Introduction to Compositional Semantics

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1 Sense vs. denotation

One way of talking about the meaning of linguistic expressions is in terms of its **denotation** (what the expression points to in the actual world). This way of talking about meaning is useful because talking about the **sense** (what the linguistic expression actually expresses) of a word can be quite complicated. For example, how would you describe what the word *blue* means in words? 'A color between violet and green'? 'An effect of light with a wavelength between 450 and 500 nm'? This comes close to describing the sense of *blue*, but it's still hard to say. But here's the deal: whatever *blue* is, the sunny sky above us has that property, the ocean has that property, sapphire has that property, your jeans have that property, and your neighbor's flashy electric blue sports car has that property too. So again, another way of talking about meaning is in terms of what it *denotes*, or points to in the actual world.

2 Sets of things

So then, we can characterize the denotation of (certain) adjectives, nouns, and intransitive verbs (i.e., verbs that don't take objects) in this way (the double brackets '$[\![\quad]\!]$' is a short hand for 'the denotation of [the linguistic expression within the brackets]') :

(1) a. $[\![\text{blue}]\!]$ = the set of all blue things in the actual world
  b. $[\![\text{bird}]\!]$ = the set of all birds in the actual world
  c. $[\![\text{snore}]\!]$ = the set of all things that snore in the actual world

You might find it counterintuitive that verbs denote a set of individuals too. But think about it this way: if someone asked “Who snores?”, you can answer “they do” and point to the people that have this snoring property.

We will treat the denotation of names (like *Linda*) to be the particular individual that that particular name is pointing to. In other words, the denotation of *Linda* is the individual Linda in the actual world, and the denotation of *Ai* is the individual Ai in the actual world. Sometimes the denotation of a name is abbreviated as the initial letter of the name.

(2) a. $[\![\text{Linda}]\!]$ = the individual Linda in the actual world

b. $[\![\text{Linda}]\!] = \mathbf{l}$

3 Denotation of sentences

We know what words denote now; how about sentences? What do they denote? One property of a sentence is that it has a **truth value**; you can't say that *Linda* is true or that *snores* is false, but you can say that *Linda snores* is true or false (depending on the actual circumstances). So we can think of the denotation of sentences as its truth value: either the abstract value TRUE or the abstract value FALSE.

(3) a. $[\![\text{One plus one equals two}]\!]$ = TRUE

b. $[\![\text{One plus one equals three}]\!]$ = FALSE

We know enough about the world often times that we know whether a sentence is true or false (like the above examples); but the more realistic view is that we don't know everything about everything in the world. Take the sentence *My neighbor's cat's liver weighs 326 grams* for example: is this true or false? The sentence surely has a truth value, but we don't have enough world knowledge to actually determine the truth value. This is why we often express the denotation of a sentence as its **truth condition** (the circumstances that yield a certain truth value):

(4) $[\![\text{My neighbor's cat's liver weighs 326 grams}]\!]$ =

TRUE if my neighbor's cat's liver weighs 326 grams in the actual world,

FALSE otherwise

This means that even the denotation of sentences like the ones in (3) can be expressed as a truth condition too (e.g., 'TRUE if one plus one equals two in the actual world, FALSE otherwise'); we just happen to have the knowledge to determine what the truth value is in these cases.

4 Functions

Let's revisit the denotation of intransitive verbs and names.

(5) a. ⟦snore⟧ = the set of all things that snore in the actual world

b. ⟦Linda⟧ = the individual Linda in the actual world

What we're doing when we build a sentence is building the meaning of the sentence **compositionally**: we get the denotation of the sentence *Linda snores* by putting together the denotation of the word *snores* and the denotation of the word *Linda*. How do we accomplish that? Here's another way of thinking about sets of things: a set of things can tell you if something is a member of that set or not. For example, *snore* denotes the set of snorers: this set is able to tell me whether it is true or false that *Linda snores* (i.e., is she in this set or not?). Another way of phrasing this idea is that the denotation of *snore* an take in an individual x, and give you TRUE if x is in the snore set, and FALSE if it isn't. This verb bridges between the denotation of a name (an individual) and the denotation of a sentence (a truth value): it is a **function** that takes in an individual and spits out a truth value.

A function is anything that has a unique output for any input. Think of a meat grinder. A meat grinder is a function because it takes in something (e.g., beef steak, pork chop) as an input and reliably yields a unique output (e.g., ground beef, ground pork, respectively). Language is like this too! We can talk about the denotation of verbs, adjectives, and nouns as sets OR as functions; it's two ways of talking about the same thing.

5 Lambdas

Here is a formal way of talking about functions as a part of linguistic meaning. This is called **lambda calculus**. (6) repeats the denotation of *snore* in set talk. (7) is the denotation of the same word *snore*, just written as a function in lambda notation.

(6) $[\![\text{snore}]\!]$ = the set of all things that snore in the actual world

(7) $[\![\text{snore}]\!] = \lambda x[SNORE(x)]$

There are three parts to a basic function in lambda notation. (i) A λ (lambda) in a formula indicates a function: something that requires an input. (ii) The thing immediately after the λ is the INPUT. We'll use the variable x to represent an individual x. Variables are a place holder; this x could eventually be Linda, Teddy, Ai — whatever individual that we feed into this function. (iii) The stuff inside the square brackets is the OUTPUT. This will be the truth condition of the sentence. Read $SNORE(x)$ as 'x is in the *snore* set (i.e., x snores)'.

Linguistically, a function corresponds to things that are "incomplete". For example, the denotation of *snore* isn't "complete" until it combines with the denotation of an individual. Wouldn't it be weird if someone just said "snores"? You'd ask them, "WHO snored?" — you want that subject. The λx of $\lambda x[SNORE(x)]$ says 'hey, give me a subject!'.

Let's calculate the denotation of the sentence *Linda snores* step-by-step, using lambda notation. Here are our lexical entries — the denotation of the individual words that we're about to combine. These are a representation of what we store in our heads as the meaning of each word. The idea is that we don't store the meaning of entire sentences in our heads; just the meaning of individual words and their rules about what they can and can't combine with.

(8) *Lexical entries:*

a. $[\![\text{snore}]\!] = \lambda x[SNORE(x)]$

b. $[\![\text{Linda}]\!] = \mathbf{l}$

Let's start the computation now. First, we want to **apply** the *snore* function to *Linda* (we apply a function to its input), so we want to indicate which word is the function and which word is the word being fed into the function. We do this by showing the double brackets and parentheses.

(9) $[\![\text{Linda snores}]\!]$

$= [\![\text{snores}]\!]([\![\text{Linda}]\!])$

"$[\![\alpha]\!]([\![\beta]\!])$" should be read as 'the denotation of α applied to the denotation of β. In other words, the denotation of β is the input to the denotation of α (which should be a function). The input is in the parentheses.

We'll ignore tense for the sake of simplicity. So we'll drop the present tense *-s* altogether:

(10) $[\![\text{Linda snores}]\!]$
$$= [\![\text{snores}]\!]([\![\text{Linda}]\!])$$
$$= [\![\text{snore}]\!]([\![\text{Linda}]\!])$$

Now, we start to convert the double brackets into what the words within them actually denote. Let's start with the denotation of the name *Linda.* That would just be the individual Linda in the actual world (via our lexical entry), which we will abbreviate as **l**:

(11) $[\![\text{Linda snores}]\!]$
$$= [\![\text{snores}]\!]([\![\text{Linda}]\!])$$
$$= [\![\text{snore}]\!]([\![\text{Linda}]\!])$$
$$= [\![\text{snore}]\!](\mathbf{l})$$

Now we give the denotation of the verb, via our lexical entry:

(12) $[\![\text{Linda snores}]\!]$
$$= [\![\text{snores}]\!]([\![\text{Linda}]\!])$$
$$= [\![\text{snore}]\!]([\![\text{Linda}]\!])$$
$$= [\![\text{snore}]\!](\mathbf{l})$$
$$= \lambda x[SNORE(x)](\mathbf{l})$$

Now we apply this lambda function to its input, **l**. The λx says 'give me an individual', so we go and look in the parentheses to make sure that we have an individual. Check. Remember that the stuff inside the square brackets ($SNORE(x)$) is the output. What we're calculating is whether it's true or false that Linda snores; this means this x in the output is going to be Linda, or **l**. So substitute the x with **l**.

(13) $[\![\text{Linda snores}]\!]$

$= [\![\text{snores}]\!]([\![\text{Linda}]\!])$

$= [\![\text{snore}]\!]([\![\text{Linda}]\!])$

$= [\![\text{snore}]\!](\mathbf{l})$

$= \lambda x[SNORE(x)](\mathbf{l})$

$= SNORE(l)$

Notice that the λx disappeared in this step. This is because a λ indicates a function; it says that it's incomplete. Once we put **l** into the function, it *is* complete; we don't have a function anymore, so we shouldn't have a lambda once **l** goes in.

We're almost done. Currently, $SNORE(l)$ just reads as 'the individual Linda is in the *snore* set in the actual world'. What was this a condition for again? Right, the truth value of the sentence — remember that what we were calculating in the first place was the denotation of the sentence *Linda snores*, which should be a truth value. So we're going to add the final touch and state this last step as a truth condition:

(14) $[\![\text{Linda snores}]\!]$ (final result, full steps)

$= [\![\text{snores}]\!]([\![\text{Linda}]\!])$

$= [\![\text{snore}]\!]([\![\text{Linda}]\!])$

$= [\![\text{snore}]\!](\mathbf{l})$

$= \lambda x[SNORE(x)](\mathbf{l})$

$=$ TRUE if $SNORE(l)$, FALSE otherwise

6 Summary

In this chapter, we have explored meaning as being denotations: language helps us hook up what we're saying to what's happening in the actual world. Furthermore, we derived the denotation of sentences compositionally using lambda calculus. In class, we will talk further about the denotation of transitive verbs (e.g., *frighten*, as in *Linda frightened Teddy*) both in terms of sets and functions.