with $\triangle = r^2 - 2mr + a^2$ and $\Sigma = r^2 + a^2 \cos^2 \theta$. The Kerr-Newman black hole linearised with respect to its electric charge q is described by metric (1) plus an electromagnetic test field having the components

$$A_t = \frac{qr}{\Sigma} \quad A_r = 0 \quad A_\theta = 0 \quad A_\varphi = -\frac{qar \sin^2 \theta}{\Sigma}. \tag{2}$$

The area of the Kerr-Newman black hole has the expression

$$\mathcal{A}(m, a, q) = 4\pi \left[\left(m + \sqrt{m^2 - a^2 - q^2} \right)^2 + a^2 \right] \tag{3}$$

which reduces to

$$\mathcal{A}(m, a, q) \approx 4\pi \left[2m^2 + 2m\sqrt{m^2 - a^2} - \frac{q^2 m}{\sqrt{m^2 - a^2}} - q^2 \right] \tag{4}$$

for a Kerr-Newman black hole linearised with respect to its electric charge q. In thermodynamics of the black hole, the entropy S_{BH} is given by

$$S_{BH}(m, j, q) = \frac{1}{4} \mathcal{A}(m, j/m, q). \tag{5}$$

in terms of the thermodynamical variables m, j and q.

On the symmetry axis of metric (1), outside the outer horizon $r_+ = m + \sqrt{m^2 - a^2}$, we consider a charged object with a mass μ, an electric charge e, an entropy S and a radius R whose the own gravitational field is negligeable and the electromagnetic field generated by the charge e is a test field. By making use of a quasi-static assumption, we restrict ourselves to the case where the charged object is at rest. We suppose that its total energy $\mathcal{E}$ with respect to a stationary observer at infinity coincides with the one of a massive point charge located at $r = r_0$ and $\theta = 0$ with $r_0 > r_+$. Obviously, the dimension R of this object is taken as the proper length along the symmetry axis of metric (1). This proper length ℓ from the outer horizon to the position r_0 for $\theta = 0$ has the expression

$$\ell = \int_{r_+}^{r_0} \frac{\sqrt{r^2 + a^2}}{\sqrt{r^2 - 2mr + a^2}} dr. \tag{6}$$

The total energy $\mathcal{E}$ of a massive point charge is the sum of the energy W_{mass} of the mass μ, the electrostatic energy W_{ext} of the charge e in the exterior electromagnetic field (2) and the electrostatic self-energy W_{self} of the charge e. The electrostatic self-force f^i_{self} exerted on the point charge by its self-field has the expression [10, 11]

$$f^i_{self} = \frac{e^2 m r_0}{(r_0^2 + a^2)^2} \delta^i_1 \tag{7}$$

in Fermi coordinates at the position of the point charge. We can easily determined W_{self} so that it yields self-force (7); we find

$$W_{self} = \frac{1}{2} \frac{e^2 m}{r_0^2 + a^2}. \tag{8}$$