1 Households

Problem:

$$\begin{cases} U = \sum_t \beta^t \left\{\log\left(c_t\right) + \theta \log\left(1-n_t\right)\right\} \\ c_t + b_{t+1} = (r_t+1)b_t + (1-\tau_t^w)w_t n_t + f_t + g_t \end{cases} \tag{1}$$

Lagrangian:

$$\mathcal{L} = \sum_t \beta^t \left\{\log\left(c_t\right) + \theta \log\left(1-n_t\right)\right\} + \lambda_t \left[(r_t+1)b_t + (1-\tau_t^w)w_t n_t + f_t + g_t - c_t - b_{t+1}\right] \tag{2}$$

Derivatives:

$$\mathcal{L}'_{c_t} = \frac{1}{c_t} - \lambda_t \tag{3}$$

$$\mathcal{L}'_{b_{t+1}} = -\lambda_t - \beta(r_{t+1}+1)\lambda_{t+1} \tag{4}$$

$$\mathcal{L}'_{n_t} = \theta\frac{-1}{1-n_t} + \lambda_t(1-\tau_t^w)w_t \tag{5}$$

FOC:
Euler equation:

$$\frac{1}{c_t} = \beta(r_{t+1}+1)\frac{1}{c_{t+1}} \tag{6}$$

Labour supply:

$$\frac{\theta}{1-n_t} = \frac{(1-\tau_t^w)w_t}{c_t} \tag{7}$$

2 Firms

Problem:

$$\begin{cases} \pi_t = \sum_t \beta^{t+1} \frac{f_t}{c_{t+1}} \\ f_t = p_t\left[c_t\right] - w_t n_t \\ y_t = k_t^\alpha (a_t n_t)^{1-\alpha} \\ i_t = k_{t+1} - (1-\delta)k_t \\ c_t + i_t = y_t \end{cases} \tag{8}$$

Lagrangian:

$$\mathcal{L} = \sum_t \beta^{t+1} \frac{\left[k_t^\alpha (a_t n_t)^{1-\alpha} - k_{t+1} + (1-\delta)k_t\right] - w_t n_t}{c_{t+1}} \tag{9}$$

Derivatives:

$$\mathcal{L}'_{n_t} \propto (1-\alpha)\, k_t^{\alpha} (a_t)^{1-\alpha} n_t^{-\alpha} - w_t \tag{10}$$

$$\mathcal{L}'_{k_{t+1}} = \frac{-1}{c_{t+1}} + \beta \frac{\left[\alpha k_{t+1}^{\alpha-1} (a_{t+1} n_{t+1})^{1-\alpha} + (1-\delta)\right]}{c_{t+2}} \tag{11}$$

FOC:

$$\frac{1}{c_{t+1}} = \beta \frac{\left[\alpha k_{t+1}^{\alpha-1} (a_{t+1} n_{t+1})^{1-\alpha} + (1-\delta)\right]}{c_{t+2}} \tag{12}$$

Labour demand:

$$(1-\alpha)\, k_t^{\alpha} (a_t)^{1-\alpha} n_t^{-\alpha} = w_t \tag{13}$$

3 Government

Problem:

$$g_t = \tau_t^w w_t n_t + b_{t+1} - (r_t + 1) b_t \tag{14}$$

$$\tau_t^w = \overline{\tau} + \varepsilon_t^{\tau_t^w} \tag{15}$$

4 Shocks

$$a_t = e^{z_t} \tag{16}$$

$$z_t = \rho z_{t-1} + \varepsilon_t^z \tag{17}$$

5 Steady state

$a_t = 1$

Euler:

$$r = \frac{1}{\beta} - 1 \tag{18}$$

Labour market:

$$(1-\alpha)\, k^{\alpha} n^{-\alpha} = \frac{c\theta}{(1-n)(1-\overline{\tau})} \tag{19}$$

$$\phi_{SS} = \frac{(\frac{1}{\beta} - 1 + \delta)}{\alpha}^{(\frac{1}{1-\alpha})} \tag{20}$$

$$\Omega_{SS} = \phi_{SS}^{(1-alpha)} - \delta \tag{21}$$

$$\mu_{SS} = \frac{(1-\alpha)}{\theta(1-\overline{\tau})}\phi_{SS}^{(-alpha)} \tag{22}$$

$$k = \frac{\mu_{SS}}{\Omega_{SS} + \mu_{SS}\phi_{SS}} \tag{23}$$

$$c = \Omega_{SS}k \tag{24}$$

$$n = \phi_{SS}k \tag{25}$$

6 Dynare Code

```
var c k n y w z tau i;
varexo e_tau e_a;

parameters alpha beta theta delta tauHat rho r phiSS omegaSS muSS;

model;
1/c = beta /c(+1) * (alpha * k^(alpha - 1) *
exp((1-alpha)*z(+1)) * n(+1)^(1-alpha) + 1 - delta);
c * theta / ((1 - n)*(1- tau)) = (1-alpha) *
k(-1)^(alpha) * exp((1-alpha)*z) * n^(-alpha);
c + i = y;
y = k(-1)^(alpha)*(exp(z)* n)^(1-alpha);
i = k-(1-delta)*k(-1);
z = rho * z(-1) + e_a;
tau = tauHat + e_tau;
w = (1-alpha) * k(-1)^(alpha) * (exp(z)*n)^(1-alpha);
end;

alpha   = 0.4;
```

```
beta    = 0.99;
delta   = 0.02388;
theta     = 1.75;
tauHat = 0.13;
rho     = 0.95;
sigma   = (0.007/(1-alpha));
r = 1/0.99-1;
phiSS = ((1/beta-1+delta)/alpha)^(1/(1-alpha));
omegaSS = (phiSS)^(1-alpha)-delta;
muSS = (1-alpha)/(theta*(1-tauHat))*phiSS^(-alpha);

initval;
k = muSS/(omegaSS + muSS * phiSS);
c = omegaSS * k;
n = phiSS * k;
z = 0;
e_tau = 0;
e_a = 0;
end;
steady;


shocks;
var e_a = sigma^2;
end;


check;

stoch_simul(order = 1);
```