MeV, $Tr[\lambda_i \lambda_j] = 2\delta_{ij}$, $UU^\dagger = f_\pi^2$), with non-vanishing *vev*'s in the long-distance limit,

$$\chi_0 \;:=\; \frac{\delta}{\delta J_\chi} \ln W[J_\chi, K_U, K_U^\dagger] \;=\; \frac{\langle\, 0 \,|\, \chi \,|\, 0 \,\rangle}{\langle\, 0 \,|\, 0 \,\rangle} \;\neq\; 0 \tag{7}$$

$$U_0 \;:=\; \left(\frac{\delta}{\delta K_U^\dagger} + \frac{\delta}{\delta K_U} \right) \ln W[J_\chi, K_U, K_U^\dagger] \;=\; \frac{\langle\, 0 \,|\, U + U^\dagger \,|\, 0 \,\rangle}{\langle\, 0 \,|\, 0 \,\rangle} \;\neq\; 0 \;, \tag{8}$$

and an effective action

$$\begin{aligned} \Gamma[\chi, U, U^\dagger] \;&\equiv\; \ln W[J_\chi, K_U, K_U^\dagger] \;-\; \int d^4 r \left\{ J_\chi \chi \;+\; U^\dagger K_U \;+\; K_U^\dagger U \right\} \\ &=\; \int d^4 r \left\{ -\, V(\chi, U) \;+\; \frac{1}{2} \, (\partial_\mu \chi)(\partial^\mu \chi) \;+\; \frac{1}{4} \, Tr \left[\, (\partial_\mu U)(\partial^\mu U^\dagger) \right] \;+\; \ldots \right\} . \end{aligned} \tag{9}$$

Consequently the Lagrangian in (6) is given by

$$\mathcal{L}[\chi, U, U^\dagger] \;=\; \frac{1}{2} \, (\partial_\mu \chi)(\partial^\mu \chi) \;+\; \frac{1}{4} \, Tr \left[\, (\partial_\mu U)(\partial^\mu U^\dagger) \right] \;-\; V(\chi, U) \;, \tag{10}$$

with a potential V that has been constructed [12, 13] on the basis of constraints which arise from the scale and chiral symmetry properties of the excact QCD Lagrangian, namely,

$$\begin{aligned} V(\chi, U) \;&=\; b \, \left[\frac{1}{4} \, \chi_0^4 \;+\; \chi^4 \, \ln \left(\frac{\chi}{e^{1/4} \chi_0} \right) \right] \;+\; \frac{1}{4} \left[1 \;-\; \left(\frac{\chi}{\chi_0} \right)^2 \right] \; Tr \left[(\partial_\mu U)(\partial^\mu U^\dagger) \right] \\ &\quad +\, c \, Tr \left[\, \hat{m}_q (U + U^\dagger) \right] \, \left(\frac{\chi}{\chi_0} \right)^3 \;+\; \frac{1}{2} \, m_0^2 \, \phi_0^2 \, \left(\frac{\chi}{\chi_0} \right)^4 \quad . \end{aligned} \tag{11}$$

Here the parameter b is related to the conventional bag constant B by

$$B \;=\; b \, \frac{\chi_0^4}{4} \; . \tag{12}$$

Furthermore, c is a constant of mass dimension 3, $m_q \;=\; \mathrm{diag}(m_u, m_d, m_s)$ is the light quark mass matrix, and m_0^2 is an extra U(1)-breaking mass term for the ninth pseudoscalar meson ϕ_0 (which we will disregard in the following). In the chiral limit, this potential has a minimum when $\langle \chi \rangle = \chi_0$ and equals the vacuum pressure B at $\langle \chi \rangle = 0$.

2.3 The intermediate regime $L \approx L_c$

Having established a field theory framework for the two regions $L \ll L_c$ and $L \gg L_c$, the crucial issue is now the intermediate range. Clearly there must be a dynamical interpolation around $L \approx L_c$ from the short-range to the long-range description. We propose here the following approach. Let us first consider the long-range domain, i.e. the physical vacuum characterized by χ_0, and introduce into the vacuum an excitation of small space-time extent.