to the weakly first-order nature of the QCD phase transition at finite temperature. The essence is that (16) enforces color charge confinement due to the fact that a color electric charge creates a displacement $\vec{D}_a = \kappa_L \vec{E}_a$, where $E_a^k = F_a^{0k}$, with energy $\frac{1}{2}\int d^3 r D_a^2/\kappa_L$ which becomes infinite at large r for non-zero total charge.

Similarly, absolute confinement can be ensured also for quarks by coupling the quark fields to the χ field through

$$\mu_L(\chi) \;=\; \mu_0 \left(\frac{1}{\kappa_L(\chi)} \;-\; 1\right) \;=\; \frac{\mu_0\,(L\,\chi)^2}{(L_0\chi_0)^2 - (L\chi)^2}\;, \tag{17}$$

where μ_0 is a constant of mass dimension one that we will set equal to 1 GeV. This form reflects that the quark mass term $\mu_L(\chi)$ in (14) is induced by non-perturbative gluon interactions, rather than being an independent quantity, as is suggested by an explicit calculation [11] of the quark self-energy involving the gluon propagator in the presence of the collective field χ. It has been shown [10] that the dynamical mass $\mu_L(\chi)$ leads to an effective confinement potential with the masses of the quarks at small L approximately equal to the current masses, but at large L when $\langle\chi\rangle \to \chi_0$ it generates an infinite asymptotic quark mass,

$$\mu_L(0) \;=\; 0 \;, \qquad\qquad \mu_L(\chi_0) \;=\; \infty \;. \tag{18}$$

It is evident from (16)-(18) that $\mathcal{L}[A^\mu,\psi,\overline{\psi},\chi]$ given by (14) vanishes in the short-distance limit ($L\to 0$, $\langle\chi\rangle\to 0$), whereas in the long-distance limit it suppresses the propagation of colored gluon and quark fluctuations, and interpolates smoothly between the two extremes. The typical functional forms of $\kappa_L(\chi)$ and $\mu_L(\chi)$ are illustrated in Fig. 1.

2.4 The scale-dependent generating functional for the effective theory

Let us now summarize and combine the three contributions of Secs. 2.1-2.3 into a single action integral, and write down the resulting generating functional as an effective description covering the full range $0 < L < \infty$ and depending implicitly on the scale L as defined by (3):

$$\begin{aligned} W_L[J,\eta,\overline{\eta},J_\chi,K_U,K_U^\dagger] \;&=\; \int \mathcal{D}A^\mu \mathcal{D}\psi\mathcal{D}\overline{\psi}\mathcal{D}\chi\mathcal{D}U\mathcal{D}U^\dagger \,\det\mathcal{F} \\ &\times\;\exp\left\{ i\int d^4 r\left(\mathcal{L}[A^\mu,\psi,\overline{\psi}] \;+\; \mathcal{L}_L[A^\mu,\psi,\overline{\psi},\chi] \;+\; \mathcal{L}[\chi,U,U^\dagger]\right.\right. \\ &\qquad\left.\left. +\; J_{\mu,a}A_a^\mu \;+\; \overline{\psi}\eta \;+\; \overline{\eta}\psi J_\chi\chi \;+\; U^\dagger K_U \;+\; K_U^\dagger U\right)\right\}\;, \end{aligned} \tag{19}$$