1. MERGE-SORT is an example of a *divide-and conquer* algorithm. The algorithm sorts an input sequence S of numbers using the following steps:

   - *divide*: split S into two parts of roughly equal length,
   - *conquer*: sort the subsequences recursively,
   - *combine*: merge the two sorted subsequences to produce the sorted output.

   In pseudocode:

   Algorithm 1: MERGE-SORT(S, p, r)

   if $p < r$ **then**
       $q \leftarrow \lfloor (p+r)/2 \rfloor$
       MERGE-SORT(S, p, q)
       MERGE-SORT(S, $q+1$, r)
       MERGE(S, p, q, r)

   (a) Explain the role of p and r in the algorithm specification. What values should they have upon initial invocation of the algorithm?

   (b) Write MERGE. It was claimed in class that MERGE can be implemented to run in $\Theta(r - p + 1)$ time. How does your implementation compare?

   (c) Implement MERGE-SORT in a language of your choice.

      i. Write a driver that invokes it with $S = (5, 2, 4, 6, 1, 3)$.

      ii. Build a container with 10^6 random numbers. Sort it using your implementation.

2. Consider the function Li_2 defined for $|z| \le 1$ by

$$\mathrm{Li}_2(z) := \sum_{n=1}^{\infty} \frac{z^n}{n^2} = z + \frac{z^2}{2^2} + \frac{z^3}{3^2} + \frac{z^4}{4^2} + \cdots$$

   (a) In the language of your choice, implement a procedure `dilog`(z, n) that computes the sum of the first n terms of the series for the given floating point number z.

   (b) Implement a procedure `sdilog`(Z, n) that accepts a sequence Z of floating point numbers and computes the sum

$$\texttt{sdilog}(Z, n) := \sum_{z \in Z} \texttt{dilog}(z, n)$$

   (c) Build a cost model for `sdilog` as a function of n and $m := \mathrm{length}(Z)$. Assume that additions and subtractions cost c_+ each, multiplications and divisions cost $c_\times$ each, and that raising a number to the nth power costs $c_\star$.