ACM/CS 114
Parallel algorithms for scientific applications

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Functional decomposition

- *functional decomposition* determines the fine grain parallel tasks by partitioning the problem into semi-independent tasks that can be executed in parallel
- our numerical integration examples fall in this category
  - partitioning identified the finest grain work unit as the evaluation of the integrand; no need for fancy domain decomposition
  - little or no communication/synchronization is required among the tasks, i.e. the are embarrassingly parallel
  - coarsening consists of grouping fine grain tasks into larger work units in a straight forward manner
  - the mapping of the coarse grain tasks onto processing units is trivial
- in general, the computations involved in carrying out the coarse tasks are computationally equivalent
  - the computation is *self balancing*
  - or there is no need for sophisticated load balancing
- scalability and parallel efficiency are determined by the particulars of the problem, such as inherent limitations on the largest problem size of interest

Monte Carlo integration

- let f be sufficiently well behaved in a region $\Omega \subset \mathbb{R}^n$ and consider the integral

$$I_\Omega(f) = \int_\Omega dx f \tag{1}$$

- the *Monte Carlo* method approximates the value of the integral in Eq. 1 by sampling f at random points in Ω
- let X_N be such a sample of N points; then the Monte Carlo estimate is given by

$$I_\Omega(f; X_N) = \Omega \cdot \langle f \rangle = \Omega \frac{1}{N} \sum_{x \in X_N} f(x) \tag{2}$$

  where $\langle f \rangle$ is the sample mean of f, and Ω is used as a shorthand for the volume of the integration region.
- the approximation error falls like $1/\sqrt{N}$
  - rather slow
  - but dimension independent!

Implementation strategy

- computer implementations require a pseudo-random number generator to build the sample
- most generators return numbers in $(0, 1)$ so
  - find a box B that contains Ω
  - generate n numbers to build a point in the unit $\mathbb{R}^n$ cube
  - stretch and translate the unit cube onto B
- the integration is restricted to Ω by introducing

$$\Theta_\Omega = \begin{cases} 1 & x \in \Omega \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

to get

$$I_\Omega(f) = \int_B dx\, \Theta_\Omega f \tag{4}$$

Recasting Monte Carlo integration

- there are now two classes of points in the sample X_N
  - those in Ω
  - and the rest
- let $\tilde{N}$ be the number of sample points in Ω; Eq. 2 becomes

$$I_\Omega(f; X_N) = \Omega \frac{1}{\tilde{N}} \sum_{x \in X_{\tilde{N}}} f(x) \tag{5}$$

- let B be the volume of the sampling box; observe that the volume of the integration region can be approximated by

$$\Omega = \frac{\tilde{N}}{N} B \tag{6}$$

and the sum over the points $x \in X_{\tilde{N}}$ can be extended to the entire sample X_N by using the filter Θ_Ω

$$I_\Omega(f; X_N) = B \frac{1}{N} \sum_{x \in X_N} \Theta_\Omega f(x) \tag{7}$$

Requirements

- to summarize, the Monte Carlo approximation is computed using

$$I_\Omega(f; X_N) = B \frac{1}{N} \sum_{x \in X_N} \Theta_\Omega f(x) \tag{8}$$

- using
  - an implementation of the function f to be integrated over Ω
  - an n-dimensional box B that contains Ω
  - a good pseudo-random number generator to build the sample $X_N \in B$
  - a routine to test points $x \in X_N$ and return `false` if they are exterior to Ω and `true` otherwise
- to sum the values of the integrand on points interior to Ω, and scale by the volume of the bounding box B over the sample size N
- essentially a reduction, similar to our other examples
  - should be straightforward to implement in parallel
  - see homework assignment

Pseudo-random number generators

- essential when solving problems using stochastic methods
- most generators are terrible, e.g. `libc`
  - always check that you are getting the statistics you expect
  - `/dev/random` is better, but not portable and produces integers only
- get the GNU scientific library
  - from **`http://www.gnu.org/software/gsl`**, or your OS distribution
  - broad scope, extensive documentation
  - thread safe
  - pick `RANLUX` or something similar
- there are algorithms that use uniformly distributed random numbers to generate numbers of any distribution function

Monte Carlo estimate of π

```
1  #include <cmath>
2  #include <iostream>
3  #include <gsl/gsl_rng.h>
4
5  int main(int, char*[]) {
6     // the number of points in the sample
7     const long N = (long) 1.0e7;
8     // point counters
9     long interiorPoints = 0, totalPoints = 0;
10
11    // create the random number generator
12    gsl_rng * generator = gsl_rng_alloc(gsl_rng_ranlxs2);
13
14    // integrate by sampling some number of times
15    for (long i=0; i<N; ++i) {
16       // create a random point
17       double x = gsl_rng_uniform(generator);
18       double y = gsl_rng_uniform(generator);
19       // check whether it is inside the unit quarter circle
20       if ((x*x + y*y) <= 1.0) { // no need to waste time computing the square root
21          // update the interior point counter
22          interiorPoints++;
23       }
24       // update the total number of points
25       totalPoints++;
26    }
27
28    // print the results
29    std::cout << "pi: " << 4.*((double)interiorPoints)/totalPoints << std::endl;
30
31    return 0;
32 }
```

Functional decomposition and load balancing

- *load balancing* refers to a class of algorithms that attempt to provide optimal or near optimal solutions to the *task scheduling* problem
  - enormous and diverse literature
  - broad applicability
    - computer systems: operating systems, parallel computing, distributed computing
    - theoretical computer science
    - operations research
    - and many other application domains, perhaps disguised
- *scheduling* is a closely related problem: determine the order in which a set of tasks should run
- problems that parallelize effectively using functional decomposition tend to be *self scheduling* and *self balancing*
- abstract the essentials of the approach and construct a load balancing technique
  - static and semi-static load balancing
  - self scheduling
  - distributed task queues

Overview of load balancing

- information relevant to work distribution
  - number of tasks: is it fixed or are tasks added as the work progresses?
  - task costs: when is the cost known? what is the cost distribution among tasks?
  - task inter-dependencies: can they be run in any order? when do we find out?
  - locality: should some tasks be scheduled close to each other, i.e. on the same processor, or a nearby one? when does this information become available?
- there is a spectrum of available solution depending on how information about task details becomes available
  - static: all necessary information is available initially
    - off-line algorithms: run before any real computation starts
  - semi-static: some information available, context changes slowly
    - off-line algorithms produce acceptable results in most cases
  - dynamic: little or no information is available at the outset
    - on-line algorithms, based on code instrumentation to enable decisions

Static and semi-static load balancing

- common static cases:
  - dense matrix algorithms: LU factorizations, etc.
  - most computations on a regular grid: FFT
  - sparse matrix-vector multiplication: when graph partitioned
- common semi-static cases:
  - particle and particle-in-cell methods
    - where the main problem is locality as particles move from one cell to another
  - computations structured as tree traversals
  - dynamic grids that change slowly, e.g. after many time steps

Self scheduling

- task manager:
  - maintain a central pool of tasks that are ready to be scheduled
  - once a processor completes its current task, assign it a new one form the pool
  - if a computation of a task generates more, add them to the pool
- works well when
  - tasks have no dependencies on each other
  - tasks are very weakly inter-dependent (but not studied extensively)
  - task cost in not known
  - locality is not important
- do not schedule the fine grain parallel tasks directly; coarsen first
  - larger grains reduce the task queue management overhead
  - smaller grains even out the finish times
- variations:
  - fixed grain size: like our quadrature implementation
  - guided self scheduling: start out with coarse grains and refine as you approach the end of the queue
  - distributed task queues: details and an important use case next time