

**LINEAR ALGEBRA AND CALCULUS
(Common to All Branches)****Time: 3 Hours****Max Marks: 6x10 = 60**

Answer ONE Question from each Unit

Each Question Carry **10 Marks**

All parts of the Question must be answered at one place

UNIT-I

1. a) Determine the values of a for which the system has **exactly one Solution (Unique solution)**.

$$x + 2y - 3z = 4$$

$$3x - y + 5z = 2$$

$$4x + y + (a^2 - 14)z = a + 2$$

5M

- b) Find the number of free variables and solve the system of equation by Gauss Elimination method:

$$Z_3 + Z_4 + Z_5 = 0, -Z_1 - Z_2 + 2Z_3 - 3Z_4 + Z_5 = 0, Z_1 + Z_2 - 2Z_3 - Z_5 = 0,$$

$$2Z_1 + 2Z_2 - Z_3 + Z_5 = 0$$

5M**(OR)**

- 2 a)

Find the relations among p , q and r for which the rank of $A = \begin{bmatrix} 1 & 1 & 1 \\ p & q & r \\ p^2 & q^2 & r^2 \end{bmatrix}$ is 2.

5M

- b) Investigate for what values of λ , μ the simultaneous equations

$$\begin{aligned} x + y + z &= 6, \\ x + 2y + 3z &= 10, \\ x + 2y + \lambda z &= \mu, \end{aligned}$$

5M

have **an infinite number of solutions.**

UNIT-II

- 3.

Find a matrix P that diagonalizes $A = \begin{bmatrix} -1 & 7 & -1 \\ 0 & 0 & 0 \\ 0 & 15 & -2 \end{bmatrix}$, and then compute A^{11} .

10M**(OR)**

- 4.

Find all values of k for which the quadratic form of $5x_1^2 + x_2^2 + kx_3^2 + 4x_1x_2 - 2x_1x_3 - 2x_2x_3$ is positive definite.

10M**UNIT-III**

- 5.

Evaluate $\iint_E \frac{xy(x+y)^2}{(x^2+y^2)} dx dy$, Where E is bounded by $y = 0$, $y = x$, $x^2 + y^2 = a^2$ the first quadrant.

10M**(OR)**

- 6.

Evaluate $\iiint_E 6z^2 dV$ where E is the region below $4x + y + 2z = 10$ in the first octant.

10M

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UNIT-IV

7. Evaluate $\int_0^{\frac{\pi}{2}} (\sin x)^{2n-1} (\cos x)^{2m-1} dx$, using β and γ functions. **10M**

(OR)

8. Evaluate $\int_0^1 x^n (\ln x)^m dx$ using β and γ functions **10M**

UNIT-V

9. Find the values of a, b so that the surfaces $ax^2 - byz = (a + 2)x$ and $4x^2y + z^3 = 4$ may intersect at orthogonally at the point $(1, -1, 2)$. **10M**

(OR)

10. Find $\text{div}(\nabla\phi)$ where $\phi = x^2yz + 4xz^2$ at $(1, -2, -1)$. **10M**

UNIT-VI

11. Evaluate $\int_S F \cdot N ds$ where $F = 18zi - 12j + 3yk$ and S is the part of the plane $2x + 3y + 6z = 12$ located in the first octant. **10M**

(OR)

12. Verify Gauss's divergence theorem to evaluate $F = (x^3 - yz)i - 2x^2yj + zk$ over the Surface of the cube bounded by the coordinate planes $x = y = z = a$. **10M**