

The singleton action from the supermembrane¹

Gianguido Dall'Agata², Davide Fabbri², Christophe Fraser³,
Pietro Fré², Piet Termonia² and Mario Trigiante⁴

² Dipartimento di Fisica Teorica, Università di Torino, via P. Giuria 1, I-10125 Torino,
Istituto Nazionale di Fisica Nucleare (INFN) - Sezione di Torino, Italy

³ Wellcome Trust Centre for the Epidemiology of Infectious Diseases, University of Oxford,
South Parks Road, OXFORD OX1 3PS, UK.

⁴ Department of Physics, University of Wales Swansea, Singleton Park,
Swansea SA2 8PP, United Kingdom

Abstract

We derive the free $Osp(8|4)$ singleton action by sending the $M2$ brane to the Minkowski boundary of an $AdS_4 \times \mathcal{M}_7$ background. We do this by means of the solvable Lie algebra parametrization of the coset space. We also give some comments on singleton actions from membranes on $AdS_4 \times G/H$ backgrounds.

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1 Introduction

For about a year now, there has been a revival of interest in supergravity vacua of the form

$$AdS \times \mathcal{M}, \tag{1.1}$$

where AdS is an anti-de Sitter space and \mathcal{M} a compact Einstein manifold. This revival started after Maldacena's conjecture [1] of duality between Kaluza–Klein supergravity theories in the bulk of anti-de Sitter space and conformal field theories on the boundary,

$$\text{KK on } AdS \text{ / CFT on } \partial AdS. \tag{1.2}$$

Since the proposal by Gubser, Klebanov, Polyakov and Witten for this duality [2], some work is being done with the scope of testing this conjecture of holography. This has brought the whole research area of branes and AdS representations from the eighties back alive.

Of particular interest in testing this AdS/CFT duality is the singleton problem. Representations of AdS algebras have been studied extensively in the past [3]. They are characterized by a lowest energy and the total angular momentum. Most of them have a Poincaré analogue, except for some ultra-short representations which are referred to as singleton representations. The most considerable property of these singletons is that they can not be formulated as a field theory on the bulk of the AdS space. Yet, singleton actions exist, but can only be formulated if the singleton fields are restricted to live on the boundary of AdS . It has also been known for a long time how to get these singleton actions and what they describe: namely, they describe the small fluctuations of a brane at *the end of the universe* [4]. All this has been known for about ten years. Still, the singleton has not been constructed explicitly from the brane until recently. It is clear that an explicit realization of this singleton at the end of the universe is an important ingredient for testing the AdS/CFT conjecture.

Thus, to find the singleton from the super membrane one has to do the following:

1. Consider the super membrane action that is invariant with respect to κ supersymmetry.
2. Expand this action around a classical solution.
3. Send it to infinity.

As far as the first step is concerned, membrane actions in an explicit background of $AdS_4 \times S^7$ have been constructed by our collaboration [5] and others [6]. Here, we will overview the construction of the membrane action and the derivation the singleton action in this background, as it was done in [5]. However, before carrying out this programme let us point out that the singleton theory that is retrieved on this space is quite trivial and doesn't yield a proper test for the AdS/CFT correspondence. Therefore one has to consider the singleton problem on other non-trivial backgrounds. Suitable backgrounds for this are given if one replaces the sphere by other coset manifolds G/H . A complete classification of these backgrounds for $D = 11$ is already known and these spaces have been thoroughly studied in the eighties [7]. They are in one-to-one correspondence with the

G/H Freund–Rubin compactifications. Also, the number of preserved supersymmetries N are known.

Of all these coset spaces, the case of the round and squashed seven spheres are the best known (corresponding to $N = 8$ [8] and $N = 1$ near horizon supergravities) but in the eighties the Kaluza–Klein spectra have been systematically derived also for all the other solutions using the technique of harmonic expansions [9]. The organization of these spectra in supermultiplets is known not only for the round S^7 [10] but also for the case of supersymmetric M^{pqr} spaces

$$M^{pqr} \equiv \frac{SU(3) \times SU(2) \times U(1)}{SU(2) \times U(1) \times U(1)},$$

where $p, q, r \in \mathbb{Z}$ define the embedding of the $U(1)^2$ factor of H in G . For $p = q = \text{odd}$ we have $N = 2$, in all the other (non supersymmetric) cases we have $N = 0$. The $N = 2$ multiplet structure was obtained in [11]. At present a group in Torino [12] is doing the harmonic analysis on the so-called Stiefel and N^{010} manifolds as well.

Since much is known and will be known about these spaces, they constitute an excellent laboratory to make direct checks of the holographic correspondence.

Let’s now clarify the qualitative difference between the seven sphere and the other G/H spaces. For a G/H space that admits N supersymmetries, the isometry group is factorized as follows:

$$G = G' \otimes SO(N),$$

where $SO(N)$ is the R -symmetry of the orthosymplectic algebra $Osp(N|4)$, while the factor G' is the gauge-group of the vector multiplets. Correspondingly the three-dimensional world-volume action of the CFT must have the following superconformal symmetry:

$$Osp(N|4) \times G', \tag{1.3}$$

where G' is a *flavor group*. In the maximal case the harmonics on S^7 are labeled only by R -symmetry representations while in the lower susy case they depend both on R labels and on representations of the gauge/flavour group G' . The structure of $Osp(8|4)$ supermultiplets determines completely their R -symmetry representation content so that the harmonic analysis becomes superfluous in this case. The eigenvalues of the internal laplacians which determine *the Kaluza–Klein masses* of the $Osp(8|4)$ graviton multiplets or, in the conformal reinterpretation of the theory, the *conformal weights* of the corresponding primary operators, are already fixed by supersymmetry and need not be calculated. In this sense the correspondence (1.2) is somewhat trivial in the maximal susy case: once the superconformal algebra has been identified with the super-isometry group $Osp(8|4)$ the correspondence between conformal weights and Kaluza–Klein masses is simply guaranteed by representation theory of the superalgebra. On the other hand in the lower susy case the structure of the $Osp(N|4)$ supermultiplets fixes only their content in $SO(N)$ representations while the Kaluza–Klein masses, calculated through harmonic analysis depend also on G' labels. In this case the holographic correspondence yields a definite prediction on the conformal weights that, as far as superconformal symmetry is concerned would be arbitrary. Explicit verification of these predictions would provide a much more stringent proof of the holographic correspondence and yield a deeper insight in its inner working. However in order to set up such a direct verification one has to solve a problem that was left open in Kaluza–Klein supergravity: the singleton problem.

In the remainder of this text we will restrict ourselves to the membrane on the seven sphere. The case of the cosets with lower supersymmetry is currently under investigation.

To avoid any confusion, we would like to stress here that we call singleton field theory the flat space limit of the free field theory of [13, 14]. We point out that, since we are going to find a theory living on a three-dimensional Minkowski space rather than on $S^2 \times S^1$, we have no scalar mass term which was instead required in [13, 14] for conformal invariance. We will see that it can also be derived as the theory living on the solitonic M2 brane.

2 The supermembrane on $AdS_4 \times S^7$

We consider the space $AdS_4 \times S^7$, with given metric,

$$d\tilde{s}^2 = \rho^2 (-dt^2 + dx^2 + dw^2) + \frac{1}{\rho^2} d\rho^2 + 4d\Omega_7^2, \quad (2.1)$$

with coordinates,

$$\begin{cases} \rho \in]0, \infty[\\ t, w, x \in]-\infty, \infty[\end{cases},$$

and $d\Omega_7^2$ is the metric of the seven sphere. This is the near horizon geometry of the $M2$ brane [15]. Moreover in [16] it was shown that this is a stable quantum vacuum of the 11D supergravity. The AdS superspace is defined as the following coset

$$AdS^{(8|4)} = \frac{Osp(8|4)}{SO(1,3) \otimes SO(8)}$$

and it is spanned by the four coordinates of the AdS_4 manifold and by eight Majorana spinors (i.e. they have 32 real components) parametrizing the fermionic generators of the superalgebra.

This space can be described by means of a super solvable Lie algebra parametrization. To see what this solvable Lie algebra parametrization is, let's have a look at the familiar "Union Jack" root diagram of C_2 , which is the complexification of $SO(2,3)$ shown in figure 1. The fermionic supercharges form a square weight diagram within this figure, and the supertranslation algebra is then simply that the anticommutator of two fermions is given by vector addition of the corresponding weights in the diagram. The diagram can in fact be seen as a projection of the full $Osp(8|4)$ root diagram, since the $SO(8)$ roots lie on a perpendicular hyperplane, and so on this diagram they would be at the centre.

It is now easy to see that the generators in the box $\{S_{\pm}, \sigma_{\pm}, \sigma_{\perp}, D\}$, form a super solvable Lie algebra with non-zero (anti) commutation relations,

$$\{S, S\} \sim \sigma, \quad [D, S] \sim S, \quad [D, \sigma] \sim \sigma, \quad (2.2)$$

since its second derivative is zero. The coset representative is now obtained by exponentiating the product of these generators with the coordinates of the coset space. We choose to write the coset representative as $L = L_F L_B$ with

$$\begin{aligned} L_B &= \exp(\rho D) \exp(\sqrt{2}x\sigma_{\perp} + t(\sigma_{+} + \sigma_{-}) + w(\sigma_{+} - \sigma_{-})), \\ L_F &= \exp(\theta_1^A S_1^A + \theta_2^A S_2^A), \end{aligned} \quad (2.3)$$

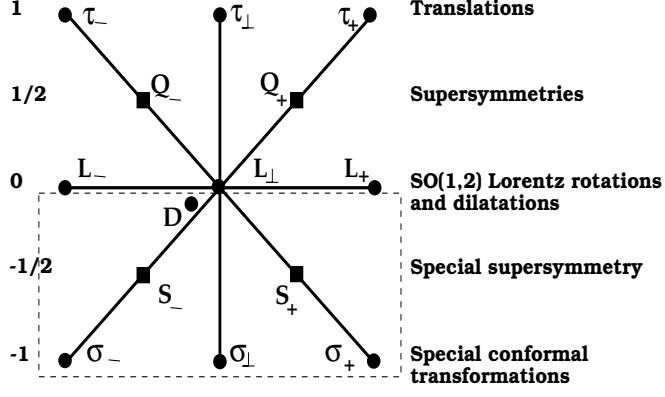


Figure 1: The root diagram of $SO(2,3)$. The bosonic weights are represented by circles, and the fermionic weights by squares. The dilatation charge of horizontal planes in the diagram are on the left, while the worldvolume theory interpretation of the planes of generators are labelled to the right. The supersolvable algebra is the boxed subalgebra.

where ρ, x, t, w, θ^A are the bosonic and fermionic coordinates. From the left invariant form,

$$\begin{aligned}\Omega &= L^{-1}dL = \Omega_B + L_B^{-1}\Omega_F L_B, \\ \Omega_F &= L_F^{-1}dL_F,\end{aligned}\tag{2.4}$$

one derives the vielbeins²,

$$\begin{aligned}E^0 &= -\rho dt - \rho \bar{\theta}^A \gamma^0 d\theta^A, \\ E^1 &= \rho dw - \rho \bar{\theta}^A \gamma^1 d\theta^A, \\ E^2 &= \frac{1}{2\rho} d\rho, \\ E^3 &= \rho dx - \rho \bar{\theta}^A \gamma^3 d\theta^A,\end{aligned}\tag{2.6}$$

and

$$\psi^A = \sqrt{2e\rho} \begin{pmatrix} 0 \\ 0 \\ d\theta_1^A \\ d\theta_2^A \end{pmatrix},\tag{2.7}$$

Notice that, due to the (anti) commutation relations (2.2) of the solvable Lie algebra parametrization, the exponentiation (2.3) only contains a finite number of terms. Hence, it immediately follows that the vielbeins are at most quadratic in their anti-commuting coordinates. For a discussion on this see [5, 18].

Another convenient feature of the solvable Lie algebra parametrization is that one projects out half of the spinors. This is equivalent to the projection of the κ -symmetry operator and thus at this stage the κ symmetry has already been fixed.

² For the fermionic coordinates the following notation is understood: $\bar{\theta}^A = \theta^A \gamma^0$ and

$$\theta^A = \begin{pmatrix} \theta_1^A \\ \theta_2^A \\ 0 \\ 0 \end{pmatrix}.\tag{2.5}$$

This parametrization of the manifold gives rise to the metric (2.1).

To complete the parametrization of the target superspace one still has to define the vielbeins of the seven sphere. To do so, we call $y^{\hat{a}}$ the seven coordinates of the sphere and write in stereographic projection coordinates:

$$E^{\hat{a}} = -\delta^{\hat{a}}_{\hat{m}} \frac{dy^{\hat{m}}}{1+y^2}. \quad (2.8)$$

Beside the κ symmetry we also fix the three-dimensional world-volume diffeomorphisms imposing the static gauge choice. To obtain static solutions we have to identify

$$\xi^I \equiv (-t, w, x), \quad (2.9)$$

where $I = 0, 1, 2$, is the curved index of the brane.

For the action of the membrane we get:

$$S = 2 \int \sqrt{-\det(h_{IJ})} d^3\xi + 4! \int B, \quad (2.10)$$

with³

$$\begin{aligned} h_{IJ}(\xi) = & \frac{1}{4\rho^2} \partial_I \rho \partial_J \rho + \frac{1}{(1+y^2)^2} \partial_I y^{\hat{a}} \partial_J y_{\hat{a}} \\ & + \rho^2 \left(\eta_{IJ} - 2e\bar{\theta}^A \gamma^i \partial_{(J} \theta^A \delta_{iI)} + \bar{\theta}^A \gamma^i \partial_I \theta^A \bar{\theta}^B \gamma_i \partial_J \theta^B \right). \end{aligned} \quad (2.11)$$

The expression for B is given by

$$\begin{aligned} B = & \frac{d^3\xi}{4!4!} \left[\epsilon^{IJK} \rho^3 (\delta_I^i - \bar{\theta}^A \gamma^i \partial_I \theta^A) (\delta_J^j - \bar{\theta}^A \gamma^j \partial_J \theta^A) (\delta_K^k - \bar{\theta}^A \gamma^k \partial_K \theta^A) \frac{\epsilon_{ijk}}{3} + \right. \\ & \left. - \frac{1}{(1+y^2)} \epsilon^{IJK} \partial_I y^{\hat{a}} \eta_{A\tau_{\hat{a}}} \eta_B \rho \partial_J \bar{\theta}^A \partial_K \theta^B \right]. \end{aligned} \quad (2.12)$$

The isometries of (2) can be calculated explicitly in this parametrization. As was noted in [1, 17], they realize conformal symmetry on the brane. For example, for the dilatation one finds,

$$\delta\rho = \rho, \quad (2.13)$$

$$\delta\xi^I = -\xi^I, \quad (2.14)$$

$$\delta\theta_{\alpha}^A = -\frac{1}{2}\theta_{\alpha}^A. \quad (2.15)$$

3 The singleton action from the supermembrane

In order to retrieve the $Osp(8|4)$ singleton action we now have to do the following. First we consider a classical solution of the action (2.10),

$$\xi^I \equiv (-t, w, x), \quad \partial_I y^{\hat{a}} = 0, \quad \theta^A = 0, \quad \rho = \bar{\rho} = \text{const}, \quad (3.1)$$

³ where $i = 0, 1, 2$, $\epsilon_{012} = 1$ and $\eta_{IJ} = \text{diag}\{-++\}$. The seven-dimensional gamma matrices $\tau^{\hat{a}}$ are the generators of the $SO(7)$ Clifford algebra and η_A are the eight real killing spinors on the seven sphere. For details on the conventions see [5].

then expand the transverse coordinates to the brane around the values for this classical solution. For this we use normal coordinates. Thus we write

$$\begin{aligned}\rho &= \bar{\rho} + \alpha'^{\frac{3}{2}} \tilde{\rho}, \\ y^{\hat{a}} &= \alpha'^{\frac{3}{2}} \tilde{y}^{\hat{a}}, \\ \theta^A &= \alpha'^{\frac{3}{2}} \Theta^A,\end{aligned}\tag{3.2}$$

where $\tilde{\rho}, \tilde{y}^{\hat{a}}, \Theta^A$ represent the fluctuations and α' is the membrane tension. Thus the action (2.10) gets the following expansion as a power series in α' :

$$\mathcal{L} = \sum_{n=0}^{\infty} \alpha'^{\frac{3(n-2)}{2}} \mathcal{L}_{(n)},\tag{3.3}$$

with

$$\mathcal{L}_{(0)} = 0 = \mathcal{L}_{(1)},\tag{3.4}$$

and we are to recover the singleton action from the order 1 term

$$\mathcal{L}_{(2)} = \frac{1}{4\bar{\rho}} \eta^{IJ} \partial_I \tilde{\rho} \partial_J \tilde{\rho} + \bar{\rho} \eta^{IJ} \partial_I \tilde{y}^{\hat{a}} \partial_J \tilde{y}^{\hat{b}} \delta_{\hat{a}\hat{b}} - 2 \bar{\rho}^3 \bar{\Theta}^A \hat{\sigma}^i \partial_I \Theta^A \delta_i^I.\tag{3.5}$$

The final step is to send the brane to the boundary of AdS. The boundary of AdS lies at

$$\bar{\rho} \rightarrow \infty \quad \text{and} \quad \bar{\rho} \rightarrow 0,\tag{3.6}$$

which is a conformally compactified Minkowski space (see one of the appendices in [5]). It is already clear from the form of the action (3.5) that in order to take one of these limits one has to rescale the fields $\tilde{\rho}, \tilde{y}^{\hat{a}}$ and Θ^A . In fact, a proper analysis of the supersymmetry variation, which we do not present here but can be found in [5], shows us that these rescalings have to be done according to

$$\lambda = \bar{\rho}^{\frac{3}{2}} \Theta^A, \quad \tilde{P} = \frac{\tilde{\rho}}{\sqrt{\bar{\rho}}}, \quad \tilde{Y}^{\hat{a}} = \sqrt{\bar{\rho}} \tilde{y}^{\hat{a}}.\tag{3.7}$$

Using the notation

$$Y^A \equiv \left\{ \frac{\tilde{P}}{4}, \frac{\tilde{Y}}{2} \right\},\tag{3.8}$$

the action (3.5) becomes

$$\mathcal{L} = 4 \eta^{IJ} \partial_I Y^A \partial_J Y^A - 2 \bar{\lambda}^A \hat{\sigma}^I \partial_I \lambda^A.\tag{3.9}$$

Clearly, this action has the right form to become the singleton action. Yet, for generic values of $\bar{\rho}$ it does not. To see this, let's look at the dilatation symmetry of the action (2.10). The transformation (2.13) can only become a symmetry of the action (2.10) if we place the brane at the boundary.

So we conclude that the singleton is found after putting the brane at the boundary of the Anti-de Sitter space and that the singleton field theory describes the centre of mass degrees of freedom of the $M2$ brane.

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