where we have abbreviated $F_{q+\bar{q}} \equiv F_q + F_{\bar{q}}$, which in the present case is equal to $2F_q$, and the $\sum_f$ means summing over quark flavors $f = u, d, s, \ldots$. The left-hand sides of these equations describe the propagation of the particles in the presence of the mean field, whereas the terms on the right-hand sides represent the effects of particle creation, annihilation, and recombination. The operator $\hat{\mathcal{K}}$ on the left-hand sides is given by

$$\hat{\mathcal{K}}\, F_\alpha \;\equiv\; \left[p_\mu\, \partial_r^\mu \;+\; (\overline{M}_\alpha\, \partial_r^\mu \overline{M}_\alpha)\, \partial_\mu^p \right] F_\alpha \;, \tag{50}$$

with the first term describing the free propagation and the second term reflecting the effect of the mean field (Fig. 6a). The functions $\overline{M}_\alpha$ are the mean-field parts of the self energies M_α defined in (39). On the right-hand side the quantities $\hat{I}_{a_1,a_2,..}^{b_1,b_2,..}$ (where $\hat{I} = \hat{A}, \hat{B}, ..$) represent integral operators that incorporate the effects of the self energies in terms of the relevant amplitudes for the various processes $a_1, a_2, .. \rightarrow b_1, b_2, ..$, and that act on the phase-space densities to their right (Fig. 6b). These coupled equations reflect a probabilistic interpretation of the evolution in terms of successive branching and recombination processes, in which the changes of the particle distributions on the left-hand sides are governed by the balance of gain (+) and loss (−) terms on the right-hand sides. The different terms on the right-hand sides of eqs. (46)-(49), the contributions to the gain and loss of particles, fall into three categories:

(a) Parton multiplication through emission processes $q \rightarrow qg$, $g \rightarrow gg$ and $g \rightarrow q\bar{q}$;

(b) Parton cluster formation through recombinations $q\bar{q} \rightarrow \chi\chi$, $qg \rightarrow q\chi$, $gg \rightarrow \chi\chi$;

(c) Hadronic cluster decay either through direct conversion of the formed scalar χ excitations into hadrons h through $\chi \rightarrow h$, or via coupling to the pseudoscalar states $\chi \rightarrow U$, and the subsequent decay into hadrons, $U \rightarrow h$.

In the following subsections we explain these contributions in detail.

4.3 Parton multiplication

The integral operators $\hat{A}$ in the quark and gluon evolution equations (46) and (47) represent the changes of the parton distributions in phase space due to the perturbative cascade evolution. Explicitly,

$$\begin{aligned}
\hat{A}_q^{qg}\, F_q &= \lambda_\chi \int_0^1 dz \left[\frac{1}{z}\, F_q\left(r; \frac{x}{z}, zp_\perp^2, zp^2\right) \;-\; F_q(r; x, p_\perp^2, p^2) \right] \gamma_{q\rightarrow qg}\,(z, \epsilon)\;\; a_q(z, p^2) \\
\hat{A}_g^{q\bar{q}}\, F_g &= \lambda_\chi \int_0^1 \frac{dz}{z}\, F_g\left(r; \frac{x}{z}, zp_\perp^2, zp^2\right) \gamma_{g\rightarrow q\bar{q}}\,(z, \epsilon)\;\; a_g(z, p^2)
\end{aligned}$$