hw3_Homographies_release

February 28, 2025

1 Homework 3: Augmented Reality with Planar Homographies

For each question please refer to the handout for more details. Programming questions begin at Q2. Remember to run all cells and save the notebook to your local machine as a pdf for gradescope submission.

2	Collaborators
List	your collaborators for all questions here:
3	Q1 Preliminaries
3.1	Q1.1 The Direct Linear Transform
3.1.	1 Q1.1.1 (3 points)
How	many degrees of freedom does h have?
	x3 homography matrix H has 9 entries obviously. However, we fix one entry as a scalar multiple I represents the same transformation. So $9 - 1 = 8$
3.1.	2 Q1.1.2 (2 points)
How	γ many point pairs are required to solve \mathbf{h} ?
	te we have 8 entries equaling 8 linear equations with 8 total unknowns, then we need 4 points (each pair is 2 unknowns)
3.1.	3 Q1.1.3 (5 points)
Deri	ive \mathbf{A}_i

For each pair of corresponding points $((x_i, y_i))$ in image 1 and $((x'_i, y'_i))$ in image 2, the homography condition in homogeneous coordinates is:

$$\begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix} \equiv H \begin{bmatrix} x_i' \\ y_i' \\ 1 \end{bmatrix}, \quad \text{where } H = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix}.$$

Or, multiplied out we have:

$$\frac{h_{11}x_i'+h_{12}y_i'+h_{13}}{h_{31}x_i'+h_{32}y_i'+h_{33}}=x_i,\quad \frac{h_{21}x_i'+h_{22}y_i'+h_{23}}{h_{31}x_i'+h_{32}y_i'+h_{33}}=y_i.$$

Rearranging these into linear equations in the components of (H), each correspondence contributes a (2×9) block:

$$A_i = \begin{bmatrix} -x_i' & -y_i' & -1 & 0 & 0 & 0 & x_ix_i' & x_iy_i' & x_i \\ 0 & 0 & 0 & -x_i' & -y_i' & -1 & y_ix_i' & y_iy_i' & y_i \end{bmatrix}.$$

Stacking (A_i) for all correspondences into a bigger matrix (A) yields:

$$A h = 0, \quad \text{where} \quad h = \begin{bmatrix} h_{11} \\ h_{12} \\ h_{13} \\ h_{21} \\ h_{22} \\ h_{23} \\ h_{31} \\ h_{32} \\ h_{33} \end{bmatrix}.$$

3.1.4 Q1.1.4 (5 points)

What will be the trivial solution for \mathbf{h} ? Is the matrix \mathbf{A} full rank? Why/Why not? What impact will it have on the singular values (i.e. eigenvalues of $\mathbf{A}^T \mathbf{A}$)?

Trival solution for Ah = 0 is just that h = 0

A full rank? No, (A) is not full rank because even with the minimum (N = 4) (i.e., 8 equations total), one cannot achieve $(\operatorname{rank}(A) = 9)$. For a valid homography solution, $(\operatorname{rank}(A) = 8)$. So there is a one-dimensional null space that provides a valid nonzero solution for (h).

Singular Values Impact? - Because (A) does not have full column rank, there is at least one singular value of (A) that is zero. Therefore, in the matrix $(A^{T}A)$, there is at least one zero eigenvalue corresponding to that one-dimensional null space. This zero singular value/eigenvalue leads to the nontrivial solution for the homography (h). —

3.2 Q1.2 Homography Theory Questions

3.2.1 Q1.2.1 (5 points)

Prove that there exists a homography **H** that satisfies $\mathbf{x}_1 \equiv \mathbf{H}\mathbf{x}_2$, given two cameras separated by a pure rotation.

Given two cameras with intrinsic matrices (K_1) and (K_2) . Camera 1 observes a 3D point (X) via:

$$x_1 = K_1[I \ 0] X \implies x_1 \sim K_1 X,$$

and Camera 2 observes the *same* 3D point (X) via a pure rotation (R) (about its center) and different intrinsics (K_2):

$$x_2 \ = \ K_2[R \ 0] \, X \ \implies \ x_2 \sim K_2 \, R \, X.$$

So to show that there is a (3×3) matrix (H) such that $(x_1 \equiv Hx_2)$. We express (X) in terms of (x_2) :

$$x_2 \sim K_2 R X \implies X \sim R^{-1} K_2^{-1} x_2.$$

Then substitute back into $(x_1 = K_1 X)$:

$$x_1 \; \sim \; K_1 \Big(R^{-1} K_2^{-1} \, x_2 \Big) \; = \; \big(K_1 \, R^{-1} \, K_2^{-1} \big) \, x_2.$$

Then, the homography can be identified as:

$$H \; = \; K_1 \, R^{-1} \, K_2^{-1}.$$

3.2.2 Q1.2.2 (5 points):

Show that \mathbf{H}^2 is the homography corresponding to a rotation of 2θ .

Now suppose we have a single camera (with intrinsic matrix (K) that rotates about its center by an angle (θ) . The homography (H) mapping the first orientation to the second is:

$$H = KR(\theta)K^{-1}$$

where (R()) is the (3×3) rotation matrix for the rotation (θ) .

If we apply this rotation again (i.e., two successive rotations of (θ) , the combined rotation is $(R(2\theta) = R(\theta), R(\theta))$). Hence the homography for that combined rotation is:

$$H^2 \; = \; \left(K \, R(\theta) \, K^{-1} \right) \left(K \, R(\theta) \, K^{-1} \right) \; = \; K \, R(\theta) \, R(\theta) \, K^{-1} \; = \; K \, R(2\theta) \, K^{-1}.$$

Thus (H^2) corresponds exactly to a rotation by (2θ) .

4 Initialization

Run the following code to import the modules you'll need.

```
[1]: import os
     import numpy as np
     import cv2
     import skimage.color
     import pickle
     from matplotlib import pyplot as plt
     import scipy
     from skimage.util import montage
     import time
     PATCHWIDTH = 9
     def read_pickle(path):
         with open(path, "rb") as f:
             return pickle.load(f)
     def write_pickle(path, data):
         with open(path, "wb") as f:
             pickle.dump(data, f)
     def briefMatch(desc1,desc2,ratio):
         matches = skimage.feature.match_descriptors(desc1,desc2,
                                                      'hamming',
                                                      cross_check=True,
                                                      max ratio=ratio)
         return matches
     def plotMatches(img1,img2,matches,locs1,locs2):
         fig, ax = plt.subplots(nrows=1, ncols=1)
         img1 = cv2.cvtColor(img1, cv2.COLOR_BGR2GRAY)
         img2 = cv2.cvtColor(img2, cv2.COLOR_BGR2GRAY)
         plt.axis('off')
         skimage.feature.plot_matched_features(img1,img2,
                                                keypoints0=locs1,keypoints1=locs2,
                                               matches=matches,ax=ax,
                                                matches_color='r',only_matches=True)
         plt.show()
         return
     def makeTestPattern(patchWidth, nbits):
         np.random.seed(0)
         compareX = patchWidth*patchWidth * np.random.random((nbits,1))
         compareX = np.floor(compareX).astype(int)
         np.random.seed(1)
```

```
compareY = patchWidth*patchWidth * np.random.random((nbits,1))
    compareY = np.floor(compareY).astype(int)
    return (compareX, compareY)
def computePixel(img, idx1, idx2, width, center):
    halfWidth = width // 2
    col1 = idx1 % width - halfWidth
    row1 = idx1 // width - halfWidth
    col2 = idx2 % width - halfWidth
    row2 = idx2 // width - halfWidth
    return 1 if img[int(center[0]+row1)][int(center[1]+col1)] <

→img[int(center[0]+row2)][int(center[1]+col2)] else 0
def computeBrief(img, locs):
    patchWidth = 9
    nbits = 256
    compareX, compareY = makeTestPattern(patchWidth,nbits)
    m, n = img.shape
   halfWidth = patchWidth//2
    locs = np.array(list(filter(lambda x: halfWidth <= x[0] < m-halfWidth and_{\sqcup})
 ⇔halfWidth <= x[1] < n-halfWidth, locs)))</pre>
    desc = np.array([list(map(lambda x: computePixel(img, x[0], x[1], __
 →patchWidth, c), zip(compareX, compareY))) for c in locs])
    return desc, locs
def corner_detection(img, sigma):
    # fast method
    result_img = skimage.feature.corner_fast(img, n=PATCHWIDTH, threshold=sigma)
    locs = skimage.feature.corner_peaks(result_img, min_distance=1)
    return locs
def loadVid(path):
    # Create a VideoCapture object and read from input file
    # If the input is the camera, pass O instead of the video file name
    cap = cv2.VideoCapture(path)
    # get fps, width, and height
    fps = cap.get(cv2.CAP_PROP_FPS)
```

```
width = cap.get(cv2.CAP_PROP_FRAME_WIDTH)
height = cap.get(cv2.CAP_PROP_FRAME_HEIGHT)
# Append frames to list
frames = []
# Check if camera opened successfully
if cap.isOpened()== False:
    print("Error opening video stream or file")
# Read until video is completed
while(cap.isOpened()):
    # Capture frame-by-frame
    ret, frame = cap.read()
    if ret:
        #Store the resulting frame
        frames.append(frame)
    else:
        break
# When everything done, release the video capture object
cap.release()
frames = np.stack(frames)
return frames, fps, width, height
```

5 Download data

Download the required data and setup the results directory. If running on colab, DATA_PARENT_DIR must be DATA_PARENT_DIR = '/content/' Otherwise, use the local directory of your choosing. Data will be downloaded to DATA_PARENT_DIR/hw3_data and a subdirectory DATA_PARENT_DIR/results will be created.

```
[3]: # Only change this if you are running locally
# Default on colab: DATA_PARENT_DIR = '/content/'

# Data will be downloaded to DATA_PARENT_DIR/hw3_data
# A subdirectory DATA_PARENT_DIR/results will be created

DATA_PARENT_DIR = 'content'

if not os.path.exists(DATA_PARENT_DIR):
    raise RuntimeError('DATA_PARENT_DIR does not exist: ', DATA_PARENT_DIR)
```

```
if not os.path.exists(RES_DIR):
  os.mkdir(RES_DIR)
  print('made directory: ', RES_DIR)
#paths different files are saved to
# OPTIONAL:
# feel free to change if funning locally
ROT_MATCHES_PATH = os.path.join(RES_DIR, 'brief_rot_test.pkl')
ROT_INV_MATCHES_PATH = os.path.join(RES_DIR, 'ec_brief_rot_inv_test.pkl')
AR_VID_FRAMES_PATH = os.path.join(RES_DIR, 'q_3_1_frames.npy')
AR_VID_FRAMES_EC_PATH = os.path.join(RES_DIR, 'q_3_2_frames.npy')
HW3 SUBDIR = 'hw3 data'
DATA_DIR = os.path.join(DATA_PARENT_DIR, HW3_SUBDIR)
ZIP_PATH = DATA_DIR + '.zip'
if not os.path.exists(DATA_DIR):
  !wget 'https://www.andrew.cmu.edu/user/hfreeman/data/16720 spring/hw3 data.
 ⇔zip' -0 $ZIP_PATH
  !unzip -qq $ZIP_PATH -d $DATA_PARENT_DIR
made directory: content/results
--2025-02-27 22:46:56--
https://www.andrew.cmu.edu/user/hfreeman/data/16720 spring/hw3 data.zip
Resolving www.andrew.cmu.edu (www.andrew.cmu.edu)... 128.2.42.53
Connecting to www.andrew.cmu.edu (www.andrew.cmu.edu) | 128.2.42.53 | :443...
connected.
HTTP request sent, awaiting response... 200 OK
Length: 36434294 (35M) [application/zip]
Saving to: 'content/hw3_data.zip'
content/hw3_data.zi 100%[===========] 34.75M 8.26MB/s
                                                                     in 4.6s
2025-02-27 22:47:01 (7.48 MB/s) - 'content/hw3_data.zip' saved
[36434294/36434294]
```

6 Q2 Computing Planar Homographies

RES_DIR = os.path.join(DATA_PARENT_DIR, 'results')

6.1 Q2.1 Feature Detection and Matching

6.1.1 Q2.1.1 (5 points):

How is the FAST detector different from the Harris corner detector that you've seen in the lectures? Can you comment on its computation performance compared to the Harris corner detector?

6.2 FAST is far simpler and quicker than Harris computationally because Harris requires computing partial derivates, comparing eigenvalues, etc. FAST on the other hand with its threshold-based decision by examining a circle of pixels around a point just sees if those pixels are all brighter or darker than the center pixel by a threshold can be implemented with short-circuit logic of a decision tree.

6.2.1 Q2.1.2 (5 points):

How is the BRIEF descriptor different from the filterbanks you've seen in the lectures? Could you use any one of the those filter banks as a descriptor?

The BRIEF descriptor is different from filterbanks because it uses pairwise intensity comparisons at pre-determined locations around a feature point yielding a binary string which is far far smaller and faster than the filterbank descriptors that produce high-dimensional responses.

Could we use a filter bank as a descriptor? - You could technically take the set of filter responses around a keypoint and treat that as a feature descriptor.

- However, matching that higher dimensional descriptor is much more computationally difficult than matching the BRIEF descriptor with the Hamming distance

6.2.2 Q2.1.3 (5 points):

Describe how the Hamming distance and Nearest Neighbor can be used to match interest points with BRIEF descriptors. What benefits does the Hamming distance have over a more conventional Euclidean distance measure in our setting?

Since the Hamming distance between two binary strings is the count of positions (bits) for which they differ, you can compute the BRIEF descriptor for a feature point in an image, and then, you can compute the BRIEF descriptors for all the key features of a 2nd image. Then, you can find the k number of nearest matching features determined by the smallest hamming distance of those features from the feature in the first image according to some threshold.

The benefit of comparing the hamming distance over the Euclidean distance is that the bitwise XOR operation followed by a population count is very fast on modern CPUs, can be stored very compactly, and robust to noise due to the changes caught by the changes in bits.

6.2.3 Q2.1.4 (10 points):

Implement the function matchPics()

```
[4]: def matchPics(I1, I2, ratio, sigma):

"""

Match features across images

Input
```

```
I1, I2: Source images (RGB or Grayscale uint8)
ratio: ratio for BRIEF feature descriptor
sigma: threshold for corner detection using FAST feature detector
Returns
matches: List of indices of matched features across I1, I2 [p x 2]
locs1, locs2: Pixel coordinates of matches [N x 2]
# ===== your code here! =====
# TODO: Convert images to GrayScale
# Input images can be either RGB or Grayscale uint8 (0 -> 255). Both need
# to be supported.
# Input images must be converted to normalized Grayscale (0.0 -> 1.0)
# skimage.color.rqb2qray may be useful if the input is RGB.
# 1) Convert images to grayscale, normalized to [0, 1]
# Handle both RGB and Grayscale (uint8) inputs
if I1.ndim == 3: # likely RGB
    I1_gray = skimage.color.rgb2gray(I1)
else: # already single channel
    I1_gray = I1.astype(np.float32) / 255.0
if I2.ndim == 3: # likely RGB
    I2_gray = skimage.color.rgb2gray(I2)
else: # already single channel
    I2_gray = I2.astype(np.float32) / 255.0
# TODO: Detect features in both images
locs1 = corner_detection(I1_gray, sigma)
locs2 = corner_detection(I2_gray, sigma)
# TODO: Obtain descriptors for the computed feature locations
desc1, locs1 = computeBrief(I1_gray, locs1)
desc2, locs2 = computeBrief(I2_gray, locs2)
# TODO: Match features using the descriptors
matches = briefMatch(desc1, desc2, ratio)
# ==== end of code ====
return matches, locs1, locs2
```

```
[6]: def displayMatched(I1, I2, ratio, sigma):
         Displays matches between two images
         Input
         I1, I2: Source images
         ratio: ratio for BRIEF feature descriptor
         sigma: threshold for corner detection using FAST feature detector
         print('Displaying matches for ratio: ', ratio, ' and sigma: ', sigma)
         # ===== your code here! =====
         # TODO: Use matchPics and plotMatches to visualize your results
         print('Displaying matches for ratio:', ratio, 'and sigma:', sigma)
         # Match features using matchPics
         matches, locs1, locs2 = matchPics(I1, I2, ratio, sigma)
         # Plot the matches using plotMatches
         plotMatches(I1, I2, matches, locs1, locs2)
         plt.show()
         # ==== end of code ====
```

Visualize the matches Use the cell below to visualize the matches. The resulting figure should look similar (but not necessarily identical) to Figure 2.

Feel free to play around with the images and parameters. Please use the original images when submitting the report.

Figure 2 parameters:

```
image1_name = "cv_cover.jpg"
image1_name = "cv_desk.png"
ratio = 0.7
sigma = 0.15
```

```
[7]: # Feel free to play around with these parameters
# BUT when submitting the report use the original images
image1_name = "cv_cover.jpg"
image2_name = "cv_desk.png"
ratio = 0.7
sigma = 0.15

image1_path = os.path.join(DATA_DIR, image1_name)
```

```
image2_path = os.path.join(DATA_DIR, image2_name)
image1 = cv2.imread(image1_path)
image2 = cv2.imread(image2_path)

#bgr to rgb
if len(image1.shape) == 3 and image1.shape[2] == 3:
    image1 = cv2.cvtColor(image1, cv2.COLOR_BGR2RGB)

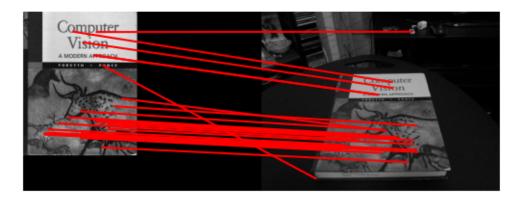
if len(image2.shape) == 3 and image2.shape[2] == 3:
    image2 = cv2.cvtColor(image2, cv2.COLOR_BGR2RGB)

displayMatched(image1, image2, ratio, sigma)
```

Displaying matches for ratio: 0.7 and sigma: 0.15 Displaying matches for ratio: 0.7 and sigma: 0.15

/tmp/ipykernel_132536/2274933143.py:60: DeprecationWarning: Conversion of an array with ndim > 0 to a scalar is deprecated, and will error in future. Ensure you extract a single element from your array before performing this operation. (Deprecated NumPy 1.25.)

return 1 if img[int(center[0]+row1)][int(center[1]+col1)] <
img[int(center[0]+row2)][int(center[1]+col2)] else 0</pre>



6.2.4 Q2.1.5 (10 points):

Experiment with different sigma and ratio values. Conduct a small ablation study, and include the figures displaying the matched features with various parameters in your write-up. Explain the effect of these two paremeters respectively.

Explain the effect of these two parameters:

The ratio parameter clearly identifies the number of features while the sigma parameter details the

threshold necessary to be considered a match. As you can see that there are far more lines as we increase the ratio parameter and some of those lines disappear as false matches are removed with the higher sigma threshold

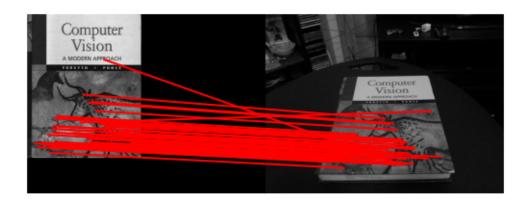
```
[8]: image1_name = "cv_cover.jpg"
     image2_name = "cv_desk.png"
     image1_path = os.path.join(DATA_DIR, image1_name)
     image2_path = os.path.join(DATA_DIR, image2_name)
     image1 = cv2.imread(image1 path)
     image2 = cv2.imread(image2_path)
     #bgr to rgb
     if len(image1.shape) == 3 and image1.shape[2] == 3:
       image1 = cv2.cvtColor(image1, cv2.COLOR_BGR2RGB)
     if len(image2.shape) == 3 and image2.shape[2] == 3:
       image2 = cv2.cvtColor(image2, cv2.COLOR_BGR2RGB)
     # ===== your code here! =====
     # Experiment with different sigma and ratio values.
     # Use displayMatches to visualize.
     # Include the matched feature figures in the write-up.
     ratios = [0.6, 0.7, 0.8]
     sigmas = [0.05, 0.1, 0.15]
     for r in ratios:
         for s in sigmas:
             print(f"\n--- Matching with ratio={r}, sigma={s} ---")
             displayMatched(image1, image2, r, s)
     # ==== end of code ====
```

```
--- Matching with ratio=0.6, sigma=0.05 ---
Displaying matches for ratio: 0.6 and sigma: 0.05
Displaying matches for ratio: 0.6 and sigma: 0.05

/tmp/ipykernel_132536/2274933143.py:60: DeprecationWarning: Conversion of an array with ndim > 0 to a scalar is deprecated, and will error in future. Ensure you extract a single element from your array before performing this operation.

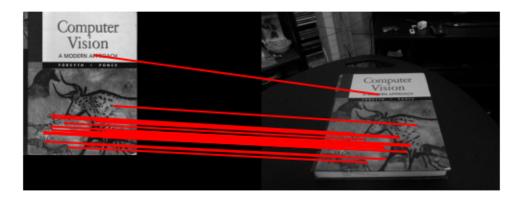
(Deprecated NumPy 1.25.)

return 1 if img[int(center[0]+row1)][int(center[1]+col1)] <
img[int(center[0]+row2)][int(center[1]+col2)] else 0
```



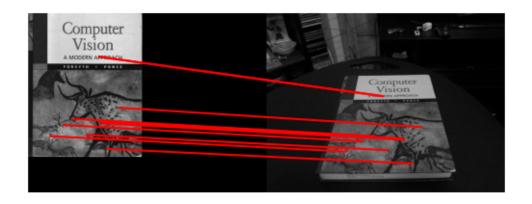
--- Matching with ratio=0.6, sigma=0.1 ---

Displaying matches for ratio: 0.6 and sigma: 0.1 Displaying matches for ratio: 0.6 and sigma: 0.1



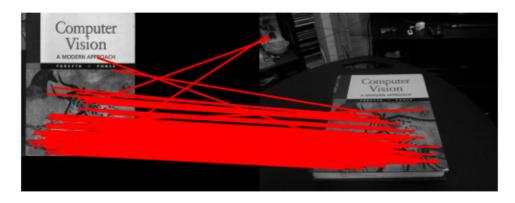
--- Matching with ratio=0.6, sigma=0.15 ---

Displaying matches for ratio: 0.6 and sigma: 0.15 Displaying matches for ratio: 0.6 and sigma: 0.15



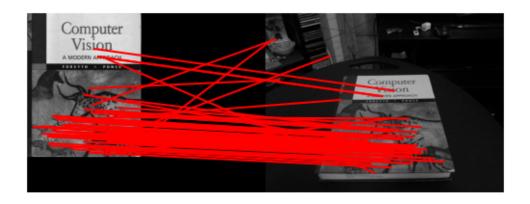
--- Matching with ratio=0.7, sigma=0.05 ---

Displaying matches for ratio: 0.7 and sigma: 0.05 Displaying matches for ratio: 0.7 and sigma: 0.05



--- Matching with ratio=0.7, sigma=0.1 ---

Displaying matches for ratio: 0.7 and sigma: 0.1 Displaying matches for ratio: 0.7 and sigma: 0.1



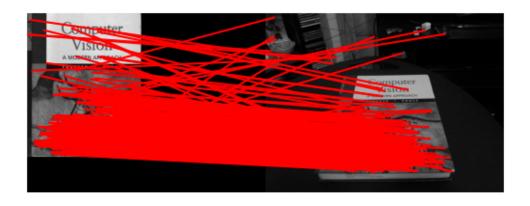
--- Matching with ratio=0.7, sigma=0.15 ---

Displaying matches for ratio: 0.7 and sigma: 0.15 Displaying matches for ratio: 0.7 and sigma: 0.15



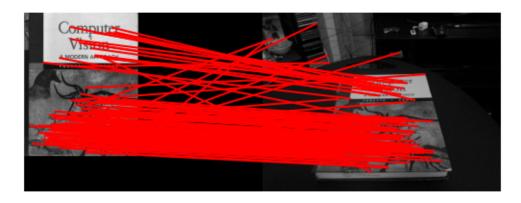
--- Matching with ratio=0.8, sigma=0.05 ---

Displaying matches for ratio: 0.8 and sigma: 0.05 Displaying matches for ratio: 0.8 and sigma: 0.05



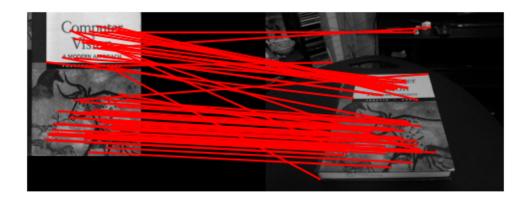
--- Matching with ratio=0.8, sigma=0.1 ---

Displaying matches for ratio: 0.8 and sigma: 0.1 Displaying matches for ratio: 0.8 and sigma: 0.1



--- Matching with ratio=0.8, sigma=0.15 ---

Displaying matches for ratio: 0.8 and sigma: 0.15 Displaying matches for ratio: 0.8 and sigma: 0.15



6.2.5 Q2.1.6 (10 points):

Implement the function briefRot

```
[12]: import scipy.ndimage
      def briefRot(min_deg, max_deg, deg_inc, ratio, sigma, filename):
          Tests Brief with rotations.
          Input
          min_deg: minimum degree to rotate image
          max_deg: maximum degree to rotate image
          deg_inc: number of degrees to increment when iterating
          ratio: ratio for BRIEF feature descriptor
          sigma: threshold for corner detection using FAST feature detector
          filename: filename of image to rotate
          11 11 11
          if not os.path.exists(RES_DIR):
           raise RuntimeError('RES_DIR does not exist. did you run all cells?')
          # Read the image and convert bgr to rgb
          image_path = os.path.join(DATA_DIR, filename)
          image = cv2.imread(image_path)
          if len(image.shape) == 3 and image.shape[2] == 3:
            image = cv2.cvtColor(image, cv2.COLOR_BGR2RGB)
          match_degrees = [] # stores the degrees of rotation
          match_counts = [] # stores the number of matches at each degree of rotation
```

```
for i in range(min_deg, max_deg, deg_inc):
       print(i)
        # ===== your code here! =====
        # TODO: Rotate Image (Hint: use scipy.ndimage.rotate)
             - Typically, we set reshape=False so the image keeps the same size.
             - If the image is not square and you have issues, you might need to
               allow reshape=True, but then you must handle dimension changes.
        rotated_img = scipy.ndimage.rotate(image, i, reshape=False)
        # TODO: Match features in images
       matches, locs1, locs2 = matchPics(image, rotated_img, ratio, sigma)
        # TODO: visualizes matches at at least 3 different orientations
        # to include in your report
        # (Hint: use plotMatches)
        if i in [0, 120, 240]:
            plt.figure(figsize=(8, 6))
            plotMatches(image, rotated_img, matches, locs1, locs2)
            plt.title(f"Matches at rotation = {i}o")
           plt.show()
        # TODO: Update match_degrees and match_counts (see descriptions above)
       match degrees.append(i)
       match_counts.append(len(matches))
        # ==== end of code ====
    # Save to pickle file
   matches_to_save = [match_counts, match_degrees, deg_inc]
   write_pickle(ROT_MATCHES_PATH, matches_to_save)
def dispBriefRotHist(matches_path=ROT_MATCHES_PATH):
    # Check if pickle file exists
   if not os.path.exists(matches_path):
     raise RuntimeError('matches path does not exist. did you call briefRot?')
   # Read from pickle file
   match_counts, match_degrees, deg_inc = read_pickle(matches_path)
    # Display histogram
    # Bins are centered and separated every 10 degrees
   plt.figure()
   bins = [x - deg_inc/2 for x in match_degrees]
   bins.append(bins[-1] + deg_inc)
   plt.hist(match_degrees, bins=bins, weights=match_counts, log=True)
   \#plt.hist(match\_degrees, bins=[10 * (x-0.5) for x in range(37)],
 →weights=match_counts, log=True)
```

```
plt.title("Histogram of BREIF matches")
plt.ylabel("# of matches")
plt.xlabel("Rotation (deg)")
plt.tight_layout()

output_path = os.path.join(RES_DIR, 'histogram.png')
plt.savefig(output_path)
```

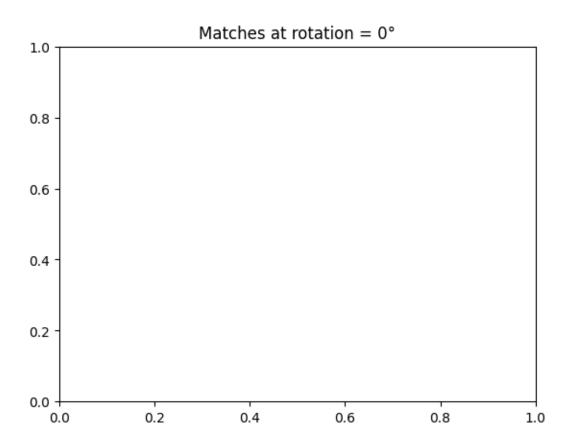
Visualize the matches under rotation See debugging tips in handout.

```
[13]: # defaults are:
      # min deq = 0
      \# max_deg = 360
      \# deq inc = 10
      # ratio = 0.7
      \# sigma = 0.15
      # filename = 'cv_cover.jpg'
      # Controls the rotation degrees
      min_deg = 0
      max_deg = 360
      deg_inc = 10
      # Brief feature descriptor and Fast feature detector paremeters
      # (change these if you want to use different values)
      ratio = 0.7
      sigma = 0.15
      # image to rotate and match
      # (no need to change this but can if you want to experiment)
      filename = 'cv_cover.jpg'
      # Call briefRot
      briefRot(min_deg, max_deg, deg_inc, ratio, sigma, filename)
```

0

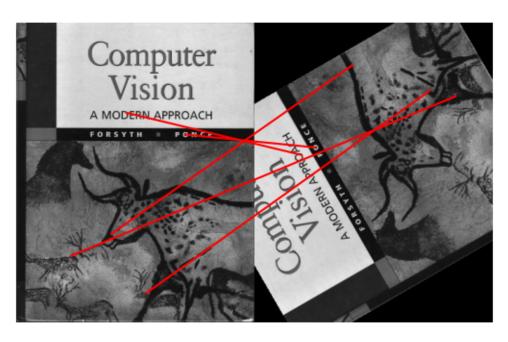
```
/tmp/ipykernel_132536/2274933143.py:60: DeprecationWarning: Conversion of an
array with ndim > 0 to a scalar is deprecated, and will error in future. Ensure
you extract a single element from your array before performing this operation.
(Deprecated NumPy 1.25.)
  return 1 if img[int(center[0]+row1)][int(center[1]+col1)] <
img[int(center[0]+row2)][int(center[1]+col2)] else 0
<Figure size 800x600 with 0 Axes>
```

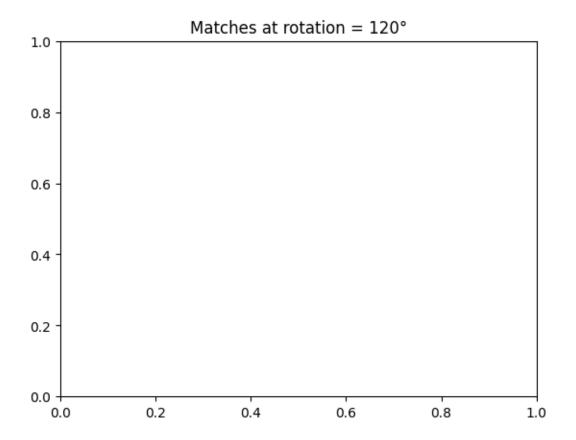




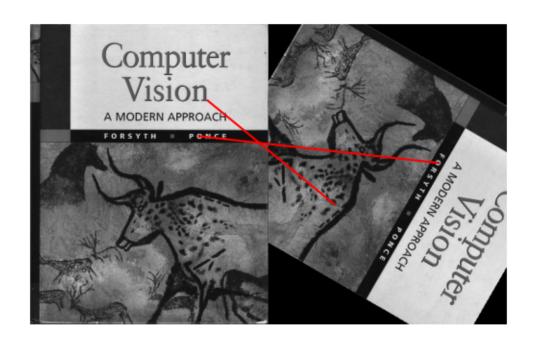
```
20
30
40
50
60
70
80
90
100
110
120
```

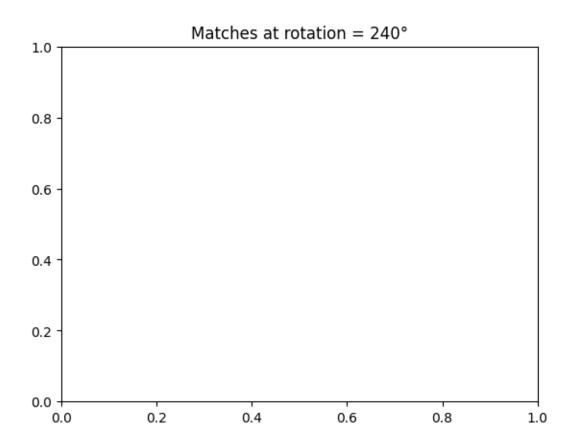
<Figure size 800x600 with 0 Axes>





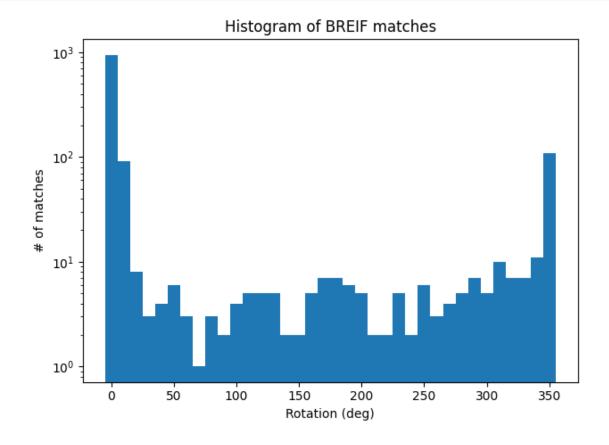
<Figure size 800x600 with 0 Axes>





Plot the histogram See debugging tips in handout.

[14]: dispBriefRotHist()



Explain why you think the BRIEF descriptor behves this way: YOUR ANSWER HERE...

6.2.6 Q2.1.7.1 (Extra Credit - 5 points):

Design a fix to make BRIEF more rotation invariant. Feel free to make any helper functions as necessary. But you cannot use any additional OpenCV or Scikit-Image functions.

```
[]: | # ===== your code here! =====
     # TODO: Define any helper functions here
     # (Feel free to put anything in its own cell)
     # TODO: Feel free to modify the inputs and the function body as necessary
     # This is only an outline
     def briefRotInvEc(min deg, max deg, deg inc, ratio, sigma, filename):
         Rotation invariant Brief.
         Input
         ____
         min_deq: minimum degree to rotate image
         max_deg: maximum degree to rotate image
         deg_inc: number of degrees to increment when iterating
         ratio: ratio for BRIEF feature descriptor
         sigma: threshold for corner detection using FAST feature detector
         filename: filename of image to rotate
         11 11 11
         if not os.path.exists(RES_DIR):
          raise RuntimeError('RES_DIR does not exist. did you run all cells?')
         #Read the image and convert bgr to rgb
         image_path = os.path.join(DATA_DIR, filename)
         image = cv2.imread(image_path)
         if len(image.shape) == 3 and image.shape[2] == 3:
           image = cv2.cvtColor(image, cv2.COLOR_BGR2RGB)
         match_degrees = [] # stores the degrees of rotation
         match_counts = [] # stores the number of matches at each degree of rotation
         for i in range(min_deg, max_deg, deg_inc):
             print(i)
             # TODO: Rotate Image (Hint: use scipy.ndimage.rotate)
             # TODO: Brief matcher that is rotation invariant
             # Feel free to define additional helper functions as necessary
             # TODO: visualizes matches at at least 3 different orientations
             # to include in your report
```

```
# (Hint: use plotMatches)

# TODO: Update match_degrees and match_counts (see descriptions above)

# Save to pickle file
matches_to_save = [match_counts, match_degrees, deg_inc]
write_pickle(ROT_INV_MATCHES_PATH, matches_to_save)

# ==== end of code ====
```

Visualize your implemented function

```
[]: min_deg = 0
    max_deg = 360
    deg_inc = 10
    filename = 'cv_cover.jpg'

# ==== your code here! =====
# TODO: Call briefRotInvEc and visualize

# ==== end of code ====
```

Plot Histogram

```
[]: dispBriefRotHist(matches_path=ROT_INV_MATCHES_PATH)
```

Compare the histograms with an without rotation invariance. Explain your rotation invariant design and how you selected any parameters that you used: YOUR ANSWER HERE...

6.2.7 Q2.1.7.2 (Extra Credit - 5 points):

Design a fix to make BRIEF more scale invariant. Feel free to make any helper functions as necessary. But you cannot use any additional OpenCV or Scikit-Image functions.

```
[]: # ===== your code here! =====

# TODO: Define any helper functions here
# (Feel free to put anything in its own cell)

# TODO: Modify the inputs and the function body as necessary
def briefScaleInvEc(ratio, sigma, filename):

#Read the image and convert bgr to rgb
image_path = os.path.join(DATA_DIR, filename)
image = cv2.imread(image_path)
if len(image.shape) == 3 and image.shape[2] == 3:
```

```
image = cv2.cvtColor(image, cv2.COLOR_BGR2RGB)
   match_scales = [] # stores the scaling factors
   match_counts = [] # stores the number of matches at each scaling factor
   for i in [1]:
       # Scale Image
       image_scale = cv2.resize(image,(int(image.shape[1]/(2**i)),
                                        int(image.shape[0]/(2**i))),
                                 interpolation = cv2.INTER_AREA)
        # TODO: Brief matcher that is scale invariant
        # Feel free to define additional helper functions as necessary
        # Compare to regular matchPics
       matches_orig, locs1_orig, locs2_orig = matchPics(image,
                                                         image_scale,
                                                         ratio, sigma)
       print('plotting non-scale invariant scale: ', 2**i)
       plotMatches(image, image_scale, matches_orig, locs1_orig,
                    locs2 orig)
       print('plotting scale-invariant: ', 2**i)
       plotMatches(image, image_scale, matches, locs1, locs2)
# ==== end of code ====
```

Visualize your implemented function

```
[]: # ===== your code here! =====

# TODO: Call briefScaleInvEc and visualize

# You may change any parameters and the function body as necessary

filename = 'cv_cover.jpg'

ratio = 0.7
sigma = 0.15

briefScaleInvEc(ratio, sigma, filename)
# ==== end of code ====
```

Explain your scale invariant design and how you selected any parameters that you used: YOUR ANSWER HERE...

6.3 Q2.2 Homography Computation

6.3.1 Q2.2.1 (15 Points):

Implement the function computeH

```
[39]: def computeH(x1, x2):
         Compute the homography between two sets of points
         Input
          x1, x2: Sets of points
         Returns
          _____
         H2to1: 3x3 homography matrix that best transforms x2 to x1
         if x1.shape != x2.shape:
             raise RuntimeError('number of points do not match')
          # ===== your code here! =====
         # TODO: Compute the homography between two sets of points
          # Ensure x1, x2 are in inhomogeneous form (N x 2)
         # If they're already (N x 2), do nothing; if (N x 3), divide by last column.
         def to_inhomogeneous(pts):
             if pts.shape[1] == 3: # homogeneous form
                 pts = pts[:, :2] / pts[:, 2, np.newaxis]
             return pts
         x1 = to_inhomogeneous(x1)
         x2 = to_inhomogeneous(x2)
         # Number of correspondences
         N = x1.shape[0]
          # Build matrix A for DLT
         A = \Gamma
         for i in range(N):
             X1, Y1 = x1[i, 0], x1[i, 1]
             X2, Y2 = x2[i, 0], x2[i, 1]
              # Each correspondence contributes two rows
             A.append([ -X2, -Y2, -1, 0, 0, X1*X2, X1*Y2, X1])
             A.append([ 0, 0, -X2, -Y2, -1, Y1*X2, Y1*Y2, Y1])
         A = np.array(A)
```

```
# Solve for h using SVD
# We want the singular vector (last column of V) corresponding to the
smallest singular value
_, _, Vt = np.linalg.svd(A)
h = Vt[-1] # last row of V^T -> last column of V

# Reshape h to 3x3
H2to1 = h.reshape(3, 3)

# Normalize so bottom-right entry is 1 (or so the matrix has unit norm)
# This is a common convention, not strictly mandatory
if H2to1[-1, -1] != 0:
    H2to1 /= H2to1[-1, -1]
# ==== end of code ====
return H2to1
```

6.3.2 Q2.2.2 (10 points):

Implement the function computeH norm

```
[40]: def computeH_norm(x1, x2):
          Compute the homography between two sets of points using normalization
          Input
          ----
          x1, x2: Sets of points
          Returns
          H2to1: 3x3 homography matrix that best transforms x2 to x1
          # ===== your code here! =====
          # TODO: Compute the centroid of the points
          c1 = np.mean(x1, axis=0)
          c2 = np.mean(x2, axis=0)
          # TODO: Shift the origin of the points to the centroid
          x1\_shifted = x1 - c1
          x2\_shifted = x2 - c2
          # TODO: Normalize the points so that the largest distance from the
          # origin is equal to sqrt(2)
          dist1 = np.sqrt(np.sum(x1_shifted**2, axis=1))
          max_dist1 = np.max(dist1) if len(dist1) > 0 else 0
```

```
s1 = np.sqrt(2) / max_dist1 if max_dist1 != 0 else 1.0
# Compute scale for x2
dist2 = np.sqrt(np.sum(x2_shifted**2, axis=1))
max_dist2 = np.max(dist2) if len(dist2) > 0 else 0
s2 = np.sqrt(2) / max_dist2 if max_dist2 != 0 else 1.0
# TODO: Similarity transform 1
''' [ s1 0 -s1*c1x ]
     [ 0 s1 -s1*c1y]
     [ 0 0 1 ] '''
T1 = np.array([
    [s1, 0, -s1 * c1[0]],
    [0, s1, -s1 * c1[1]],
    [0,
        0, 1]
])
# TODO: Similarity transform 2
T2 = np.array([
    [s2, 0, -s2 * c2[0]],
    [0, s2, -s2 * c2[1]],
        0, 1]
    [0,
])
# Convert x1, x2 into homogeneous coordinates for transformation
x1_{hom} = np.hstack([x1, np.ones((x1.shape[0], 1))])
x2_{hom} = np.hstack([x2, np.ones((x2.shape[0], 1))])
# Apply similarity transforms
x1_normalized = (T1 @ x1_hom.T).T
x2\_normalized = (T2 @ x2\_hom.T).T
# Now they are normalized. Extract inhomogeneous parts
x1_norm_inh = x1_normalized[:, :2] / x1_normalized[:, [2]]
x2_norm_inh = x2_normalized[:, :2] / x2_normalized[:, [2]]
# TODO: Compute homography
H_norm = computeH(x1_norm_inh, x2_norm_inh)
# TODO: Denormalization
# Denormalize: H2to1 = T1^{-1} * H_norm * T2
T1_inv = np.linalg.inv(T1)
H2to1 = T1_inv @ H_norm @ T2
# ==== end of code ====
return H2to1
```

6.3.3 Q2.2.3 (25 points):

Implement RANSAC

```
[57]: def computeH_ransac(locs1, locs2, max_iters, inlier_tol):
          Estimate the homography between two sets of points using ransac
          Input
          locs1, locs2: Lists of points
          max_iters: the number of iterations to run RANSAC for
          inlier_tol: the tolerance value for considering a point to be an inlier
          Returns
          bestH2to1: 3x3 homography matrix that best transforms locs2 to locs1
          inliers: indices of RANSAC inliers
          11 11 11
          # ===== your code here! =====
          # TODO:
          # Compute the best fitting homography using RANSAC
          # given a list of matching points locs1 and loc2
          # Initialize
          bestH2to1 = None
          best_inliers = np.array([], dtype=int)
          num_points = locs1.shape[0]
          # Basic check
          if num_points < 4:</pre>
              raise ValueError("Need at least 4 correspondences to compute a
       ⇔homography")
          for _ in range(max_iters):
              # 1) Randomly sample 4 correspondences
              sample_idxs = np.random.choice(num_points, 4, replace=False)
              x1_sample = locs1[sample_idxs]
              x2_sample = locs2[sample_idxs]
              # 2) Compute the homography mapping x2 \rightarrow x1
              # If computeH_norm is provided, use it for better stability:
              H = computeH_norm(x1_sample, x2_sample)
              # Or if only unnormalized version is available, call computeH:
              \# H = computeH(x1\_sample, x2\_sample)
```

```
# 3) Project all locs2 using this H
    locs2 hom = np.hstack((locs2, np.ones((num_points, 1)))) # Nx3
    projected = (H @ locs2_hom.T).T
                                                              # Nx3
    # Convert back to inhomogeneous
    projected_inhom = projected[:, :2] / (projected[:, [2]] + 1e-12)
    # 4) Compute distances to locs1
         If distance < inlier tol, the point is considered an inlier
    dists = np.linalg.norm(locs1 - projected_inhom, axis=1)
    inliers = np.where(dists < inlier_tol)[0]</pre>
    # 5) Keep track of the best H (largest number of inliers so far)
    if len(inliers) > len(best_inliers):
        best_inliers = inliers
        bestH2to1 = H
# ==== end of code ====
return bestH2to1, best_inliers
```

6.3.4 Q2.2.4 (10 points):

Implement the function compositeH

```
[56]: def compositeH(H2to1, template, img):
          Returns the composite image.
          Input
          H2to1: Homography from image to template
          template: template image to be warped
          img: background image
          Returns
          _____
          composite imq: Composite image
          11 11 11
          # ===== your code here! =====
          # TODO: Create a composite image after warping the template image on top
          # of the image using the homography
          # 1) Warp the template image to the same size as 'img'
          # The output width/height should match 'img'
              # --- Ensure that both template and img are 3-channel (RGB/BGR) ---
          def to_3channel(arr):
```

```
if arr.ndim == 2:
           return cv2.cvtColor(arr, cv2.COLOR GRAY2RGB)
       elif arr.ndim == 3 and arr.shape[2] == 4: # RGBA
           return arr[:, :, :3]
      return arr
  template = to_3channel(template)
  img = to_3channel(img)
  h_img, w_img = img.shape[:2]
  warped_template = cv2.warpPerspective(template, H2to1, (w_img, h_img))
  # 2) Create a mask of the warped template where it is non-zero
       So we can overlay it onto the background 'img'
  mask = np.sum(warped_template, axis=2) > 0 # shape (h_img, w_img)
  # 3) Make a copy of the background
  composite_img = img.copy()
  \# 4) For all pixels where the warped template is non-zero, replace in the \sqcup
\hookrightarrow background
  composite img[mask] = warped template[mask]
  # ==== end of code ====
  return composite_img
```

Implement the function warpImage

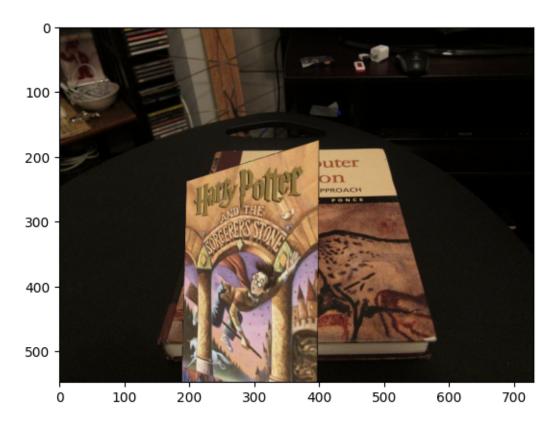
```
# ===== your code here! =====
  if cv_cover.ndim == 2:
      cv_cover_color = cv2.cvtColor(cv_cover, cv2.COLOR_GRAY2RGB)
  elif cv_cover.shape[2] == 4: # has alpha?
      cv_cover_color = cv_cover[:, :, :3]
  else.
      cv_cover_color = cv_cover.copy()
  # 2) For feature matching, we can pass grayscale images to matchPics.
  # If cv_cover is already grayscale, fine. If it's color, we can convert.
  if cv cover.ndim == 3:
      cv_cover_gray = cv2.cvtColor(cv_cover_color, cv2.COLOR_RGB2GRAY)
  else:
      cv_cover_gray = cv_cover # it was already grayscale
  cv_desk_gray = cv2.cvtColor(cv_desk, cv2.COLOR_RGB2GRAY)
  # TODO: match features between cv_desk and cv_cover using matchPics
  matches, locs_desk, locs_cv = matchPics(cv_desk_gray, cv_cover_gray, ratio,_
⇔sigma)
  # TODO: Scale matched pixels in cv cover to size of hp cover
  pts_desk = locs_desk[matches[:, 0]] # shape (p, 2) in cv_desk
  pts_cv = locs_cv[matches[:, 1]] # shape (p, 2) in cv_cover
  # 3) Compute scale factors to map cv_cover -> hp_cover
  h_hp, w_hp = hp_cover.shape[:2]
  h_cv, w_cv = cv_cover.shape[:2]
  scale_x = w_hp / w_cv
  scale_y = h_hp / h_cv
  # 4) Scale pts2 to match hp_cover size
  pts_hp = pts_cv.astype(np.float32).copy()
  pts_hp[:, 0] *= scale_x
  pts_hp[:, 1] *= scale_y
  # 5) Compute homography by RANSAC (pts2_scaled -> pts1)
  bestH2to1, _ = computeH_ransac(pts_desk, pts_hp, max_iters, inlier_tol)
  # 6) Warp hp_cover onto cv_desk using the homography
  composite_img = compositeH(bestH2to1, hp_cover, cv_desk)
  # ==== end of code ====
  plt.imshow(composite_img)
  plt.show()
```

Visualize composite image

```
[64]: # defaults are:
# ratio = 0.7
# sigma = 0.15
# max_iters = 600
# inlier_tol = 1.0

# (no need to change this but can if you want to experiment)
ratio = 0.7
sigma = 0.15
max_iters = 600
inlier_tol = 1.0

warpImage(ratio, sigma, max_iters, inlier_tol)
```



6.3.5 Q2.2.5 (10 points):

Conduct ablation study with various max_iters and inlier_tol values. Plot the result images and explain the effect of these two parameters respectively.

```
[60]: # ===== your code here! =====

# Experiment with different max_iters and inlier_tol values.

# Include the result images in the write-up.

max_iters_list = [500, 1000, 2000]
inlier_tol_list = [1.0, 2.0, 4.0]

for m in max_iters_list:
    for tol in inlier_tol_list:
        print(f"\n--- RANSAC with max_iters={m}, inlier_tol={tol} ---")
        warpImage(ratio=0.7, sigma=0.15, max_iters=m, inlier_tol=tol)

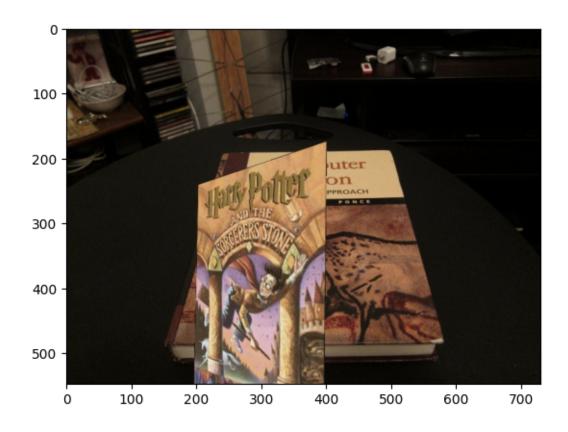
# ==== end of code ====
```

--- RANSAC with max_iters=500, inlier_tol=1.0 ---

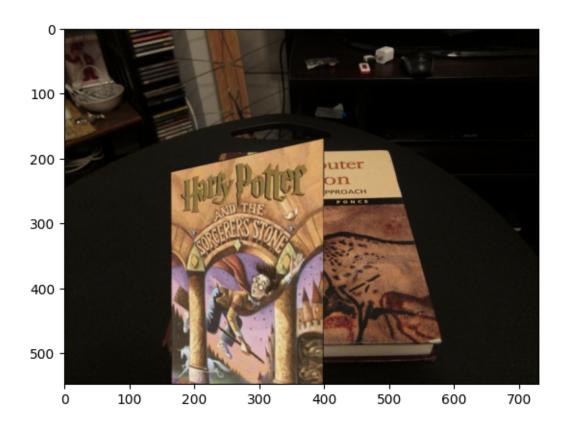
/tmp/ipykernel_132536/2274933143.py:60: DeprecationWarning: Conversion of an array with ndim > 0 to a scalar is deprecated, and will error in future. Ensure you extract a single element from your array before performing this operation. (Deprecated NumPy 1.25.)



--- RANSAC with max_iters=500, inlier_tol=2.0 ---



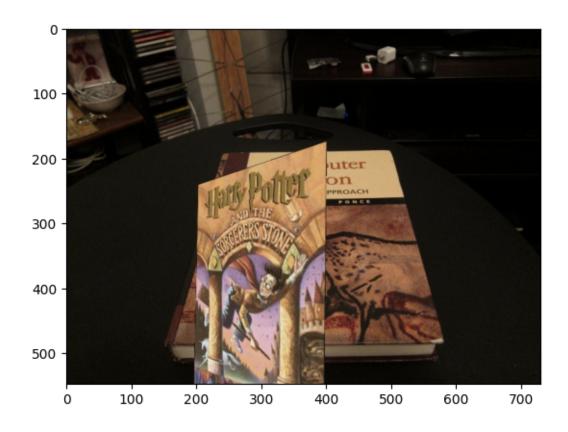
--- RANSAC with max_iters=500, inlier_tol=4.0 ---



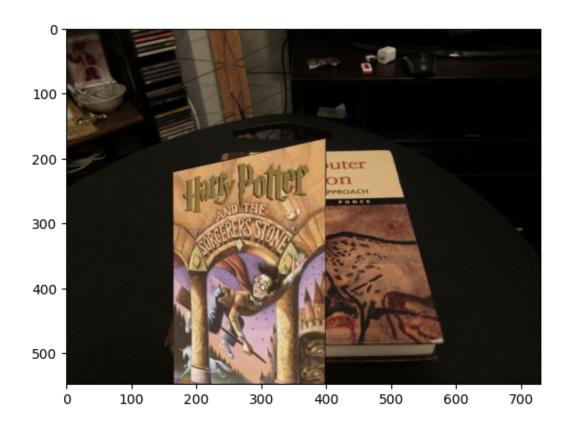
--- RANSAC with max_iters=1000, inlier_tol=1.0 ---



--- RANSAC with max_iters=1000, inlier_tol=2.0 ---



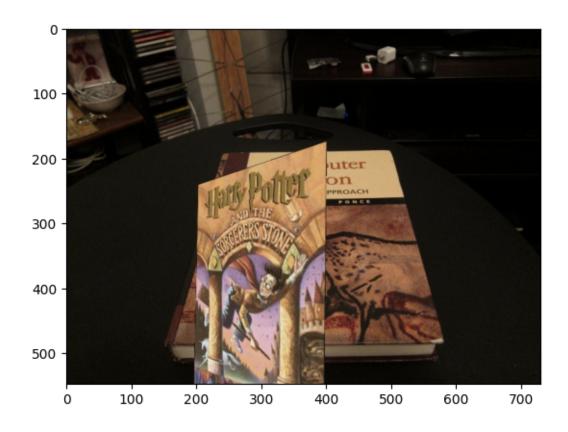
--- RANSAC with max_iters=1000, inlier_tol=4.0 ---



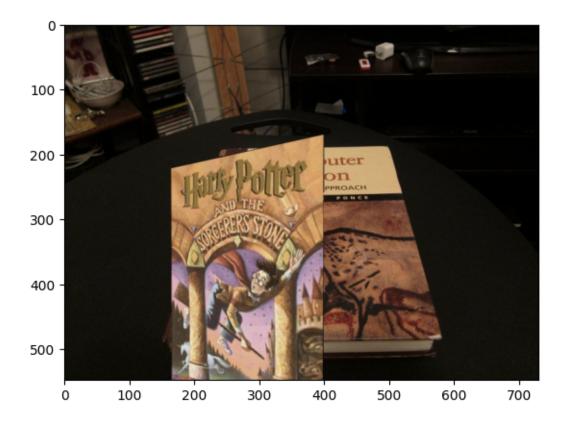
--- RANSAC with max_iters=2000, inlier_tol=1.0 ---



--- RANSAC with max_iters=2000, inlier_tol=2.0 ---



--- RANSAC with max_iters=2000, inlier_tol=4.0 ---



Explain the effect of max_iters and inlier_tol: It doesn't perfectly work above, but obviously the max iterations doesn't have near as much effect as the inlier tolerance which begins to turn the HP cover to be more in plane with the CV cover by allowing the warp to make greater and more matches matches

7 Q3 Create a Simple Panorama

7.1 Q3.1 Create a panorama (10 points):

Implement the function createPanorama

```
[65]: def createPanorama(left_im, right_im, ratio, sigma, max_iters, inlier_tol):

"""

Create a panorama augmented reality application by computing a homography and stitching together a left and right image.

Input

----
left_im: left image right_im: right image
```

```
ratio: ratio for BRIEF feature descriptor
  sigma: threshold for corner detection using FAST feature detector
  max_iters: the number of iterations to run RANSAC for
  inlier tol: the tolerance value for considering a point to be an inlier
  Returns
  panorama_im: Stitched together panorama
  11 11 11
  # ===== your code here! =====
  # TODO: match features between images
  # This can be done using matchPics, cpselect, or any other function.
  matches, locs_left, locs_right = matchPics(left_im, right_im, ratio, sigma)
  pts_left = locs_left[matches[:, 0]] # shape (p, 2)
  pts_right = locs_right[matches[:, 1]] # shape (p, 2)
  # TODO: Get homography by RANSAC using computeH_ransac
  bestH2to1, _ = computeH_ransac(pts_left, pts_right, max_iters, inlier_tol)
  # TODO: Stich together the two images
  # Requires the use of cv2.warpPerspective
  h left, w left = left im.shape[:2]
  h_right, w_right = right_im.shape[:2]
  pano_w = w_left + w_right
  pano_h = max(h_left, h_right)
  # 5) Warp the right image to the left image's coordinate system
  warped_right = cv2.warpPerspective(right_im, bestH2to1, (pano_w, pano_h))
  # 6) Combine: overlay the left image on top of the warped right image
       (Many advanced methods do alpha blending. We'll do a simple overlay
⇔here.)
  panorama_im = warped_right.copy()
  panorama_im[0:h_left, 0:w_left] = left_im
  # ==== end of code ====
  return panorama_im.astype(np.uint8)
```

Visualize Panorama Make sure to use your own images and include them as well as the result in the report.

```
[66]: left_im_path = os.path.join(DATA_DIR, 'pano_left.jpg')
left_im = skimage.io.imread(left_im_path)
right_im_path = os.path.join(DATA_DIR, 'pano_right.jpg')
```

```
right_im = skimage.io.imread(right_im_path)

# Feel free to adjust as needed
ratio = 0.7
sigma = 0.15
max_iters = 600
inlier_tol = 1.0

panorama_im = createPanorama(left_im, right_im, ratio, sigma, max_iters, usinlier_tol)

plt.imshow(panorama_im)
plt.axis('off')
plt.show()
```

