In the remainder of this text we will restrict ourselves to the membrane on the seven sphere. The case of the cosets with lower supersymmetry is currently under investigation.

To avoid any confusion, we would like to stress here that we call singleton field theory the flat space limit of the free field theory of [13, 14]. We point out that, since we are going to find a theory living on a three–dimensional Minkowski space rather than on $S^2 \times S^1$, we have no scalar mass term which was instead required in [13, 14] for conformal invariance. We will see that it can also be derived as the theory living on the solitonic M2 brane.

2 The supermembrane on $AdS_4 \times S^7$

We consider the space $AdS_4 \times S^7$, with given metric,

$$d\tilde{s}^2 = \rho^2 \left(-dt^2 + dx^2 + dw^2\right) + \frac{1}{\rho^2} d\rho^2 + 4d\Omega_7^2 \,, \tag{2.1}$$

with coordinates,

$$\begin{cases} \rho \in \,]0, \infty[\\ t, w, x \in \,]-\infty, \infty[\end{cases} ,$$

and $d\Omega_7^2$ is the metric of the seven sphere. This is the near horizon geometry of the $M2$ brane [15]. Moreover in [16] it was shown that this is a stable quantum vacuum of of the $11D$ supergravity. The AdS superspace is defined as the following coset

$$AdS^{(8|4)} = \frac{OSp(8|4)}{SO(1,3) \otimes SO(8)}$$

and it is spanned by the four coordinates of the AdS_4 manifold and by eight Majorana spinors (i.e. they have 32 real components) parametrizing the fermionic generators of the superalgebra.

This space can be described by means of a super solvable Lie algebra parametrization. To see what this solvable Lie algebra parametrization is, let's have a look at the familiar "Union Jack" root diagram of C_2, which is the complexification of $SO(2,3)$ shown in figure 1. The fermionic supercharges form a square weight diagram within this figure, and the supertranslation algebra is then simply that the anticommutator of two fermions is given by vector addition of the corresponding weights in the diagram. The diagram can in fact be seen as a projection of the full $Osp(8|4)$ root diagram, since the $SO(8)$ roots lie on a perpendicular hyperplane, and so on this diagram they would be at the centre.

It is now easy to see that the generators in the box $\{S_\pm, \sigma_\pm, \sigma_\perp, D\}$, form a super solvable Lie algebra with non–zero (anti) commutation relations,

$$\{S, S\} \sim \sigma \,, \qquad [D, S] \sim S \,, \qquad [D, \sigma] \sim \sigma \,, \tag{2.2}$$

since its second derivative is zero. The coset representative is now obtained by exponentiating the product of these generators with the coordinates of the coset space. We choose to write the coset representative as $L = L_F L_B$ with

$$\begin{aligned} L_B &= \exp(\rho D) \exp(\sqrt{2} x \sigma_\perp + t(\sigma_+ + \sigma_-) + w(\sigma_+ - \sigma_-)) \,, \\ L_F &= \exp\left(\theta_1^A S_1^A + \theta_2^A S_2^A\right) \,, \end{aligned} \tag{2.3}$$