

# Tapping Sources of Mathematical (Big) Data

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- ▶ My Domain of Application is Math (no e.g. protocol verification)

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  - ▶ OAF: the Open Archive of Formalizations (<http://oaf.mathhub.info>)
  - ▶ OEIS: "Conjecturing relations between Sequences"  
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- ▶ I am looking for good GOFAI Ph.D. students (maybe even DLFAI)

# 1 Background: Towards a Math Digital Library

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- ▶ **Mathematics** plays a fundamental role in Science, Technology, and Engineering  
*(learn from Math, apply for STEM)*
- ▶ Mathematical knowledge is rich in content, sophisticated in structure, and technical in presentation!



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  - ▶ there are **120.000 journal articles per year** in pure/applied math, **3.5 Million overall**
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- ▶ We need to preserve this heritage and make it accessible to working mathematicians!
- ▶ The EUDML Project digitized large amounts of European Journals
- ▶ The (US) National Research Council issued a Plan/Report for a “World Digital Heritage Library of Mathematics” [DLC<sup>+</sup>14].
  - ▶ Form a non-profit organization IMKT *(Sloan grant for founding)*
  - ▶ digitize, standardize, and **semanticize** math content *(~ added value services)*
  - ▶ Collaborate with Publishers/Organizations *(to obtain rights)*
- ▶ The International Mathematical Union (IMU) chartered a WG to bring this about.

# Background: Mathematical Documents

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*(learn from Math, apply for STEM)*
- ▶ Mathematical knowledge is rich in content, sophisticated in structure, and technical in presentation,
- ▶ its conservation, dissemination, and utilization constitutes a challenge for the community and an attractive line of inquiry.
- ▶ Challenge: How can/should we do mathematics in the 21<sup>st</sup> century?
- ▶ Mathematical knowledge and objects are transported by documents
- ▶ Three levels of electronic documents:
  0. printed (for archival purposes) (~90%)
  1. digitized (usually from print) (~50%)
  2. presentational: encoded text interspersed with presentation markup (~20%)
  3. semantic: encoded text with functional markup for the meaning (~≤0.1%)  
transforming down is simple, transforming up needs humans or AI.
- ▶ Observation: Computer support for access, aggregation, and application is (largely) restricted to the semantic level.
- ▶ This talk: How do we do maths and math documents at the semantic level?

# But there is more Math Knowledge than Documents

- ▶ There are large mathematical data bases
  - ▶ Zentralblatt Math: the first resource in Maths (<http://zbmath.org>)
  - ▶ MathSciNet: Mathematical Reviews (<http://www.ams.org/mathscinet/>)
  - ▶ LMFDB: *L*-functions & Modular Forms (<http://lmfdb.org>)
  - ▶ OEIS: Open Encyclopedia of Integer Sequences (<http://oeis.org>)
  - ▶ FindStat: Combinatorics Statistics Finder (<http://findstat.org>)
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- ▶ **Idea:** Some of this information is already in a semantic/machine-actionable form.
- ▶ **Problems:** licenses, representations, versioning, GUIs, system APIs, ...
- ▶ **Idea:** To arrive at a core DML start at Math DBs and
  - ▶ specify open licenses ↵ data commons
  - ▶ standardize representations ↵ knowledge commons
  - ▶ even in maths, data changes ↵ support versioning
  - ▶ system APIs ↵ collaborate on content, compete on services

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- ▶ **OpenDreamKit:** EU Project 2015-2019 ↵ Math Virtual Research Environment  
Computer Algebra, HPC, MathUI, KWARC (<http://opendreamkit.org>)

# Zentralblatt Math: the first resource in Maths

The screenshot shows the zbMATH website interface. At the top, there is a navigation bar with links to various services like AAR, LICS Newsletter, Haskell Info P..., The Math..., FindStat..., MR: Search, and the zbMATH logo. Below the navigation bar is a secondary menu with links to Services, News, MathWeb, Jacobs, AG, Lists, TRAC, Rotary, Haus, Most Visited, and SPP Semantic Arg... On the right side of the top bar, there are icons for a search bar, a magnifying glass, and other site functions. The main content area features the zbMATH logo with the tagline "the first resource for mathematics". Below the logo is a horizontal menu with tabs for Documents, Authors, Journals, Classification, Software, and Formulae. To the right of this menu is a "Structured Search" button and a search input field containing "Search for documents". To the right of the search field are buttons for "Fields" and "Operators". Below the search area is a "Help" link. A "Welcome to the zbMATH interface." message is displayed, followed by a "Explore our new features:" section. This section contains a bulleted list of three items: 1. Feedback to improve document assignments can be given directly at the author profile pages or via this link (if you already know the author ID). 2. The formula facet in the structured search now allows for a free combination of formula search with other query types. Formula queries can now also be refined by filters. At the bottom of the page, there are logos for FIZ Karlsruhe, The European Mathematical Society, and Heidelberg Academy of Sciences and Humanities.

# MathSciNet: Mathematical Reviews

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# LMFDB: *L*-functions & Modular Forms

The screenshot shows the LMFDB homepage with a red 'BEYOND' watermark over the logo. The main navigation bar includes links for Services, News, MathWeb, Jacobs, AG, Lists, TRAC, Rotary, Haus, Most Visited, SPP Semantic Arg., Feedback, Hide Menu, and Login.

**Introduction and more**

- Introduction
- Features
- Tutorial
- Map
- Contact
- Citation
- Future Plans

Degree: 1 2 3 4

First zeros  $\zeta$  zeros

Operations

**Modular Forms**

GL(2)	Classical	Maass
Hilbert		

GL(3) Maass

Other Siegel

**Varieties**

Curves

- Elliptic:
  - /Q
  - /NumberFields
- Genus 2:
  - /Q

**A Database**

The LMFDB is an extensive database of mathematical objects arising in Number Theory.

Sample lists: L-functions, Elliptic curves, Maass forms, Tables of zeros, Number fields.

**Hall of Fame**

- Riemann zeta function
- Cubic Field of discriminant -23
- Ramanujan  $\Delta$  function and its L-function
- First Rank 4 Elliptic curve and its L-function

**Search and Browse**

Search for objects with specific properties, or browse categories.

Browse: L-functions, Modular forms, Elliptic curves, Number fields.

**Explore and Learn**

The LMFDB makes visible the connections predicted by the Langlands program. Knowls offer background information when you need it.

**Visualize Data**

Explore individual plots or view distributions of various objects.

Examples: GL(4) Level one Maass forms, Isogeny graph of elliptic curve 102.c.

**Code and Open Software**

Download the data, download the code, or see how the data were generated.

Github SageMath Pari/GP

Some rights reserved

# MGP: Math Genealogy Project

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www.genealogy.math.ndsu.nodak.edu/id.p... AAR LICS Newsletter Haskell Info P... The M... Services News MathWeb Jacobs AG Lists TRAC Rotary Haus

## Mathematics Genealogy Project



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A service of the [NDSU Department of Mathematics](#), in association with the [American Mathematical Society](#).

### Michael Kohlhase

[MathSciNet](#)

Ph. D. [Universität des Saarlandes](#) 1994 

Dissertation: *A Mechanization of Sorted Higher-Order Logic Based on the Resolution Principle*

Mathematics Subject Classification: 68—Computer science

Advisor 1: [Jörg H. Siekmann](#)  
Advisor 2: [Peter Bruce Andrews](#)  
Advisor 3: [Frank Pfenning](#)

Students:  
Click [here](#) to see the students listed in chronological order.

Name	School	Year	Descendants
<a href="#">Christoph Benzmueller</a>	Universität des Saarlandes	1999	
<a href="#">Immanuel Normann</a>		2008	
<a href="#">Florian Rabe</a>	Jacobs University Bremen	2008	

According to our current on-line database, Michael Kohlhase has 3 [students](#) and 3 [descendants](#).  
We welcome any additional information.

If you have additional information or corrections regarding this mathematician, please use the [update form](#). To submit students of this mathematician, please use the [new data form](#), noting this mathematician's MGP ID of 136649 for the advisor ID.

Kohlhase: Tapping Sources of Mathematical (Big) Data Search About MGP Links FAQs Posters 9 Submit Data Contact  FRIEDRICH-ALEXANDER UNIVERSITÄT ERLANGEN-NÜRNBERG

# OEIS: Open Encyclopedia of Integer Sequences

The screenshot shows a web browser window displaying the OEIS homepage. The address bar shows 'oeis.org'. The page title is 'THE ON-LINE ENCYCLOPEDIA OF INTEGER SEQUENCES®'. Below it, a banner states 'founded in 1964 by N. J. A. Sloane'. A search bar contains the sequence '1,2,3,6,11,23,47,106,236'. Navigation links include 'Search', 'Hints', 'Welcome', and 'Video'. A footer section lists languages supported by the site.

This site is supported by donations to [The OEIS Foundation](#).

**THE ON-LINE ENCYCLOPEDIA  
OF INTEGER SEQUENCES®**

founded in 1964 by N. J. A. Sloane

Enter a sequence, word, or sequence number:

1,2,3,6,11,23,47,106,236

Search [Hints](#) [Welcome](#) [Video](#)

For more information about the Encyclopedia, see the [Welcome](#) page.

**Languages:** English Shqip العربية Bangla Български Català 中文 (正體字, 簡化字 (1), 簡化字 (2)) Hrvatski Čeština Dansk Nederlands Esperanto Eesti سُلْطَن Suomi Français Deutsch Ελληνικά ગુજરાતી ମୁଖ୍ୟ Magyar Igbo Bahasa Indonesia Italiano 日本語 ଶ୍ରୀଲଙ୍କା ହାନ୍କ୍ୟ Korean Lietuvių ମାତ୍ରା ବୋକମି ନ୍ୟାନ୍ସକ୍ ପାଲ୍କି ପୋର୍ତ୍ଗୁସ୍ Romană Русский Српски Slovenčina Español Svenska Tagalog ຖ່າວັນທິ ຖຸກ ຖຸກ ຖຸກ ຖຸກ Tiếng Việt Cymraeg

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oeis.org

# Take-Home Message: Digital Libraries for Maths

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- ▶ But it needs integration, aggregation, and versioning
- ▶ Licensing is a major stumbling block

## 2 Converting the arXiv

# The arXMLiv Project: arXiv to semantic XML

- ▶ **Idea:** Develop a large corpus of knowledge in *OMDoc/PhysML*
  - ▶ to get around the chicken-and-egg problem of MKM
  - ▶ corpus-linguistic methods for semantics recovery (linguists interested)
- ▶ **Definition 2.1 (The Cornell Preprint arXiv)** (<http://www.arxiv.org>)  
Open access to ca. 850K e-prints in Physics, Mathematics, Computer Science and Quantitative Biology.
- ▶ **Definition 2.2 (The arXMLiv Project)** (<http://arxmliv.kwarc.info>)
  - ▶ use Bruce Miller's  $\text{\LaTeX} \text{XML}$  to transform to XHTML+MathML
  - ▶ extend to  $\text{\LaTeX} \text{XML}$  daemon (RESTful web service) (<http://latexml.mathweb.org>)
  - ▶ we have an automated, distributed build system (ca. Q2CPU-years)
  - ▶ create ca. 12K  $\text{\LaTeX} \text{XML}$  binding files (8 Jacobs students help)
  - ▶ use MathWebSearch to index XML version (realistic search corpus)
- ▶ More semantic information will enable more added-value services, e.g.
  - ▶ filter hits by model assumptions (expanding, stationary, or contracting universe)
  - ▶ use linguistic techniques to add the necessary semantics

# Why reimplement the TeX parser? I

- ▶ **Problem:** The TeX parser can change the tokenizer while at runtime (`\catcode`)
- ▶ **Example 2.3 (Obfuscated TeX)** David Carlisle posted the following, when someone claimed that word counting is simple in TeX/LaTeX

```
\let~\catcode~'76~'A13~'F1~'j00~'P2jdefA71F~'7113jdefPALLF
PA~'FwPA~; FPAZZFLaLPA//71F71iPAHHFLPAzzFenPASSFthP; A$$FevP
A@@FfPARR717273F737271P; ADDFRgniPAWW71FPATTFvePA**FstRsamP
AGGFRRuoPAqq71.72.F717271PAYY7172F727171PA??Fi*LmPA&&71jfi
Fjfi71PAVVVFjbigskipRPWGAUU71727374 75,76Fjpar71727375Djifx
:76 jelse&U76jfiPLAKK7172F71I7271PAXX71FVLnOSeL71SLRyadR@oL
RrhC?yLRurtKFeLPFovPgaTLtReRomL; PABB71 72,73: Fjif.73. jelse
B73:jfiXF71PU71 72,73:PWs;AMM71F71diPAJJFRdriPAQQFRsreLPAI
I71Fo71dPA!!FRgiePBt'eI@ ITLqdrYmu.Q.,Ke;vz vzLqppip.Q.,tz;
;Lql.IrsZ.eap,qn.i.i.eLIMaesLdRcna,;!;h htLqm.MRasZ.ilk,%
s$;z zLqs'.ansZ.Ymi,/sx ;LYegseZRyal,@i;@ TLRlogdLrDsW,@;G
LcYlaDLbJsW,SWXJW ree @rzchLhzsW,;WERcesInW qt.'oL.Rtrul;e
doTsW,Wk;Rri@stW aHAHHFndZPpqar.tridgeLinZpe.LtYer.W,:jbye
```

When formatted by TeX, this leads to the full lyrics of “The twelve days of christmas”. When formattet by LaTeXML, it gives

# Why reimplement the T<sub>E</sub>X parser? II

```
<song>
  <verse>
    <line>On the first day of Christmas my true love gave to me</line>
    <line>a partridge in a pear tree.</line>
  </verse>
  <verse>
    <line>On the second day of Christmas my true love gave to me</line>
    <line>two turtle doves</line>
    <line>and a partridge in a pear tree.</line>
  </verse>
  <verse>
    <line>On the third day of Christmas my true love gave to me</line>
    <line>three french hens</line>
    <line>two turtle doves</line>
    <line>and a partridge in a pear tree.</line>
  </verse>
  <verse>
    <line>On the fourth day of Christmas my true love gave to me</line>
    <line>four calling birds</line>
    <line>three french hens</line>
    <line>two turtle doves</line>
    <line>and a partridge in a pear tree.</line>
  </verse>
  ...
...
```

# Why reimplement the $\text{\TeX}$ parser? III

- ▶ But the real reason is: that we can take advantage of the semantics in the  $\text{\LaTeX}$ .
- ▶  $\text{\LaTeX}\text{XML}$  does not need to expand macros, we can tell it about XML equivalents.
- ▶ **Example 2.4 (Recovering the Semantics of Proofs)**

Add the following magic incantation to `amsthm.sty.ltxml` ( $\text{\LaTeX}\text{XML}$  binding)

```
DefEnvironment('{proof}', "<xhtml:div class='proof'>#body</xhtml:div>");
```

The ar $\text{MLiv}$  approach: Try to cover most packages and classes in the arXiv  
(Jacobs undergrads' intro to research)

# Future Plans for arXMLiv

- ▶ **State:** L<sup>A</sup>T<sub>E</sub>X-to-XHTML+MathML Format Conversion works (65% success)
- ▶ **Over the summer:** Bump up success rate to 75%, daily downloads, web site, instrumentation,...
- ▶ **Soon:** Integrate user-level quality control (integrate JS feedback into html)
- ▶ **starting Fall:** Extend post-processing by linguistic methods for semantic analysis
  - ▶ build semantics blackboard/database for linguistic information (rdf triples)
  - ▶ extend build system for arbitrary XML2BB processes
  - ▶ invite the linguists over (they leave semantics results in BB)
  - ▶ harvest the semantics BB to get OMDoc representations

# Current and Possible Applications

- ▶ the arxmliv build system <http://arxmliv.kwarc.info>
- ▶ the transformation web service <http://tex2xml.kwarc.info>
- ▶  $\text{\LaTeX}$ XML daemon to avoid perl and  $\text{\LaTeX}$  startup times (Deyan Ginev)
  - ▶ keep  $\text{\LaTeX}$ XML alive as a daemon that can process multiple files/fragments (patch memory leaks)
  - ▶ a  $\text{\LaTeX}$ XML client just passes files/fragments along (10/s to 100/s)
- ▶ embedding/editing  $\text{\LaTeX}$  in web pages <http://tex2xml.kwarc.info/test>
- ▶ a MathML version of the arXiv allows vision-impaired readers to understand the texts
- ▶ generalization search (need to know sentence structure for detecting universal variables)
- ▶ semantic search by academic discipline or theory assumption (need discourse structure)
- ▶ development of scientific vocabularies (over the past 18 years; drink from the source)

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- ▶ Lots of potential Applications
  - ▶ Formula Search ([arxivsearch.mathweb.info](http://arxivsearch.mathweb.info), <https://zbmath.org/formulae/>)
  - ▶ screenreaders for quantity expressions (semantics extraction, annotation)
  - ▶ applicable theorem search (need to identify the universal/existential/constant identifiers)
  - ▶ machine translation (need a handle on the math terminology (large, dynamic))

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- ▶ I am sceptical of DLAI autoformalization (surprise me!)

### 3 OAF: Assembling a Global Resource of Formalizations

# OAF: Open Archive of Formalizations: Motivation

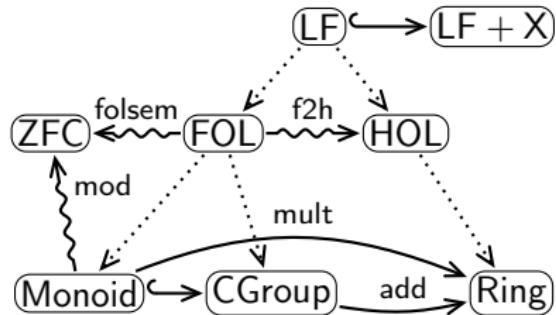
- ▶ **Idea1 (OAF):** Assemble all theorem prover libraries in a common synergy space
- ▶ **Observation:** Formal/symbolic systems and their libraries are non-interoperable
  - ▶ differing, mutually incompatible foundations (e.g., set theory, higher-order logic, constructive type theory, etc.),
  - ▶ library formats, and library structures,
- ▶ **Consequence:** Too much work is spent developing
  - ▶ basic libraries for mathematics in each system.
  - ▶ library organization features (e.g., distribution, browsing, search, change management) for each library format.
- ▶ **Problem:** All these investments bind resources that could be used to improve the core functionality of the systems and the scope of the libraries.
- ▶ **Idea2 (QAF = QED reloaded):** System and tool chain for all of formal maths!

# OAF Architecture

- ▶ **Idea (OAF)**: Assemble all theorem prover libraries in a common synergy space
- ▶ **Problem**: Different systems have different, mutually incompatible logical/mathematical foundations (optimize different aspects)
- ▶ **Observation**: need a system with multiple foundations  $\leadsto$  foundational pluralism
- ▶ **Definition 3.1** A **foundation** (of mathematics) consists of
  - ▶ a **foundational language** (aka. logic, e.g. first-order logic or the CIC)
  - ▶ a **foundational theory** (e.g. axiomatic set theory)
- ▶ **Idea1**: treat logics as mathematical theories themselves (metalogical frameworks)
- ▶ **Idea2**: relate logics in a theory graph via logic transformations (LATIN)

# Representing Logics and Foundations as Theories

- ▶ Logics and foundations represented as MMT theories (in the sample graph)

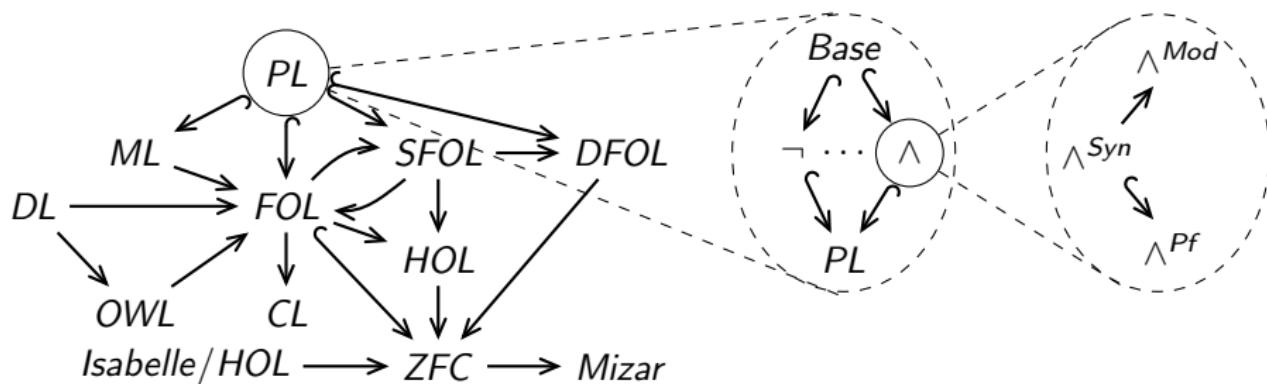


Meta-relation between theories – special case of inclusion (meta<sup>\*</sup>-level)

- ▶ Uniform Meaning Space: morphisms between formalizations in different logics become possible via meta-morphisms.
- ▶ **Remark 3.2** Semantics of logics as views into foundations, e.g., folsem.
- ▶ **Remark 3.3** Models represented as views into foundations
- ▶ **Example 3.4**  $\text{mod} := \{G \mapsto \mathbb{Z}, \circ \mapsto +, e \mapsto 0\}$  interprets Monoid in ZFC.

# The LATIN Logic Atlas

- ▶ **Definition 3.5** The LATIN project (Logic Atlas and Integrator) develops a logic atlas, its home page is at <http://latin.omdoc.org>.
- ▶ **Idea:** Provide a standardized, well-documented set of theories for logical languages, logic morphisms as theory morphisms.



- ▶ **Technically:** Use MMT as a representation language **logics-as-theories**
- ▶ Integrate logic-based software systems via views.
- ▶ **State:** About 1000 modules (theories and morphisms) written in MMT/LF [RS09]

# MMT a Module System for Mathematical Content

- ▶ MMT: Universal representation language for formal mathematical/logical content
- ▶ Implementation: MMT API with generic
  - ▶ module system for math libraries, logics, foundations
  - ▶ parsing + type reconstruction + simplification
  - ▶ IDE
  - ▶ change management
- ▶ Continuous development since 2007 ( $> 30000$  lines of Scala code)
- ▶ Close relatives:
  - ▶ LF, Isabelle, Dedukti: but flexible choice of logical framework
  - ▶ Hets: but declarative logic definitions

# Exports from Proof Assistants

- ▶ General Approach: Export library as MMT projects, store in MathHub
- ▶ Library Export Architecture: (this seems to work sustainably)
  - ▶ System-near export (e.g. to XML or JSON) as part of system code
  - ▶ aggregate, into OMDoc/MMT in MMT API system.
- ▶ Current state of the collection effort:
  - ▶ Mizar: set theoretical (initial export done (with Josef Urban))
  - ▶ HOL Family: HOL Light, HOL4, Isabelle, TPS (initial export done (Rabe/Kaliszyk))
  - ▶ Coq or Matita: type theoretical (Work with Sacerdoti Coen ongoing)
  - ▶ IMPS: heterogeneous method (Partial Export Done)
  - ▶ PVS: rich foundational language (Müller/Owre)
  - ▶ TPTP: mostly first-order ATP problems
  - ▶ Computer Algebra Signatures: GAP, Sage (Konovalov/Pfeiffer/Thierry)
  - ▶ Specware, OEIS, MetaMath, ... (experimental)

## ► Example 3.6 (Search in the MMT API/MathHub)

Enter Java regular expressions to filter based on the URI of a declaration

Namespace

Theory

Name

Enter an expression over theory <http://code.google.com/p/hol-light/source/browse/trunk>

$\$x,y,p: x \text{ MOD } p = y \text{ MOD } p$

Use  $\$x,y,z:$ query to enter unification variables.

Search

type of **MOD\_EQ**

$\vdash \forall m:\text{num} . \forall n:\text{num} . \forall p:\text{num} . \forall q:\text{num} . m = n + q * p \implies m \text{ MOD } p = n \text{ MOD } p$

type of **MOD\_MULT\_ADD**

$\vdash \forall m:\text{num} . \forall n:\text{num} . \forall p:\text{num} . (m * n + p) \text{ MOD } n = p \text{ MOD } n$

# Goal: Towards Library Integration

- ▶ Refactor exports to introduce modularity
- ▶ 2 options
  - ▶ systematically during export (e.g., one theory for every HOL type definition)
  - ▶ heuristic or interactive MMT API-based refactoring
- ▶ Collect correspondences between concepts in different libraries (heuristically or interactively)
- ▶ Relate isomorphic theories across languages
- ▶ Use partial morphisms to translate libraries

## Take-Home Message: OAF

- ▶ There is a wealth of formal mathematics out there  
(diversity?)
- ▶ Unfortunately, it is segregated into 20+ silos  
(need foundational pluralism)
- ▶ System-specific part of the exporter must be part of the exporting system

- ▶ There is a wealth of formal mathematics out there (diversity?)
- ▶ Unfortunately, it is segregated into 20+ silos (need foundational pluralism)
- ▶ System-specific part of the exporter must be part of the exporting system
- ▶ integration of heterogeneous libraries necessary (DLAI?)

## 4 The OEIS as a Mathematical Resource

## 4.1 The OEIS: Online Encyclopedia of Integer Sequences

# OEIS: Open Encyclopedia of Integer Sequences

- ▶ **Definition 4.1** An **integer sequence** is a function  $s: \mathbb{N} \rightarrow \mathbb{Z}$ .
- ▶ **Applications:** Every parametric phenomenon that can be counted
- ▶ **Example 4.2** [A000944](#): Number of polyhedra (or 3-connected simple planar graphs) with  $n$  nodes  $(0, 0, 0, 1, 2, 7, 34, 257, 2606, \dots)$
- ▶ **Example 4.3** [A001222](#): Number of prime divisors of  $n$  counted with multiplicity  $(0, 1, 1, 2, 1, 2, 1, 3, 2, 2, 1, 3, 1, 2, 2, \dots)$
- ▶ **Example 4.4** [A031214](#): First elements in all OEIS sequences (in order)  $(1, 1, 1, 0, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, \dots)$
- ▶ **Intuition:** If phenomena grow with the same sequence  $\leadsto$  related?
- ▶ **Idea:** Collect many integer sequences (Neil Sloane 1965  $\leadsto$  OEIS)
  - ▶ started as a book: *A Handbook of Integer Sequences* 1973 (2372 sequences)
  - ▶ online since 1994 (16.000 sequences  $\leadsto$  <http://oeis.org>)
  - ▶ OEIS Foundation: 2009 (Creative Commons License)
  - ▶ today:  $\sim 275.000$  sequences

# OEIS Data Representation

- ▶ One “record” per sequence with fields including
  - ▶ Identifier: A??????
  - ▶ start values
  - ▶ name (maybe with short explanation)
  - ▶ author
  - ▶ references to papers
  - ▶ program code
  - ▶ **Formulae** (Mathematica, Pari, . . . )

All in ASCII files keyed by one-letter line prefixes.

- ▶ **Example 4.5 (Fibonacci Numbers)** %I A000045 M0692 N0256  
%S A000045 0,1,1,2,3,5,8,13,21,34,55,89,144,233,377,610,987  
%N A000045 Fibonacci numbers:  $F(n) = F(n-1) + F(n-2)$  with  $F(0) = 0$  and  $F(1) = 1$ .  
%D A000045 V. E. Hoggatt, Jr., Fibonacci and Lucas Numbers. Houghton, Boston, MA, 1969.  
%F A000045  $F(n) = ((1+\sqrt{5})^n - (1-\sqrt{5})^n) / (2^n * \sqrt{5})$   
%F A000045 G.f.:  $\sum_{n \geq 0} x^n * \prod_{k=1..n} (k+x)/(1+k*x)$ . — Paul D. Hanna  
%F A000045 This is a divisibility sequence; that is, if  $n$  divides  $m$ , then  $a(n)$  divides  $a(m)$   
%A A000045 N. J. A. Sloane, Apr 30 1991

## 4.2 OEIS Semantification

# Parsing the OEIS format

- ▶ Formulae have no prescribed format (look good to the editors)
- ▶ But they are sufficiently regular (on average) to allow parsing
  - ▶ infix operators, e.g. the + symbol in  $m+n$ .
  - ▶ suffix operators, e.g. the ! symbol in  $n!$ .
  - ▶ prefix operators (with or without brackets), e.g. sin in  $\sin(x)$  or  $\sin x$ .
  - ▶ infix relation symbols, e.g. the < symbol in  $x < 2$ .
  - ▶ binding operators, e.g. the  $\forall$  symbol in  $\forall x. x^2 > 0$ .
- ▶ and some OEIS idioms like G.f. or g.f. for “generating function”.
- ▶ Problem: open-ended set of primitives, e.g. sqrt, ^, sum/ $\Sigma$  and prod/ $\Pi$
- ▶ Ambiguity: ASCII formulae have multiple plausible readings, e.g.
  - ▶ implicit multiplication/application:  $a(x+y)$  or  $\ln x$
  - ▶ elided brackets/precedences:  $\sin x$  or even  $\sin x/y$
- ▶ Delineating Formulae/Text:

Note that  $\text{ppzeta}(s) = \sum_{\{p \text{ prime}\}} 1/(p^s - 1)$  and  
 $\text{ppzeta}(s) = \sum_{\{k=1\}^{\{\infty\}}} \text{primezeta}(k*s)$ .  
— Franklin T. Adams-Watters, Sep 11 2005.

# The Generated OMDoc

```
<omdoc xmlns:dc="http://purl.org/dc/elements/1.1/">
  <theory id="A000045">
    <metadata>
      <dc:creator>N. J. A. Sloane</dc:creator>
      <dc:title>Fibonacci numbers</dc:title>
    </metadata>
    <symbol name="seq"/>
    <assertion>
      <!-- OpenMath for  $\forall n. \text{seq}(n) = \frac{(1+\sqrt{5})^n - (1-\sqrt{5})^n}{2^n\sqrt{5}}$  -->
      <OMBIND>
        <OMS cd="arith" name="forall"/>
        <OMBVAR> <OMV name="n"/> </OMBVAR>
        <OMA>
          <OMS cd="arith" name="equal"/>
          <OMA><OMS name="seq"/><OMV name="n"/></OMA>
          :
        </OMA>
      </OMBIND>
    </assertion>
```

- ▶ Implementation as an extension of the MMT System (2000 LoC)
- ▶ Formula parsing via the Scala PackRat framework (**left recursive linear parsing**)
- ▶ available at <https://svn.kwarc.info/repos/MMT/src/mmt-oeis/>
- ▶ OEIS corpus:
  - ▶ 223.866 formula lines, The formula parser succeeds on 201384 (or 90%).
  - ▶ Out of that, 196515 (or 97.6%) contain mathematical expressions.
  - ▶ **remaining problems:** connectives, formula delineation
- ▶ What does the 90% mean?  $\leadsto$  parser accepts formula
- ▶ Manual Evaluation: 40 randomly selected parsed formulae evaluated  $\leadsto$  85% semantically correct
- ▶ Need to scale evaluation  $\leadsto$  involve OEIS editors (see below)

## 4.3 Applications

# Application: Math (Formula) Search

- We have a Math Search Engine: MathWebSearch
  - Harvest Formulae ↔ convert OpenMath to MathML
  - index them in MWS (together with full text).
  - formula converter daemon

(employ it)

(for user input)

The screenshot shows the OEIS MathSearch interface. At the top, there is a green header bar with the text "OEIS MathSearch". Below it, a message states: "The OEIS MathSearch system is a text-and-math search engine for the On-line Encyclopedia of Integer Sequences based on MathWebSearch and MMT. To report an issue click [here](#). You can see existing issues [here](#)."

The main search interface has a "Search Text" input field containing the expression  $((1+x)^n - (1-x)^n) / y$ . To the right of the input field are buttons for "Examples" and "Search". Below the input field is a navigation bar with buttons for "«", "1", "2", "3", and "»".

Below the search interface, there are three search results displayed in boxes:

- MathHub.info : A001045.omdoc**

Title: A022103.omdoc  
OEIS Link: <https://oeis.org/A022103>  
Show substitutions

... , math4 . math5 . G.f.: math6 . 
$$[a(x)] = \left( \frac{(1+\sqrt{5})^n + (1-\sqrt{5})^n}{(2^{2n} \times \sqrt{5})} \right) + \left( \frac{0 + (1+\sqrt{5})(n-1) - (1-\sqrt{5})(n-1)}{(2^{(n-2)} \times \sqrt{5})} \right)$$
 Offset math8 . math9 . -AI

Hakanson (hakuu(AT)gmail.com)[...]
- MathHub.info : A022103.omdoc**

Title: A022103.omdoc  
OEIS Link: <https://oeis.org/A022103>  
Show substitutions

... , math4 . math5 . G.f.: math6 . 
$$[a(x)] = \left( \frac{(1+\sqrt{5})^n + (1-\sqrt{5})^n}{(2^{2n} \times \sqrt{5})} \right) + \left( \frac{0 + (1+\sqrt{5})(n-1) - (1-\sqrt{5})(n-1)}{(2^{(n-2)} \times \sqrt{5})} \right)$$
 Offset math8 . math9 . -AI

Hakanson (hakuu(AT)gmail.com)[...]
- MathHub.info : A088138.omdoc**

- Demo: <http://oeissearch.mathweb.org/>

# Application: Standardizing Input in OEIS

- ▶ **Problem:** 400 OEIS submissions per week (three out of 60 editors really active)  
quality of submissions often low (including syntax)

# Application: Standardizing Input in OEIS

- ▶ **Problem:** 400 OEIS submissions per week (three out of 60 editors really active)  
quality of submissions often low (including syntax)
- ▶ **Idea:** Parse before submitting (use a normative parser)

The screenshot shows a software interface for managing OEIS submissions. At the top, there is a navigation bar with buttons for 'MEP', 'Home', and 'Report issue'.

**Editor Panel:** This panel contains fields for 'Text format' (set to 'OEIS') and 'Examples' (set to 'Number of trees with n unlabeled nodes.'), along with a note about the generating function for A000055. Below these is a 'PRESENT ▶' button.

**Presenter Panel:** This panel displays the OEIS entry for A000055. It includes the definition  $G.f. : A(x) = \left(1 + T(x) - \frac{T(x)^2}{2}\right) + \frac{\frac{T(x^2)}{2}}{x}$ , where  $T(x) = x + x^2 + 2x^3 + \dots$  is the g.f. for A000081.

**Error log Panel:** This panel lists several errors and warnings:

- Warning Unknown symbol `A`
- Warning Unknown symbol `T`
- Warning Unknown symbol `A000081`
- Info No stex:srcref attribute for <theory name="A000055">
- Info No stex:srcref attribute for <OMOBJ >

**Bottom Navigation Bar:** The bar includes links for 'Kohlhase', 'Tapping Sources of Mathematical (Big) Data', '34', 'AITP 2017', and the FAU logo.

- ▶ Demo: <http://ash.eecs.jacobs-university.de:9090/>

## 4.4 Finding Relations between Sequences

# Relations between Sequences

- ▶ Understanding relations between sequences is a genuine mathematical concern.
- ▶ **State of the Art:** Matching initial segments of sequences.
- ▶ **Example 4.6** [Ste04] found 117 conjectures proves 100.
- ▶ **Problem:** Sampling limited data gives only conjectures. (need proof)
- ▶ **Example 4.7**  $\lfloor \frac{2n}{\log(2)} \rfloor$  and  $\lceil \frac{2}{2^{1/n}-1} \rceil$  agree for 777451915729367 terms but are not equal [Slo12].
- ▶ **Idea:** use the formulae from the OEIS instead.
  - ▶ they are exact and peer-reviewed (relations found will be theorems)
  - ▶ we have about 50k generating functions (powerful, compact, structured representations)

# Generating Functions for Integer Sequences

- ▶ **Definition 4.8** Let  $s := (a_n)_{n \in \mathbb{N}}$  be an integer sequence, then we call  $g_s(x) := \sum_{i=0}^{\infty} a_i x^i$  the **ordinary generating function** of  $s$ .
- ▶ **Example 4.9** The sequence  $A000012 = 1, 1, 1, 1, 1, \dots$  can be represented as  $1 + x + x^2 + \dots = \frac{1}{1-x}$
- ▶ represent an infinite sequence finitely (cf. **Kolmogorov complexity**)
- ▶ There are other generating functions: exponential generating functions, Lambert series, Bell series, and Dirichlet series. (use only ordinary ones for now)
- ▶ **Operations on Generating Functions:** induce to operations on the sequences.
  - ▶ constant factor:  $c \cdot g_s = g_{c \cdot s}$ .
  - ▶ shift:  $x^n \cdot g_s(x) = g_s(x+n)$ .
  - ▶ ... partial fraction decomposition, differentiation, integration, ...
- ▶ **Idea:** systematically search for relations on the generating functions in the OEIS induced by such operations

# Relation Finding Experiment

- ▶ **Experiment:** search for relations on  $\sim 50\,000$  OEIS generating functions
  - ▶ Method 1: const, shift, sort (sanity check; expect known relations)
  - ▶ Method 2: ... partial fraction decomposition, differentiation, integration, ...
  - ▶ Method 3: See Enxhell's B.Sc thesis [Luz16]
- ▶ **Implementation:** import parsed equations into MMT, normalize/transform by Sage, hash, compare.

# Relation Finding Experiment

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- ▶ **Implementation:** import parsed equations into MMT, normalize/transform by Sage, hash, compare.
- ▶ **Example 4.13 (from Method 1)**  $A001478(n) = -A000027(n)$ . (\pm \text{Id on } \mathbb{N})
- ▶ **Example 4.14 (from Method 2)** accepted in <https://oeis.org/A001787>:

$$A001787(n) = \frac{n}{6} A007283(n)$$

- ▶ **Example 4.15 (from Method 2)** accepted in <https://oeis.org/A037532>:

$$A037532(n) = \frac{5}{57} A049347(n-1) + \frac{3}{57} A049347(n) + \frac{29}{171} A000420(n) - \frac{2}{9}$$

# Relation Finding Experiment

- ▶ **Experiment:** search for relations on  $\sim 50\,000$  OEIS generating functions
  - ▶ Method 1: const, shift, sort (sanity check; expect known relations)
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  - ▶ Method 3: See Enxhell's B.Sc thesis [Luz16]
- ▶ **Implementation:** import parsed equations into MMT, normalize/transform by Sage, hash, compare.
- ▶ **Example 4.16 (from Method 1)**  $A001478(n) = -A000027(n)$ . (±Id on  $\mathbb{N}$ )
- ▶ **Example 4.17 (from Method 2)** accepted in <https://oeis.org/A001787>:

$$A001787(n) = \frac{n}{6} A007283(n)$$

- ▶ **Example 4.18 (from Method 2)** accepted in <https://oeis.org/A037532>:

$$A037532(n) = \frac{5}{57} A049347(n-1) + \frac{3}{57} A049347(n) + \frac{29}{171} A000420(n) - \frac{2}{9}$$

- ▶ two out of three randomly picked OEIS submissions were accepted by Neil Sloane (third one not deemed to be interesting enough)
- ▶ OEIS acceptance prompted immediate human submission of trivial corollaries

# Relation Finding Experiment: Overall Results

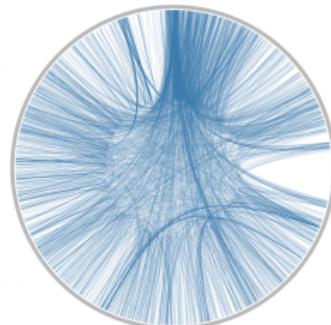
- ▶ **Results:** before recent parser enhancements.

Parsed Generating Functions	43 005
SageMath verified Generating Functions	16 065
Parsed Ordinary Generating Functions	35 953
SageMath verified Ordinary Generating Functions	13 400
Method 1 relations	4 859
Sequences in Method 1 relations	853
Method 2 relations	297 284 646
Method 2 relations without normalization	66 427

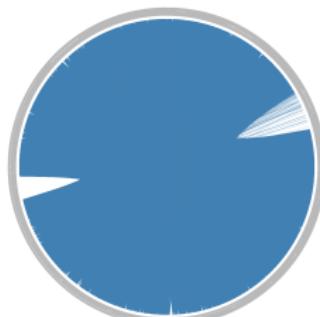
**Caveat:**  $G = A + B + C$  is counted  $4^3$  times (trivial variants)

- ▶ **Results:**

current realations



one B.Sc.



## Take-Home Message: OEIS

- ▶ OEIS is an interesting corpus (mostly Data  $\hat{=}$  facts about special individuals)
- ▶ OEIS grows steadily (6000 submitters, 300sub/week, 150 accepted by human editors))
- ▶ It is definitely not formal (but the GF are after parsing)
- ▶ induced GF database allows deriving new theorems

## Take-Home Message: OEIS

- ▶ OEIS is an interesting corpus (mostly Data  $\hat{=}$  facts about special individuals)
- ▶ OEIS grows steadily (6000 submitters, 300sub/week, 150 accepted by human editors))
- ▶ It is definitely not formal (but the GF are after parsing)
- ▶ induced GF database allows deriving new theorems
- ▶ Need a Theorem Appreciator for automated submission  
(DLAI?)

## Take-Home Message again (If I managed to get here)

- ▶ I only go GOFAI (Good Old-fashioned AI aka. Logic)
- ▶ My Domain of Application is Math (no e.g. protocol verification)
- ▶ no DLAI (applying Deep Learning to everything)
- ▶ BUT we have a lot of interesting Data
  - ▶ OAF: the Open Archive of Formalizations (<http://oaf.mathhub.info>)
  - ▶ arXMLiv preprints and ZBMath Abstracts (licensing problems)
  - ▶ OEIS: "Conjecturing relations between Sequences" ([https://github.com/eluzhnica/\\*](https://github.com/eluzhnica/*))
- ▶ Could use DLAI help (but not in ATP improvements)
- ▶ I am looking for good GOFAI Ph.D. students (maybe even DLAI)



Ingrid Daubechies, Clifford A. Lynch, Kathleen M. Carley, Timothy W. Cole, Judith L. Klavans, Yann LeCun, Michael Lesk, Peter Olver, Jim Pitman, and Zhihong Xia.

*Developing a 21st Century Global Library for Mathematics Research.*  
THE NATIONAL ACADEMIES PRESS, 2014.



Arif Jinha.

Article 50 million: an estimate of the number of scholarly articles in existence.  
*Learned Publishing*, 23(3):258–263, 2010.



Enxhell Luzhnica.

Formula semantification and automated relation finding in the OEIS.  
B. sc. thesis, Jacobs University Bremen, 2016.



Peder Olesen Larsen and Markus von Ins.

The rate of growth in scientific publication and the decline in coverage provided by science citation index.

*Scientometrics*, 84(3):575–603, 2010.



F. Rabe and C. Schürmann.

A Practical Module System for LF.

In J. Cheney and A. Felty, editors, *Proceedings of the Workshop on Logical Frameworks: Meta-Theory and Practice (LFMTP)*, volume LFMTP'09 of



Neil J. A. Sloane.

The on-line encyclopedia of integer sequences.

<http://neilsloane.com/doc/eger.pdf>, 2012.



Ralf Stephan.

State of 100 conjectures from the oeis.

<http://www.ark.in-berlin.de/conj.txt>, 2004.