double counting, because the introduction of the scale L and the behaviour of the L-dependent coupling functions $\kappa_L(\chi)$ and $\mu_L(\chi)$ truncate the dynamics of the elementary fields A^μ, ψ to the short-distance, high-momentum regime ($L \ll L_c$), whereas the effective description in terms of the collective fields χ, U covers the complementary long-range, low-energy domain ($L \gg L_c$). Accordingly, a quark or gluon is either considered a colored short-range fluctuation (parton) or it is part of a complex bound state (hadron), but not both.

c) The presence of the non-linear coupling function $\kappa_L(\chi)$, which also enters $\mu_L(\chi)$ via (17), means that the sum $\Delta\mathcal{L} \equiv \mathcal{L}[\chi, U] + \mathcal{L}[A^\mu, \psi, \overline{\psi}, \chi]$ in (20) is non-renormalizable. However, there is no need for explicit renormalization, because the composite fields χ and U are already interpreted as effective degrees of freedom with loop corrections implicitly included in $\Delta\mathcal{L}$, and it would be double counting to add them again. Moreover, as mentioned in item b) above, the low-energy domain of $\mathcal{L}[\chi, U]$ is by construction bounded from above by the onset of the high-energy regime described by $\mathcal{L}[A^\mu, \psi, \overline{\psi}]$. The characteristic scale L_c that separates the two domains, therefore, provides an 'ultra-violet' cut-off for $\mathcal{L}[\chi, U]$, and at the same time an 'infra-red' cut-off for $\mathcal{L}[A^\mu, \psi, \overline{\psi}]$.

2.5 Analogies with QCD at finite temperature

We close this Section with pointing out some immediate phenomenological implications: the particular form (11) of the potential V as a function of χ, as well as the functions κ_L and μ_L that couple short- and long-range regimes, play a central role in dynamical processes where the scale L changes with time. The effect of (14) can be interpreted as a scale-(L-)dependent modification δV, which adds to the (L-independent) potential V, eq. (11),

$$\mathcal{V}(L) \;:=\; V(\chi, U) \;+\; \delta V(L, \chi)\;, \tag{21}$$

$$\delta V(L, \chi) \;=\; \frac{(L\chi)^2}{4\,(L_0\chi_0)^2}\, F_{\mu\nu,a} F_a^{\mu\nu} \;+\; \frac{\mu_0\,(L\chi)^2}{(L_0\chi_0)^2 \,-\, (L\chi)^2}\, \overline{\psi}_i\, \psi_i\;. \tag{22}$$

where we used eqs. (16) and (17). We emphasize again that L is not an input parameter, but rather is determined by the space-time dependent separation of the colored quanta. In Sec. 4, we will specify how to determine the variation of the variable L.

In view of (22), one has $\delta V \propto O(L^2)$, therefore it is suggestive that the variable L plays a similar role as the temperature T in finite-temperature QCD, where the correction