NAZARBAYEV UNIVERSITY, PHYSICS DEPARTMENT

Assignment 3

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TASK GIVEN

1. Extend your unit cell by applying multiple of them from the previous assignment. Construct a bulk band structure $\omega(k_B)$
2. Given that you construct a scattering matrix from the previous problem, terminate it from the endpoints. It is called a Ribbon problem.

SOLUTION 1

The easiest way I can think for solving such problem is to apply a recursive functions. Initially, I'll make a scattering matrix for $(n-1)$ unit cell structure and combine it with another unit cell. Finally, I should get a nxn matrix. See Fig.

$$\begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_{2n} \end{pmatrix} = M^{n-1} \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_{2n} \end{pmatrix} \tag{0.1}$$

While for the another unit cell

$$\begin{pmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \end{pmatrix} = S \begin{pmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{pmatrix} \tag{0.2}$$

At the interface of these structures, we have $B_1 = a_{2n}$ and $A_1 = b_{2n}$. By combining equations (0.1) and (0.2) we can now get $(2n+2)x(2n+1)$ matrix.

$$\begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_{2N+2} \end{pmatrix} = M^N \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_{2N+2} \end{pmatrix} \tag{0.3}$$

For simplicity, let us use notation $M = M^n$
The Bloch matrix $\tilde{B}$ can be found using relations

$$b_{2n+1} = a_{2n} e^{i\mathbf{k_b}\cdot\mathbf{a}}$$
$$b_{2n} = a_{2n+1} e^{-i\mathbf{k_b}\cdot\mathbf{a}}$$

where n varies from 1 to N. At the lower and upper boundaries

$$b_{2N+2} = a_1 e^{iN\mathbf{k_b}\cdot\mathbf{b}}$$
$$b_1 = a_{2N+2} e^{-iN\mathbf{k_b}\cdot\mathbf{b}}$$

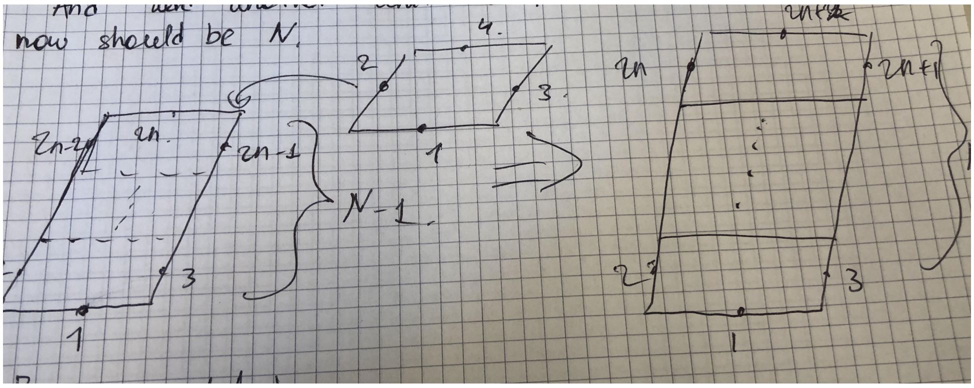

Figure 0.1: Our recursive strategy.

The bulk band structure, namely $-\log|det(M-\tilde{B})|$ represented in the figure below

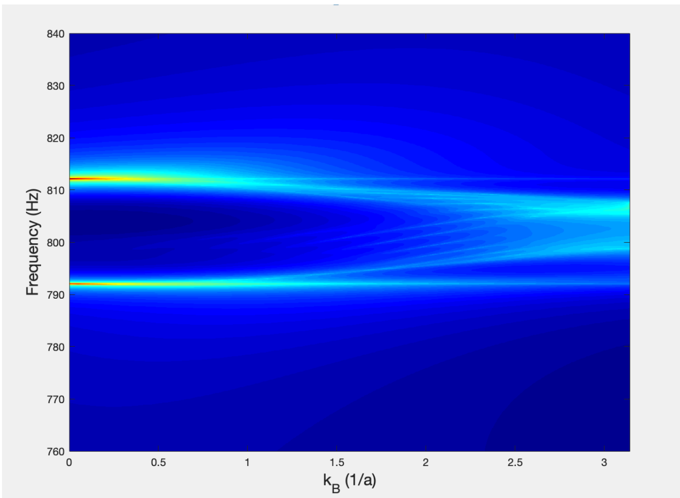

Figure 0.2: Bulk band structure for no bias velocity.

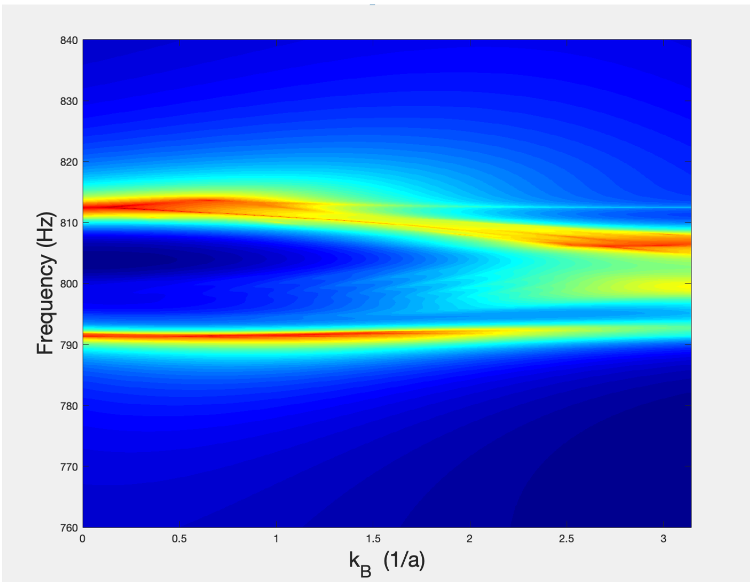

Figure 0.3: Bulk band structure for $\frac{\nu}{D\gamma} = 0.2$.

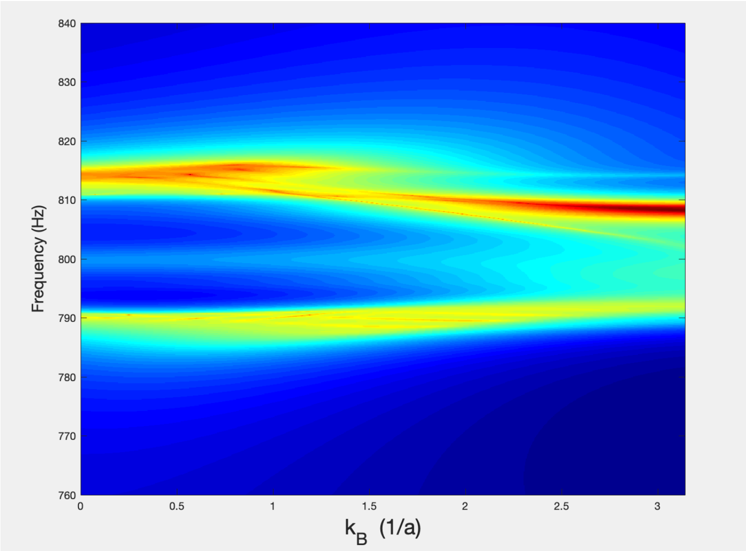

Figure 0.4: Bulk band structure for $\frac{\upsilon}{D\gamma} = 0.5$.

SOLUTION 2

Here in Ribbon problem, let's say that we have a same unit cell and we just need to equate lower and upper bands. To be exact

$$\begin{pmatrix} b_{1'} \\ b_1 \\ \vdots \\ b_{2N} \\ b_{2'} \end{pmatrix} = M \begin{pmatrix} a_{1'} \\ a_1 \\ \vdots \\ a_{2N} \\ a_{2'} \end{pmatrix} \tag{0.4}$$

By using the relations $b_{1'} = a_{1'}$ and $b_{2'} = a_{2'}$, we should transfer it to $2Nx2N$ matrix

$$\begin{pmatrix} b_1 \\ \vdots \\ b_{2N} \end{pmatrix} = T \begin{pmatrix} a_1 \\ \vdots \\ a_{2N} \end{pmatrix} \tag{0.5}$$

After using "simple" calculations, we can actually find the coefficients of T matrix

$$T_{ij} = M_{ij} + \frac{M_{i2'}[M_{2'j}(1 - M_{1'1'}) + M_{2'1'}M_{1'j}] + M_{i1'}[M_{1'j}(1 - M_{2'2'}) + M_{1'2'}M_{2'j}]}{(1 - M_{1'1'})(1 - M_{2'2'}) + M_{1'2'}M_{2'1'}} \tag{0.6}$$

Below, bulk band structure for Ribbon problem are represented

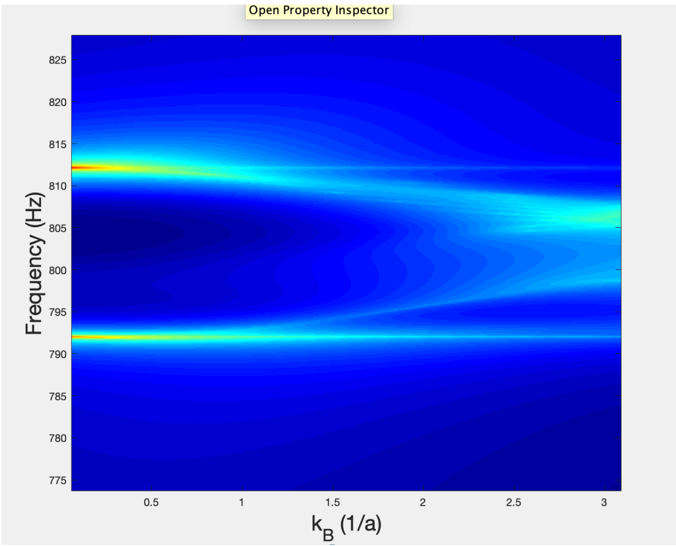

Figure 0.5: Bulk band structure for (Ribbon problem) for no bias velocity.

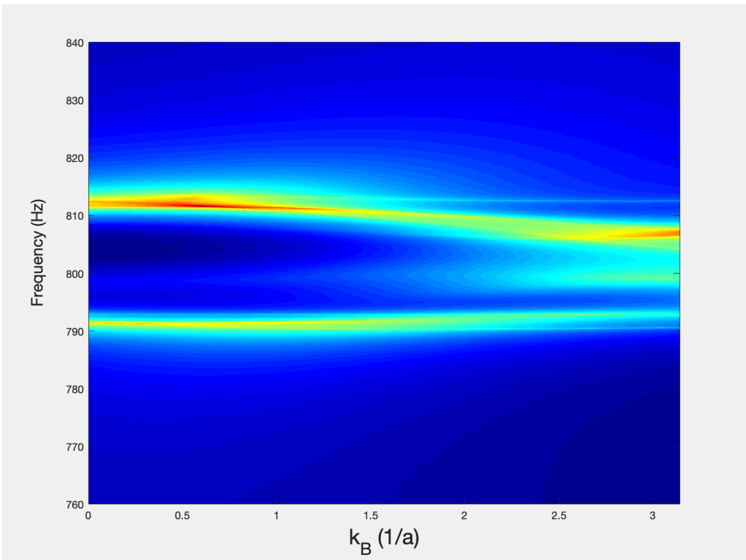

Figure 0.6: Bulk band structure for (Ribbon problem) for $\frac{v}{D\gamma} = 0.2$.

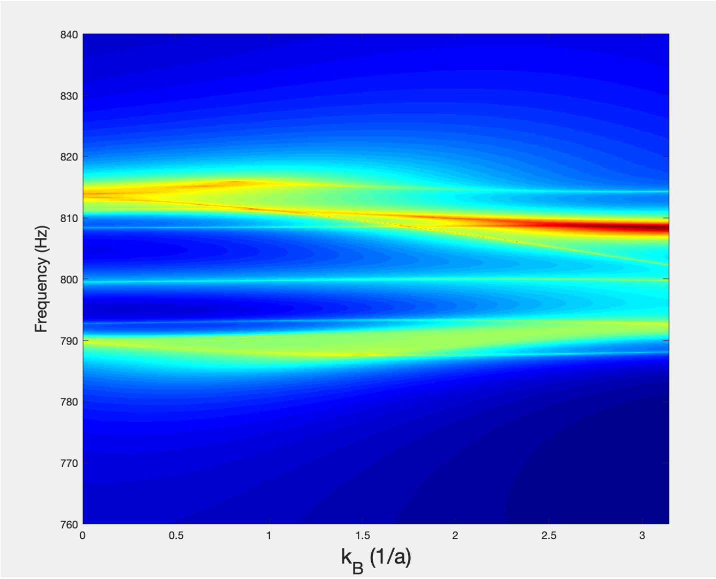

Figure 0.7: Bulk band structure for (Ribbon problem) for $\frac{v}{D\gamma} = 0.5$.

References

[1] Romain Fleury, MS. *Breaking Temporal Symmetries in Metamaterials and Metasurfaces*, August 2015.

[2] Charles Kittel. *Introduction to Solid State Physics*, 1914.

[3] Alexander B. Khanikaev, Romain Fleury, S. Hossein Mousavi and Andrea Alu. *Topologically robust sound propagation in an angular-momentum-biased graphene-like resonator lattice*, Nat. Comm. 2015