Field Review 3

April 15, 2018

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b)

proposition 1. *The polynomial $x^{p^n} - x$ is precisely the product of all the distinct irreducible polynomials in $F_p[x]$ of degree d where d runs through all divisors of n.*

Proof. Check the notes □

With this proposition, we can get the number of number of irreducible polynomials of degree n. Check the book in page 587-588

So here we have

$$\psi(6) = \frac{1}{6}[\mu(1)p^6 + \mu(2)p^3 + \mu(3)p^2 + \mu(6)p] \tag{1}$$

$$= \frac{1}{6}[p^6 - p^3 - p^2 + p] \tag{2}$$

$\Rightarrow \# \, of \, \beta = 6\psi(6) = p^6 - p^3 - p^2 + p$

Even though I believe this is the correct answer, if we look back to Q39, we should get the conclusion that the number of such $\beta = p^n - p^{n-1}$. Which is less than this, what's wrong with my previous proof?

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Since $x^{3470} - 1 = (x^{10} - 1)^{7^3}$, let $f(x) = (x^{10} - 1)^{7^3}$ Suppose f(x) splits in Field F_p^n

$$\Rightarrow f(x) | x^{p^n} - x \tag{3}$$

$$\tag{4}$$

if we can find the smallest n such that $f(x)|x^{p^n} - x$ then we can just say F_{p^n} is the splitting field of f(x) since in finite fields, fields with same degree are isomorphic. Then we can see that n = 4.

Notice I never assumed $x^{10} - 1$ is irreducible, so we don't need to worry about if 10 divides n

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Let p be prime and Let m and n be integers such that $m, n \geq 2$. Suppose that $f(x) \in F_p[x]$ is monic irreducible of degree n. Let d be the number of distinct irreducible polynomials in $F_{p^m}[x]$ occurring in a factorization of $f(x)$ in $F_{p^m}[x]$. Express d as a function of m and n. Justify your answer.