Electrophysiological behavior of active myocardium: Implementation of the monodomain model

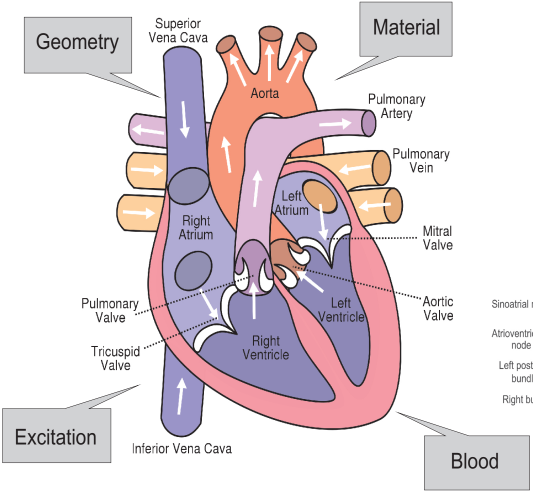
Stefanie Heyden

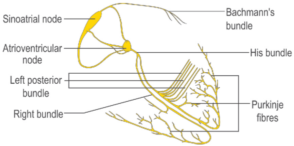
Graduate Aerospace Laboratories
Computational Solid Mechanics Group
California Institute of Technology

April 25th, 2012

Introduction

Modeling aspects





Adaptation of:
wikipedia.org/wiki/File:Diagram_of_the_human _heart_(cropped).svg
commons.wikimedia.org/wiki/File:Conductionsystemoftheheart.png

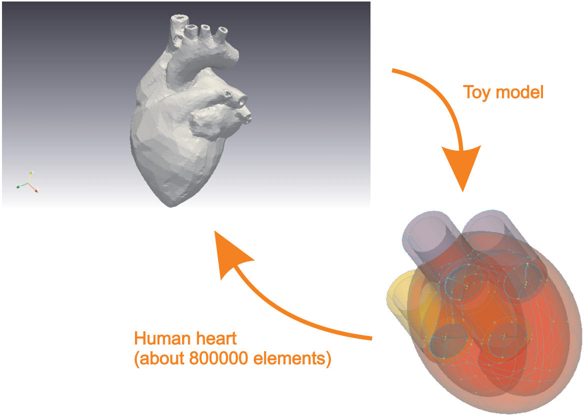
Introduction

Applications

international.transmedics.com/wt/page/PROCEED_II

Introduction

Geometrical Modeling



Parallelization:
unstructured mesh $\rightarrow$ load balancing
electrophysiology $\rightarrow$ fine computational grids, small time steps

Computational methods

Governing equations

Monodomain model:

$$\frac{\partial s}{\partial t} = F(s, v, t),\ x \in H$$

$$\frac{\lambda}{1+\lambda} \nabla \cdot (\mathbf{M_i} \nabla v) = \frac{\partial v}{\partial t} + I_{ion}(v, s),\ x \in H$$

$$\mathbf{n} \cdot (\mathbf{M_i} \nabla v) = 0,\ x \in \partial H$$

H: reference domain, v: transmembrane potential
I_{ion}: ionic current across the membrane, s: gate variable,
M_i: intracellular conductivity tensor

FitzHugh-Nagumo model:

$$I_{ion}(v, s) = k(v - v_{rest})(v - v_{th})(v - v_{peak}) - k(v - v_{rest})s$$

$$\frac{\partial s}{\partial t}(v, s) = l(v - v_{rest} - b\, s)$$

Computational methods

Operator Splitting

For $t_n < t \leq t_n + \theta\Delta t$: $v(t_n + \theta\Delta t) = v_\theta^n,\ s(t_n + \theta\Delta t) = s_\theta^n$

$$\frac{\partial v}{\partial t} = -I_{ion}(v, s), \qquad \frac{\partial s}{\partial t} = F(v, s) \qquad \rightarrow RK4$$

For $t_n < t \leq t_n + \Delta t$: $v(t_n) = v_\theta^n, v(t_n + \Delta t) = v_\theta^{n+1}$

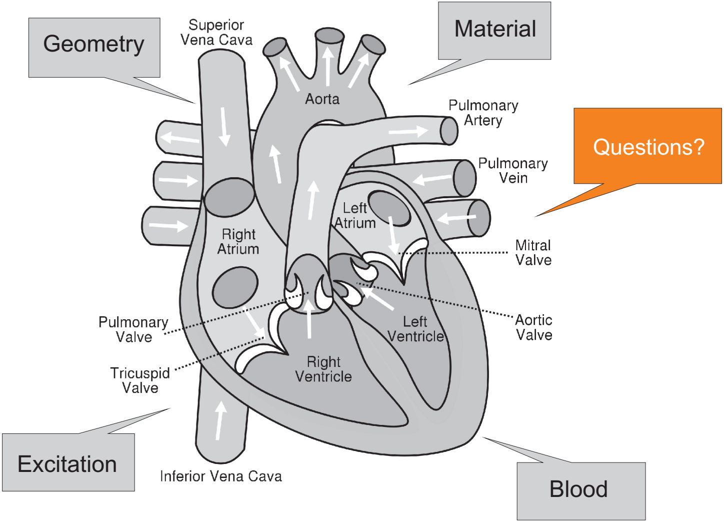
$$\frac{\partial v}{\partial t} = \frac{\lambda}{1+\lambda}\nabla \cdot (\mathbf{M_i}\nabla v) \rightarrow ForwardEuler + FE$$

For $t_n + \theta\Delta t < t \leq t_n + \Delta t$: $v(t_n + \theta\Delta t) = v_\theta^{n+1},\ s(t_n + \theta\Delta t) = s_\theta^n$

$$\frac{\partial v}{\partial t} = -I_{ion}(v, s), \qquad \frac{\partial s}{\partial t} = F(v, s) \qquad \rightarrow RK4$$

Parallelization:
operator splitting $\rightarrow$ parallelization for subsequent solution steps
grid update $\rightarrow$ m MPI tasks spawning n threads

Questions



Adaptation of:
wikipedia.org/wiki/File:Diagram_of_the_human _heart_(cropped).svg

Computational methods

Weak form and discrete formulation

PDE-Part: Time discretization

$$\frac{v^{n+1}-v^n}{\Delta t} = \theta\left(\frac{\lambda}{1+\lambda}\nabla\cdot(\mathbf{M_i}\nabla v^{n+1})\right) + \left((1-\theta)\frac{\lambda}{1+\lambda}\nabla\cdot(\mathbf{M_i}\nabla v^{n})\right)$$

PDE-Part: Weak formulation ($\gamma = \frac{\Delta t\,\lambda}{1+\lambda}$)

$$\sum_{j=1}^{n} v_j^{n+1}\int_H \Phi_j\Phi_i dx + \theta\gamma\sum_{j=1}^{n} v_j^{n+1}\int_H \mathbf{M_i}\nabla\Phi_j\cdot\Phi_i dx =$$

$$\sum_{j=1}^{n} v_j^{n}\int_H \Phi_j\Phi_i dx - (1-\theta)\gamma\sum_{j=1}^{n} v_j^{n}\int_H \mathbf{M_i}\nabla\Phi_j\nabla\Phi_i dx$$

Computational methods

Weak form and discrete formulation

Obtained general form:

$$A_{ij}v_j = b_i$$

Using isoparametric tetrahedral elements:

$$\begin{pmatrix} \Phi_{,r} \\ \Phi_{,s} \\ \Phi_{,t} \end{pmatrix} = \mathbf{J} \begin{pmatrix} \Phi_{,x} \\ \Phi_{,y} \\ \Phi_{,z} \end{pmatrix}, \quad \mathbf{J} = \begin{pmatrix} x_{,r} & y_{,r} & z_{,r} \\ x_{,s} & y_{,s} & z_{,s} \\ x_{,t} & y_{,t} & z_{,t} \end{pmatrix}$$

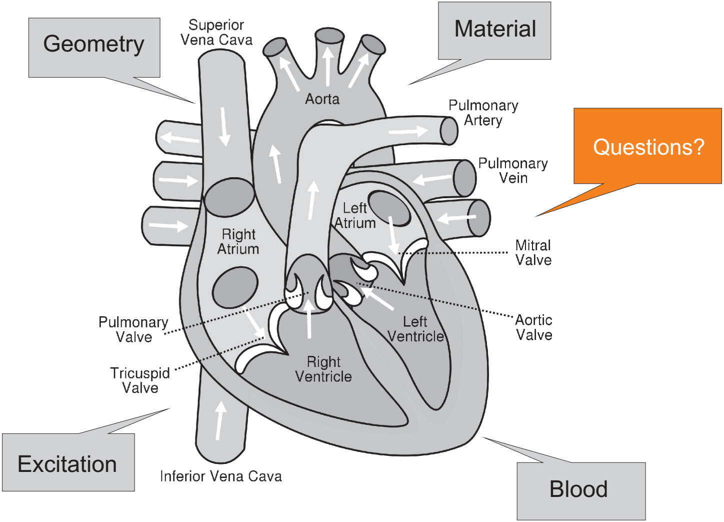
$$\Phi_1 = 1 - r - s - t$$

$$\Phi_2 = r$$

$$\Phi_3 = s$$

$$\Phi_4 = t$$

Questions



Adaptation of:
wikipedia.org/wiki/File:Diagram_of_the_human _heart_(cropped).svg