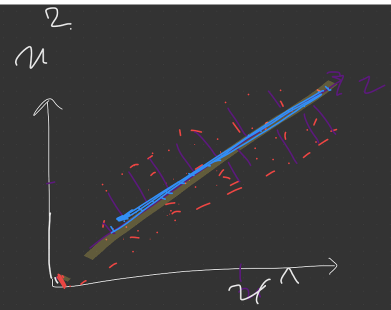
# Untitled note




$$\sum_{i=1}^{n} (x - \bar{x})^2 = V(x^1$$

$$Cov(x^1, x^2) = \sum_{i=1}^{n} (x^1 - \bar{x}^1)(x^2 - \bar{x}^2)$$

After centering

we can assume

$$\bar{X}^1 = 0$$

$$\bar{X}^2 = 0$$

| $i$ | $X^1$ | $X^2$ | $X^3$ |
|---|---|---|---|
| $x$ | | | |
| $x$ | | | |
| $\vdots$ | | | |

$$V(X^1) = \sum_{i=1}^{n} x_i^{1\,2}$$

$$= \langle X^1, X^1 \rangle$$

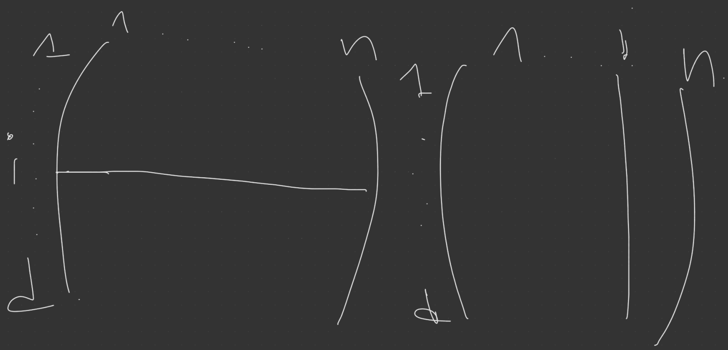
$$= \|X^1\|^2$$

$$= \underline{X^{1^T} X^1}$$

$$\mathrm{Cov}(X^i, X^j) = \underline{X^{i^T} X^j}$$

$$V(X) = \begin{array}{c|cccc} & 1 & 2 & \cdots & d \\ \hline 1 & V(1) & \mathrm{Cov}(1,2) & & \mathrm{Cov}\ \text{--} \\ \vdots & \mathrm{Cov}(2,1) & V(2) & & \\ \vdots & & & \ddots & \\ d & & & & V(d) \end{array}$$

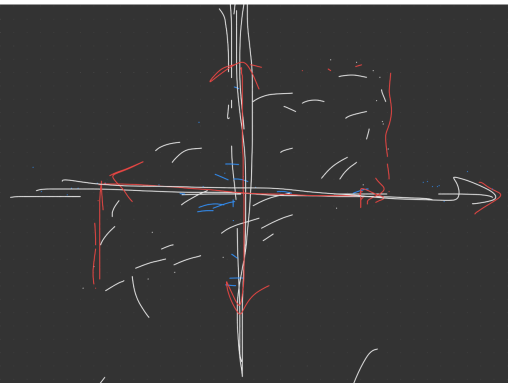
$$V(X) = \underline{{}^{d}_{n}X^T \, {}_{n}X^{d}}$$



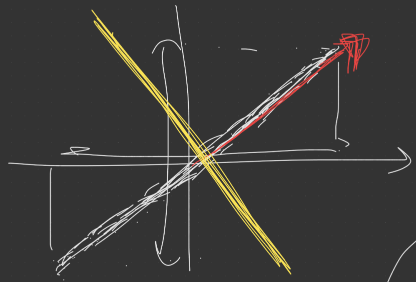
$$\begin{pmatrix} 1 & & & 0 \\ & 1 & & \\ & & 1 & \\ 0 & & & 1 \end{pmatrix}$$

$$d = 2 \qquad \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$V(X) = \begin{pmatrix} 1 & 0.5 \\ 0.5 & 1 \end{pmatrix}$$



$$V(X) = \begin{pmatrix} 1 & -0.99 \\ -0.99 & 1 \end{pmatrix}$$



$$\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = 2 \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

eigenvalue

eigenvector

$X^T X$

↓

eigenvectors are the PCA components

↓

their importance is given by their eigenvalues

$$\boxed{e_1 = \sum_{i=} c_i x^i}$$