Electrophysiological behavior of active myocardium: Implementation of the monodomain model

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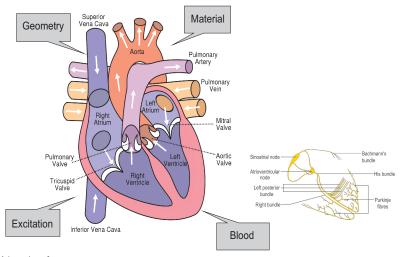
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Adaptation of:

wikipedia.org/wiki/File:Diagram_of_the_human _heart_(cropped).svg commons.wikimedia.org/wiki/File:Conductionsystemoftheheart.png

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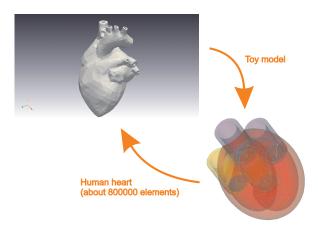
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international.transmedics.com/wt/page/PROCEED_II

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Parallelization: unstructured mesh \to load balancing electrophysiology \to fine computational grids, small time steps

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Monodomain model:

$$\frac{\partial s}{\partial t} = F(s, v, t), x \in H$$

$$\frac{\lambda}{1 + \lambda} \nabla \cdot (\mathbf{M_i} \nabla v) = \frac{\partial v}{\partial t} + I_{ion}(v, s), x \in H$$

$$\mathbf{n} \cdot (\mathbf{M_i} \nabla v) = 0, x \in \partial H$$

H: reference domain, v: transmembrane potential I_{ion} : ionic current across the membrane, s: gate variable, M_i : intracellular conductivity tensor

FitzHugh-Nagumo model:

$$I_{ion}(v,s) = k(v - v_{rest})(v - v_{th})(v - v_{peak}) - k(v - v_{rest})s$$

$$\frac{\partial s}{\partial t}(v,s) = l(v - v_{rest} - bs)$$

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For
$$t_n < t \le t_n + \theta \Delta t$$
: $v(t_n + \theta \Delta t) = v_{\theta}^n$, $s(t_n + \theta \Delta t) = s_{\theta}^n$
$$\frac{\partial v}{\partial t} = -I_{ion}(v, s), \qquad \frac{\partial s}{\partial t} = F(v, s) \longrightarrow RK4$$

For
$$t_n < t \le t_n + \Delta t$$
: $v(t_n) = v_{\theta}^n$, $v(t_n + \Delta t) = v_{\theta}^{n+1}$

$$\frac{\partial v}{\partial t} = \frac{\lambda}{1+\lambda} \nabla \cdot (\mathbf{M_i} \nabla v) \rightarrow \textit{ForwardEuler} + \textit{FE}$$

For
$$t_n + \theta \Delta t < t \le t_n + \Delta t$$
: $v(t_n + \theta \Delta t) = v_{\theta}^{n+1}$, $s(t_n + \theta \Delta t) = s_{\theta}^n$

$$\frac{\partial v}{\partial t} = -I_{ion}(v,s), \qquad \frac{\partial s}{\partial t} = F(v,s) \qquad \to RK4$$

Parallelization:

operator splitting \to parallelization for subsequent solution steps grid update $\to m$ MPI tasks spawning n threads

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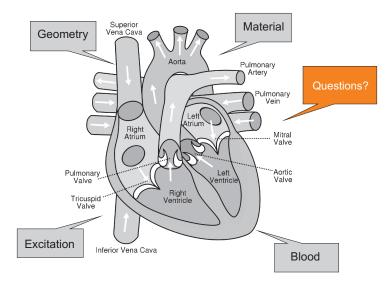
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$$\begin{split} \frac{v^{n+1} - v^n}{\Delta t} &= \theta \left(\frac{\lambda}{1 + \lambda} \nabla \cdot (\mathbf{M_i} \nabla v^{n+1}) \right) \\ &+ \left((1 - \theta) \frac{\lambda}{1 + \lambda} \nabla \cdot (\mathbf{M_i} \nabla v^n) \right) \end{split}$$

PDE-Part: Weak formulation $(\gamma = \frac{\Delta t \lambda}{1+\lambda})$

$$\sum_{j=1}^{n} v_{j}^{n+1} \int_{H} \Phi_{j} \Phi_{i} dx + \theta \gamma \sum_{j=1}^{n} v_{j}^{n+1} \int_{H} \mathbf{M_{i}} \nabla \Phi_{j} \cdot \Phi_{i} dx =$$

$$\sum_{j=1}^{n} v_{j}^{n} \int_{H} \Phi_{j} \Phi_{i} dx - (1-\theta) \gamma \sum_{j=1}^{n} v_{j}^{n} \int_{H} \mathbf{M_{i}} \nabla \Phi_{j} \nabla \Phi_{i} dx$$

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Obtained general form:

$$A_{ij}v_j=b_i$$

Using isoparametric tetrahedral elements:

$$\begin{pmatrix} \Phi_{,r} \\ \Phi_{,s} \\ \Phi_{,t} \end{pmatrix} = \mathbf{J} \begin{pmatrix} \Phi_{,x} \\ \Phi_{,y} \\ \Phi_{,z} \end{pmatrix}, \quad \mathbf{J} = \begin{pmatrix} x_{,r} & y_{,r} & z_{,r} \\ x_{,s} & y_{,s} & z_{,s} \\ x_{,t} & y_{,t} & z_{,t} \end{pmatrix}$$

$$\Phi_{1} = 1 - r - s - t$$

$$\Phi_{2} = r$$

$$\Phi_{3} = s$$

$$\Phi_{4} = t$$

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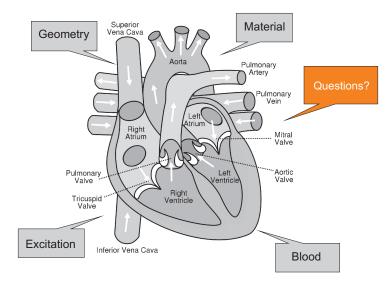
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