In this assignment, we will use Monte Carlo integration to compute an approximation to the value of π by computing the area of the upper right quadrant of a unit circle centered at the origin, as shown in Fig. 1.

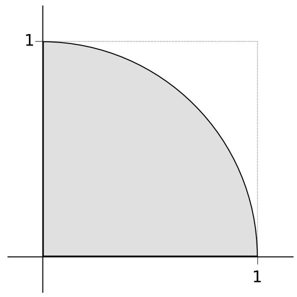

Figure 1: Estimating π by computing the area of a quadrant of the unit circle

You can get an estimate for $\pi/4$ by generating random points in the unit box and counting the fraction that fall within the shaded quarter circle.

1. The effectiveness of computing using stochastic techniques is intimately related to the properties of the pseudo-random generator used.
   (a) Choose a random number generator and use it to fill a container with 10 numbers between 0 and 1. Do you get the same 10 numbers every time you run the program? How can you seed your generator so that each execution of your program produces the same sequence? How can you seed it so that each execution produces different random numbers?
   (b) Use your generator to fill a container with 10^6 numbers between 0 and 1. Partition the interval $[0, 1]$ into 10 bins of equal width and build a frequency histogram, i.e. plot the number of times random numbers from your container fall within each bin. What is the average occupancy of the bins? What is the standard deviation?
   (c) Repeat the above steps using 100 bins.
2. Compute the Monte Carlo estimate for π from Fig. 1.
   (a) Implement the sequential version. How many points are required to get the error to drop below 10^{-3}. Below 10^{-5}?
   (b) Implement a parallel version using threads. Make sure that your random number generator is thread safe.
   (c) Implement a parallel version using `MPI`. Make sure that you seed your generator properly so that each process gets a different sequence of random numbers.
3. Monte Carlo integration converges to the answer very slowly. However, it has the great advantage that its convergence properties are not affected by the dimensionality of the problem. Adapt one of your parallel implementations to compute another estimate for π using the volume of the positive octant of a unit ball centered at the origin. How many points in three dimensions does it take to get the error to drop below 10^{-5}?