Some Extensions of Neural Machine Translation for Auto-formalization of Mathematics

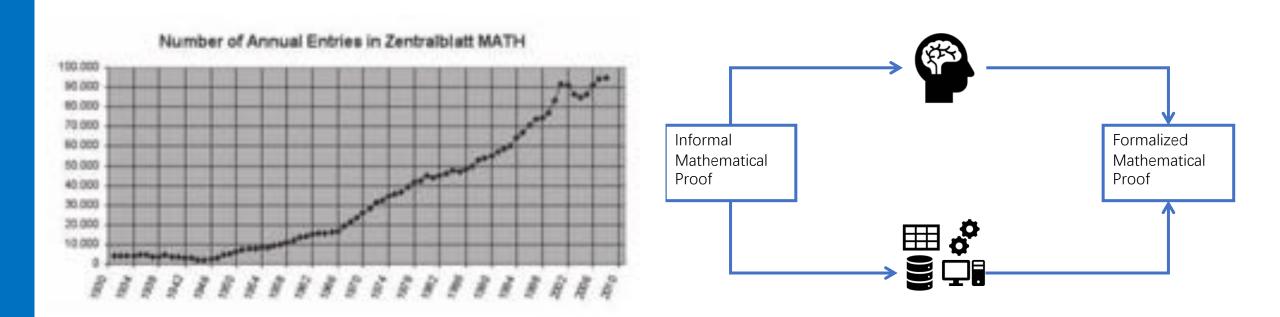
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AITP 2019 – Obergurgl, Austria April 11, 2019

Overview

- Auto-Formalization with Deep Learning
- Universal Approximation
- Supervised NMT (Luong et al.)
- Unsupervised NMT (Lample et al.)
- NMT with Type Elaboration
- Summary

Auto-Formalization with Deep Learning



Universal Approximation

Theorem 2. Let σ be any continuous sigmoidal function. Then finite sums of the form

$$G(x) = \sum_{j=1}^{N} \alpha_j \sigma(y_j^{\mathsf{T}} x + \theta_j)$$

are dense in $C(I_n)$. In other words, given any $f \in C(I_n)$ and $\varepsilon > 0$, there is a sum, G(x), of the above form, for which

$$|G(x) - f(x)| < \varepsilon$$
 for all $x \in I_n$.

G. Cybenko 89 - Approximation by Superpositions of a Sigmoidal Function

- Default: two-layer LSTM with attention.
- Lots of configurable hyper-parameters:

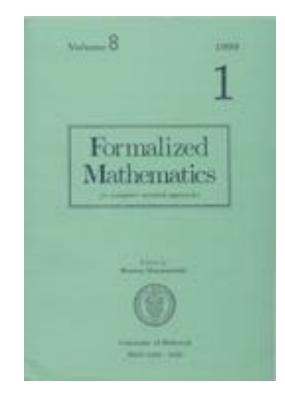
(Attention, Layers, Unit Size, Unit Type, Residual, Encoding, Optimizers, etc)

- Formal abstracts of *Formalized mathematics*, which are generated latex from Mizar (v8.0.01_5.6.1169)
- 1,056,478 pairs of Latex–Mizar sentences in 90:10.

```
theorem P0:
    for L being right_zeroed non empty addLoopStr,
        S,T being Subset of L
    st 0.L in T holds S c= S + T

Formalized
Mathematics

Now we state the propositions:
(1) Let us consider a right zeroed, non empty additive loop structure L, and subsets S, T of L. If 0<sub>L</sub> ∈ T, then S ⊆ S + T.
```



```
Latex

If $ X \mathrel { = } { \rm the ~ } { { \rm carrier } ~ { \rm
of } ~ { \rm } } } { A _ { 9 } } $ and $ X $ is plane , then $ { A
_ { 9 } } $ is an affine plane .

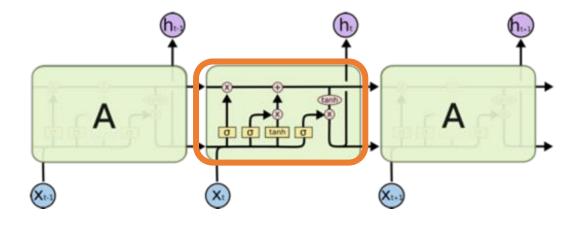
Mizar

X = the carrier of AS & X is being_plane implies AS is AffinPlane;
```

```
Latex
    If $ { s _ { 9 } } $ is convergent and $ { s _ { 8 } } $ is a
    subsequence of $ { s _ { 9 } } $ , then $ { s _ { 8 } } $ is
    convergent .

Mizar
    seq is convergent & seq1 is subsequence of seq implies seq1 is
    convergent;
```

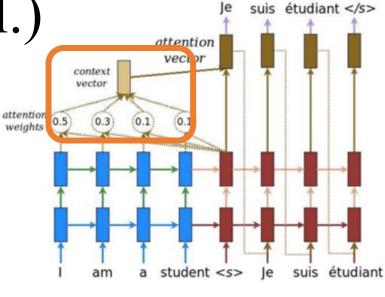
• Memory-cell unit types



Parameter	Final Test Perplexity	Final Test BLEU	Identical Statements (%)	Identical No-overlap (%)
LSTM	3.06	41.1	40121 (38.12%)	6458 (13.43%)
GRU	3.39	34.7	37758 (35.88%)	5566 (11.57%)
Layer-norm LSTM	11.35	0.4	11200 (10.64%)	1 (0%)

Table 5. Evaluation on type of memory cell (attention not enabled)

• Attention

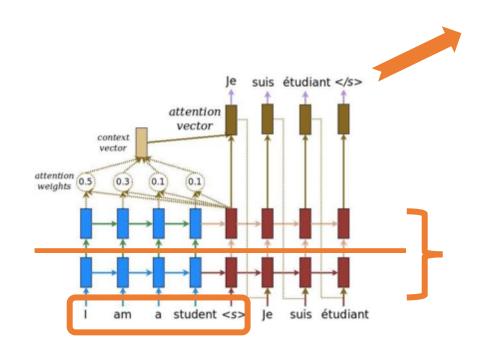


Parameter	Final Test Perplexity	Final Test BLEU	Identical Statements (%)	Identical No-overlap (%)
No Attention	3.06	41.1	40121 (38.12%)	6458 (13.43%)
Bahdanau	3	40.9	44218 (42.01%)	8440 (17.55%)
Normed Bahdanau	1.92	63.5	60192 (57.19%)	18057 (37.54%)
Luong	1.89	64.8	60151 (57.15%)	18013 (37.45%)
Scaled Luong	2.13	65	60703 (57.68%)	18105 (37.64%)

Table 6. Evaluation on type of attention mechanism (LSTM cell)

Supervised NMT

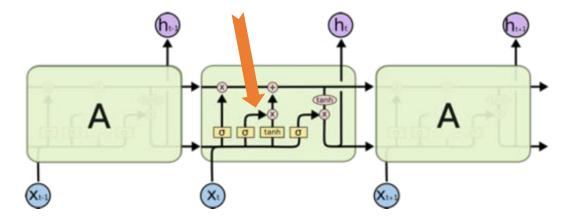
• Residuals, layers, etc.



Parameter	Final Test	Final Test		Identical	
	Perplexity	BLEU	Statements (%)	No-overlap (%)	
2-Layer	3.06	41.1	40121 (38.12%)	6458 (13.43%)	
3-Layer	2.10	64.2	57413 (54.55%	16318 (33.92%)	
4-Layer	2.39	45.2	49548 (47.08%)	11939 (24.82%)	
5-Layer	5.92	12.8	29207 (27.75%)	2698 (5.61%)	
6-Layer	4.96	20.5	29361 (27.9%)	2872 (5.97%)	
2-Layer Residual	1.92	54.2	57843 (54.96%)	16511 (34.32%)	
3-Layer Residual	1.94	62.6	59204 (56.25%)	17396 (36.16%)	
4-Layer Residual	1.85	56.1	59773 (56.79%)	17626 (36.64%)	
5-Layer Residual	2.01	63.1	59259 (56.30%)	17327 (36.02%)	
6-Layer Residual	NaN	0	0 (0%)	0 (0%)	
2-Layer Adam	1.78	56.6	61524 (58.46%)	18635 (38.74%)	
3-Layer Adam	1.91	60.8	59005 (56.06%)	17213 (35.78%)	
4-Layer Adam	1.99	51.8	57479 (54.61%)	16288 (33.86%)	
5-Layer Adam	2.16	54.3	54670 (51.94%)	14769 (30.70%)	
6-Layer Adam	2.82	37.4	46555 (44.23%)	10196 (21.20%)	
2-Layer Adam Res.	1.75	56.1	63242 (60.09%)	19716 (40.97%)	
3-Layer Adam Res.	1.70	55.4	64512 (61.30%)	20534 (42.69%)	
4-Layer Adam Res.	1.68	57.8	64399 (61.19%)	20353 (42.31%)	
5-Layer Adam Res.	1.65	64.3	64722 (61.50%)	20627 (42.88%)	
6-Layer Adam Res.	1.66	59.7	65143 (61.90%)	20854 (43.35%)	
2-Layer Bidirectional	2.39	69.5	63075 (59.93%)	19553 (40.65%)	
4-Layer Bidirectional	6.03	63.4	58603 (55.68%)	17222 (35.80%)	
6-Layer Bidirectional	2	56.3	57896 (55.01%)	16817 (34.96%)	
2-Layer Adam Bi.	1.84	56.9	64918 (61.68%)	20830 (43.30%)	
4-Layer Adam Bi.	1.94	58.4	64054 (60.86%)	20310 (42.22%)	
6-Layer Adam Bi.	2.15	55.4	60616 (57.59%)	18196 (37.83%)	
2-Layer Bi. Res.	2.38	24.1	47531 (45.16%)	11282 (23.45%)	
4-Layer Bi. Res.	NaN	0	0 (0%)	0 (0%)	
6-Layer Bi. Res.	NaN	0	0 (0%)	0 (0%)	
2-Layer Adam Bi. Res.	1.67	62.2	65944 (62.66%)	21342 (44.37%)	
4-Layer Adam Bi. Res.	1.62	66.5	65992 (62.70%)	21366 (44.42%)	
6-Layer Adam Bi. Res.	1.63	58.3	66237 (62.93%)	21404 (44.50%)	

Table 7. Evaluation on various hyperparameters w.r.t. layers

• Unit dimension in cell



Parameter	Final Test Perplexity	Final Test BLEU	Identical Statements (%)	Identical No-overlap (%)	Training Time (hrs.)
128 Units	3.06	41.1	40121 (38.12%)	6458 (13.43%)	1
256 Units	1.59	64.2	63433 (60.27%)	19685 (40.92%)	3
512 Units	1.6	67.9	66361 (63.05%)	21506 (44.71%)	5
1024 Units	1.51	61.6	69179 (65.73%)		11
2048 Units	2.02	60	59637 (56.66%)	16284 (33.85%)	31

Table 8. Evaluation on number of units

	Identical Statements	0	≤ 1	≤ 2	≤ 3
Best Model	69179 (total)	65.73%	74.58%	86.07%	88.73%
- 1024 Units	22978 (no-overlap)	47.77%	59.91%	70.26%	74.33%
Top-5 Greedy Cover	78411 (total)	74.50%	82.07%	87.27%	89.06%
- 1024 Units - 4-Layer Bi. Res. - 512 Units - 6-Layer Adam Bi. Res. - 2048 Units	28708 (no-overlap)	59.68%	70.85%	78.84%	81.76%
Top-10 Greedy Cover - 1024 Units - 4-Layer Bi. Res. - 512 Units - 6-Layer Adam Bi. Res. - 2048 Units - 2-Layer Adam Bi. Res. - 256 Units - 5-Layer Adam Res. - 6-Layer Adam Res. - 2-Layer Bi. Res.	80922 (total)	76.89%	83.91%	88.60%	90.24%
	30426 (no-overlap)	63.25%	73.74%	81.07%	83.68%
Union of All 39 Models	83321 (total)	79.17%	85.57%	89.73%	91.25%
	32083 (no-overlap)	66.70%	76.39%	82.88%	85.30%

Table 9. Coverage w.r.t. a set of models and edit distances

• But generates gibberish when we tried arbitrary LaTeX statements on the trained model... 🕾

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5 the_subsets_of_cord ( the_subsets_of_cord ( POS ) , stges 6 ) is inisident
 9 Let 1 T I be the Arens-Fort space
18 From [ [ Arens-Fort Space is not Locally Connected ] ] , $ T $ is not a locally connected space
                                                                                                                                             the subsets of cord ( the subsets of cord ( 6 ) , signs 6 ) is being homeomythise
12 The result fullows from [ [ Components and Quasi-components of Areno-Port Space are Equal ] ]
                                                                                                                                             7 func Toller :
                                                                                                                                             # ex G be locally_directed OrderSortedSign st not non-constituted-PinSeus non-empty daubleLongStr
12 C and 3
13 Let 5 M . 1 - Vieft C ( A . 1 , d . 1 ) Vright ) 5 and 5 M . 2 - Vieft C ( A . 2 , d . 2 ) Vright
                                                                                                                                                For V be Reflaguence of the subsets of used C the subsets of cord C V 3 3 st not the subsets of
                                                                                                                                                cord ( G ) is closed_under_lines for f be function as f be function at f . f = f )
      ) I be metric spaces :
34 Let 5 F : A _ 1 No A _ 2 5 be a mageing from 5 A _ 3 5 to 5 A _ 2 5 :
                                                                                                                                              9 Let T be GAMFinSpace :
15 Let 5 a Vin A _ 1 5 be a point in 5 A _ 1 5
                                                                                                                                             10 the subsets of cord C the subsets of cord C $1.5 . T 3 is non Cl TopSpace
16 This is proved in [ [ Metric Space Continuity by Spatlan-Delta ] ] :
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17 C god i Turmu 3
18 This is proved in [ [ Metric Space Continuity by Open Reli ] ] :
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19 ( ged )
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39 This is proved in [ [ Metric Space Continuity by Heighborhood ] ] .
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Z1 ( und )
22 Let 5 G 5 and 5 H 5 be topological groups .
                                                                                                                                             15 rigms.Heas is terminating ;
23 Let 5 F : 6 Vos H 5 be a norphise .
                                                                                                                                            17 func .1. ( 528 ) → Element of L ;
24 Let its image ] ] 5 \approximate { in } \inft ( ( f ) \right ) 5 be [ [ Definition : Housdorff S
                                                                                                                                            18 rigno. Hees is terwinoting :
                                                                                                                                            19 func Yoler :
25 Then its kernel ] ] $ Ver \left ( ( F ) \right) is is [ [ Befinition : Closed Set ( Topology )
                                                                                                                                            20 signs. Mean is terminoting :
      closed in $ 6.5
                                                                                                                                            Zt func Yoller :
25 by ( [ Image of Group Hammorghian is Subgroup ] ] . 1 Vaperutarrane ( in ) Vieft ( ( f ) Vright
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    ) i is a group .
                                                                                                                                            23 Let f , G be Morphise ;
27 Let 5 a 5 be the identity of 5 R 5 .
                                                                                                                                            24 c[18]: for f be function at (f - g) is the subsets of and for f be further of the subsets a
28 by [ [ Topological Group is Houndorff iff Identity is Closed ] ] . 1 \left \ { { a } \right \ }
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                                                                                                                                            25 for f being function at (f - f) is the subsets of cord holds f (+ 6):
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     ) i is [ [ definition : Closed Set ( ispology ) I closed in i i i .
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30 ( ged )
                                                                                                                                            28 the advets of cord ( the advets of cord ( f , %) ) > opi & a - b is closed under lines & a
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32 Let 5.5 \subseteq A.5 be a subset of 5.A.5.
                                                                                                                                            29 f continuous f in C f 1 th a 3 . a & C f 1 th f 3 . C a - f 3 = 15 the_adverts_of_cond ( G ) . (
 $3 then $ 5.5 is an open set of 5 is 5.
                                                                                                                                                - 43311
36 From the definition of standard discrete metric 1 \(\text{first}\) i. \(\g\) \(\text{Vie } \(\text{i} \) \(\text{d} \) \(\text{Vieft} \((\text{i} \) \, \(\text{y} \) \(\text{Vieft} \) \((\text{i} \) \) \((\text{i} \) \) \(\text{Vieft} \) \((\text{i} \) \(\text{i} \) \(\text{i} \) \(\text{i} \) \((\text{i} \) \) \(\text{i} \) \((\text{i} \) \) \((\text{i} \) \) \((\text{i} \) \(\text{i} \) \((\text{i} \) \((\text{i} \) \) \((\text{i} \) \(\text{i} \) \((\text{i} \) \((\text{i} \) \) \((\text{i} \) \) \((\
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MIRITORY
                                                                                                                                            30 S is open set , S is open set holds S is open set holds S is open of M ;
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37 Land ( coses ) 1
38 Let 5 Negation No. No., E of 3 5 be such that 5 0 = Negation No. 2 5 .
                                                                                                                                            35 W etc. 91
 59 Let 5 x Vin 5 5 .
                                                                                                                                            M x -- y
40 Let 1 8 _ Vepsilon Viefs ( ( x ) Veight ) 5 be the open 5 Vepsilon 3 -ball of 5 x 5 .
                                                                                                                                            45 Then by definition of 5 hamilton 5 and 5 if 5 5 ii. Negation Nieft ( ( x ) hright ) = Nieft \ ( )
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   # 3 Netable N 3 S
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42 Thus & \forst\ a \in 5 : 8 _ \egstlen \teft ( ( x ) \right ) \subseteq 5 $
                                                                                                                                            48 Let 8 'Sn' ( x , d ) be open array of x :
43 Mence the result by definition of upon set
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AM ( and )
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45 let 5 T = Nieft ( ( 5 , Minu ) Tright ) 3 be an injective topological space .
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48 Let $ 8 - Vieft ( ( I , Mon ' ) Vright ) $ be a retract of $ 7 $ .
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47 Then 5 E 5 to intective
                                                                                                                                            44 func Toller :
48 By definition of retract there exists a continuous 3 3 [ [ Befinition : Betraction ( Topology )
                                                                                                                                            45 Let T be injective has for 5 be injective TopSpace;
                                                                                                                                            468-6.62.003:
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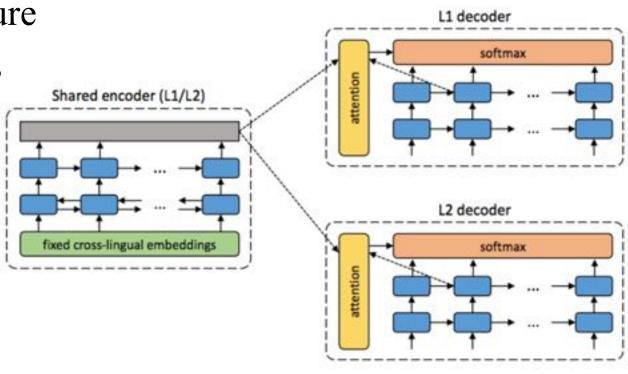
• Demo

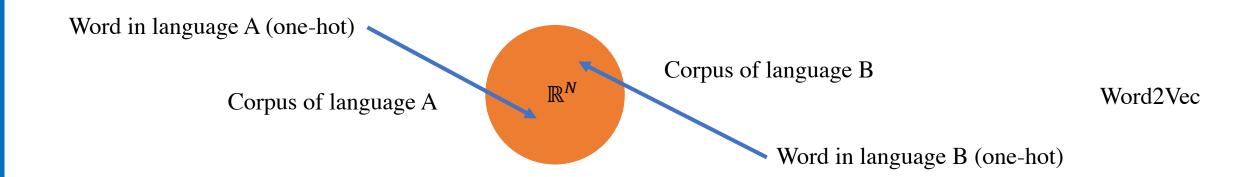
• Two monolingual corpora instead of one parallel corpora (ProofWiki - Mizar)

• Shared-encoder NMT architecture

• Fixed cross-lingual embeddings

- Word2Vec
- BPE (Byte Pair Encoding)
- Denoising and backtranslation

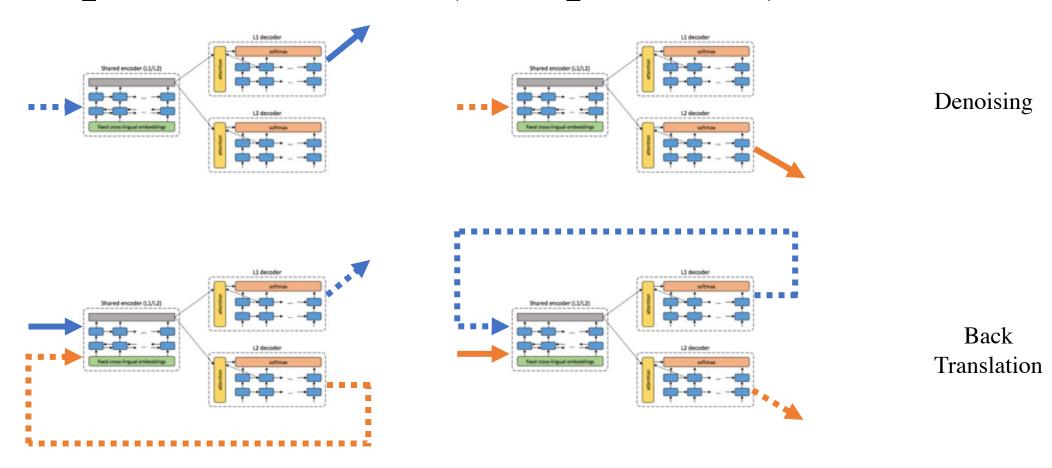




3 BPE iterations on a corpus with the word "Lower"

BPE

$$\{\text{``L''}, \text{``o''}, \text{``w''}, \text{``e''}, \text{``r''}\} \longrightarrow \{\text{``L''}, \text{``o''}, \text{``w''}, \text{``er''}\} \longrightarrow \{\text{``Low''}, \text{``er''}\}$$



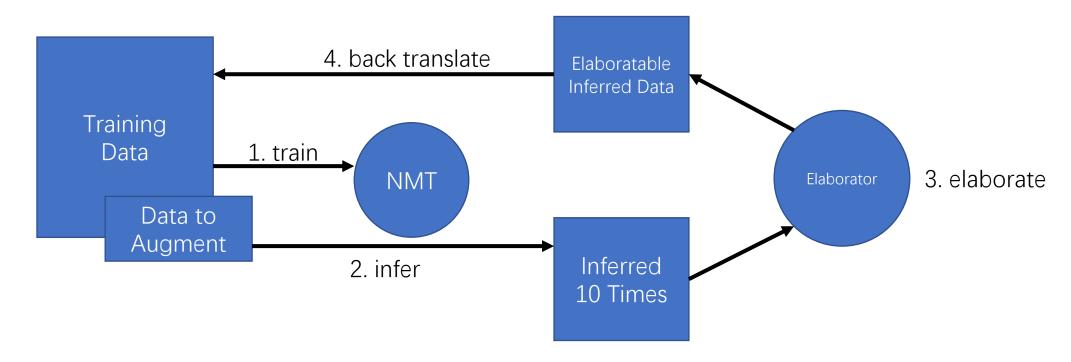
• Generating gibberish on our data... 🕾

```
I sweepler dir vite. It is a representation in ... It's
1 Bet $ \left ( (5 , \precake ) \right ) $ be a preordered set .
2 Let 5 H 5 be a non-empty subset of 5 5 5 .
3 Then 5 H 5 is directed ( 1FF ) 1 H ^ \precise 5 is directed , where 1 H ^ \precise 1 detectes the lower closure of set
& Let us assume that $ H $ is directed
$ Let $ x , y \in K A \grecole $
$ By definition of lower closure $ hextats is 1.5 in H : is horecasts is 1.5 and $ hextats y 1.5 in H : is horecasts y 1.5.
7 By definition of directed subset $ Nextsts a \lin K i x " \precisio x \lind y " \precise a $
$ By definition of reflexivity $ x \preckie x $
9 By definition of lower choose $ a \in H A \precein $
This by definition of transitivity $ \unklass a \lin 6 ^ \underlinesis : a \underlinesis a \lin 4 \underlinesis
31 Thus by definition $ H A 'grecole $ is directed'.
SE ( god i lemme )
13 Let us assume that $ N A torocole $ is directed
34 Let Sx , y \in H S
35 By definition of reflexivity $ x \grecole x \land y \grecole y $.
36 By definition of lower closure $ x , y \ln H > \precise $
IF By definition of directed subset I 'variate a 'lin H * 'grecole : x 'grecole a Viand y 'precole a I
IN By definition of lower closure $ \estats a ? \ln H | a \precate a ? $
39 This by definition of transitivity $ harists a " \lin ti : a \gracele a " \linds y \gracele a " $
20 This by definition $ H $ is directed
Z1 ( ged )
-/Unsupervise#0/90//http://pone/infer.en
2 Non take
3 Thus
4 1 heoder - 3
$ Let $ \ank : 5 _ 1 \to 5 _ 2 $ be an isometry .
6 Then 5 T 5 is a paracompact space
Flet $ \Left \Longle ( r _ n ) \right \rongle $ be a sequence ] ] of [ [ Definition : Complex Number | complex numbers
# By assumption 5 or 515 h 5
9 Let $ X 5 be a subset of $ 5 5
M Let $ 18 A n 5 be the 5 n 5 -dimensional fluctideon space.
III and so $ if \authorize X $ .
III Let S Y = W / C Lette 3 S .
35 Let Sx , y \in V S
14 The proof proceeds in two stages.
15 The result follows
36 Let S x , y \in V S
IF the have that $ \lieft ( { 0 , 0 } \right ) \in U \cop \left \ ( { \left ( { 0 , 0 } \right ) } \right \ ) \right \ ) $
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20 Ne have that $ \left ( { 0 , 0 } \right ) \right ) \in U \cup \left \ { { \left ( { 0 , 0 } \right ) \ \right \ } \right \ } \ \right \ } }
21 Then for all $ 0 Vin Venthcol C $ . $ 0 Your H $ is open in $ T _ H $ .
spit, fin-en, teut, txt.
-/braupervised#1/##I/mspf/gare/infer.en* 211, 1272C
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• Demo

NMT with Type Elaboration

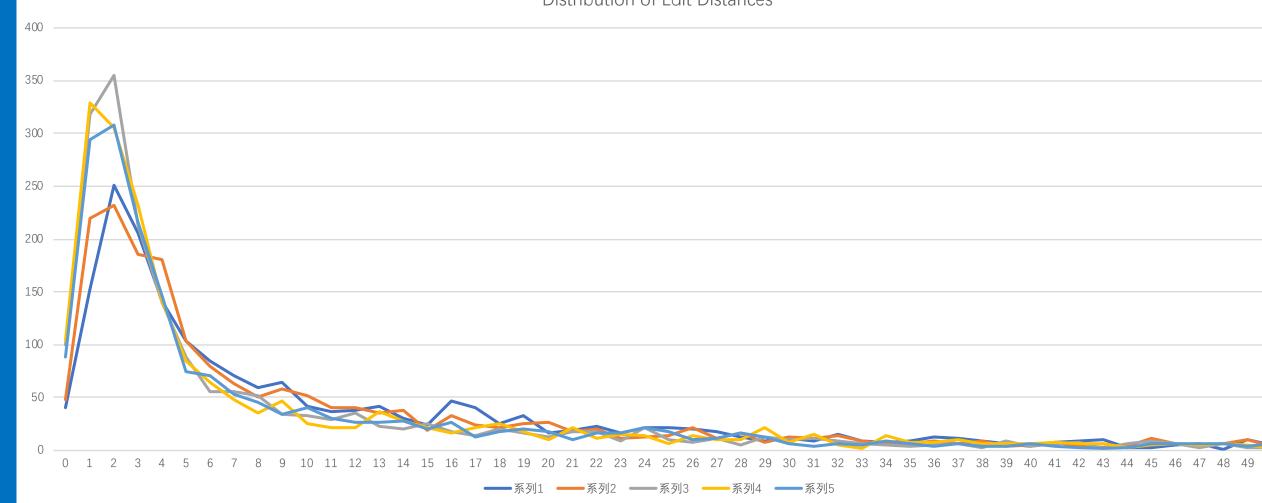
- Still Luong's NMT, but with Mizar -> TPTP (prefix format) as data.
- Augment our data through type elaboration and iterative training.



• Performance stabilizes after a few iterations... ©

NMT with Type Elaboration

Distribution of Edit Distances



Summary

- For auto-formalization, we hit a wall with NMT techniques with limited data.
- Focus on obtaining high-quality data.
- This is still a direction worth going as manual translation is too costly.

Thanks

All historical orientation is only living when we learn to see what is ultimately essential is due to our own interpreting in the free rethinking by which we gain detachment from all erudition.

Martin Heidegger – The Metaphysical Foundations of Logic