mer (latter) is defined by $\Delta_d(i) = (\Delta_{i,i+x} - \Delta_{i,i+y})/2$ ($\Delta_s(i) = (\Delta_{i,i+x} + \Delta_{i,i+y})/2$). Following a standard procedure we expand the free energy with respect to Δ_d and Δ_s.[17] The resulting GL energy is given after taking a continuum limit

$$\begin{aligned} F = & \int d^3x[\sum_{j=d,s}\{\tilde{a}_j(T)|\Delta_j|^2 + \beta_d|\Delta_j|^4 + K_j|\mathbf{D}\Delta_j|^2\} \\ & +\gamma_1|\Delta_d|^2|\Delta_s|^2 + \tfrac{1}{2}\gamma_2(\Delta_d^{*2}\Delta_s^2 + \Delta_d^2\Delta_s^{*2}) \\ & +\tilde{K}\{(D_x\Delta_d)^*(D_x\Delta_s) - (D_y\Delta_d)^*(D_y\Delta_s) \\ & +c.c.\} + \frac{1}{8\pi}(\nabla\times\mathbf{A})^2] \end{aligned} \tag{2.2}$$

where $\vec{D} = \nabla - i(2\pi/\Phi_0)\vec{A}$ with $\Phi_0(= hc/2e)$ being the standard flux quantum. The coefficients in F are given as

$$\begin{aligned} \alpha_j = & \frac{3J}{4}\big(1 - \frac{3J}{8N}\sum_k \frac{\tanh(\xi_k/2T)}{\xi_k}\omega_j^2(k)\big) \\ \beta_j = & \Big(\frac{3J}{4}\Big)^4\frac{1}{N}\sum_k I(\xi_k)\omega_j^4(k) \\ \gamma = & \Big(\frac{3J}{4}\Big)^4\frac{1}{N}\sum_k I(\xi_k)\omega_d^2(k)\omega_s^2(k) \\ K_j = & W_F^2\frac{9J^2}{32N}\sum_k\frac{f"(\xi_k)}{\xi_k}\sin^2 k_x\omega_j^2(k) \\ \tilde{K} = & W_F^2\frac{9J^2}{32N}\sum_k\frac{f"(\xi_k)}{\xi_k}\sin^2 k_x\omega_d(k)\omega_s(k) \end{aligned} \tag{2.3}$$

where $j = d$ or s, $\gamma_1 = 2\gamma$, $\gamma_2 = \gamma/2$, $\omega_d(k) = \cos k_x - \cos k_y$ and $\omega_s(k) = \cos k_x + \cos k_y$. Here

$$\begin{aligned} I(\xi_k) = & \frac{1}{2\xi_k^2}\Big[f'(\xi_k) + \frac{1}{2\xi_k}\tanh(\frac{\xi_k}{2T})\Big] \\ W_F = & t\delta + \frac{3}{8}J\chi_F \end{aligned} \tag{2.4}$$

and $f(\xi_k)$ is the Fermi distribution function.

The surface energy at the junction is calculated under the assumption of a specularly reflecting surface. We consider a planar interface parallel to the c-axis as shown in Fig.2. In Fig.2 the left and the right hand side are the same d-wave superconductors described by the $t-J$ model. Here the crystalline a-axis of the left hand side (L) is normal to the interface, while that in the right hand side (R) is taken as a free parameter, denoted as φ ($0 \le \varphi \le \pi$). (We can treat an S/D-junction, where the left side is an s-wave superconductor, in a similar way. We consider this case in § 5.) In this configuration the important effects are associated mainly with the OP on the right hand side, so that we will simply represent the left hand side by a single d-wave order parameter Δ_0 only.

The transmission and the reflection of electrons at the interface (I) may be described by the following Hamiltonian,

$$\begin{aligned} H_I = & \sum_\sigma\sum_{k,p}\big[t_{kp}\big(f_{k\sigma}^{\dagger(L)}f_{p\sigma}^{(R)} + f_{p\sigma}^{\dagger(R)}f_{k\sigma}^{(L)}\big) \\ + & \ r_{kp}\big(f_{k\sigma}^{\dagger(R)}f_{p\sigma}^{(R)} + f_{p\sigma}^{\dagger(R)}f_{k\sigma}^{(R)}\big)\big] \end{aligned} \tag{2.5}$$

where $f_{k\sigma}^{(L)}$ ($f_{k\sigma}^{(R)}$) is the spinon operator for the left (right) side, and the matrix elements for tunneling (t_{kp}) and the reflection (r_{kp}) are taken to be real. Treating H_I in a second-order perturbation theory we get the surface free energy F_I to lowest order in Δ's,

$$\begin{aligned} F_I = & \int_I dS\big[\sum_{i,j=\{d,s\}} g_{ij}(\varphi)\Delta_i^*\Delta_j \\ + & \sum_{i=\{d,s\}} t_i(\varphi)(\Delta_0^*\Delta_i + \Delta_0\Delta_i^*)\big]. \end{aligned} \tag{2.6}$$

The first term originates from the reflection of the Cooper pairs at the interface and the second term represents the coupling between the two sides ($g_{ij} = g_{ji}$). For the D/D-junction composed of the same superconductors, the coefficients in eq.(2.6) are given as

$$\begin{aligned} t_d = & \Big(\frac{3J}{4}\Big)^2\sum_{kp} t_{kp}^2 J_1(\xi_k,\xi_p)\omega_d(k)\omega_d(p) \\ t_s = & \Big(\frac{3J}{4}\Big)^2\sum_{kp} t_{kp}^2 J_1(\xi_k,\xi_p)\omega_d(k)\omega_s(p) \\ g_d = & \Big(\frac{3J}{4}\Big)^2\sum_{kp}\big[r_{kp}^2 J_1(\xi_k,\xi_p)\omega_d(k)\omega_d(p) \\ + & \ (r_{kp}^2 + t_{kp}^2)J_2(\xi_k,\xi_p)\omega_d(k)^2\big] \\ g_s = & \Big(\frac{3J}{4}\Big)^2\sum_{kp}\big[r_{kp}^2 J_1(\xi_k,\xi_p)\omega_s(k)\omega_s(p) \\ + & \ (r_{kp}^2 + t_{kp}^2)J_2(\xi_k,\xi_p)\omega_s(k)^2\big] \\ g_{ds} = & \Big(\frac{3J}{4}\Big)^2\sum_{kp}\big[r_{kp}^2 J_1(\xi_k,\xi_p)\omega_s(k)\omega_d(p) \\ + & \ (r_{kp}^2 + t_{kp}^2)J_2(\xi_k,\xi_p)\omega_d(k)\omega_s(k)\big] \end{aligned} \tag{2.7}$$

with

$$\begin{aligned} J_1(\xi_k,\xi_p) = & \frac{1}{\xi_k^2 - \xi_p^2}\big(\frac{\tanh(\frac{\xi_k}{2T})}{2\xi_k} - \frac{\tanh(\frac{\xi_p}{2T})}{2\xi_p}\big) \\ J_2(\xi_k,\xi_p) = & \frac{2\xi_k}{\xi_k - \xi_p}I(\xi_k) \\ + & \ \frac{\xi_p}{(\xi_k+\xi_p)(\xi_k-\xi_p)^2}\big(\frac{\tanh(\frac{\xi_k}{2T})}{\xi_k} - \frac{\tanh(\frac{\xi_p}{2T})}{\xi_p}\big). \end{aligned} \tag{2.8}$$

The J_1 terms represent the usual tunneling and the reflection processes of a Cooper pair. On the other hand, the J_2 terms give different types of reflection processes where one of the particle consisting of a Cooper is reflected at the interface, while the other one tunnels to the opposite side (see in the Appendix for the derivation of the surface terms and the interpretation of J_2).

Here we follow the method of Ref.[18,19] in taking the angular dependences of t_{kp} and r_{kp}, which are consistent