18

CODE: 18BST101 **SET-2**

ADITYA INSTITUTE OF TECHNOLOGY AND MANAGEMENT, TEKKALI
(AUTONOMOUS)
I B.Tech I Semester Supplementary Examinations, April, 2022

LINEAR ALGEBRA AND CALCULUS
(Common to All Branches)

Time: 3 Hours **Max Marks: 60**

Answer ONE Question from each Unit
All Questions Carry Equal Marks
All parts of the Question must be answered at one place

UNIT-I

1. a) Investigate for what values of a, b the equations $x + y + z = 3$, $x + 2y + 2z = 6$, $x + ay + 3z = b$ will have a unique solution — 6M

   b) Reduce the given matrix into Echelon form and hence find the rank of the matrix — 6M

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & -4 \\ 2 & 3 & 5 & -5 \\ 3 & -4 & -5 & 8 \end{bmatrix}$$

(OR)

2. Calculate the eigen values and its corresponding eigen vectors of the matrix — 12M

$$\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

UNIT-II

3. Find the Maximum and Minimum values of $f(x,y) = x^3 + 3xy^2 - 3x^2 - 3y^2 + 4$ — 12 M

(OR)

4. a) If $U = x \log(xy)$, where $x^3 + y^3 + 3xy + 1$ find du/dx — 6 M

   b) If $u = x + y + z$, $uv = y + z$, $uvw = z$ then S.T $\frac{\partial(x,y,z)}{\partial(u,v,w)} = u^2 z$ — 6M

UNIT-III

5. Find the perimeter of the cardioid $r = a(1+\cos\theta)$,also show that the upper half of the cardioid $r = a(1+\cos\theta)$ is bisected by the line $\theta = \pi/3$. 12M

(OR)

6. Find the volume of the solid generated by the revolution of the area bounded by the curves $y^2 = ax^3$ and $x^2 = ay^3$ about x- axis 12M

UNIT-IV

7. Using double integration , determine the area of the region bounded by the curves $y^2 = 4ax$, $x + y = 3a$ and the line $y = 0$. 12 M

(OR)

8. Evaluate the double integral $\int_0^2 \int_x^{\sqrt{2x-x^2}} (x^2+y^2)\,dx\,dy$ by changing into polar coordinates. 12M

UNIT-V

9. a) Find the angle between the surfaces $x^2+ y^2+ z^2 = 9$, $Z = x^2 + y^2 -3$ at the point (2,-1,2) 6M

   b) Show that the vector $(x^2 - yz)\bar{i} + (y^2 - zx)\bar{j} + (z^2 - xy)\bar{k}$ is irrotational and find its scalar potential 6M

(OR)

10. Verify Green's theorem in a plane for $\oint (3x^2 - 8y^2)dx + (4y - 6xy)\,dy$, where C is the region bounded by x = 0 and y = 0 and x+y =1 12M

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AR16

CODE: 16BS1001 **SET-1**

ADITYA INSTITUTE OF TECHNOLOGY AND MANAGEMENT, TEKKALI

(AUTONOMOUS)

I B.Tech I Semester Supplementary Examinations, April, 2022

ENGINEERING MATHEMATICS – I

(Common to All Branches)

Time: 3 Hours **Max Marks: 70**

Answer ONE Question from each Unit

All Questions Carry Equal Marks

All parts of the Question must be answered at one place

UNIT-I

1. a) Solve $(Cos\, x - x\, Cos\, y)dy - (Sin\, y + y\, Sin\, x)dx = 0$. 7M

b) Show that family of curves$x^2 + 4y^2 = c_1$ and $y = c_2x^4$ are (mutually) orthogonal (to each other). 7M

(OR)

2. a) Solve $y' + y = e^{e^x}$. 7M

b) If a substance cools from 370 k to330 k in 10 minutes, when the temperature of the surrounding air is 290 k, find the temperature of the substance after 40 minutes. 7M

UNIT-II

3. a) Solve $(D^2 + 2D + 1)y = 2e^{3x} + Sin\, 2x$. 7M

b) Solve $x^2y'' - 3xy' + 3y = 0$ with $y(1) = 0, y'(1) = -2$. 7M

(OR)

4. a) Solve $(D^3 - 3D^2 - 6D + 8)y = xe^x$. 7M

b) Solve $(D^2 + 1)y = Csc\, x$by the method of variation of parameters. 7M

UNIT-III

5. a) Use Taylor's theorem to expand$f(x, y) = x^2 + xy + y^2$ in powers of $(x - 1)$and $(y - 2)$. 7M

b) Calculate $\frac{\partial(u,v)}{\partial(r,\theta)}$ if $u = 2axy, v = a(x^2 - y^2)$ where $x = r\, Cos\, \theta, y = r\, Sin\, \theta$. 7M

(OR)

6. Find the maximum and minimum values of $f(x, y) = x^4 + y^4 - x^2 - y^2 + 1$. 14M

UNIT-IV

7. a) Evaluate $\iint_D (x^2 + y^2)dxdy$ where D is the region bounded by $y = x, y = 2x$ and $x = 1$ in the first quadrant. 7M

b) Find the total mass of the region in the cube $0 \le x \le 1, 0 \le y \le 1, 0 \le z \le 1$ with density at any point given by xyz. 7M

(OR)

8. a) Change the order of integration and evaluate $\int_0^1 \int_{x^2}^{2-x} xy\, dydx$. 7M

b) Evaluate $\int_0^1 \int_0^x \int_0^{x+y} (x + y + z)dzdydx$. 7M

UNIT-V

9. a) Determine the directional derivative of $f = xy^2 + yz^3$ at the point (2, –1, 1) in the direction of the vector i + 2j + 2k. 7M

b) Prove that $\bar{A} = (2x^2 + 8xy^2z)i + (3x^3y - 3xy)j - (4y^2z^2 + 2x^3z)k$ is not solenoidal but $\bar{B} = xyz^2\bar{A}$ is solenoidal. 7M

(OR)

10. Verify the divergence theorem for$\bar{A} = 2x^2yi + y^2j + 4xz^2k$ taken over the region in the first octant bounded by the cylinder $y^2 + z^2 = 9$and the plane x = 2 14M

AR13

CODE: 13BS1001 **SET-1**

ADITYA INSTITUTE OF TECHNOLOGY AND MANAGEMENT, TEKKALI
(AUTONOMOUS)

I B.Tech I Semester Supplementary Examinations, April, 2022

ENGINEERING MATHEMATICS - I
(Common to All Branches)

Time: 3 Hours **Max Marks: 70**

PART-A

ANSWER ALL QUESTIONS **[1 x 10 = 10 M]**

1. a) Solve $(x^2-ay)dx - (ax - y^2)dy = 0$
   b) State Newton's law of cooling.
   c) Find the roots of the auxiliary equation $D^2-D+1=0$
   d) If $f(D) = D^3-1$ then find $\frac{1}{f(D)}e^{2x}$.
   e) If $x= u(1-v)$, $y=uv$, find $\frac{\partial(x,y)}{\partial(u,v)}$
   f) Find the stationary points of $f(x,y) = x-y-xy$
   g) Evaluate $\int_0^1\int_1^2 x^2y\,dxdy$
   h) Change into polar coordinates $\int_0^1\int_0^{\sqrt{1-y^2}} dydx$.
   i) Find ∇f at (1,1,1) where $f = x^2+y^2+z^2$
   j) State Green's theorem in a plane .

PART-B

Answer one question from each unit **[5x12=60M]**

UNIT-I

2. a) Form the differential equation from $y = a\,e^{2x} + be^{-3x} + ce^{x}$ by eliminating a, b and c. [6M]
   b) Solve $\frac{dy}{dx} = y\,tanx - y^2secx$. [6M]

(OR)

3. a) Solve $y\sin 2x\,dx - (1+y^2+\cos^2x)dy = 0$ [6M]
   b) Solve $(1+y^2)\frac{dy}{dx} = tan^{-1}y - x$. [6M]

UNIT-II

4. a) Solve $(D-2)^2\,y = e^{2x} + \sin 2x$. [6M]
   b) Solve $(D^2-2D+1)y = e^x / x^2$ by method of variation of parameters. [6M]

(OR)

5. a) Solve $(D^2+2)\,y = x^2e^{3x} + \cos 2x$ [6M]
   b) Solve $(D^2 + 2D +1)y = x\cos x + e^{-x}$ [6M]

UNIT-III

6. a) Find the Taylor series expansion of $f(x,y) = e^x \sin y$ in powers of x and y up to the terms of third degree [6M]
   b) If $u = f(r, s, t)$ and $r = x/y$, $s = y/z$, $t = z/x$ prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$. [6M]

(OR)

7. a) If $u = x^2+y^2+z^2$, $v = xy+yz+zx$, $w = x+y+z$ then find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$. [6M]
   b) Find the Maximum and Minimum values of $f(x,y) = x^3+y^3-3axy$ [6M]

UNIT-IV

8. Evaluate by changing the order of integration $\int_0^1 \int_{x^2}^{2-x} xy\, dxdy$. [12M]

(OR)

9. a) Evaluate $\int_0^1 \int_{y^2}^1 \int_0^{1-x} x\, dzdxdy$. [6M]

b) Evaluate $\iint r^3 drd\theta$ over the area included between the circles r = 2sinθ and r = 4sinθ [6M]

UNIT-V

10. a) Find the directional derivative of f(x,y,z) = xy^2+yz^3 at the point (2,-1,1) in the direction of $\bar{\imath}+2\bar{\jmath}+\bar{k}$. [6M]

b) Show that $\nabla^2 r^m = m(m+1)r^{m-2}$ where $r^2 = x^2+y^2+z^2$. [6M]

(OR)

11. Verify Green's theorem for $\oint (xy + y^2)dx + x^2 dy$ where C is bounded by y=x and $y = x^2$. [12M]

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