This parametrization of the manifold gives rise to the metric (2.1).

To complete the parametrization of the target superspace one still has to define the vielbeins of the seven sphere. To do so, we call $y^{\hat{a}}$ the seven coordinates of the sphere and write in stereographic projection coordinates:

$$E^{\hat{a}} = -\delta^{\hat{a}}{}_{\hat{m}} \frac{dy^{\hat{m}}}{1+y^2}. \tag{2.8}$$

Beside the κ symmetry we also fix the three–dimensional world–volume diffeomorphisms imposing the static gauge choice. To obtain static solutions we have to identify

$$\xi^I \equiv (-t, w, x), \tag{2.9}$$

where $I = 0, 1, 2$, is the curved index of the brane.

For the action of the membrane we get:

$$S = 2\int \sqrt{-\det(h_{IJ})}\, d^3\xi + 4!4! \int B\,, \tag{2.10}$$

with[3]

$$\begin{aligned} h_{IJ}(\xi) &= \frac{1}{4\rho^2}\partial_I\rho\partial_J\rho + \frac{1}{(1+y^2)^2}\partial_I y^{\hat{a}}\partial_J y_{\hat{a}} \\ &\quad + \rho^2\left(\eta_{IJ} - 2e\bar{\theta}^A\gamma^i\partial_{(J}\theta^A\delta_{i\,I)} + \bar{\theta}^A\gamma^i\partial_I\theta^A\ \bar{\theta}^B\gamma_i\partial_J\theta^B\right). \end{aligned} \tag{2.11}$$

The expression for B is given by

$$\begin{aligned} B &= \frac{d^3\xi}{4!4!}\Big[\epsilon^{IJK}\rho^3(\delta_I{}^i - \bar{\theta}^A\gamma^i\partial_I\theta^A)(\delta_J{}^j - \bar{\theta}^A\gamma^j\partial_J\theta^A)(\delta_K{}^k - \bar{\theta}^A\gamma^k\partial_K\theta^A)\frac{\epsilon_{ijk}}{3} + \\ &\quad - \frac{1}{(1+y^2)}\epsilon^{IJK}\partial_I y^{\hat{a}}\ \eta_A\tau_{\hat{a}}\eta_B\ \rho\partial_J\bar{\theta}^A\partial_K\theta^B\Big]. \end{aligned} \tag{2.12}$$

The isometries of (2) can be calculated explicitly in this parametrization. As was noted in [1, 17], they realize conformal symmetry on the brane. For example, for the dilatation one finds,

$$\delta\rho = \rho, \tag{2.13}$$

$$\delta\xi^I = -\xi^I, \tag{2.14}$$

$$\delta\theta^A_\alpha = -\frac{1}{2}\theta^A_\alpha\,. \tag{2.15}$$

3 The singleton action from the supermembrane

In order to retrieve the $Osp(8|4)$ singleton action we now have to do the following. First we consider a classical solution of the action (2.10),

$$\xi^I \equiv (-t, w, x), \quad \partial_I y^{\hat{a}} = 0, \quad \theta^A = 0, \quad \rho = \bar{\rho} = \text{const}\,, \tag{3.1}$$

[3] where $i = 0, 1, 2$, $\epsilon_{012} = 1$ and $\eta_{IJ} = diag\{-++\}$. The seven–dimensional gamma matrices $\tau^{\hat{a}}$ are the generators of the $SO(7)$ Clifford algebra and η_A are the eight real killing spinors on the seven sphere. For details on the conventions see [5].