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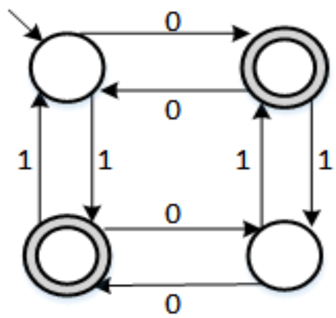
Assignment 3 – CPS420

Group 27

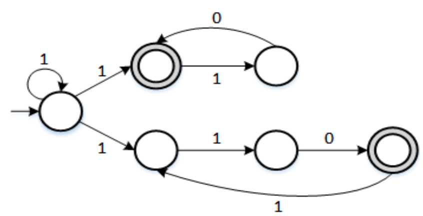
PART A: Recognizing Automata

1) **Give a precise, concise, and unambiguous English description of the language accepted by this automation:**



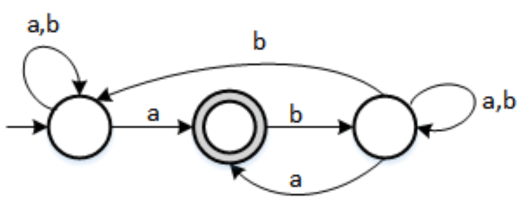
There are four states in this image with there being two final states. To get to the two final states we start from the initial state and there are two possible paths; 1 or 0 that can be taken to get to the two final states. Same way after you reach both these final states there are again two more path; 0 or 1 that can be taken to get to the next state. You can visit each path at least once and it is possible to return to the initial state after going around all the paths at least once. This automata represents all strings that have the same amount of 0's and 1's.

2) **Given a regular expression describing the language accepted by the automation below. This regular expression should be is as simple as possible.**



$((1^{+} (10)^{*}) \mid (1^{+}10 (110)))$

3) **Given a regular expression describing the language accepted by the automation below. This regular expression should be is as simple as possible.**



((a|b)* a (b (a|b)* (a|b (a|b)* a)) *)

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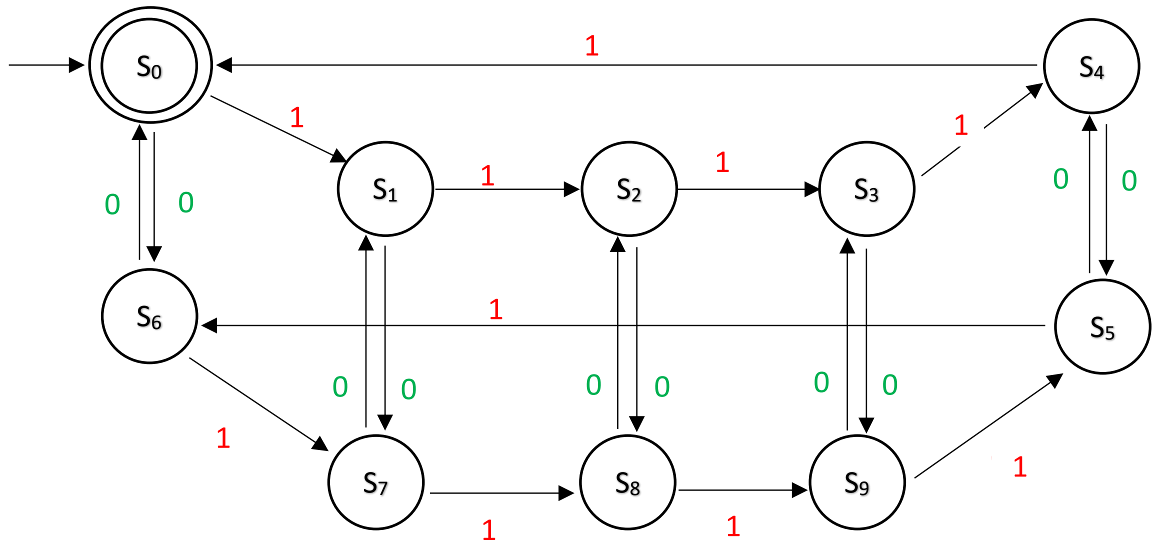
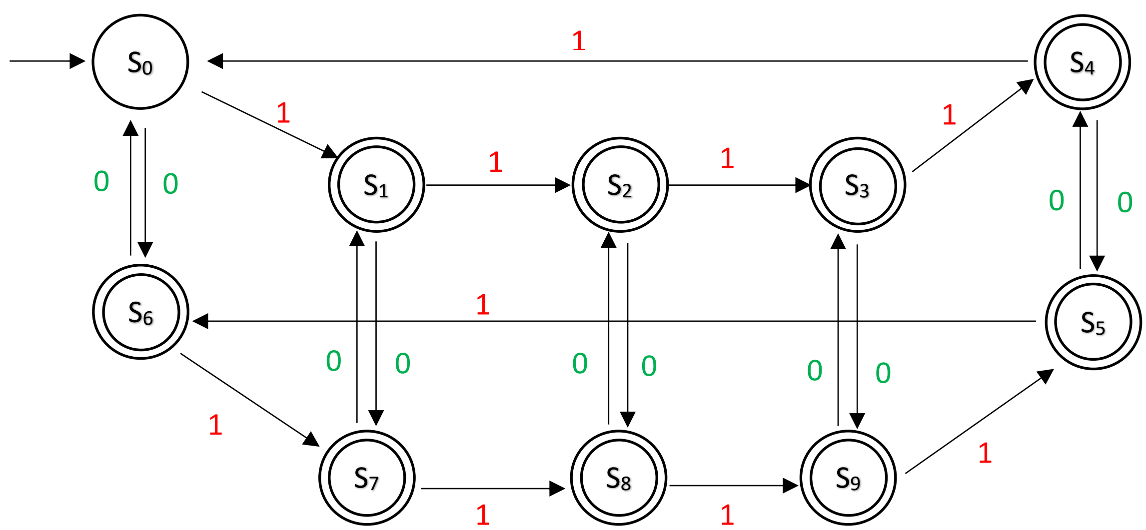
PART B: Automata Construction

1) **Draw a DFA which accepts the following language over the alphabet of {0,1}: the set of all strings such that the number of 0s is divisible by 2 and the number of 1s is divisible by 5. Your DFA must handle all intput strings in {0,1}*.**



2) **Draw a DFA which accepts the following language over the alphabet of {0, 1}: the set of all strings such that the number of 0s is not divisible by 2 or the number of 1s is not divisible by 5. Your DFA must handle all input strings in {0, 1}*.**
(Hint: look at solution of previous question)



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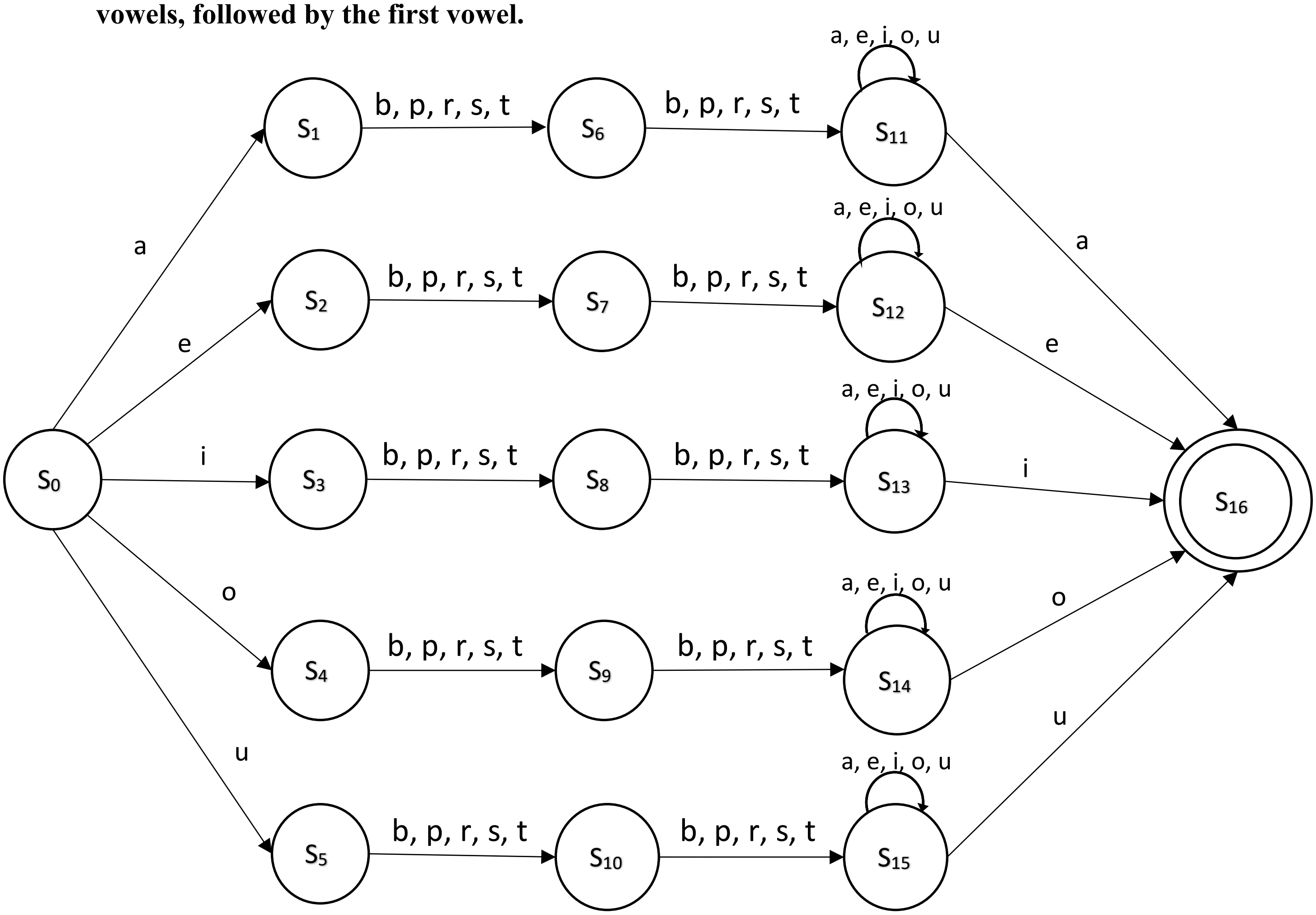
3) **Draw the simplest possible NFA which accepts the following language over the alphabet of {a, e, i, o, u, b, p, r, s, t} : the set of strings which start with a vowel, then two consonants, then 0 or more vowels, followed by the first vowel.**



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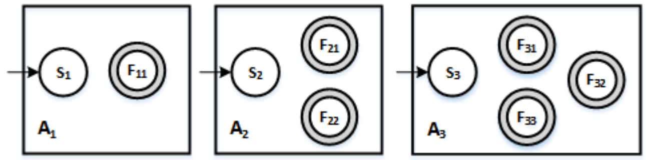
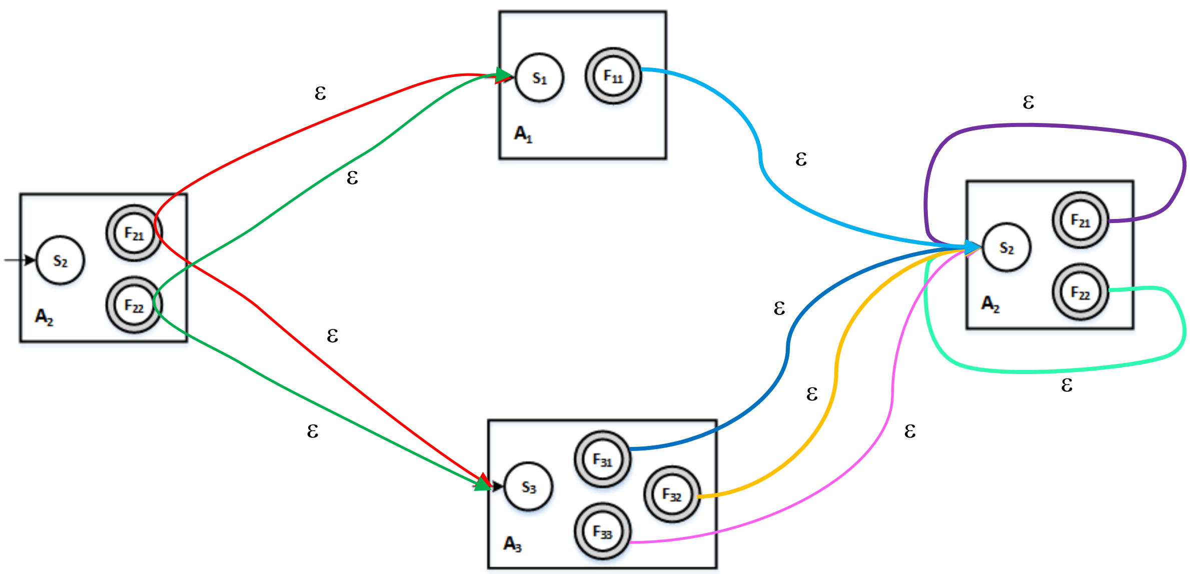
4) Assuming that you have the following three automata, A_1, A_2, A_3 recognizing the languages L_1, L_2. L_3 respectively, shown here as black boxes:



a. Draw a NFA that recognizes the language L_2 (L_1 | L_3) L_2*



b. What is the starting state of A?

The starting state of A is A2.

c. What are A's final states?

The final state are F21, F22, F11, F31, F33, F32.

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PART C: Counting

1. **Only 8 of the 24 semi finalist will end up in the final race. Based solely on the number of athletes per country in the semi-finals and not on their relative running abilities, what is the expected value of:**

   a. **The number of runners from JAM in the final race?**

To find the expected value of the number of JAM runners in the final race, we use the following formula:

$$\sum_{k=1}^{n} a_k p_k = a_1 p_1 + a_2 p_2 + \cdots + a_n p_n$$

Here a_k = number of experiments and p_k = probability of the outcome
Therefore, a_k = number of runners selected for the final race
p_k = probability of JAM runners selected for the final race
There are 4 JAM runners in total of 24 runners. So,
$p_k = \frac{4}{24} = \frac{1}{6}$ and $a_k = 8$
Substituting these values in the formula for expected value,
$\sum_{k=1}^{n} a_k p_k = a_1 p_1 + a_2 p_2 + \cdots + a_n p_n$
$= 8 \times \frac{1}{6} = 1.33 \sim 1$ runner

Hence, the expected value of number of JAM runners in the final race is 1.33.

   b. **The number of runners from TTO in the final race?**

To find the expected value of the number of TTO runners in the final race, we use the following formula:

$$\sum_{k=1}^{n} a_k p_k = a_1 p_1 + a_2 p_2 + \cdots + a_n p_n$$

Here a_k = number of experiments and p_k = probability of the outcome
Therefore, a_k = number of runners selected for the final race
p_k = probability of TTO runners selected for the final race
There are 3 TTO runners in total of 24 runners. So,
$p_k = \frac{3}{24} = \frac{1}{8}$ and $a_k = 8$
Substituting these values in the formula for expected value,
$\sum_{k=1}^{n} a_k p_k = a_1 p_1 + a_2 p_2 + \cdots + a_n p_n$
$= 8 \times \frac{1}{8} = 1$ runner

Hence, the expected value of number of TTO runners in the final race is 1.

c. The number of runners from NED in the final race?

To find the expected value of the number of NED runners in the final race, we use the following formula:

$$\sum_{k=1}^{n} a_k p_k = a_1 p_1 + a_2 p_2 + \cdots + a_n p_n$$

Here a_k = number of experiments and p_k = probability of the outcome
Therefore, a_k = number of runners selected for the final race
p_k = probability of NED runners selected for the final race
There is 1 NED runner in total of 24 runners. So,
$p_k = \frac{1}{24}$ and $a_k = 8$
Substituting these values in the formula for expected value,

$$\sum_{k=1}^{n} a_k p_k = a_1 p_1 + a_2 p_2 + \cdots + a_n p_n$$
$$= 8 \times \frac{1}{24} = 0.33 \sim 0 \text{ athlete}$$

Hence, the expected value of number of NED runners in the final race is 0.33.

2. The following 8 athletes end up in the final race: 3 from USA, 3 from JAM, 1 from TTO, 1 from NED (Netherlands). As you know, 3 of these runners will end up on the podium with a gold (G), silver (S), or bronze (B) medal.

a. How many possible outcomes will there eventually be on the podium (by individual runner)?

There are 8 runners and 3 positions for medals: Gold, Silver and Bronze. Since repetition is not allowed, we use the formula for permutations:

$$P(n, k) = \frac{n!}{(n-k)!}$$

Substituting values of the number of players and number of medals,

$$P(8, 3) = \frac{8!}{(8-3)!} = \frac{8!}{5!} = 336$$

b. How many possible podium outcomes by country are there? (in other words we are asking you to figure out how many possible combinations of medals by country there can be) Your answer should not be derived by listing all the possibilities one by one. Instead you should derive your answer by reasoning with known formulas for permutations and combinations

<u>To find all possible outcomes of runners from different countries:</u>
There are 4 countries in total and 3 podiums. Choosing 2 countries for the podiums requires a permutation and therefore, we use the formula for permutations:

$$P(n, k) = \frac{n!}{(n-k)!}$$

Substituting values of the number of players and number of medals,

$$P(4, 3) = \frac{4!}{(4-3)!} = \frac{4!}{1!} = 24$$

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<u>To find all possible outcomes of countries whose total runners can take up all podiums available:</u>
There are two countries, USA and JAM who have runners ≥ number of podiums.

<u>To find all possible outcomes of 2 runners from one country and 1 from another:</u>

Number of countries with more than 2 runners = 2 (USA and JAM)
Number of medals that can be rewarded = 3
Number of countries who can win the last medal left after the 2 runners from same country = 3
Therefore, 2 x 3 x 3 = 18

Hence, number of possible outcomes by countries = 24 + 2 + 18 = 44

c. **Based solely on the number of athletes per country in the finals and not on their relative running abilities:**
   i. **What is the probability that NED will earn at least one medal?**

Probability NED will earn at least one medal = $1-\frac{P(7,3)}{P(8,3)}=1-\frac{\frac{7!}{4!}}{\frac{8!}{5!}}=1-\frac{210}{336}=\frac{126}{336}=\frac{3}{8}$

   ii. **What is the probability that TTO will earn at least one medal?**

Probability TTO will earn at least one medal = $1-\frac{P(7,3)}{P(8,3)}=1-\frac{\frac{7!}{4!}}{\frac{8!}{5!}}=1-\frac{210}{336}=\frac{126}{336}=\frac{3}{8}$

   iii. **What is the probability that JAM will earn at least one medal?**

Probability JAM will earn at least one medal = $1-\frac{P(5,3)}{P(8,3)}=1-\frac{\frac{5!}{2!}}{\frac{8!}{5!}}=1-\frac{210}{336}=\frac{276}{336}=\frac{23}{28}$

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3. Calculate the probability that an Olympic sprinter who tests positive has actually used performance enhancing drugs. For this exercise, define the following events:

- **T^+ : getting an AAF on a test (i.e. testing positive)**
- **T^- : not getting an AAF on a test (i.e. testing negative)**
- **U : using a performance enhancing drug (i.e. being user)**
- **C : not using a performance enhancing drug (i.e. being clean)**

Define Terms:

$P(T^+) = 0.9\%$
$P(T^-) = 99.1\%$
$P(T^+ \mid U) = 95\%$
$P(T^- \mid U) = 5\%$
$P(T^+ \mid C) = 0.5\%$
$P(T^- \mid C) = 99.5\%$

Calculate P(U) :

$$
\begin{aligned}
P(T^+) &= P(T^+ \cap U) + P(T^+ \cap C) \\
&= P(T^+ \mid U) \times P(U) + P(T^+ \mid C) \times P(C) \\
&= (95\% \times P(U)) + (0.5\% \times (1 - P(U))) \\
0.9\% &= 95\%P(U) + 0.5\% - 0.5\%P(U) \\
P(U) &= \frac{0.9\% - 0.5\%}{94.5\%} \\
&= \frac{0.4\%}{94.5\%} \\
&\approx 0.42\%
\end{aligned}
$$

Calculate P(C) :

$$
\begin{aligned}
P(T^-) &= P(T^- \cap U) + P(T^- \cap C) \\
&= P(T^- \mid U) \times P(U) + P(T^- \mid C) \times P(C) \\
&= P(T^- \mid U) \times (1 - P(C)) + P(T^- \mid C) \times P(C) \\
99.1\% &= (5\% - 5\%P(C)) + 99.5\%P(C) \\
P(C) &= \frac{94.1\%}{94.5\%} \\
&\approx 99.58\%
\end{aligned}
$$

Find $P(U \mid T^+)$:

According to Bayes' Theorem

$$
\begin{aligned}
P(U|T^+) &= \frac{P(T^+ \mid U) \times P(U)}{P(T^+)} \\
P(U|T^+) &= \frac{95\% \times 0.42\%}{0.9\%} \\
P(U|T^+) &\approx 44.33\%
\end{aligned}
$$

Therefore, the probability that an Olympic sprinter who tests positive has actually performed the test is 44.3%