4.6 Method of solution by Monte Carlo simulation

We can now solve the set of evolution equations (46)-(49) by means of a real-time simulation in full phase space using the computational methods of Ref. [38]. One starts from an initial phase-space density of partons, which in the case of a jet-initiating $q\bar{q}$ pair with invariant mass Q is

$$F_{q+\bar{q}}(t=0,\vec{r};x,p_\perp^2,p^2) \;=\; \delta^3(\vec{r})\,\delta(\vec{r}^2-Q^2)\,\delta(x-1)\,\delta(p_\perp^2)\,\delta(p^2-Q^2)\;, \tag{78}$$

where we choose the $q\bar{q}$ center-of-mass frame as our reference frame, and we sum over all quark flavors f weighted with a factor $w_f = e_f^2/n_f(Q^2)\sqrt{1-4m_f^2/Q^2}\,\theta\left(Q^2-4m_f^2\right)$, that accounts for the electromagnetic charge and mass threshold of the initial $q\bar{q}$ pair produced by the photon (or Z^0).

The parton shower development is then followed in a cascade simulation (for details see [24, 38]) in the center-of-mass frame: The system of particles is evolved in discrete time steps, here taken as $\Delta t = 0.01\ fm$, in coarse-grained 7-dimensional phase-space with cells $\Delta\Omega = \Delta^3 r\Delta^3 p\Delta M^2$. The partons propagate along classical trajectories until they interact, i.e., decay (branching) or recombine (cluster formation). Similarly, the formed clusters travel along straight lines until they decay into hadrons. The corresponding probabilities and time scales of interactions are sampled stochastically from the relevant probability distributions according to Secs. 4.3-4.5. At any time $t > 0$ we can extract the phase-space densities (38), $F_\alpha(t,\vec{r},\vec{p},p^2) = dN_\alpha(t)/d^3rd^4p$ of the particle species $\alpha = q, g, \chi, U$, and with these phase-space profiles we can then calculate, using the formulae (40) and (41), the associated local pressure $P(r)$, particle density $n(r)$ and energy density $\varepsilon(r)$ for each species individually, these being the quantities that characterize the macroscopic state of the system at t and $\vec{r}$.

With this concept, we can trace the space-time evolution of the parton-hadron conversion process self-consistently: at each time step, any "hot" off-shell parton is allowed to decay into "cooler" partons, with a probability determined by its virtuality and life time. Also in each step, every parton and its nearest spatial neighbor are considered as defining a fictious space-time bubble with invariant radius L, as defined by (65). By comparing the local pressure of partons $P_{qg}(t,\vec{r},L)$ with the pressure $P_\chi(t,\vec{r},L)$ that such a pre-hadronic