Section 2 warmups

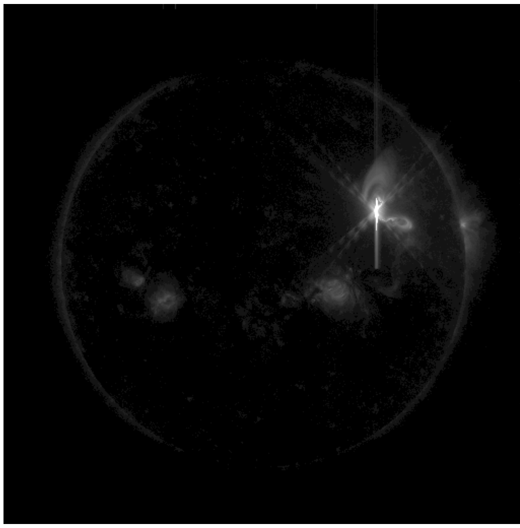
```
py3-10-ml-env-py3.10jrob@Jonathan-Laptop:~/cmu_grad/computer_vision$ cd HW0/numpy/
py3-10-ml-env-py3.10jrob@Jonathan-Laptop:~/cmu_grad/computer_vision/HW0/numpy$ python run.py --allwarmups
Running w1
Running w2
Running w3
Running w4
Running w5
Running w6
Running w7
Running w8
Running w9
Running w10
Running w11
Running w12
Running w13
Running w14
Running w15
Running w16
Running w17
Running w18
Running w19
Running w20
Ran warmup tests
20/20 = 100.0
```

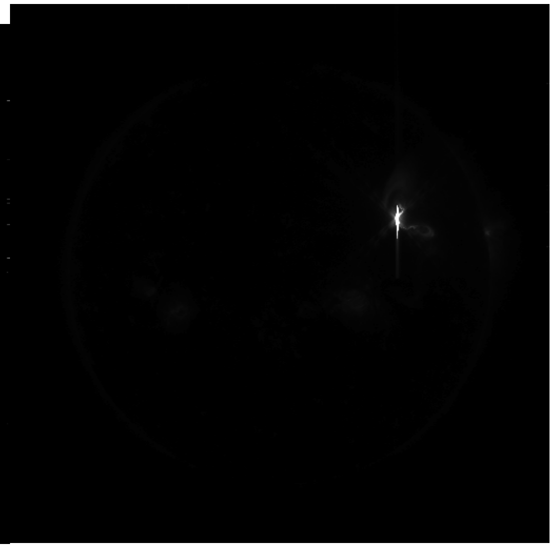
Section 2 tests

```
py3-10-ml-env-py3.10jrob@Jonathan-Laptop:~/cmu_grad/computer_vision/HW0/numpy$ python run.py --alltests
Running t1
Running t2
Running t3
Running t4
Running t5
Running t6
Running t7
Running t8
Running t9
Running t10
Running t11
Running t12
Running t13
Running t14
Running t15
Running t16
Running t17
Running t18
Running t19
Running t20
Ran all tests
20/20 = 100.0
py3-10-ml-env-py3.10jrob@Jonathan-Laptop:~/cmu_grad/computer_vision/HW0/numpy$
```

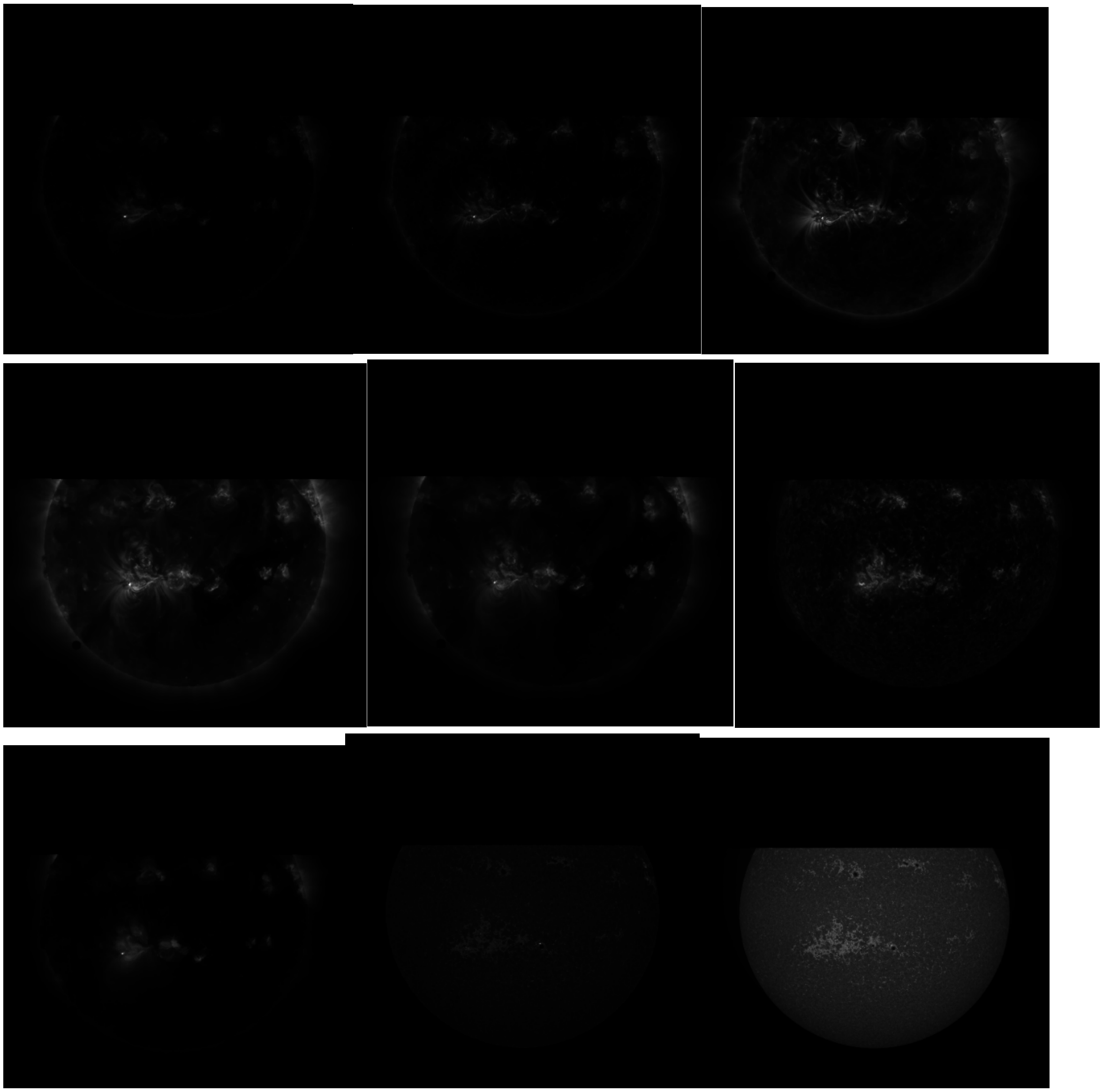
Section 3

3.1:

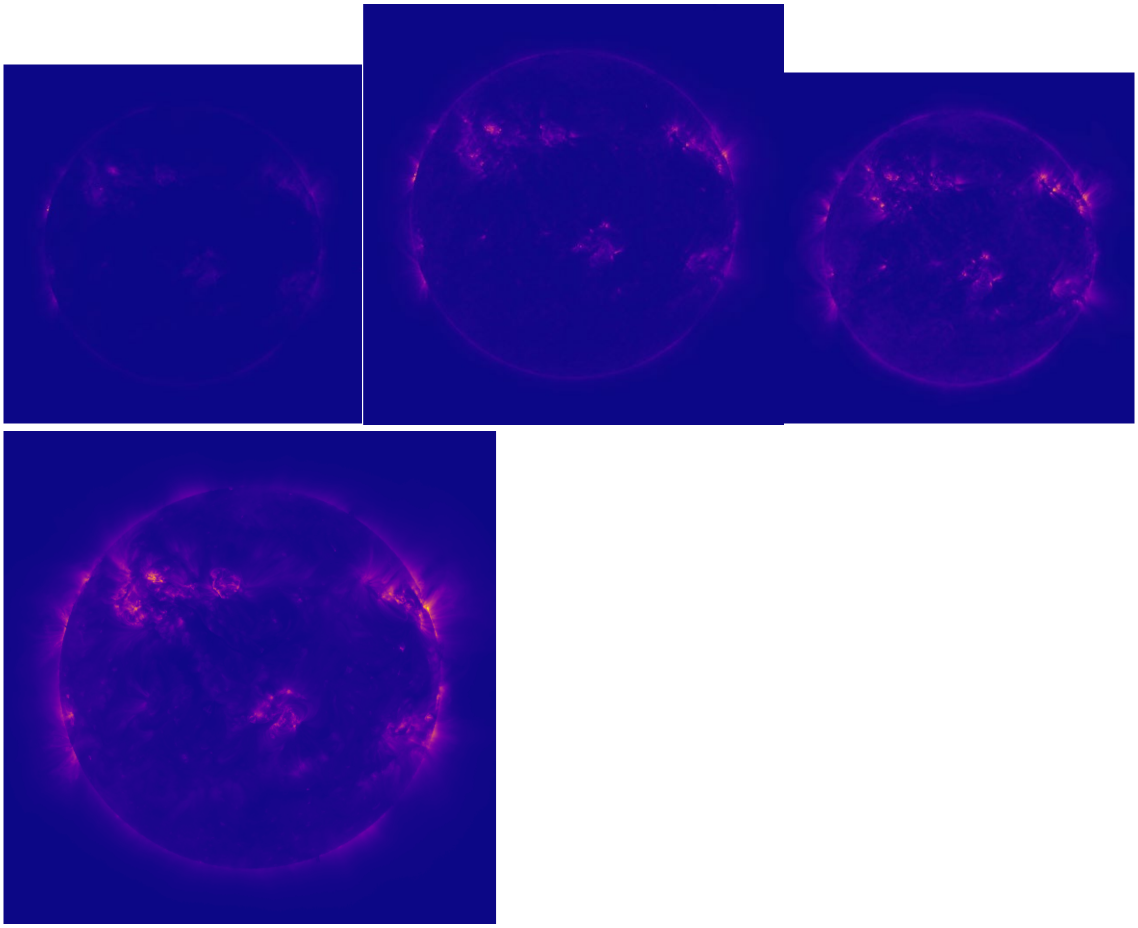




3.2



3.3



Code:

Section 2

Warmups.py:

```
import numpy as np


def w1(X):
    """
    Input:
    - X: A numpy array


    Returns:
    - A matrix Y such that Y[i, j] = X[i, j] * 10 + 100


    Hint: Trust that numpy will do the right thing
    """
    return X * 10 + 100


def w2(X, Y):
    """
    Inputs:
    - X: A numpy array of shape (N, N)
    - Y: A numpy array of shape (N, N)


    Returns:
    A numpy array Z such that Z[i, j] = X[i, j] + 10 * Y[i, j]
```

```
    Hint: Trust that numpy will do the right thing
    """
    return X + 10 * Y

def w3(X, Y):
    """
    Inputs:
    - X: A numpy array of shape (N, N)
    - Y: A numpy array of shape (N, N)

    Returns:
    A numpy array Z such that Z[i, j] = X[i, j] * Y[i, j] - 10

    Hint: By analogy to +, * will do the same thing
    """
    return X * Y - 10

def w4(X, Y):
    """
    Inputs:
    - X: Numpy array of shape (N, N)
    - Y: Numpy array of shape (N, N)

    Returns:
    A numpy array giving the matrix product X times Y
```

```
    Hint:
    1. Be careful! There are different variants of *, @, dot
    2.  a = [[1,2],
             [1,2]]
        b = [[2,2],
             [3,3]]
        a * b = [[2,4],
                 [3,6]]
    Is this matrix multiplication?

    """
    return X @ Y


def w5(X):
    """
    Inputs:
    - X: A numpy array of shape (N, N) of floating point numbers

    Returns:
    A numpy array with the same data as X, but cast to 32-bit integers

    Hint: Check .astype() !
    """
    return X.astype(np.int32)


def w6(X, Y):
```

```python
    """
    Inputs:
    - X: A numpy array of shape (N,) of integers
    - Y: A numpy array of shape (N,) of integers

    Returns:
    A numpy array Z such that Z[i] = float(X[i]) / float(Y[i])

    """
    return X.astype(np.float64)/Y.astype(np.float64)

def w7(X):
    """
    Inputs:
    - X: A numpy array of shape (N, M)

    Returns:
    - A numpy array Y of shape (N * M, 1) containing the entries of X in row
      order. That is, X[i, j] = Y[i * M + j, 0]

    Hint:
    1) np.reshape
    2) You can specify an unknown dimension as -1
    """
    return X.reshape(-1,1)
```

```
def w8(N):
    """
    Inputs:
    - N: An integer

    Returns:
    A numpy array of shape (N, 2N)

    Hint: The error "data type not understood" means you probably called
    np.ones or np.zeros with two arguments, instead of a tuple for the shape
    """
    return np.ones((N,2*N))

def w9(X):
    """
    Inputs:
    - X: A numpy array of shape (N, M) where each entry is between 0 and 1

    Returns:
    A numpy array Y where Y[i, j] = True if X[i, j] > 0.5

    Hint: Try boolean array indexing
    """
    return X > 0.5

def w10(N):
```

```
    """
    Inputs:
    - N: An integer

    Returns:
    A numpy array X of shape (N,) such that X[i] = i

    Hint: np.arange
    """
    return np.arange(N)

def w11(A, v):
    """
    Inputs:
    - A: A numpy array of shape (N, F)
    - v: A numpy array of shape (F, 1)

    Returns:
    Numpy array of shape (N, 1) giving the matrix-vector product Av
    """
    return A.dot(v)

def w12(A, v):
    """
    Inputs:
    - A: A numpy array of shape (N, N), of full rank
```

```
    - v: A numpy array of shape (N, 1)

    Returns:
    Numpy array of shape (N, 1) giving the matrix-vector product of the inverse
    of A and v: A^-1 v
    """
    return np.linalg.inv(A).dot(v)

def w13(u, v):
    """
    Inputs:
    - u: A numpy array of shape (N, 1)
    - v: A numpy array of shape (N, 1)

    Returns:
    The inner product u^T v

    Hint: .T
    """
    return u.T @ (v)

def w14(v):
    """
    Inputs:
    - v: A numpy array of shape (N, 1)
```

```
    Returns:
    The L2 norm of v: norm = (sum_i^N v[i]^2)^(1/2)
    You MAY NOT use np.linalg.norm
    """
    return np.sqrt(np.sum(v**2))


def w15(X, i):
    """
    Inputs:
    - X: A numpy array of shape (N, M)
    - i: An integer in the range 0 <= i < N


    Returns:
    Numpy array of shape (M,) giving the ith row of X
    """
    return X[i]


def w16(X):
    """
    Inputs:
    - X: A numpy array of shape (N, M)


    Returns:
    The sum of all entries in X


    Hint: np.sum
```

```
    """
    return np.sum(X)

def w17(X):
    """
    Inputs:
    - X: A numpy array of shape (N, M)

    Returns:
    A numpy array S of shape (N,) where S[i] is the sum of row i of X

    Hint: np.sum has an optional "axis" argument
    """
    return np.sum(X, axis=1)

def w18(X):
    """
    Inputs:
    - X: A numpy array of shape (N, M)

    Returns:
    A numpy array S of shape (M,) where S[j] is the sum of column j of X

    Hint: Same as above
    """
    return np.sum(X, axis=0)
```

```
def w19(X):
    """
    Inputs:
    - X: A numpy array of shape (N, M)

    Returns:
    A numpy array S of shape (N, 1) where S[i, 0] is the sum of row i of X

    Hint: np.sum has an optional "keepdims" argument

    *** keepdims=True simply makes it (N,1) for a 2D answer rather than (N,)
    """
    return np.sum(X, axis=1, keepdims=True)

def w20(X):
    """
    Inputs:
    - X: A numpy array of shape (N, M)

    Returns:
    A numpy array S of shape (N, 1) where S[i] is the L2 norm of row i of X
    """

    return np.sqrt(np.sum(X**2, axis=1, keepdims=True))
```

Tests.py:

```
import numpy as np


def t1(L):
    """
    Inputs:
    - L: A list of M numpy arrays, each of shape (1, N)


    Returns:
    A numpy array of shape (M, N) giving all inputs stacked together


    Par: 1 line
    Instructor: 1 line


    Hint: vstack/hstack/dstack, no for loop
    """
    return np.vstack(L)


def t2(X):
    """
```

Inputs:

- X: A numpy array of shape (N, N)

Returns:

Numpy array of shape (N,) giving the eigenvector corresponding to the

smallest eigenvalue of X

Par: 5 lines

Instructor: 3 lines

Hints:

1) np.linalg.eig

2) np.argmin

3) Watch rows and columns!

"""

eig_vals, eig_vecs = np.linalg.eig(X) # returns an Eig object of ([eigenvalues], [eigenvectors])

eig_val_min = np.argmin(eig_vals) # selects the INDEX of the smallest argument in the 1D array of Eigenvalues

'''

Eigenvectors is an (N,N) matrix where each column (,n) is an eigenvector rather than the rows

eig_vecs[eig_val_min] returns the row which is incorrect while eig_vecs[: , eig_val_min] returns the column

'''

```python
    # Use the index of the minimum to return the corresponding eigenvector
    return eig_vecs[: ,eig_val_min]

def t3(X):
    """
    Inputs:
    - A: A numpy array of any shape

    Returns:
    A copy of X, but with all negative entires set to 0

    Par: 3 lines
    Instructor: 1 line

    Hint:
    1) If S is a boolean array with the same shape as X, then X[S] gives an
      array containing all elements of X corresponding to true values of S
    2) X[S] = v assigns the value v to all entires of X corresponding to
      true values of S.
    """
    return np.where(X < 0, 0, X)

def t4(R, X):
    """
    Inputs:
    - R: A numpy array of shape (3, 3) giving a rotation matrix
```

- X: A numpy array of shape (N, 3) giving a set of 3-dimensional vectors

Returns:

A numpy array Y of shape (N, 3) where Y[i] is X[i] rotated by R

Par: 3 lines

Instructor: 1 line

Hint:

1) If v is a vector, then the matrix-vector product Rv rotates the vector

by the matrix R.

2) .T gives the transpose of a matrix

Why use R.T instead of R?

Rotation matrices are typically orthogonal, meaning that to rotate vectors,

you should multiply by the transpose of the rotation matrix

This correctly applies the rotation to each row vector in X (vector-matrix multiplication)

Rotation matrices typically assume column vectors when they are defined mathematically.

However, in numerical programming, row vectors (each row being a vector) are often used,

requiring a transpose operation for correct application.

So in matrix multiplication, if you were to do nothing, you'd be applying stuff row-wise, but by transposing

```
    you apply the second matrix to the first matrix column wise, thus achieving the desired
transformation

    """
    return X @ R.T

def t5(X):
    """
    Inputs:
    - X: A numpy array of shape (N, N)

    Returns:
    A numpy array of shape (4, 4) giving the upper left 4x4 submatrix of X
    minus the bottom right 4x4 submatrix of X.

    Par: 2 lines
    Instructor: 1 line

    Hint:
    1) X[y0:y1, x0:x1] gives the submatrix
      from rows y0 to (but not including!) y1
      from columns x0 (but not including!) x1
    """
    return X[:4, :4] - X[-4:,-4:]

def t6(N):
```

```
    """
    Inputs:
    - N: An integer

    Returns:
    A numpy array of shape (N, N) giving all 1s, except the first and last 5
    rows and columns are 0.

    Par: 6 lines
    Instructor: 3 lines
    """
    a = np.ones((N, N))  # Create an NxN matrix filled with 1s
    a[:5, :] = 0  # Set the first 5 rows to 0
    a[-5:, :] = 0  # Set the last 5 rows to 0
    a[:, :5] = 0  # Set the first 5 columns to 0
    a[:, -5:] = 0  # Set the last 5 columns to 0
    return a

def t7(X):
    """
    Inputs:
    - X: A numpy array of shape (N, M)

    Returns:
    A numpy array Y of the same shape as X, where Y[i] is a vector that points
    the same direction as X[i] but has unit norm.
```

```
    Par: 3 lines
    Instructor: 1 line

    Hints:
    1) The vector v / ||v||| is the unit vector pointing in the same direction
       as v (as long as v != 0)
    2) Divide each row of X by the magnitude of that row
    3) Elementwise operations between an array of shape (N, M) and an array of
       shape (N, 1) work -- try it! This is called "broadcasting"
    4) Elementwise operations between an array of shape (N, M) and an array of
       shape (N,) won't work -- try reshaping

    The row magnitudes would be the L2 norm. So executing the solution from the
    last of the warmups, storing them in an (N,1) array and then dividing X by
    that array would work, but np.linalg.norm already executes this.

    np.linalg.norm(axis =1, keepdims=True) executes row-wise and keeps as a 2D array
    """
    return X / np.linalg.norm(X, axis=1, keepdims=True)

def t8(X):
    """
    Inputs:
    - X: A numpy array of shape (N, M)
```

```
    Returns:
    A numpy array Y of shape (N, M) where Y[i] contains the same data as X[i],
    but normalized to have mean 0 and standard deviation 1.

    Par: 3 lines
    Instructor: 1 line

    Hints:
    1) To normalize X, subtract its mean and then divide by its standard deviation
    2) Normalize the rows individually
    3) You may have to reshape
    """
    row_mean = X.mean(axis=1, keepdims=True)
    row_std = X.std(axis=1, keepdims=True)
    return (X - row_mean) / row_std


def t9(q, k, v):
    """
    Inputs:
    - q: A numpy array of shape (1, K) (queries)
    - k: A numpy array of shape (N, K) (keys)
    - v: A numpy array of shape (N, 1) (values)

    Returns:
    sum_i exp(-||q-k_i||^2) * v[i]
```

```
    Par: 3 lines
    Instructor: 1 ugly line

    Hints:
    1) You can perform elementwise operations on arrays of shape (N, K) and
      (1, K) with broadcasting
    2) Recall that np.sum has useful "axis" and "keepdims" options
    3) np.exp and friends apply elementwise to arrays

    *** First conduct the sums of the Euclidean distances followed by
    exponent of the negative of the distances sums, THEN the sum of that * V
    """

    dist_sum = np.sum((q-k)**2, axis=1, keepdims=True)
    return np.sum(np.exp(-dist_sum)*v)


def t10(Xs):
    """
    Inputs:
    - Xs: A list of length L, containing numpy arrays of shape (N, M)

    Returns:
    A numpy array R of shape (L, L) where R[i, j] is the Euclidean distance
    between C[i] and C[j], where C[i] is an M-dimensional vector giving the
    centroid of Xs[i]
```

```
    Par: 12 lines
    Instructor: 3 lines (after some work!)

    Hints:
    1) You can try to do t11 and t12 first
    2) You can use a for loop over L
    3) Distances are symmetric
    4) Go one step at a time
    5) Our 3-line solution uses no loops, and uses the algebraic trick from the
       next problem.
    """
    # Compute the centroid aka mean of rows
    ''' Killer way to execute this is using what I'm calling an array comprehension
        Basically a list comprehension inside np.array instantion'''
    centroids_of_arrays = np.array([np.mean(X, axis=0) for X in Xs]) # row-wise centroids aka
means
    # Use equations below
    dist_of_centroids = np.sum(centroids_of_arrays**2, axis=1, keepdims=True)
    Distance_squared = dist_of_centroids + dist_of_centroids.T - 2 * centroids_of_arrays @
centroids_of_arrays.T
    return np.sqrt(np.maximum(Distance_squared, 0))


def t11(X):
    """
    Inputs:
    - X: A numpy array of shape (N, M)
```

Returns:

A numpy array D of shape (N, N) where D[i, j] gives the Euclidean distance

between X[i] and X[j], using the identity

||x - y||^2 = is the distance squared so square root that

||x||^2 + ||y||^2 - 2x^T y is what to focus on

*** And y = X.T

Par: 3 lines

Instructor: 2 lines (you can do it in one but it's wasteful compute-wise)

Hints:

1) What happens when you add two arrays of shape (1, N) and (N, 1)?

2) Think about the definition of matrix multiplication

3) Transpose is your friend

4) Note the square! Use a square root at the end

5) On some machines, ||x||^2 + ||x||^2 - 2x^Tx may be slightly negative,

   causing the square root to crash. Just take max(0, value) before the

   square root. Seems to occur on Macs.

"""

```
# Square X in preparation for full equation

X_squared = np.sum(X**2, axis=1, keepdims=True)

Distance_squared = X_squared + X_squared.T - 2 * X @ X.T

return np.sqrt(np.maximum(Distance_squared, 0))
```

```
def t12(X, Y):
    """
    Inputs:
    - X: A numpy array of shape (N, F)
    - Y: A numpy array of shape (M, F)

    Returns:
    A numpy array D of shape (N, M) where D[i, j] is the Euclidean distance
    between X[i] and Y[j].

    Par: 3 lines
    Instructor: 2 lines (you can do it in one, but it's more than 80 characters
        with good code formatting)

    Hints: Similar to previous problem
    """
    X_squared = np.sum(X**2, axis=1, keepdims=True)
    Y_squared = np.sum(Y**2, axis=1, keepdims=True).T
    return np.sqrt(np.maximum(X_squared + Y_squared - 2 * X @ Y.T, 0))

def t13(q, V):
    """
    Inputs:
    - q: A numpy array of shape (1, M) (query)
    - V: A numpy array of shape (N, M) (values)
```

```
    Return:
    The index i that maximizes the dot product q . V[i]

    Par: 1 line
    Instructor: 1 line

    Hint: np.argmax

    *** Dimensions:
      V.T (transpose of V) will have shape (M, N).
      q @ V.T results in shape (1, N).

      And we need (N,1) so do it like below
      where (N,M) * (M, 1)
    """
    return np.argmax(V @ q.T)

def t14(X, y):
    """
    Inputs:
    - X: A numpy array of shape (N, M)
    - y: A numpy array of shape (N, 1)

    Returns:
    A numpy array w of shape (M, 1) such that ||y - Xw||^2 is minimized
```

```
    Par: 2 lines
    Instructor: 1 line

    Hint: np.linalg.lstsq or np.linalg.solve

    *** Finding the optimal weight vecor that minimizes the squared error
      AKA a linear least squares problem and np.linalg.lstsq aka least squares

    """
    return np.linalg.lstsq(X,y, rcond=None)[0]

def t15(X, Y):
    """
    Inputs:
    - X: A numpy array of shape (N, 3)
    - Y: A numpy array of shape (N, 3)

    Returns:
    A numpy array C of shape (N, 3) such C[i] is the cross product between X[i]
    and Y[i]

    Par: 1 line
    Instructor: 1 line

    Hint: np.cross
    """
```

```
    return np.cross(X,Y)


def t16(X):
    """
    Inputs:
    - X: A numpy array of shape (N, M)

    Returns:
    A numpy array Y of shape (N, M - 1) such that
    Y[i, j] = X[i, j] / X[i, M - 1]
    for all 0 <= i < N and all 0 <= j < M - 1

    Par: 1 line
    Instructur: 1 line

    Hints:
    1) If it doesn't broadcast, reshape or np.expand_dims
    2) X[:, -1] gives the last column of X

    *** expand_dims solution:  X[:, :-1] / np.expand_dims(X[:, -1], axis=1)
    Probably best solution: X[:, :-1] / X[:, -1, np.newaxis]
    """
    return X[:, :-1] / X[:, -1].reshape(-1, 1)


def t17(X):
    """
```

```
    Inputs:
    - X: A numpy array of shape (N, M)

    Returns:
    A numpy array Y of shape (N, M + 1) such that
        Y[i, :M] = X[i]
        Y[i, M] = 1

    Par: 1 line
    Instructor: 1 line

    Hint: np.hstack, np.ones

    *** hstack adds columns while vstack adds rows
    """
    # Basically just use np.ones of shape, X number of rows to create an array of size(1,M) with the value 1 and hstack it to X
    return np.hstack((X, np.ones((X.shape[0], 1))))


def t18(N, r, x, y):
    """
    Inputs:
    - N: An integer
    - r: A floating-point number
    - x: A floating-point number
    - y: A floating-point number
```

Returns:

A numpy array I of floating point numbers and shape (N, N) such that:

I[i, j] = 1 if ||(j, i) - (x, y)|| < r

I[i, j] = 0 otherwise

Par: 3 lines

Instructor: 2 lines

Hints:

1) np.meshgrid and np.arange give you X, Y. Play with them. You can also do it without them, but np.meshgrid and np.arange are easier to understand.

2) Arrays have an astype method

"""

X, Y = np.meshgrid(np.arange(N), np.arange(N))

return (np.sqrt((X - x)**2 + (Y - y)**2) < r).astype(float)

def t19(N, s, x, y):

"""

Inputs:

- N: An integer

- s: A floating-point number

- x: A floating-point number

- y: A floating-point number

Returns:

```
    A numpy array I of shape (N, N) such that
    I[i, j] = exp(-||(j, i) - (x, y)||^2 / s^2)


    Par: 3 lines
    Instructor: 2 lines
    """
    X, Y = np.meshgrid(np.arange(N), np.arange(N))
    return np.exp(-((X - x)**2 + (Y - y)**2) / s**2)


def t20(N, v):
    """
    Inputs:
    - N: An integer
    - v: A numpy array of shape (3,) giving coefficients v = [a, b, c]


    Returns:
    A numpy array of shape (N, N) such that M[i, j] is the distance between the
    point (j, i) and the line a*j + b*i + c = 0


    Par: 4 lines
    Instructor: 2 lines


    Hints:
    1) The distance between the point (x, y) and the line ax+by+c=0 is given by
      abs(ax + by + c) / sqrt(a^2 + b^2)
      (The sign of the numerator tells which side the point is on)
```

```
  2) np.abs

  ***
  """
  X, Y = np.meshgrid(np.arange(N), np.arange(N))
  return np.abs(v[0] * X + v[1] * Y + v[2]) / np.sqrt(v[0]**2 + v[1]**2)
```

Visualize:

```
import os
import numpy as np
import matplotlib.pyplot as plt
import cv2

def colormapArray(X, colors):
  """
  Basically plt.imsave but return a matrix instead

  Given:
    a HxW matrix X
    a Nx3 color map of colors in [0,1] [R,G,B]
  Outputs:
    a HxW uint8 image using the given colormap. See the Bewares
  """
```

```
    X_normalized = (X - np.nanmin(X)) / (np.nanmax(X) - np.nanmin(X))
    X_normalized = np.clip(X_normalized, 0, 1)
    indices = (X_normalized * (len(colors) - 1)).astype(int)
    colormapped_image = (colors[indices] * 255).astype(np.uint8)
    return colormapped_image

if __name__ == "__main__":
    # Solve 3.1: Nonlinear correction and visualization
    data2 = np.load("mysterydata/mysterydata2.npy")
    corrected_sqrt = np.sqrt(data2)
    corrected_log1p = np.log1p(data2)
    plt.imsave("mysterydata2_sqrt.png", corrected_sqrt[:, :, 0], cmap='gray')
    plt.imsave("mysterydata2_log1p.png", corrected_log1p[:, :, 0], cmap='gray')
    print("Saved corrected images for mysterydata2.npy")

    # Solve 3.2: Handling NaN and Inf values in mysterydata3.npy
    data3 = np.load("mysterydata/mysterydata3.npy")
    finite_fraction = np.mean(np.isfinite(data3))
    print(f"Fraction of finite values: {finite_fraction}")

    if finite_fraction < 1:
        data3_cleaned = np.nan_to_num(data3, nan=np.nanmin(data3),
posinf=np.nanmax(data3), neginf=np.nanmin(data3))
    else:
        data3_cleaned = data3
```

```
for i in range(9):
    vmin, vmax = np.nanmin(data3_cleaned[:, :, i]), np.nanmax(data3_cleaned[:, :, i])
    plt.imsave(f"vis3_{i}.png", data3_cleaned[:, :, i], vmin=vmin, vmax=vmax, cmap='gray')
print("Saved cleaned images for mysterydata3.npy")

# Solve 3.3: Custom colormap visualization for mysterydata4.npy
data4 = np.load("mysterydata/mysterydata4.npy")
colors = np.load("mysterydata/colors.npy")
for i in range(9):
    colormap_image = colormapArray(data4[:, :, i], colors)
    cv2.imwrite(f"vis4_{i}.png", cv2.cvtColor(colormap_image, cv2.COLOR_RGB2BGR))
print("Saved colormap applied images for mysterydata4.npy")
```