2. FIELD THEORY FRAMEWORK

As explained in the introduction, the vacuum state in QCD may be pictured as a color-dielectric medium characterized by long-range order parameters. Consider the vacuum expectation values (*vev*'s) of the normal-ordered products of color-singlet and Lorentz scalar (pseudoscalar) functions χ (U) [8, 15]

$$\langle\, 0 \,|\, \chi(F_a^{\mu\nu}) \,|\, 0 \,\rangle \quad ; \qquad\qquad \langle\, 0 \,|\, U(\psi_i, \overline{\psi}_j) \,|\, 0 \,\rangle \;, \tag{1}$$

where $F_a^{\mu\nu}$ is the usual $SU(3)$ field strength tensor, ψ, $\overline{\psi}$ the quark fields, and $\chi(0)$, $U(0)$ is set to be zero. For instance, χ can be $f_{abc}F^a_{\mu\nu}F^b_{\nu\lambda}F^c_{\lambda\mu}$ or $(F^a_{\mu\nu}F^a_{\mu\nu})^2$, or other combinations. Similarly, U may be composed of $Tr[(\overline{\psi}_i\psi_j)^2]$, $Tr[(\overline{\psi}_i\overline{\psi_j}\psi_k\psi_l)^2]$, etc.. These *vev*'s are physical quantities that characterize the structure of the QCD vacuum [16] and are related to the measurable gluon and quark condensates, respectively.

Clearly, if we take the long wavelength limit $L \rightarrow \infty$ and simultanously let the coupling strength g_s among the fields tend to zero, we have in general the non-comutativity of the double limits

$$\begin{aligned} \lim_{L\to\infty}\lim_{g_s\to 0} \langle\, 0 \,|\, \chi(F_a^{\mu\nu}) \,|\, 0 \,\rangle \;=\; 0 \quad &\neq \quad \lim_{g_s\to 0}\;\lim_{L\to\infty} \langle\, 0 \,|\, \chi(F_a^{\mu\nu}) \,|\, 0 \,\rangle \\ \lim_{L\to\infty}\lim_{g_s\to 0} \langle\, 0 \,|\, U(\psi_i, \overline{\psi}_j) \,|\, 0 \,\rangle \;=\; 0 \quad &\neq \quad \lim_{g_s\to 0}\;\lim_{L\to\infty} \langle\, 0 \,|\, U(\psi_i, \overline{\psi}_j) \,|\, 0 \,\rangle \;\;, \end{aligned} \tag{2}$$

with $\langle 0|F_a^{\mu\nu}|0\rangle = 0 = \langle 0|\psi_i|0\rangle$. Eq. (2) is a pure quantum phenomenon and a typical property of phase transitions. It implies that there is long-range order in the vacuum which can be characterized by the operator functions χ and U.

In order to embody this concept into a field theory formulation, let us define the *distance measure* L for the space-time separation between two points r and r' ($r^\mu = (t, \vec{r})$):

$$L \;:=\; \sqrt{(r - r')_\mu (r - r')^\mu} \;, \tag{3}$$

and introduce a *characteristic length scale* L_c that separates short distance ($L \ll L_c$) and long range ($L \gtrsim L_c$) physics in QCD. The scale L_c can be associated with the confinement length of the order of a hadron radius, as we will specify more precisely later.