ACM/CS 114
Parallel algorithms for scientific applications

Michael A. G. Aïvázis

California Institute of Technology

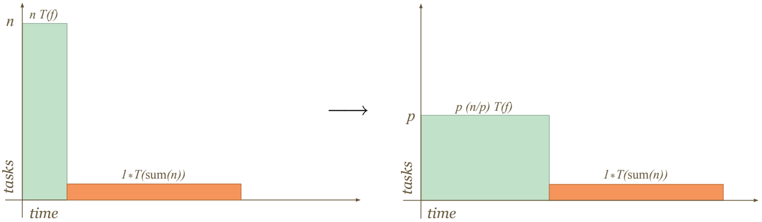
Winter 2010

Time, parallelism and computational work

- recall our embarrassingly parallel reduction:
  - given a function f and a sequence of numbers S of length N, evaluate

$$s = \sum_{i=0}^{N-1} f(S_i)$$

- initial parallelism profile for a simple mapping, assuming that
  - the computation of $f(S)$ is the parallel task
  - the summation is sequential



- shaded area is w, the *computational work*

Metrics: speedup and efficiency

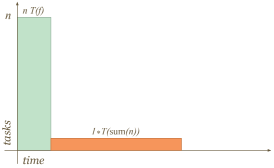
- let
  - T_1 be the sequential execution time on one processor
  - T_p be the parallel execution time on p processors
- define
  - *speedup*:
    $$\sigma := T_1/T_p$$
  - *efficiency*:
    $$\eta := T_1/(pT_p)$$
  - related through $\eta = \sigma/p$ and $\sigma = \eta p$
- pseudo-theorems: $\sigma \leq p$ and $\eta \leq 1$
  - but *speedup anomalies* can occur if resources increase with p causing an increase in the effective computation rate
  - example: for large enough p, your problem may fit entirely in the L2 cache
  - *sweet spots* like that abound; the craftsman knows how to
    - implement the solution in a portable manner
    - expose enough controls to be able to tune the implementation to a given architecture

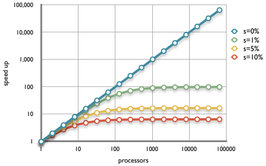
The bad news: Amdahl's law

- consider a solution that consists of two parts
  - a serial fraction s with $0 \le s \le 1$
  - a p-fold parallel fraction $1 - s$
- for a fixed problem size, Amdahl's law relates T_p, σ and η to T_1 and s

$$\begin{aligned} T_p &= sT_1 + (1-s)T_1/p \\ \sigma &= \frac{p}{sp + (1-s)} \\ \eta &= \frac{1}{sp + (1-s)} \end{aligned}$$

- with corollaries $\sigma_\infty = \frac{1}{s}$ and $\eta_\infty = 0$





Beating Amdahl's law

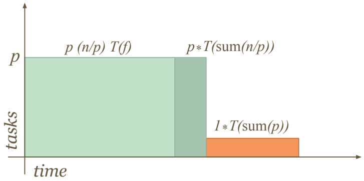
- Amdahl's law holds if either
  - the problem size is fixed
  - the serial fraction s is not a function of p
- *weak scaling*: let the problem size grow with p
  - larger computers are used to solve larger problems
  - the effective serial fraction *decreases* with problem size
  - the right scaling metric would be constant, or properly bounded, as $p \to \infty$
- *isoefficiency*
  - how rapidly must problem size grow so that η is constant as p increases?
  - since $\eta = T_1/(pT_p)$, constant efficiency implies

    $$T_1 = c(pT_p)$$

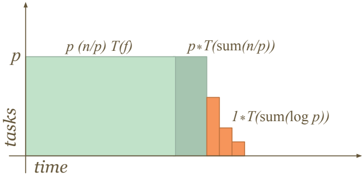
    for some constant c
  - T_1 measures the sequential work, so the above relation determines your implementation's *isoefficiency function*

Algorithmic improvements

- getting smarter is the best way to improve σ and η
  - reduce the sequential fraction s
  - what are the effects on communication and locality?
- parallelize the partial sums

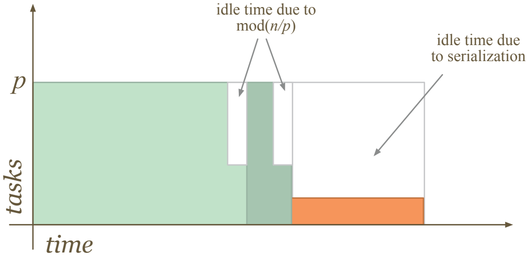


- parallelize the final sum using a *reduction tree*



Load balance

- non-optimal task distributions show up as *load imbalance*

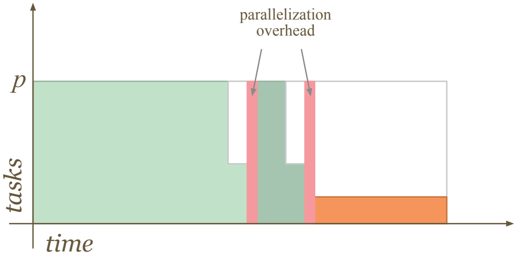


- excessive coarsening tends to increase load imbalance
- so can inappropriate mapping
- synchronization also causes load imbalance (see later slide)
- new upper bound for the speedup

$$\sigma \leq \frac{w_1}{\max_p (w_p + \text{idle})}$$

Parallelization overhead

- there is always some extra work that is not present in the sequential implementation

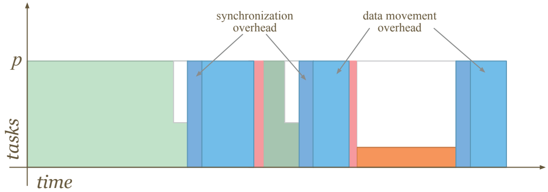


- orchestration, management, bookkeeping

$$\sigma \leq \frac{w_1}{\max_p (w_p + \text{idle} + \text{overhead})}$$

Communication and synchronization costs

- communication is required for data movement and synchronization



- the cost is modeled by

$$T_c = \lambda + \beta L$$

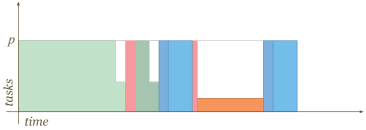
where the *latency* λ measures the communication startup cost, β is the bandwidth of the interconnect and L is the message length in *words*

- the speedup is now bounded by

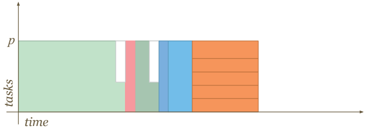
$$\sigma \leq \frac{w_1}{\max_p (w_p + \text{idle} + \text{overhead} + \text{comm})}$$

Reducing communication costs

- multiple strategies
- coördinating placement of work and the associated data to minimize inter-process dependencies



- trading memory for efficiency by replicating data
- trading cpu for efficiency by doing redundant work



- improving communication efficiency by tuning the cost factors
  - communication frequency, message size, contention, architecture specific optimizations

Optimizing speedup and efficiency

- the goal is to minimize the denominator

$$\sigma \leq \frac{w_1}{\max_p(w_p + \text{idle} + \text{overhead} + \text{comm})}$$

  - but its parts are in tension: minimizing one happens at the expense of another
- fine grain decomposition and intelligent mapping tend to minimize load imbalance at the cost of increased communication
  - coarser grains imply larger message size and fewer synchronization events
  - for many problems communication costs decrease as surface to volume
- naïve static partitioning reduces redundant work but cause load imbalance

The good news

- the basic work unit of a parallel algorithm may be more efficient (and better performing) than the sequential equivalent
  - only a small fraction of typical problems fits in L2 cache
  - single node performance *requires* partitioning
  - just like the parallel implementation
  - don't be surprised by the poor quality of your sequential version after you see your parallel implementation
- communication can be interleaved with computation
  - better algorithms on today's complicated memory hierarchies
- parallel algorithms may lead to better sequential ones
  - e.g. parallel search may explore configuration space more effectively