Prover9 Unleashed: Automated Configuration for Enhanced Proof Discovery

Kristina Aleksandrova[2] Jan Jakubův[1,2] Cezary Kaliszyk[2]

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[1]Czech Technical University in Prague, Prague, Czech Republic
[2]University of Innsbruck, Innsbruck, Austria

Outline

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Prover9: Legendary Theorem Prover

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- . . . based on resolution and paramodulation.
- Not developed since 2009 but . . .
- . . . still actively and successfully used by its users.
- . . . still used in the CASC competition (as a reference).

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- Question: Can Prover9 still compete with state-of-the-art?

Prover9: Proof Search Strategy Configuration

- Supports more than 100 basic options
  . . . (booleans, numerical, categorical).

```
set(binary_resolution).
set(expand_relational_defs).
clear(back_subsume).
assign(order, kbo).
assign(not_weight, -2).
assign(or_weight, 200).
assign(prop_atom_weight, 9999).
```

Prover9: Given Clause Selection Configuration

- Prover9 processes one given clause a time.
- Allows to define which clauses should be preferred,
  . . . using priority queues where clauses are sorted (age/weight)

```
list(given_selection).
   part(queue1, high, age   ,  weight < 500 & -horn  ) = 50.
   part(queue2, high, weight,  depth < 15            ) = 50.
   part(queue3, low , weight,  -false                ) = 5.
   part(rest  , low , random,  all                   ) = 1.
end_of_list.
```

Prover9: Action Rules

- action rules: Change options values on the fly,
  . . . when certain event occur.

```
list(actions).
  generated=5000  -> assign(pick_given_ratio, 4).
end_of_list.
```

Prover9: Keep/Delete Rules

- keep/delete rules: Delete generated clauses (incomplete),
  . . . satisfying a delete condition unless kept is satisfied.

```
list(delete).
    weight > 20 & variables > 4.
    weight > 30.
end_of_list.

list(keep).
    horn & variables<3.
end_of_list.
```

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- During evolution, different grackle species developed different bill sizes to be able to feed on different types of nutrients.
- This decreases competition between different species and increases the chances of their survival.

Grackle: Algorithm Overview

- Successor/generalization of BliStr/Tune tools.
- Grackle utilizes existing algorithm configuration frameworks
  - ParamILS: Iterative Local Search (Hutter et al.)
  - SMAC3: Bayesian Optimization (Lindauer et al.)

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- Grackle output:
    - portfolio of strategies complementary on input problems

Grackle: Invent Portfolio of Strategies

Repeat the following:

1. Evaluate all strategies on all problems P
2. Select one strategy S to be improved
3. Specialize strategy S for the problems where it performs best
4. Go to 1

Terminate when:

- all strategies has been improved, or . . .
- time limit is reached.

Grackle: Encoding advanced Prover9 features

To describe a given clause selection queue:

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part(queue1, high, age , weight < 500 & -horn ) = 50.
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we introduce a set of parameters

while fixing the maximal count of parts in the condition (j)

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6. H^{ratio}: the queue selection ratio

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Similarly, we encode other Prover9 advanced features:

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This gives rise to 2 Grackle domains:

- default domain: only basic options (value-based)
- full domain: basic + advanced features

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- The 10 best strategies are evaluated on testing problems,
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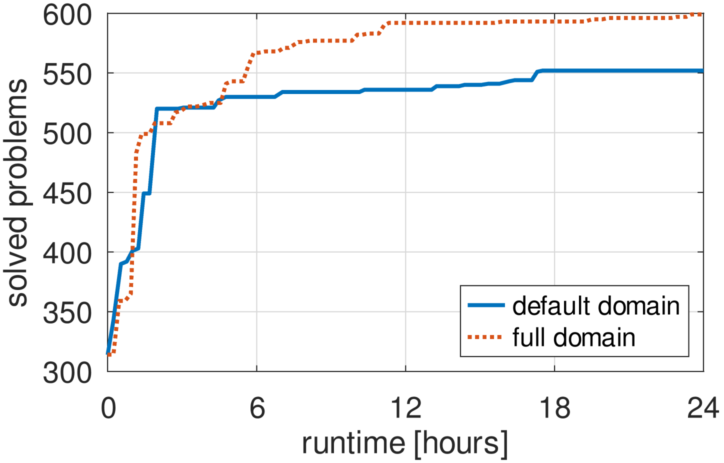
- This portfolio is compared with E and Vampire's portfolios
  ...with a compatible time limit of 300 seconds.

Grackle run on AIM train problems



Results on AIM test problems

prover	*solves*
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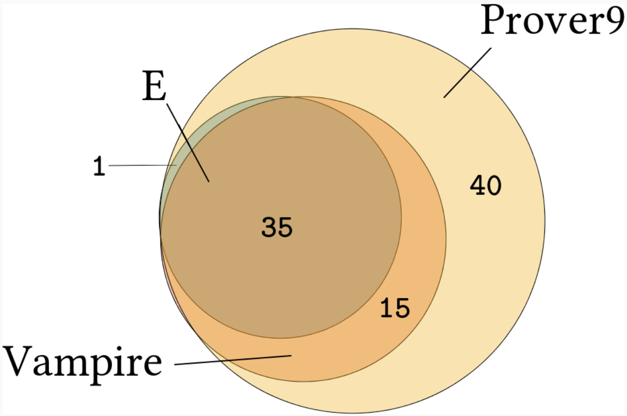
Results on 200 AIM testing problems



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Initial Evaluation on Whole TPTP

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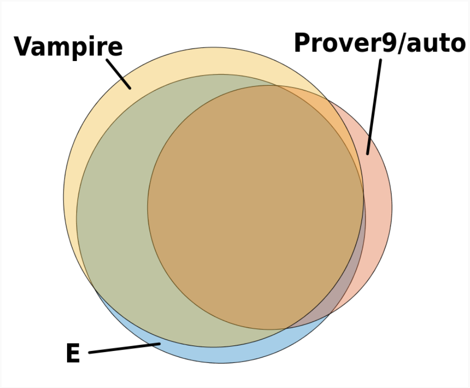

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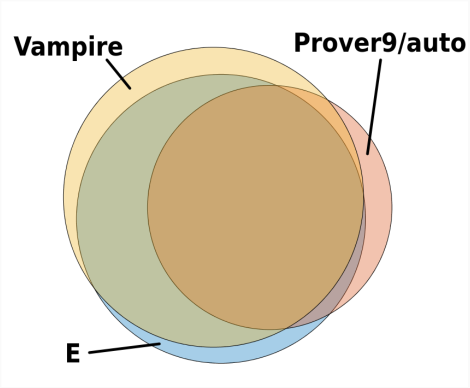
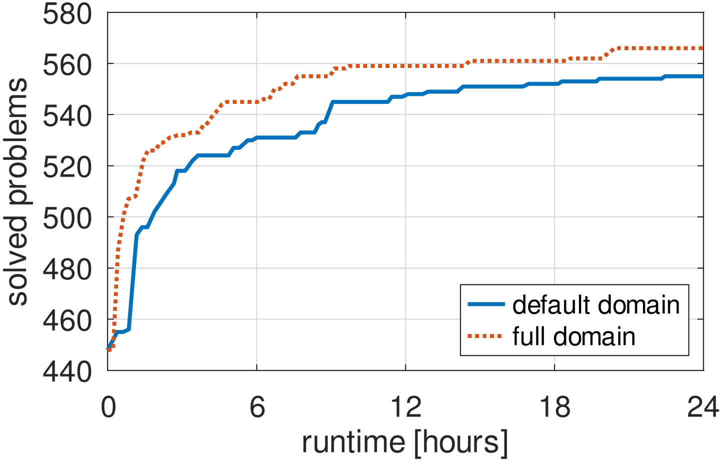


- A: On TPTP/NUM category.

Grackle run on TPTP/NUM problems



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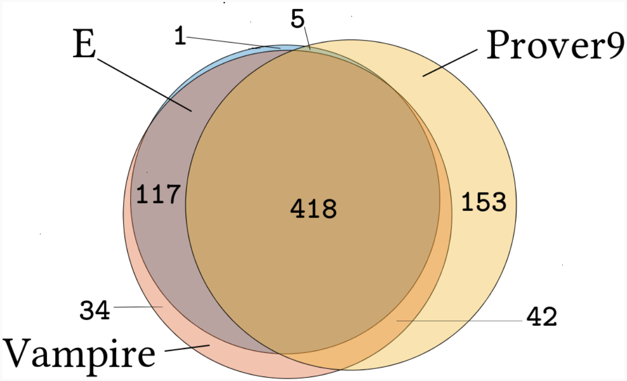

Thank you for listening.

Future work:

- Run Grackle for Prover9 on other TPTP categories.
- Intelligent problem-based strategy selection.
- Grackle strategy invention with Prover9 hints.

Grackle is available at GitHub:

`http://github.com/ai4reason/grackle`

Questions?