For instance, imagine the creation of a $q\bar{q}$ pair with invariant mass $Q \simeq L^{-1} \gg L_c^{-1}$ by a time-like virtual photon from e^+e^- annihilation. The insertion of such a localized excitation ("hot spot") modifies the vacuum and we assume that the corresponding change in the action integral $S \equiv \int d^4r \mathcal{L}[\chi, U, U^\dagger]$ in (6) can be evaluated sufficiently accurately to second order as

$$\begin{aligned}
\delta S &= \frac{1}{2} \int d^4r \left\{ \left\langle 0 \left| \frac{\delta^2 \mathcal{L}[\chi, U, U^\dagger]}{\delta F_{\mu\nu,a} \delta F_b^{\mu\nu}} \right| 0 \right\rangle F_{\mu\nu,a} F_b^{\mu\nu} \, \delta_{ab} \; + \; \left\langle 0 \left| \frac{\delta^2 \mathcal{L}[\chi, U, U^\dagger]}{\delta \psi_i \delta \overline{\psi}_j} \right| 0 \right\rangle \psi_i \overline{\psi}_j \, \delta_{ij} \right\} \\
&= \int d^4r \left\{ -\frac{\kappa_L(\chi)}{4} \; F_{\mu\nu,a} F_a^{\mu\nu} \; - \; \mu_L(\chi) \overline{\psi}_i \, \psi_i \right\} \; , \qquad (13)
\end{aligned}$$

where κ_L and μ_L refer to the appropriate *vev*'s. Note that this change δS in the action preserves local gauge invariance. We also remark that this ansatz implicitly assumes that the elementary gluon ($F_{\mu\nu}$) and quark fields ($\psi, \overline{\psi}$) couple directly only to the scalar field χ, but not to the pseudoscalar field U. The dynamics of U is solely driven by its coupling to χ through the potential $V(\chi, U)$, eq. (11).

On the other hand, we know that the short-range properties at $L \ll L_c$ of our $q\bar{q}$ excitation are not affected by the long-range correlations. Thus, here we can use (5) with perturbative methods, since the quanta are asymptotically free and $\langle \chi \rangle = 0 = \langle U + U^\dagger \rangle$. Thus we can combine (5) and the effect of (13) by adding to $\mathcal{L}[A^\mu, \psi, \overline{\psi}]$ and $\mathcal{L}[\chi, U, U^\dagger]$ the following contribution that carries an explicit scale- (L-)dependence:

$$\mathcal{L}_L[A^\mu, \psi, \overline{\psi}, \chi] \; = \; \int d^4r \left\{ \frac{1}{4} \Big(1 - \kappa_L(\chi) \Big) \; F_{\mu\nu,a} F_a^{\mu\nu} \; - \; \mu_L(\chi) \, \overline{\psi}_i \, \psi_i \; - \; \Big(1 - \kappa_L(\chi) \Big) \; \xi_a(A) \right\} \; , \qquad (14)$$

where the third term in the integrand is necessary to maintain local gauge invariance. It remains to specify the form of the functions $\kappa_L(\chi)$ and $\mu_L(\chi)$. Since κ_L has to satisfy the boundary conditions [15]

$$\kappa_L(0) \; = \; 1 \;\; , \qquad\qquad \kappa_L(\chi_0) \; = \; 0 \; , \qquad (15)$$

and is constrained to be a Lorentz-invariant color-singlet function of scale dimension zero, a minimal possibility is

$$\kappa_L(\chi) \; = \; 1 \; - \; \left(\frac{L \, \chi}{L_0 \, \chi_0} \right)^2 \; . \qquad (16)$$

It turns out that the particular form of $\kappa_L(\chi)$ is not crucial as long as the properties (15) are satisfied [17], because parton-hadron conversion is quite rapid, as we see later, being related