## Relieving User Effort for the Auto Tactic in Coq with Machine Learning

Lasse Blaauwbroek

Czech Technical University in Prague Radboud University in Nijmegen

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An exposition of different proof styles in Coq

An exposition of different proof styles in Coq

should be directed at me and me alone

any ridicule regarding proof styles

auto. reflexivity. constructor. econstructor. intros. cbn. constructor.
\* eapply typing\_abs. intros. cbn. apply typing\_merge2. eapply typing\_apply
-- apply typing\_inter\_intro.
++ eapply typing\_abs. intros. cbn. apply typing\_merge1.
++ eapply typing\_abs. intros. cbn. apply typing\_merge2.
 \*\* apply typing\_merge1. eapply typing\_var. apply M.add\_2.
 \*\* apply typing\_merge2. eapply typing\_var. apply M.add\_1.
 \*\* constructor.

eapply typing\_inter\_elim2. eapply typing\_var. apply M.add\_2.

eapply typing\_inter\_elim1. eapply typing\_var. apply M.add\_1. auto.

- eapply typing\_abs. intros. cbn. eapply typing\_app. apply typing\_union\_el

apply typing\_union\_intro1. eapply typing\_var. apply M.add\_1.

eapply typing\_abs. intros. cbn. apply typing\_merge1.

+ apply typing\_inter\_intro.

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- eapply typing\_abs. intros. cbn. eapply typing\_app. apply typing\_union\_el
  + apply typing\_inter\_intro.
   \* eapply typing\_abs. intros. cbn. apply typing\_merge1.
  - apply typing\_union\_intro1. eapply typing\_var. apply M.add\_1.
     auto. reflexivity. constructor. econstructor. intros. cbn. constructor.
    - \* eapply typing\_abs. intros. cbn. apply typing\_merge2. eapply typing\_a
    - -- apply typing\_inter\_intro.

      ++ eapply typing\_abs. intros. cbn. apply typing\_merge1.
      - ++ eapply typing\_abs. intros. cbn. apply typing\_merge2.
        - \*\* apply typing\_merge1. eapply typing\_var. apply M.add\_2.
          \*\* apply typing\_merge2. eapply typing\_var. apply M.add\_1.
    - \*\* constructor.
      -- eapply typing\_inter\_elim2. eapply typing\_var. apply M.add\_2.
  - -- constructor.
  - + eapply typing\_inter\_elim1. eapply typing\_var. apply M.add\_1. auto.

typing\_union\_intro1. eapply typing\_var. apply M.add\_1. auto. reflexivity. constructor. econstructor. intros. cbn. constructor. eapply typing\_abs. intros. cbn. apply typing\_merge2. eapply typing\_a apply typing\_inter\_intro. eapply typing\_abs. intros. cbn. apply typing\_merge1. eapply typing\_abs. intros. cbn. apply typing\_merge2. apply typing\_merge1. eapply typing\_var. apply M.add\_2. apply typing\_merge2. eapply typing\_var. apply M.add\_1. constructor. eapply typing\_inter\_elim2. eapply typing\_var. apply M.add\_2. constructor.

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easy to read

easy to step through

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tedious to maintain

tedious to write

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auto. reflexivity. constructor. econstructor. intros. cbn. constructor.

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apply typing\_union\_intro1. eapply typing\_var. apply M.add\_1.

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+ apply typing\_inter\_intro.

constructor.

firstorder; destruct (compare\_spec t0 n); auto; destruct H6; rewrite H6 in

[rewrite H0, H4 | rewrite H | rewrite H1]; try destruct leb; auto.

rewrite H8 in H0; firstorder.

[case\_eq (leb n t0) | case\_eq (leb t0 n)]; intro; try contradiction;

```
split; induction T; cbn in *; auto; unfold compare, ge, flip in *.
firstorder; (rewrite tree_forall_occurs in H, H1;
            [| apply le_q_compatible | apply ge_eq_compatible]);
```

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less maintenance effort

difficult to read

difficult to write

short to write

rewrite H8 in H0; firstorder.

```
have [-> | nzU] := eqVneq U 0%VS.
  by right=> [[e []]]; rewrite memv0 => ->.
pose X := vbasis U; pose feq f1 f2 := [tuple of map f1 X ++ map f2 X].
have feqL f i: tnth (feq _ f _) (lshift _ i) = f X'_i.
  set v := f : rewrite (tnth_nth_v) /= nth_cat_size_map_size_tuple.
 by rewrite ltn_ord (nth_map 0) ?size_tuple.
apply: (iffP (vsolve_eqP _ _ _)) => [[e Ue id_e] | [e [Ue _ id_e]]].
  suffices idUe: in U, forall u, e * u = u / u * e = u.
    exists e: split=> //; apply: contraNneg nzU => e0; rewrite -subv0.
    by apply/subvP=> u /idUe[<- ]; rewrite e0 mul0r mem0v.
  move=> u /coord_vbasis->; rewrite mulr_sumr mulr_suml.
 have: (\dim (A * M) - \dim (sum A X) < k.+1)%N by [].
  have: [/ (sumA X \le A * M)%VS, directv (sumA X) & O \setminus X].
  split; apply/eq_bigr=> i _; rewrite -(scalerAr, scalerAl); congr (_ *: _).
    by have:= id_e (lshift _ i); rewrite lfunE.
 by have:= id_e (rshift _ i); rewrite lfunE.
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  move=> u /coord vbasis->: rewrite mulr sumr mulr suml.
  have: (\dim (A * M) - \dim (sum A X) < k.+1)%N by [].
  have: [/\ (sumA X \le A * M)\%VS, directv (sumA X) & 0 \notin X].
  split; apply/eq_bigr=> i _; rewrite -(scalerAr, scalerAl); congr (_ *: _).
    by have: = id e (lshift i); rewrite lfunE.
  by have:= id_e (rshift _ i); rewrite lfunE.
                  compact and independently readable in principle
```

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 suffices idUe: in U, forall u, e * u = u / u * e = u.
    exists e; split=> //; apply: contraNneq nzU => e0; rewrite -subv0.
    by apply/subvP=> u /idUe[<- _]; rewrite e0 mul0r mem0v.
  move=> u /coord_vbasis->; rewrite mulr_sumr mulr_suml.
  have: (\dim (A * M) - \dim (sum A X) < k.+1)%N by [].
 have: [\ \ (sumA\ X \le A * M)\%VS, directv\ (sumA\ X) \& 0 \ \ notin\ X].
 split; apply/eq_bigr=> i _; rewrite -(scalerAr, scalerAl); congr (_ *: _).
    by have: = id e (lshift i); rewrite lfunE.
  by have:= id_e (rshift _ i); rewrite lfunE.
              difficult to read in practice, unsuitable for large proof states
```

tauto.

ring.

omega.

tauto. omega. ring.

generally pretty great

tauto. omega. ring.

generally pretty great

only operate in a specific domain

firstorder.

hammer.

search.

firstorder. hammer. search.

you don't have to do anything

firstorder. hammer. search.

you don't have to do anything

you will be doing nothing for a long time

# One or two crucial proof steps,

induction x; auto.

followed by human-guided automation.

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▷ Abandons the panacea of a "readable" proof▷ Instead, encodes all knowledge into lemmas

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- ▷ Abandons the panacea of a "readable" proof
- ▷ Instead, encodes all knowledge into lemmas
- ▶ Plus heuristics on how and when to use lemmas (which may also be unreadable)

(and how does it differ from any other automation styles)

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Simple, general purpose BFS/DFS proof search based on user-hints.

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Simple, general purpose BFS/DFS proof search based on user-hints.

Lemma le\_S :  $\forall xy$ , x < y  $\rightarrow$  x + 1 < x + 1.

Proof. ... Qed.

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Proof State:  $\frac{w}{z} + 1 < k * a + 1$ 

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Lemma plus\_comm :  $\forall xy$ , x + y = y + x. Proof. ... Qed.

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Hint Rewrite plus\_comm.

Proof State:  $\frac{w}{p} + z = k$ 

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Lemma plus\_comm :  $\forall xy, |x + y| = y + x$ .

Proof. ... Qed.

Hint Rewrite plus comm.

Proof State:

$$|z| =$$

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Simple, general purpose BFS/DFS proof search based on user-hints.

Lemma plus\_comm : 
$$\forall xy$$
, x + y =  $y$  + x.

Proof. ... Qed.

Hint Rewrite plus comm.

Proof State:

$$\left|\frac{p}{p}\right| = 1$$

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Simple, general purpose BFS/DFS proof search based on user-hints.

Hint Extern => tactic\_expr

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Simple, general purpose BFS/DFS proof search based on user-hints.

```
Gating
Hint Extern =>
match goal with
| gate_expr => apply lemma
| _ => fail
end
```

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Simple, general purpose BFS/DFS proof search based on user-hints.

Gating

Lemma le\_trans :  $\forall xyz, x < y \rightarrow y < z \rightarrow x < z$ 

Proof. ... Qed.

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Simple, general purpose BFS/DFS proof search based on user-hints.

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Lemma le\_trans :  $\forall xyz, x < y \rightarrow y < z \rightarrow x < z$ 

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Proof. ... Qed.

Hint Apply le\_trans.

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Simple, general purpose BFS/DFS proof search based on user-hints.

```
Gating
Lemma le trans : \forall xvz. x < v \rightarrow v < z \rightarrow x < z
Proof. ... Qed.
Hint Extern =>
match goal with
| H1 : ?x < ?y, H2 : ?y < ?z | - ?x < ?z =>
         apply le_trans H1 H1
| => fail
```

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# Rule of thumb

Keep auto branching factor below 1.5 (Otherwise auto would just be a bad version of other automation tactics)

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# Practical experience

Branching factor < 10 easily achievable with gating

Branching factor < 2 increasingly hard for complex developments

Small-data online machine learning

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Record the successful hints executed by previous auto runs  $\Gamma_1 \vdash T_1 \Rightarrow \text{Hint Apply lemma} \Rightarrow \Gamma_2 \vdash T_2$ 

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Modify the auto tactic

▷ On each branch point, execute all possible hints (expected +/- 10)

### Small-data online machine learning

Record the successful hints executed by previous auto runs  $\Gamma_1 \vdash T_1 \Rightarrow \text{Hint Apply lemma} \Rightarrow \Gamma_2 \vdash T_2$ 

Modify the auto tactic

- $\triangleright$  On each branch point, execute all possible hints (expected +/- 10)
- Order them based on machine learning
- ▷ Pray to the ML-gods that branching factor is now « 2

For each recorded triple

 $\Gamma_1 \vdash T_1 \Rightarrow \text{Hint} \ldots \Rightarrow \Gamma_2 \vdash T_2$ 

For each recorded triple  $\Gamma_1 \vdash T_1 \Rightarrow \texttt{Hint} \ldots \Rightarrow \Gamma_2 \vdash T_2$  calculate  $\Gamma_{\Delta} \vdash T_{\Delta} = \Gamma_1 \vdash T_1 - \Gamma_2 \vdash T_2$ 

For each recorded triple  $\Gamma_1 \vdash T_1 \Rightarrow \mathtt{Hint} \ldots \Rightarrow \Gamma_2 \vdash T_2$  calculate  $\Gamma_1 \vdash T_2 = \Gamma_2 \vdash T_3 = \Gamma_4 \vdash T_4 = \Gamma_5 \vdash T_5 = \Gamma_5 = \Gamma_5 \vdash T_5 = \Gamma_5 = \Gamma_5 \vdash T_5 = \Gamma_5 =$ 

 $\Gamma_{\Delta} \vdash T_{\Delta} = \Gamma_1 \vdash T_1 - \Gamma_2 \vdash T_2$ and execute a k-nearest neighbor on its features

For each recorded triple  $\begin{array}{ccc} \Gamma_1 \vdash T_1 \Rightarrow \mathtt{Hint} & \ldots & \Rightarrow \Gamma_2 \vdash T_2 \\ & \mathsf{calculate} \\ & \Gamma_\Delta \vdash T_\Delta = \Gamma_1 \vdash T_1 - \Gamma_2 \vdash T_2 \\ \\ \mathsf{and} & \mathsf{execute} & \mathsf{a} & \mathsf{k}\text{-nearest neighbor on its features} \end{array}$ 

Currently, subtraction is a pointwise textual diff for hypotheses and goal

Integrated into experimental Tactician version

#### Integrated into experimental Tactician version



# The Tactician

- ▷ ML on tactic scripts
- Seamless integration into user workflow
- Suitable for proving-in-the-large
- ▷ Alpha version available
- ∀ersion 1.0 expected soon

https://coq-tactician.github.io

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