

Solving Hard Mizar Problems with Instantiation and Strategy Invention *

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1 Introduction: Mizar, ATPs, Hammers

In this work, we prove over 3000 previously ATP-unproved Mizar/MPTP problems by using several ATP and AI methods. First, we start to experiment with the *cvc5* SMT solver which uses several instantiation-based heuristics that differ from the superposition-based systems, that were previously applied to Mizar, and add many new solutions. Then we use automated strategy invention system Grackle to develop *cvc5* strategies that largely improve *cvc5*'s performance on the hard problems. In particular, the best invented strategy solves over 14% more problems than the best previously available *cvc5* strategy. We also show that different clausification methods have a high impact on such instantiation-based methods, again producing many new solutions. In total, the methods raise the number of ATP-solved Mizar problems from 75% to above 80%. This is a new milestone over the Mizar large-theory benchmark and a large strengthening of the hammer methods for Mizar.

The Mizar Mathematical Library (MML) [1] is one of the earliest large libraries of formal mathematics, containing a wide selection of lemmas and theorems from various areas of mathematics. The MML and the Mizar system [26, 2, 15] has been used as a source of automated theorem proving (ATP) [31] problems for over 25 years, starting with the export of several Mizar articles done by the ILF system [10, 9]. Since 2003, the MPTP system [36, 37] has been used to export the MML in the DFG [16] and later TPTP [35] formats. In the earliest (2003) ATP experiments over the whole library, state-of-the-art ATPs could prove about 40% of these problems when their premises were limited to those used in the human-written Mizar proofs (the so called *bushy*¹, i.e., easier, mode).

Since 2013, a fixed version of the MML (1147) and MPTP consisting of 57880 problems has been used as a large benchmark for ATPs and related hammer [6] (large-theory) methods over Mizar [29, 21, 34, 30, 17, 8]. When using many ATP and premise-selection methods, 56.2% of the problems could be proved in [22]. This was recently raised to 75.5% [19], mainly by using the learning-guided E [32] (ENIGMA [20, 13]) and Vampire [25] (Deepire [33]) systems.

Both E and Vampire are mainly saturation-style superposition systems. In the recent years, instantiation-based systems and SMTs such as *cvc5* [3], iProver [24] and Z3 [11] are however becoming competitive even for problems that do not contain explicit theories in the SMT sense [5, 12, 14]. The problems that they solve are often complementary to those solved by the superposition-based systems.

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¹<https://tptp.org/MPTPChallenge>

2 Summary of the Involved Methods

We employ instantiation-based methods in `cvc5` to solve automatically as many hard Mizar problems as possible. Our main result is that the set of ATP-provable MPTP problems has been increased by over 3,000, from 75.5% to 80.7%. All these problems are proved by the `cvc5` system which we improve in several ways. First, we use the Grackle system [18] to automatically invent stronger strategies for MPTP. Our best strategy outperforms the previously best `cvc5` strategy by 14% and our best 7-strategy portfolio solves 8.8% more problems than the corresponding CASC portfolio. We also combine strategy development with alternative clausification methods. This turns out to have a surprisingly high impact on the instantiation-based system, contributing many new solutions. Finally, we obtain further solutions by modifying the problems with premise selection. Ultimately, these methods together double the number of the previously ATP-unproved Mizar problems solved by `cvc5` from 1,534 to 3,021. We show that the methods extend to previously unseen Mizar problems.

Grackle Strategy Invention. Grackle [18] is a system for the automated invention of a portfolio of solver strategies targeted to selected benchmark problems. A user provides a set of benchmark problems and Grackle can automatically discover a set of diverse solver strategies that maximize the number of solved benchmark problems. Grackle supports invention of good-performing strategies for several solvers, including ATP solvers E [32], Vampire [25], Lash [7], and an SMT solver Bitwuzla [27]. Support for additional solvers can be easily added by providing a parametrization of the solver strategy space, and by implementing a simple wrapper to launch the solver. In this paper, we extend Grackle to support an SMT solver `cvc5` [3], and we evaluate its capabilities on a first-order translation of Mizar problems.

Different Clausification Methods. The Mizar problems are given as TPTP [35] problems in first-order logic (FOF). For `cvc5` we translate them to the SMT2 language [4] in the theory of uninterpreted functions (UF). By default, `cvc5` converts to clausal normal form (CNF) internally but since instantiation-based heuristics seem sensitive to problem reformulation, we also experiment with external clausification. This gives us syntactically different variants of the problems and we can test whether `cvc5` benefits from such alternative ways of clausification. We use E as the external classifier and we construct two more problem variants. The first variant is produced by using E’s default clausification parameters. The second variant uses much more aggressive introduction of definitions for frequent subformulas, introducing a new definition if a subformula appears at least four times.

Effects of Premise Selection. Based on the success with problem reformulation, we perform additional experiments, this time with different premise selection methods developed in our prior work [19]. Namely, we evaluate Grackle and baseline strategies on the *bushy* variants of the problems, on the strongest GNN (graph neural network [28]) premise selection slices, and on LightGBM [23] premise selection slices. These variants were found complementary in our previous experiments [19].

Conclusions. In the end, we have solved **3,021** (21.3%) of the remaining 14,163 hard Mizar problems, raising the percentage of automatically proved Mizar problems from 75.5% to **80.7%**. This was mainly done by automatically inventing suitable instantiation-based strategies for the `cvc5` solver, using our Grackle system. Further improvements were obtained by using alternative clausifications of the problems, and also alternative premise selections. Such problem transformations have a surprisingly large effect on the instantiation-based procedures and are likely to be explored further when creating strong portfolios for such systems.

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