peratures below T_{cs} this region in the phase diagram could extend to a wider range. Obviously, the GL formulation does not allow us to access this temperature region. In our approach the ratio T_{cs}/T_{cd} is of the order 10^{-2} so that temperatures below T_{cs} are definitely far away from the range of validity of a GL theory. The low-temperature region can only be treated by methods based on Bogolyubov-de Gennes equations or quasiclassical theory which are considerably more complicated if one intends to include more realistic microscopic details.

Previously a series of phenomena were discussed, in connection with broken time reversal symmetry on interfaces. A first example are flux lines on the interface which do enclose neither integer nor half-integer multiples of the standard flux quanta, but some intermediate fractional fluxes[3,4,25]. Although the observation of fractional flux by Kirtley et al.[26] is entirely compatible with our discussion here, it is not clear whether the data could not be explained in an alternative way. The main problem for this kind of experiments lies in the requirement of comparatively long homogeneous interfaces, a condition hard to satisfy with present technology. On the other hand, it was also discussed that phase slip effects on short interfaces could be used as a test[27]. So far no experimental data are available for this type of effect. Finally an important aspect of the $\mathcal{T}$-violating state is the presence of spontaneous currents. Their magnitude is small, however, and together with screening effects they would not lead to a net magnetization. Thus only a very sensitive probe with high spatial resolution, smaller or of order London penetration depth, would be sufficient to observe this effect. Until now the only method which has successfully observed the small magnetic fields induced by a $\mathcal{T}$-violating state are muon spin rotation measurements in zero external field[28].

It is important to notice, however, that also surfaces of d-wave superconductors can yield states with locally broken time reversal symmetry. Recent experiments indicate that this type of state might be realized at low temperatures on YBCO surfaces with [110]-orientation[29]. The evidence is given by the splitting of the zero-bias anomaly in the IV-characteristics as discussed by Fogelström et al.[30]. Clearly similar effects due to the rearrangement of quasiparticle states are also expected in $\mathcal{T}$-violating interfaces as discussed by Huck et al. and could possibly be tested by spectroscopy with a scanning tunneling microscope[31].

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Appendix: Derivation of surface terms

The surface free energy F_I in §2 is derived in the following way. We describe the transmission and and the reflection of electrons at the interface by the Hamiltonian H_I in eq.(5). Then the total Hamiltonian is given by $H + H_I$. Second order perturbation theory gives the excess energy due to H_I:

$$\Delta F = -\frac{1}{2}\int_0^\beta d\tau \big\langle T_\tau H_I(\tau) H_I(0)\big\rangle_0 \qquad \text{(A·1)}$$

where $\langle \cdots \rangle_0$ denotes the average with respect to H. We substitute eq.(5) into this expression and decouple it in terms of Green's functions of spinons for both sides

$$\begin{aligned}
\Delta F &= \Delta F_1 + \Delta F_2 \\
\Delta F_1 &= -\sum_{kp} T \sum_{\epsilon_n} \big[t_{kp}^2 \big\{ G_{21}^L(p,i\epsilon_n) G_{12}^R(k,i\epsilon_n) \\
&+ G_{12}^L(p,i\epsilon_n) G_{21}^R(k,i\epsilon_n) \big\} + r_{kp}^2 G_{21}^R(p,i\epsilon_n) G_{12}^R(k,i\epsilon_n) \big] \\
\Delta F_2 &= -\sum_{kp} T \sum_{\epsilon_n} \big[t_{kp}^2 \big\{ G_{11}^L(p,i\epsilon_n) G_{11}^R(k,i\epsilon_n) \\
&+ G_{22}^L(p,i\epsilon_n) G_{22}^R(k,i\epsilon_n) \big\} \\
&+ \frac{1}{2} r_{kp}^2 \big\{ G_{11}^R(p,i\epsilon_n) G_{11}^R(k,i\epsilon_n) + G_{22}^R(p,i\epsilon_n) G_{22}^R(k,i\epsilon_n) \big\} \big]
\end{aligned} \qquad \text{(A·2)}$$

where the Green's functions are defined by

$$\begin{aligned}
G_{11}^A(k,i\epsilon_n) &= -\frac{i\epsilon_n + \xi_k}{\epsilon_n^2 + \xi_k^2 + |\Delta_k^A|^2} \\
G_{22}^A(k,i\epsilon_n) &= -\frac{i\epsilon_n - \xi_k}{\epsilon_n^2 + \xi_k^2 + |\Delta_k^A|^2} \\
G_{12}^A(k,i\epsilon_n) &= -\frac{\Delta_k^A}{\epsilon_n^2 + \xi_k^2 + |\Delta_k^A|^2} \\
G_{21}^A(k,i\epsilon_n) &= G_{12}^A(k,i\epsilon_n)^*
\end{aligned} \qquad \text{(A·3)}$$

with $A = R$ or L, $\epsilon_n = \pi T(2n+1)$, $\Delta_k^R = (3J/4)(\Delta_d \omega_d(k) + \Delta_s \omega_s(k))$ and $\Delta_k^L = (3J/4)\Delta_0 \omega_d(k)$. Now we extract the lowest order ($O(\Delta^2)$) terms. In ΔF_1 there are terms of the form $\Delta^L \Delta^R$ and $\Delta^R \Delta^R$ in the numerator. The former results in the t_i-terms, and the latter leads to the part of the g_{ij}-terms. We can also obtain the $O(\Delta^2)$ terms from the denominators of G_{11} and G_{22} in ΔF_2 . These terms are usually discarded in the discussion of the Josephson effect, since it does not depend on the phase difference of Δ's if both sides of the superconductors have only one component of the order parameter. In the present case, however, this term depends on the phase difference of Δ_d and Δ_s, and thus it is equally important as the one from ΔF_1. Neglecting terms independent of Δ, we perform the ϵ_n-summation in eq.(A2) to get the following expressions

$$\begin{aligned}
\Delta F_1 &= -\sum_{kp} J_1(\xi_k,\xi_p) \big[t_{kp}^2 \big\{ (\Delta_p^L)^* \Delta_k^R + \Delta_p^L (\Delta_k^R)^* \big\} \\
&+ r_{kp}^2 (\Delta_p^R)^* \Delta_k^R \big] \\
\Delta F_2 &= \sum_{kp} J_2(\xi_k,\xi_p)(t_{kp}^2 + r_{kp}^2) |\Delta_p^R|^2
\end{aligned} \qquad \text{(A·4)}$$