where $B = \nabla_{\tilde{x}} A_{\tilde{y}} - \nabla_{\tilde{y}} A_{\tilde{x}}$ and $A_{\tilde{x}} = A_x \cos\varphi - A_y \sin\varphi, A_{\tilde{y}} = A_x \sin\varphi + A_y \cos\varphi$. The boundary conditions at the interface are

$$\left(\frac{\partial F}{\partial(\nabla_{\tilde{x}} \Delta_i(\tilde{x}))} + \frac{\partial F_I}{\partial \Delta_i(\tilde{x})}\right)\Big|_{\tilde{x}=0} = 0 \quad (i = d, s) \qquad (4.4)$$

and

$$B(\tilde{x} = 0) = 0. \qquad (4.5)$$

Now we analyze the instability to a $\mathcal{T}$-violating state at the temperature T^*. We consider a junctions as described in Fig.2, and assume that on both sides the superconductors have the same properties, in particular, the same T_{cd}. At $T = T_c$, Δ_d and Δ_0 (d-wave) get finite, and Δ_s is induced simultaneously near the interface due to the t_s and g_{ds} term. For $\varphi = \pi/4$, t_d and g_{ds} vanish, so only the $\gamma_2$term determines the relative phase ϕ_{ds}. Since γ_2 (> 0) favors $\phi_{ds} = \pm\pi/2$, $\mathcal{T}$-breaking occurs. Thus for $\varphi = \pi/4$, we have $T^* = T_c$, namely, $\mathcal{T}$-violation should occur at T_c, the bulk superconducting transition temperature, irrespective of any other details.

Numerical calculation for the GL equations confirms this argument. We have solved GL equations (4.2) and (4.3) under the boundary conditions, eq. (4.4) and (4.5). In Fig.4 we show the spatial dependences of Δ_d, Δ_s and ϕ_{ds} for $\delta = 0.15$, i.e., so-called optimum doping. The results for other values of δ is qualitatively the same. For $\varphi = \pi/4$, ϕ_{ds} always takes a value $\pi/2$ or $-\pi/2$, once T becomes lower than T_c. As φ moves away from $\pi/4$, t_d and g_{ds} become finite and compete with γ_2, which favors $\mathcal{T}$-breaking states. Since t_d grows very rapidly as φ moves away from $\pi/4$, T^* drastically decreases as a function of $|\varphi - \pi/4|$ (see Fig.5.).

However, we should note here that at very low temperature the region of $\mathcal{T}$-violation can be much larger because of the following reason. If T is lower than T_{cs}, there is a chance to have finite Δ_s without using tunneling terms. In this case both t_d and t_s can be smaller than γ_2, with keeping Δ_s finite[21]. Unfortunately T_{cs} derived from the $t-J$ model is quite low (of the order of $10^{-3}J$ corresponding to $T_{cs}/T_{cd} \sim 10^{-2}$), so the region $T < T_{cs}$ is not accessable by the GL theory.

It has been shown that in a $\mathcal{T}$-breaking state the surface current along the interface can flow.[10, 22, 23] The condition for the minimum free energy requires the vanishing of the current normal to the junction (i.e., the first equation of (4.3)). This condition in turn leads to a finite current along the junction which induces a magnetic field. In Fig.6 the spatial distributions of the surface current and the magnetic field are shown. The extension of the current and the magnetic field are of the order of the penetration depth λ. Note that no net current is present, i.e. the integrated current vanishes[24],

$$\int_0^L d\tilde{x} J_{\tilde{y}}(\tilde{x}) = \frac{1}{4\pi}\big[B(\tilde{x}=0) - B(\tilde{x}=L)\big] = 0 \qquad (4.6)$$

where L is the length of the system ($L \gg \lambda$), and we have used eq.(4.5) .

§5. Case of S/D-junctions

So far we have considered only D/D-junctions, where both sides are the same d-wave superconductors. The S/D-junction (where the left side is replaced by a different superconductor with isotropic s-wave symmetry) can be treated similarly.

We consider a conventional superconductor for the left side of the junction. We take the band width, the chemical potential and the magnitude of the order parameter in (L) to be the same as in (R) for simplicity. Our aim here is to compare the qualitative features of D/D- and S/D-junctions, but not the quantitative comparison.

The surface GL energy can be calculated in a similar way as in § 2. The modified expressions are simply given by the replacement

$$\omega_d(k) \to 1 \qquad (5.1)$$

in eq.(2.7) for t_d and t_s. For the S/D-junction $|t_s|$ is larger than $|t_d|$ (for φ not so far apart from $\pi/4$), in contrast to the case of D/D-junctions. (Fig. 7) However, the value of $|t_s|$ in this case is much smaller than that of $|t_d|$ for D/D-junction. This is due to the same reason as we have $|t_d| \gg |t_s|$ for the D/D-junction: the factor $(\cos k_x + \cos k_y)$ is always small on the Fermi surface, and so $|t_s|$ is reduced even if the OPs of both sides have s-wave symmetry.

The phase diagram of surface states for S/D-junction is shown as a solid line in Fig.5. In Fig.8 the φ-dependence of the relative phases between Δ_d and Δ_s (Δ_0), ϕ_{ds} (ϕ_0) is also shown. We find that in the phase diagram the region of $\mathcal{T}$-breaking states is larger than for the D/D-junction. This is the case even when we assume a rather small value of $|\Delta_0|/|\Delta_d^{bulk}|$. In S/D-junction $|\Delta_s|$ can be sizable due to large $|t_s|$, and then the γ_2-term (fourth order) which favors $\mathcal{T}$-violating states can compete with the second order terms (t_d and g_{ds}) even when φ is not so close to $\pi/4$. In the case of D/D-junction, however, t_d and g_{ds} become dominant once φ moves away from $\pi/4$, leading to a very limited region of $\mathcal{T}$-violating states.

§6. Conclusions

We have discussed the S/D- and D/D-interface states using a GL-formulation derived from the slave-boson mean-field approximation for the $t-J$ model. The $\mathcal{T}$-violating interface state was found in both cases for certain orientations in agreement with previous phenomenological studies.

In addition we found that the $\mathcal{T}$-violating state is more likely to occur on an S/D- than on a D/D-interface. This distinction is mainly due to the fact that the Josephson coupling to a d-wave superconductor is generally weaker, because of the angular phase structure of the pair wave function which leads to destructive interference in the tunneling. Furthermore, the shape of the Fermi surface plays an important role, in particular, for the extended S-wave state.

Within the GL theory the range of angles φ where the $\mathcal{T}$-violating state can appear is rather small for the D/D-junction. However, it should be noted that for tem-