TEMA 1: ÁLGEBRA DE VARIABLES LÓGICAS

Algebra Booleana

Básicos

- Elemento Neutro:
 - + → 0
 - $\cdot \rightarrow 1$
- Propiedad Distributiva:
 - $a+(b\cdot c)=(a+b)\cdot(a+c)$
 - $a \cdot (b+c) = (a \cdot b) + (a \cdot c)$
- Complemento $\rightarrow \bar{a}$
 - $a + \bar{a} = 1$
 - $a \cdot \bar{a} = 0$

Teoremas

$$\mathbf{1.} \begin{cases} a+a=a \\ a \cdot a=a \end{cases}$$

$$a + 1 = 1$$

$$a \cdot 0 = 0$$

$$3. \ a = \bar{\bar{a}}$$

$$4. \begin{cases} a+ab=a \\ a(a+b)=a \end{cases}$$

5.
$$\begin{cases} a + (b+c) = (a+b) + c \\ a(bc) = (ab)c \end{cases}$$
 6.
$$\begin{cases} \overline{(a+b)} = \bar{a} \cdot \bar{b} \\ \overline{a \cdot b} = \bar{a} + \bar{b} \end{cases}$$

6.
$$\begin{cases} \overline{(a+b)} = \bar{a} \cdot \bar{b} \\ \overline{a \cdot b} = \bar{a} + \bar{b} \end{cases}$$

Conclusiones

- DE MORGAN (6) $\rightarrow \begin{cases} \overline{ab} \neq \bar{a} \cdot \bar{b} \\ \overline{a+b} \neq \bar{a} + \bar{b} \end{cases}$
- SHANNON $\rightarrow \overline{f(a,b,\cdots,n,+,\cdot)} = f(\bar{a},\bar{b},\cdots,\bar{n},\cdot,+)$

Término CANÓNICO

• Producto canónico \rightarrow MINTERM = 1

$$egin{aligned} 000 &
ightarrow ar{a}ar{b}ar{c} \ 010 &
ightarrow ar{a}bar{c} \end{aligned} \Longrightarrow & \mathbf{z} = f(a,b,c) = \Sigma(Los \cdot QueDan1) \end{aligned}$$

• Suma canónica → MAXTERM = 0

$$\begin{vmatrix}
000 \rightarrow a + b + c \\
010 \rightarrow a + \bar{b} + c
\end{vmatrix} \Longrightarrow \mathbf{z} = f(a, b, c) = \Pi(Los + QueDan0)$$

Puertas Lógicas

Generales

Puerta	fun./n	USA/EU	a	b	Z
	$z = a \cdot b$	7	0	0	0
AND	~ 4 5		0	1	0
11112	7408	- & -	1	0	0
			1	1	1
	$z = \overline{a \cdot b}$	-	0	0	1
NAND			0	1	1 1
	7400		1	1	0
		7	0	0	0
	z = a + b		0	1	1
OR			1	0	1
	7432	_ ≥ 1 -	1	1	1
		7	0	0	1
NOR	z = a + b		0	1	0
NOK	7402		1	0	0
	7402	_=	1	1	0
NOT	$z = \bar{a}$	->-			
			0		1
	7404	1	1		0

Conversiones

$\begin{array}{c} \text{AND} \\ z = a \cdot b \end{array}$	$z = \overline{\overline{a \cdot b}} = \overline{\bar{a} + \bar{b}}$
OR $z = a + b$	$z = \overline{\overline{a+b}} = \overline{\bar{a} \cdot \bar{b}}$

Exclusive OR

XO	R

XOR	XNOR
$z = \bar{a}b + a\bar{b}$	$z = \overline{\bar{a}b + a\bar{b}}$
$a = (a+b)\cdot(\bar{a}+\bar{b})$	
$z = a \oplus b$	$z = \overline{a \oplus b}$

Puerta	fun./n	USA/EU	a	b	Z
	$z = a \oplus b$	1	0	0	0
XOR	$z - u \oplus b$		0	1	1
AOIC	7486	- = 1 -	1	0	1
	7400	<u> </u>	1	1	0
	$z = \overline{a \oplus b}$	1	0	0	1
XNOR	$z = u \oplus v$		0	1	0
ANOR	74266	- - 1 = 1 ►	1	0	0
	7 1200		1	1	1

Simplificación de Funciones Lógicas

Simplificación Algebraica

$$abc \cdots + \bar{a}bc \cdots = (a + \bar{a})bc \cdots = bc \cdots$$
$$(a + b + c + \cdots) \cdot (\bar{a} + b + c + \cdots) = a\bar{a} + b + c + \cdots = b + c + \cdots$$

Tablas de Karnaugh

Cambia 1 variable lógica por celda

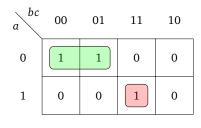
ab	0	1
0	0	1
1	2	3

abc	00	01	11	10
0	0	1	3	2
1	4	5	7	6

Mapas de Karnaugh

$$f(a, b, c) = \Sigma(0, 1, 7)$$

a	b	c	Z
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1



$$f(a,b,c) = \bar{a}\bar{b} + abc$$

- Agrupamos (tamaños):
 - $1 \rightarrow \text{Todas la variables (n)}$
 - $2 \rightarrow (n-1)$ variables
 - $4 \rightarrow (n-2)$ variables
- Mínimos grupos (nadie solo)
- Con 1 \rightarrow $f() = \Sigma(\cdot)$ MINTERM
- Con $0 \rightarrow f() = \Pi(+)$ MAXTERM

Funciones Incompletas

$$f() = \Sigma() + \Sigma_d()$$

d = don't care