

**DIFFERENTIAL EQUATIONS
(Common to All Branches)****Time: 3 Hours****Max Marks: 60**

Answer ONE Question from each Unit

All Questions Carry Equal Marks

All parts of the Question must be answered at one place

UNIT-I

1. a) Solve $(1 + x^2)dy = (e^{\tan^{-1}x} - y)dx$ 5M

b) If a substance cools from 370k to 330k in 10minutes, 5M
when the temperature of the surrounding air is 290k,
find the temperature of the substance after 40minutes.**(OR)**

2. a) Solve $(1 + xy)ydx + (1 - xy)x dy = 0$ 5M

b) Find the orthogonal trajectories of the family of 5M
confocal conics $\frac{x^2}{a^2} + \frac{y^2}{b^2 + \lambda} = 1$, where λ is the parameter**UNIT-II**

3. Solve $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = xe^{3x} + \sin 2x$ 10M

(OR)

4. Solve, by the method of variation parameters, 10M
 $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = e^x \log x$

UNIT-III5. Find Fourier series for $f(x) = x^2$ in the interval 10M
 $[-\pi, \pi]$.Hence show that $\sum \frac{1}{n^2} = \frac{\pi^2}{6}$ **(OR)**

6. Find a half range cosine series for 10M
- $$f(x) = \begin{cases} kx, & 0 \leq x \leq \frac{L}{2} \\ k(L - x), & \frac{L}{2} \leq x \leq L \end{cases}$$

UNIT-IV

7. Find the maximum and minimum values of 10M
- $$x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$$

(OR)

8. a) Find Maclaurin's series expansion for 5M
- $$e^x \log(1 + y) \text{ about } (0,0)$$

- b) The temperature T at any point (x, y, z) in space 5M
- $$T(x, y, z) = Kxyz^2 \text{ where } K \text{ is a constant. Find the highest temperature on the surface } x^2 + y^2 + z^2 = a^2$$

UNIT-V

9. a) Form the partial differential equation by eliminating 5M
- the arbitrary constants a and b from $2z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$

- b) Solve $zpq = p + q$ 5M

(OR)

10. a) Form the partial differential equation by eliminating the 5M
- arbitrary function from

$$z = (x + y) f(x^2 - y^2)$$

- b) Solve $x^2(y - z)p + y^2(z - x)q = z^2(x - y)$ 5M

UNIT-VI

11. Solve $\frac{\partial u}{\partial x} = 2 \frac{\partial u}{\partial t} + u$, where $u(x, 0) = 6e^{-3x}$ by the 10M
- method of separation of variables.

(OR)

12. A tightly stretched string of length l with fixed ends is 10M
- initially in equilibrium position. It is set vibrating by giving each point a velocity $v_0 \sin^3(\pi x/l)$. Find the displacement $y(x, t)$