$$
\begin{aligned}
\gamma_{q\to gq}(z,\epsilon) &= C_F \left(\frac{1+(1-z)^2}{z+\epsilon}\right) \\
\gamma_{g\to gg}(z,\epsilon) &= 2\,C_A \left(\frac{z}{1-z+\epsilon} + \frac{1-z}{z+\epsilon} + z(1-z)\right) \\
\gamma_{g\to q\bar{q}}(z,\epsilon) &= \frac{1}{2}\left(z^2+(1-z)^2\right) \ ,
\end{aligned} \tag{56}
$$

where $C_F = (N_c^2-1)/(2N_c) = 4/3$, $C_A = N_c = 3$. In the denominator of $\gamma_{q\to qg}$ and $\gamma_{g\to gg}$, there appears the function

$$
\epsilon \;=\; \frac{p'^{\,2} n^2}{4(p\cdot n)^2} \;\propto\; \frac{p_\perp^2}{p_z^2} \ , \tag{57}
$$

where p (p') is the momentum of the mother (daughter) parton and $p_\perp$ the relative transverse momentum of the daughter partons with respect to the mother. It arises here as a consequence of the constraint equations (35) which determine spatial uncertainty associated with the off-shellness of the partons. It effectively cuts off small-angle gluon emission by modifying the free gluon propagator $\propto z_g^{-1}$ to the form $(z_g+\epsilon)^{-1}$ (where $z_g = z$ or $z_g = 1-z$) when $p_\perp/p_z = O(1)$, that is, in branching processes with large space-time uncertainty. This ensures that the two daughter partons can be resolved as individual quanta only if they are separated sufficiently by $\Delta r_\perp \propto 1/p_\perp$ in position space, in accord with the uncertainty principle. Note that ϵ can be neglected in the terms $\propto (z_g+\epsilon)^{-1}$ in (56) for energetic gluon emission ($z_g \to 1$), but is essential in the soft regime ($z_g \to 0$). The effect of ϵ has been shown [29, 35] to result in a natural regularization of the infra-red-divergent behaviour of the branching kernels (56), due to destructive gluon interference which becomes complete in the limit $z_g \to 0$.

4.4 Parton cluster (bubble) formation

The operators $\hat{B}, \hat{C}$ in eqs. (46)-(48) represent the changes of the phase-space densities due to recombinations of two partons at r and r' to color-neutral clusters, or bubbles that arise as non-trivial structures in the vacuum because of the confinement mechanism. Their formation is determined, in analogy to the finite-temperature QCD phase transition [14], by the probability for tunnelling through the potential barrier of $\mathcal{V}$ between $\chi = 0$ and $\chi = \chi_c$ in Fig. 2, which separates perturbative and non-perturbative vacua. The associated rate of bubble formation around $L = L_c$ is generically given by an exponential probability distribution [36, 37],

$$
\pi(L) \;=\; \pi_0(L)\,\left(1 \;-\; \exp[-\Delta F\, L]\right) \ , \tag{58}
$$