

describe the space-time evolution of the Wigner transforms, whereas the constraint equations describe the “orthogonal” evolution in momentum space, and express a normalization condition imposed by unitarity and the renormalization group. In order to relate these operator equations to physically-relevant (observable) quantities, we define the *Wigner operators* $\hat{F}_\alpha(r, p)$ ($\alpha \equiv q, g, \chi, U$) in terms of the operators \mathcal{S} , \mathcal{D} , Δ , $\tilde{\Delta}$ and the self energies Σ , Π , Δ , $\tilde{\Delta}$ as follows:

$$\begin{aligned}
i\mathcal{S}_{ij}(r, p) &= \delta_{ij} (\gamma \cdot p + \Sigma) (2\pi i) \delta(p^2 - \Sigma^2) \hat{F}_q(r, p) \\
i\mathcal{D}_{ab}^{\mu\nu}(r, p) &= \delta_{ab} \varepsilon^{\mu\sigma}(p, s) \varepsilon_{\sigma}^{\nu*}(p, s) (2\pi i) \delta(p^2 - \Pi) \hat{F}_g(r, p) \\
i\Delta(r, p) &= (2\pi i) \delta(p^2 - \Xi) \hat{F}_\chi(r, p) \\
i\tilde{\Delta}(r, p) &= (2\pi i) \delta(p^2 - \tilde{\Xi}) \hat{F}_U(r, p) .
\end{aligned} \tag{37}$$

Then, by tracing over color and spin polarizations, and taking the expectation values (or, in a medium, the ensemble average) of these Wigner operators, one obtains the scalar functions

$$F_\alpha(r, p) \equiv F_\alpha(t, \vec{r}; \vec{p}, p^2 = M_\alpha^2) \quad (\alpha = q, g, \chi, U) \tag{38}$$

with

$$\begin{aligned}
F_q(r, p) &= \langle \text{Tr}[\mathcal{S}(r, p)] \rangle , & M_q^2 &= \Sigma^2(r, p) \\
F_g(r, p) &= \langle \text{Tr}[\mathcal{D}(r, p)] \rangle , & M_g^2 &= \Pi(r, p) \\
F_\chi(r, p) &= \langle \Delta(r, p) \rangle , & M_\chi^2 &= \Xi(r, p) \\
F_U(r, p) &= \langle \text{Tr}[\tilde{\Delta}(r, p)] \rangle , & M_U^2 &= \tilde{\Xi}(r, p) .
\end{aligned} \tag{39}$$

The c -number functions $F_\alpha(r, p)$ are the quantum-mechanical analogues of the classical phase-space distributions that measure the number of particles at time t in a 7-dimensional phase-space element $d^3r d^4p$. Due to the effects of the self energies, three-momentum and energy are generally independent variables, because the quanta can be off mass shell, i.e., for zero rest masses, $E^2 = \vec{p}^2 + M_\alpha^2 \neq \vec{p}^2$, where M_α^2 , eq. (39), represents the off-shellness due to the self and mutual interactions of the quanta ($M^2 = 0$ for on-shell particles). In contrast to the classical propagation of on-shell particles, the Wigner functions (37) incorporate the quantum “Zitterbewegung” even in the absence of interactions with other particles. These spatial fluctuations arise from the combination $p^2 - \partial_r^2/4$ acting on the Wigner operators in