G/H Freund–Rubin compactifications. Also, the number of preserved supersymmetries N are known.

Of all these coset spaces, the case of the round and squashed seven spheres are the best known (corresponding to $N = 8$ [8] and $N = 1$ near horizon supergravities) but in the eighties the Kaluza–Klein spectra have been systematically derived also for all the other solutions using the technique of harmonic expansions [9]. The organization of these spectra in supermultiplets is known not only for the round S^7 [10] but also for the case of supersymmetric M^{pqr} spaces

$$M^{pqr} \equiv \frac{SU(3) \times SU(2) \times U(1)}{SU(2) \times U(1) \times U(1)} ,$$

where $p, q, r \in \mathbb{Z}$ define the embedding of the $U(1)^2$ factor of H in G. For $p = q =$ odd we have $N = 2$, in all the other (non supersymmetric) cases we have $N = 0$. The $N = 2$ multiplet structure was obtained in [11]. At present a group in Torino [12] is doing the harmonic analysis on the so–called Stiefel and N^{010} manifolds as well.

Since much is known and will be known about these spaces, they constitute an excellent laboratory to make direct checks of the holographic correspondence.

Let's now clarify the qualitative difference between the seven sphere and the other G/H spaces. For a G/H space that admits N supersymmetries, the isometry group is factorized as follows:

$$G = G' \otimes SO(N) ,$$

where $SO(N)$ is the R–symmetry of the orthosymplectic algebra $Osp(N|4)$, while the factor G' is the gauge–group of the vector multiplets. Correspondingly the three–dimensional world–volume action of the CFT must have the following superconformal symmetry:

$$Osp(N|4) \times G' , \tag{1.3}$$

where G' is a *flavor group*. In the maximal case the harmonics on S^7 are labeled only by R–symmetry representations while in the lower susy case they depend both on R labels and on representations of the gauge/flavour group G'. The structure of $Osp(8|4)$ supermultiplets determines completely their R–symmetry representation content so that the harmonic analysis becomes superfluous in this case. The eigenvalues of the internal laplacians which determine *the Kaluza–Klein masses* of the $Osp(8|4)$ graviton multiplets or, in the conformal reinterpretation of the theory, the *conformal weights* of the corresponding primary operators, are already fixed by supersymmetry and need not be calculated. In this sense the correspondence (1.2) is somewhat trivial in the maximal susy case: once the superconformal algebra has been identified with the super-isometry group $Osp(8|4)$ the correspondence between conformal weights and Kaluza–Klein masses is simply guaranteed by representation theory of the superalgebra. On the other hand in the lower susy case the structure of the $Osp(N|4)$ supermultiplets fixes only their content in $SO(N)$ representations while the Kaluza–Klein masses, calculated through harmonic analysis depend also on G' labels. In this case the holographic correspondence yields a definite prediction on the conformal weights that, as far as superconformal symmetry is concerned would be arbitrary. Explicit verification of these predictions would provide a much more stringent proof of the holographic correspondence and yield a deeper insight in its inner working. However in order to set up such a direct verification one has to solve a problem that was left open in Kaluza–Klein supergravity: the singleton problem.