Chapter 19:The Kinetic Theory of Gases

(Fundamental of Physics, 10th edition)

- **Lecture- 6:** Molar specific heat at constant volume for an ideal gas molar specific heat at constant pressure for an ideal gas, Relation between **Cp** and $\mathbf{C_v}$, degrees of freedom, related problems.

Molar Specific Heat of an Ideal Gas

- We recall the definition of molar specific heat:

$$C_m = \frac{Q}{n\Delta T}$$

- Thus, we can calculate the amount of heat, ***Q*** an ideal gas needs to raise its temperature by $\boldsymbol{\Delta T}$:

$$Q = nC_m\Delta T$$

- We consider molar specific heat in two situations:

1. Molar specific heat at constant volume (for constant volume process)
2. Molar specific heat at constant pressure (for constant pressure process)

1. Molar Specific Heat at Constant Volume, C_v

Added amount of heat, $Q = nC_v\Delta T$

Work done, W= 0 [as $\Delta V=0$]

Applying first law of thermodynamics, $\Delta E_{int}= Q - W$, we get

$$\Delta E_{int}= nC_v\Delta T$$

$$\text{Thus, } C_v = \frac{\Delta E_{int}}{n\Delta T} = \frac{\frac{f}{2}\boldsymbol{nR\Delta T}}{n\Delta T} = \frac{f}{2}R$$

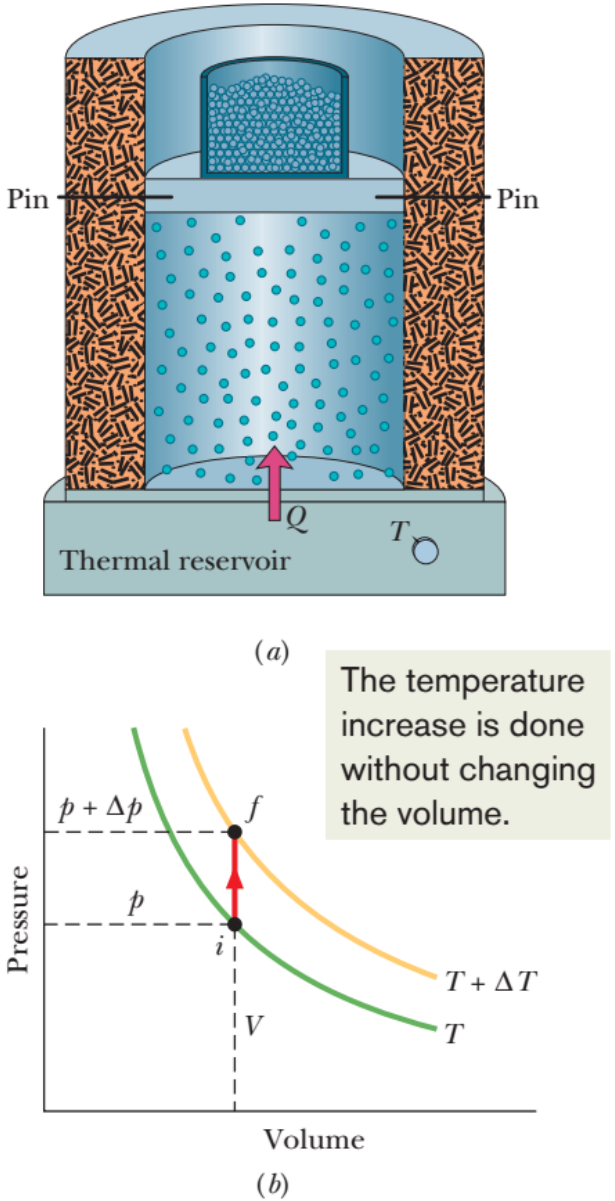


Figure 19-9 (*a*) The temperature of an ideal gas is raised from T to $T + \Delta T$ in a constant-volume process. Heat is added, but no work is done. (*b*) The process on a p-V diagram.

2. Molar Specific Heat at Constant Pressure, C_p

Added amount of heat, $Q = nC_p\Delta T$

Work done, $W = p\Delta V = nR\Delta T$ (pV=nRT)

Applying first law of thermodynamics, $\Delta E_{int} = Q - W$, we get

$$\Delta E_{int} = nC_p\Delta T - nR\Delta T$$

$$\gg nC_v\Delta T = nC_p\Delta T - nR\Delta T \quad [\Delta E_{int} = nC_v\Delta T]$$

$$\gg C_v = C_p - R \quad [\text{Divided by } n\Delta T\,]$$

$$\gg C_p = C_v + R$$

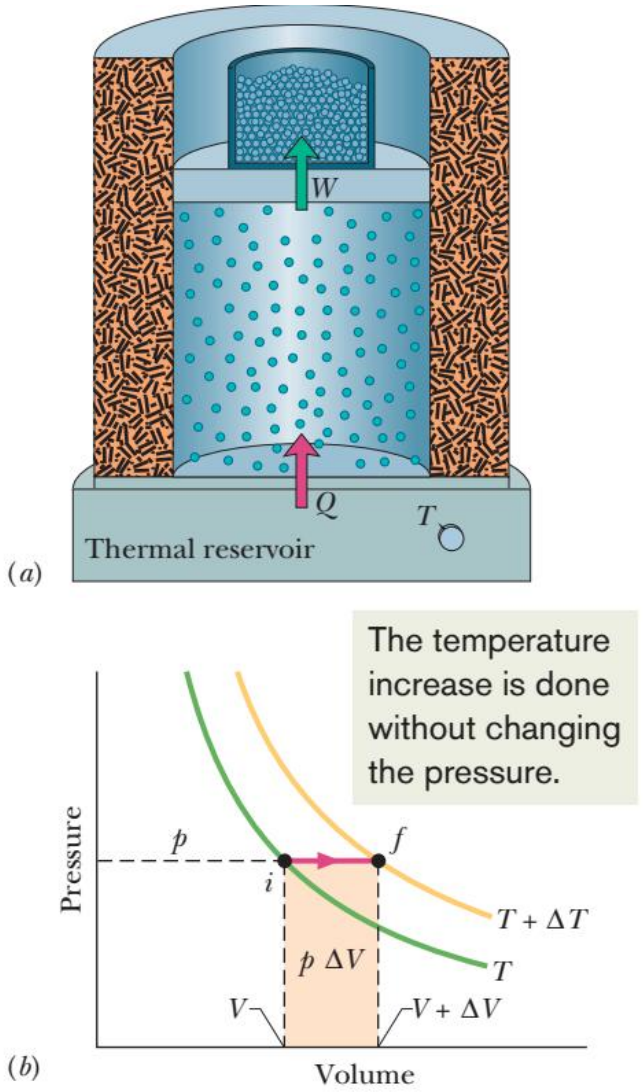


Figure 19-11 (*a*) The temperature of an ideal gas is raised from T to $T + \Delta T$ in a constant-pressure process. Heat is added and work is done in lifting the loaded piston. (*b*) The process on a p-V diagram. The work $p\,\Delta V$ is given by the shaded area.

- Thus, C_p is greater than the molar specific heat at constant volume C_V, because energy must now be supplied not only to raise the temperature of the gas but also for the gas to do work as shown in Fig.

Molar Specific Heats of Various Molecules

Table 19-3 Degrees of Freedom for Various Molecules

Molecule	Example	Degrees of Freedom		
		Translational	Rotational	Total (f)
Monatomic	He	3	0	3
Diatomic	O_2	3	2	5
Polyatomic	CH_4	3	3	6

Predicted Molar Specific Heats	
C_V (Eq. 19-51)	$C_p = C_V + R$
$\frac{3}{2}R$	$\frac{5}{2}R$
$\frac{5}{2}R$	$\frac{7}{2}R$
$3R$	$4R$

Table 19-2 Molar Specific Heats at Constant Volume

Molecule	Example	C_V (J/mol · K)
Monatomic	Ideal	$\frac{3}{2}R = 12.5$
	Real	He 12.5
		Ar 12.6
Diatomic	Ideal	$\frac{5}{2}R = 20.8$
	Real	N_2 20.7
		O_2 20.8
Polyatomic	Ideal	$3R = 24.9$
	Real	NH_4 29.0
		CO_2 29.7

47. The temperature of 2.00 mol of an ideal monatomic gas is raised 15.0 K at constant volume. What are (a) the work *W* done by the gas, (b) the energy transferred as heat *Q*, (c) the change ΔE_{int} in the internal energy of the gas, and (d) the change ΔK in the average kinetic energy per atom?

Solution: Here, ΔT= 15 K

n = 2.00 mol

ΔV = V - V = 0

$\mathbf{k = \frac{R}{N_A} = \frac{8.31\ J/mol-K}{6.023\ x\ 10^{23}/mol} = 1.38\ x\ 10^{-23}\ J/K}$

(a) W = pΔV = p(0) = 0

(b) $Q = nC_V\Delta T = n\left(\frac{3}{2}\right)R\,\Delta T = 2.00\left(\frac{3}{2}\right)8.31\,(15) = 373.95$ J

$[C_V = \left(\frac{3}{2}\right)R\,]$

$[R = 8.31\ J/mol - K]$

(c) $\Delta E_{int} = Q - W = 373.95 - 0 = 373.95$ J

(d) $\Delta K = \left(\frac{3}{2}\right)k\Delta T = \left(\frac{3}{2}\right)(1.38\ x\ 10^{-23})15 = 31.05\ x\ 10^{-23}$ J

48. When 20.9 J was added as heat to a particular ideal gas, the volume of the gas changed from 50.0 cm^3 to 100 cm^3 while the pressure remained at 1.00 atm. (a) By how much did the internal energy of the gas change? If the quantity of gas present was 2.00×10^{-3} mol, find (b) C_p and (c) C_V.

Solution:

Here, $Q = 20.9$ J

$\Delta V = (100 - 50)\ cm^3 = 50\ cm^3 = 50(10^{-2}\ m)^3 = 50 \times 10^{-6}\ m^3$

$p = 1.00\ atm = 1.01 \times 10^5\ Pa$

$n = 2.00 \times 10^{-3}$ mol

(a) $\Delta E_{int} = Q - W = Q - p\,\Delta V = 20.9 - 1 \times 10^5 (50 \times 10^{-6}) = 20.9 - 5.05 = 15.85$ J

(b) $Q = nC_p\Delta T$ [p = constant]

$C_p = \frac{Q}{n\Delta T}$

Ideal gas law, pV = nRT

$$\frac{\Delta}{\Delta T}(pV) = \frac{\Delta}{\Delta T}(nRT)$$

$$p\frac{\Delta V}{\Delta T} = nR\frac{\Delta T}{\Delta T}$$

$$p\frac{\Delta V}{\Delta T} = nR$$

$$p\Delta V = nR\,\Delta T$$

$$\frac{p\Delta V}{R} = n\,\Delta T$$

$$C_p = \frac{Q}{\frac{p\Delta V}{R}}$$

$$C_p = \frac{QR}{p\Delta V} = \frac{20.9(8.31)}{1x10^5(50x10^{-6})} = \frac{173.68}{5.0} = 34.39 \text{ J/mol-K}$$

(c) $C_p - C_V = R$

$C_V = C_p - R = 34.74 - 8.31 = 26.08$ J/mol-K

Problems for Practice

Module 19-7 The Molar Specific Heats of an Ideal Gas

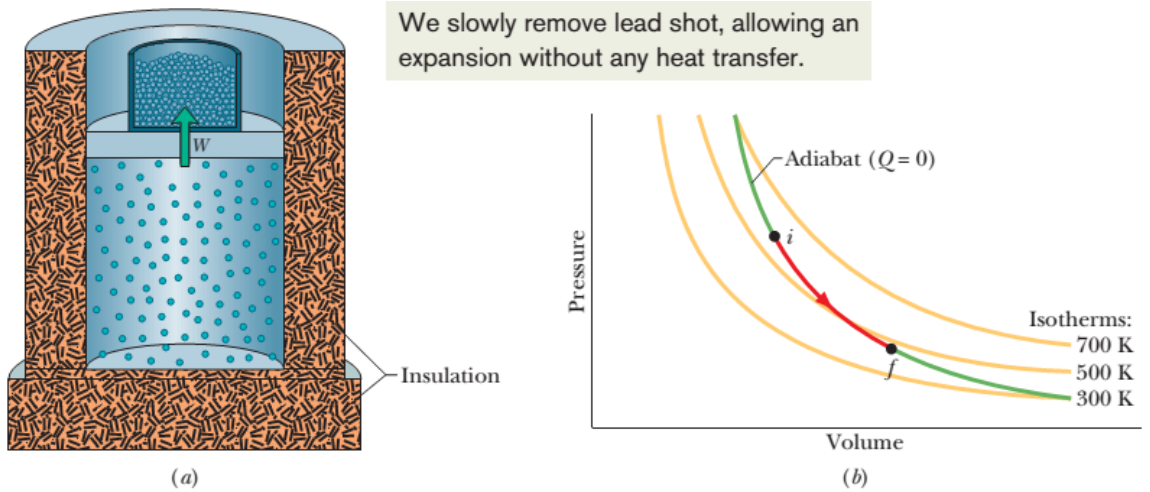
•42 What is the internal energy of 1.0 mol of an ideal monatomic gas at 273 K?

••43 GO The temperature of 3.00 mol of an ideal diatomic gas is increased by 40.0 C° without the pressure of the gas changing. The molecules in the gas rotate but do not oscillate. (a) How much energy is transferred to the gas as heat? (b) What is the change in the internal energy of the gas? (c) How much work is done by the gas? (d) By how much does the rotational kinetic energy of the gas increase?

19-9 Adiabatic expansion of an ideal gas: pV^{γ}= a constant

Where,

$$\gamma = \frac{C_p}{C_V}$$



adiabatic process, Q = 0

19-9 $TV^{\gamma-1}$ = constant for an adiabatic process:

For an adiabatic process, pV^{γ} = constant

To write an equation for an adiabatic process in terms of *T* and *V*, we use the ideal gas equation to eliminate *p*

Ideal gas equation, pV = nRT

$$p = \frac{nRT}{V}$$

$$\left(\frac{nRT}{V}\right) V^{\gamma} = \text{constant}$$

$$T\left(\frac{V^{\gamma}}{V^{1}}\right) = \frac{\text{constant}}{nR}$$ [n and R are constants]

$$TV^{\gamma-1} = \text{constant}$$

When the gas goes from an initial state *i* to a final state *f*: $T_i V_i^{\gamma-1} = T_f V_f^{\gamma-1}$

C_v, C_p and γ:

Types of Gas	$C_v = \frac{f}{2}R$	$C_p = C_v + R$	$\gamma = \frac{C_p}{C_v}$
Monoatomic	$\frac{3}{2}R$	$\frac{5}{2}R$	$\frac{5}{3} = 1.67$
Diatomic	$\frac{5}{2}R$	$\frac{7}{2}R$	$\frac{7}{5} = 1.4$
Polyatomic	$3R$	$4R$	$\frac{4}{3} = 1.33$

19-9 Work done for an ideal gas in an adiabatic process: $W = \frac{P_i V_i - P_f V_f}{\gamma - 1}$

$$W = \int_{V_i}^{V_f} p\,dV = = \int_{V_i}^{V_f} \frac{a}{V^{\gamma}}\,dV = a\int_{v_i}^{v_f} V^{-\gamma}\,dV = a\left[\frac{V^{-\gamma+1}}{-\gamma+1}\right]_{V_i}^{V_f}$$

Adiabatic process of an ideal gas: $pV^{\gamma} = a$

$$p = \frac{a}{V^{\gamma}}$$

$$P_i V_i^{\gamma} = P_f V_f^{\gamma} = a$$

$$\mathrm{W} = \frac{a}{-\gamma+1}\left[V^{-\gamma+1}\right]_{V_i}^{V_f} = \frac{a}{-\gamma+1}\left(V_f^{-\gamma+1} - V_i^{-\gamma+1}\right) = \frac{aV_f^{-\gamma+1} - aV_i^{-\gamma+1}}{-\gamma+1}$$

$$\mathrm{W} = \frac{aV_f^{-\gamma+1} - aV_i^{-\gamma+1}}{-\gamma+1} = \frac{P_f V_f^{\gamma} V_f^{-\gamma+1} - P_i V_i V_i^{-\gamma+1}}{-\gamma+1} = \frac{P_f V_f^{\gamma-\gamma+1} - P_i V_i^{\gamma-\gamma+1}}{-\gamma+1}$$

$$\mathrm{W} = \frac{P_f V_f - P_i V_i}{-\gamma+1} = \frac{-(P_i V_i - P_f V_f)}{-(\gamma-1)}$$

$$\mathrm{W} = \frac{P_i V_i - P_f V_f}{\gamma - 1}$$

55. A certain gas occupies a volume of 4.3 L at a pressure of 1.2 atm and a temperature of 310 K. It is compressed adiabatically to a volume of 0.76 L. Determine (a) the final pressure and (b) the final temperature, assuming the gas to be an ideal gas for which $\gamma = 1.4$.

Solution:

Here, $V_i = 4.3$ L

$p_i = 1.2$ atm $= 1.2 \times 10^5$ Pa

$T_i = 310$ K

$V_f = 0.76$ L

$\gamma = 1.4$

(a) pV^{γ} = constant

$$p_i V_i^{\gamma} = p_f V_f^{\gamma}$$

$$p_f = \frac{p_i V_i^{\gamma}}{V_f^{\gamma}} = p_i\left(\frac{V_i^{\gamma}}{V_f^{\gamma}}\right) = p_i\left(\frac{V_i}{V_f}\right)^{\gamma} = 1.2\times10^5\left(\frac{4.3\text{ L}}{0.76\text{ L}}\right)^{1.4} = 1.2\times10^5(11.3166) = 1.36\times10^6 \text{ Pa}$$

(b) $TV^{\gamma-1}$ = constant

$$T_i V_i^{\gamma-1} = T_f V_f^{\gamma-1}$$

$$T_f = \frac{T_i V_i^{\gamma-1}}{V_f^{\gamma-1}} = T_i\left(\frac{V_i}{V_f}\right)^{\gamma-1}$$

$$= 310\left(\frac{4.3\text{ L}}{0.76\text{ L}}\right)^{1.4-1} = 310(2.00) = 620 \text{ K}$$

Home work

54. We know that for an adiabatic process pV^{γ} = a constant. Evaluate "a constant" for an adiabatic process involving exactly 2.0 mol of an ideal gas passing through the state having exactly p = 1.0 atm and T = 300 K. Assume a diatomic gas whose molecules rotate but do not oscillate.

Solution:

Here, n = 2 mol

T = 300 K

P = 1.0 atm = 1.0×10^5 Pa

pV^{γ} = constant

$$\gamma = \frac{c_P}{c_v}$$

Diatomic gas whose molecules rotate but do not oscillate, f = 3+2 = 5

$$C_V = \left(\frac{f}{2}\right)R = \left(\frac{5}{2}\right)R$$

$$C_p - C_V = R$$

$$C_p = C_V + R = \left(\frac{5}{2}\right)R + R = \left(\frac{7}{2}\right)R$$

$$\gamma = \frac{c_P}{c_v} = \frac{\frac{7}{2}R}{\frac{5}{2}R} = \frac{7}{5} = 1.4$$