

AR18

CODE: 18BST101

SET-1

**ADITYA INSTITUTE OF TECHNOLOGY AND MANAGEMENT, TEKKALI
(AUTONOMOUS)**

I B.Tech I Semester Supplementary Examinations, February-2020

LINEAR ALGEBRA AND CALCULUS

(Common to CE, EEE, ME, ECE, CSE & IT Branches)

Time: 3 Hours

Max Marks: 60

Answer ONE Question from each Unit

All Questions Carry Equal Marks

All parts of the Question must be answered at one place

UNIT-I

1. a) Reduce the matrix $A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$ to Echelon form and find its rank 4M
- b) Solve the equations $5x+3y+7z=4$, $3x+26y+2z=9$, $7x+2y+10z=5$. 8M

(OR)

2. Find the Eigen values and Eigen vectors of the matrix 12M
- $$A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$$

UNIT-II

3. a) Verify Rolle's theorem for $f(x) = (x-a)^m(x-b)^n$ in $[a, b]$ where m, n are positive integers. 6M
- b) If $f(x)$ and $g(x)$ are respectively e^x and e^{-x} , prove that c of Cauchy's mean value theorem is the arithmetic mean of a and b. 6M.
- (OR)**
4. a) Expand $\log_e x$ in powers of $(x-1)$ and hence evaluate $\log_e 1.1$ correct to four decimal places. 6M
- b) Examine $f(x, y) = x^4 + y^4 - 2x^2 + 4xy - 2y^2$ for extreme values. 6M

UNIT-III

5. a) Find the perimeter of the loop of the curve $9ay^2 = (x - 2a)(x - 5a)^2$. 6M
b) Find the surface of the solid formed by revolving the cardioid $r=a(1+\cos\theta)$ about the initial line. 6M
- (OR)**
6. a) Find the perimeter of the cardioid $r = a(1 + \cos\theta)$. 6M
b) Find the volume of the solid generated by revolving the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ about the minor axis. 6M

UNIT-IV

7. Change the order of integration in $I = \int_0^1 \int_{x^2}^{2-x} xy dx dy$ and hence evaluate the same. 12M
- (OR)**
8. a) Evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dx dy$ by changing into polar coordinates. 6M
Hence show that $\int_0^\infty e^{-x^2} dx = \frac{\sqrt{\pi}}{2}$.
- b) Evaluate $\iiint_R (x+y+z) dz dy dx$ where R is the region bounded by the planes $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$. 6M

UNIT-V

9. Verify Green's theorem for $\int_c [(3x^2 - 8y^2)dx + (4y - 6xy)dy]$ where c is the boundary of the region bounded by $x=0, y=0, x+y=3$. 12M
- (OR)**
10. a) Find the directional derivative of $f=xy^2+yz^3$ at the point (2,-1,1) in the direction of the normal to the surface $x\log z - y^2 = -4$ at (-1,2,1). 6M
b) Prove that $\frac{\vec{r}}{r^3}$ is solenoidal. 6M

AR16

CODE: 16BS1001

SET-1

**ADITYA INSTITUTE OF TECHNOLOGY AND MANAGEMENT, TEKKALI
(AUTONOMOUS)**

I B.Tech I Semester Supplementary Examinations, February-2020

ENGINEERING MATHEMATICS – I

(Common to CE, EEE, ME, ECE, CSE & IT Branches)

Time: 3 Hours

Max Marks: 70

Answer ONE Question from each Unit

All Questions Carry Equal Marks

All parts of the Question must be answered at one place

UNIT-I

1. a Solve $(1 + y^2)dx = (\tan^{-1} y - x)dy$ 7M
- b Prove that the system of parabolas $y^2 = 4a(x + a)$ is self-orthogonal. 7M

(OR)

2. a Solve $(y \log x - 1)y dx = xdy$ 7M
- b A body is originally at $80^\circ C$ and cools down to $60^\circ C$ in 20 minutes. If the temperature of the air is $40^\circ C$, find the temperature of the body after 40 minutes. 7M

UNIT-II

3. a Solve $(D^2 + 1)y = \sin x \sin 2x$ 7M
- b Solve $(D^2 + D)y = x^2 + 2x + 4$ 7M

(OR)

4. a Solve $\frac{d^2 y}{dx^2} - 3\frac{dy}{dx} + 2y = xe^{3x} + \sin 2x$ 7M
- b Solve $\frac{d^2 y}{dx^2} + y = \operatorname{cosec} x$ by the method of variation of parameters. 7M

UNIT-III

5. a If $x = r \cos \theta$; $y = r \sin \theta$ then show that 7M
$$\frac{\partial^2 r}{\partial x^2} + \frac{\partial^2 r}{\partial y^2} = \frac{1}{r} \left[\left(\frac{\partial r}{\partial x} \right)^2 + \left(\frac{\partial r}{\partial y} \right)^2 \right]$$
- b Obtain Taylor's expansion for $\tan^{-1}(y/x)$ about (1,1) upto second degree terms 7M

(OR)

6. a Find the shortest distance from origin to the surface $xyz^2 = 2$ 7M
- b Verify if $u = 2x - y + 3z$, $v = 2x - y - z$, $w = 2x - y + z$ are functionally dependent and if so, find the relation between them. 7M

UNIT-IV

7. a By change the order of integration, evaluate $\int_0^{4a} \int_{x^2/4a}^{2\sqrt{ax}} dy dx$ 7M
- b Evaluate $\iint (x^2 + y^2) dx dy$ in the positive quadrant for which $x+y \leq 1$ 7M
- (OR)**
8. a Evaluate $\iint_R (x^2 + y^2) dx dy$, where R is the region bounded by the parabolas $y^2 = 4x$ and $x^2 = 4y$ 7M
- b Changing into polar coordinates and evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dy dx$ 7M

UNIT-V

9. a Find the directional derivative of the function $f = x^2 - y^2 + 2z^2$ at the point P(1,2,3) in the direction of the line PQ where Q is (5,0,4) 7M
- b Find the Scalar potential ϕ such that $\vec{F} = \nabla \phi$ where $\vec{F} = 2xyz^3 \vec{i} + x^2z^3 \vec{j} + 3x^2yz^2 \vec{k}$ 7M
- (OR)**
10. a Verify Green's theorem in the plane for $\oint_C (3x^2 - 8y^2) dx + (4y - 6xy) dy$ where C is the boundary of the region defined by $y = \sqrt{x}$ and $y = x^2$. 14M

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**I B.Tech I Semester Supplementary Examinations, February-2020
ENGINEERING MATHEMATICS – I**

(Common to CE, ME, CSE, IT, ECE & EEE)

Time : 3 Hours

Max. Marks : 70

PART-A

Answer all questions

[10 X 1 = 10 M]

1. a) Find the Integrating Factor of the linear differential equation $x^2 \frac{dy}{dx} + \frac{y}{x} = 2x^2$.
- b) Check the differential equation $(5x^4 + 3x^2y^2 - 2xy^3)dx + (2x^3y - 3x^2y^2 - 5y^4)dy = 0$ for exactness.
- c) Solve $(D^2 + 9)y = 0$
- d) Find the particular integral $[y_p]$ of the differential equation $(D^2 - 4)y = e^{2x}$
- e) If $u = \log \frac{x^2}{y}$, then find $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$.
- f) If $x = u(1-v)$, $y = uv$ then Find $\frac{\partial(x, y)}{\partial(u, v)}$
- g) Evaluate $\int_1^2 \int_1^3 xy^2 dx dy$
- h) Write the formula for finding Moment of Inertia of a solid about x-axis.
- i) If $\Phi = 3x^2y - y^3z^2$, then find $\text{grad } \Phi$ at $(1, -2, -1)$
- j) Find $\text{curl}(xy \mathbf{i} + yz \mathbf{j} + zx \mathbf{k})$

PART-B

Answer one question from each unit

[5 X 12 = 60 M]

UNIT - I

2. a) Solve $x^2 \frac{dy}{dx} = 3x^2 - 2xy + 1$
- b) Find the orthogonal trajectories of the family of circle $x^2 + y^2 + 2fy + 1 = 0$.

[6M + 6M]

(OR)

3. a) Solve $\frac{dy}{dx} + 2xy = e^{-x^2}$
- b) Solve $\frac{dy}{dx} + y \cos x = y^2 \sin(2x)$

[6M + 6M]

UNIT - II

4. Solve $(D^2 - 1)y = x \sin x + x^2 e^x$ [12M]

(OR)

5. Solve $\frac{d^3 y}{dx^3} + 4 \frac{dy}{dx} = \sin 2x$ [12M]

UNIT - III

6. a) Find Taylor's series expansion of $\sin(2x)$ about $x = \pi/4$

b) If $u = \frac{x+y}{x-y}$ and $\theta = \tan^{-1} x + \tan^{-1} y$, find $\frac{\partial(u, \theta)}{\partial(x, y)}$ [6M + 6M]

[6M + 6M]

(OR)

7. Find the volume of the greatest rectangular parallelepiped that can be inscribed in the ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$
 [12M]

UNIT - IV

8. a) Evaluate $\int_0^1 \int_0^{1-x} \int_0^{1-x-y} dz dy dx$

b) Find the surface area of the solid generated by revolving the arc of the parabola $x^2 = 12y$, bounded by its latus rectum about y-axis. [6M + 6M]

(OR)

9. Change the order of integration in $\int_0^1 \int_{x^2}^{2-x} xy dx dy$ and hence Evaluate the double integral. [12M]

UNIT - V

10. a) Find the angle of intersection of the spheres $x^2 + y^2 + z^2 = 39$ and $x^2 + y^2 + z^2 + 4x - 6y - 8z + 52 = 0$ at $(4, -3, 2)$.

b) Prove that $\nabla^2(r^n) = n(n+1)r^{n-2}$ where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $r = |\vec{r}|$. [6M + 6M]

(OR)

11. By Transforming into triple integral to evaluate $\int \int \int x^2 dy dz + x^2 y dy dx + x^2 z dx dy$, where the surface is the closed surface consisting the cylinder $x^2 + y^2 = a^2$ and the circular disc $z=0$ and $z=6$.

[12M]