$$\hat{A}_g^{gg}\, F_g \;=\; \lambda_\chi \int_0^1 dz \left[\frac{1}{z}\, F_g\left(r;\frac{x}{z}, zp_\perp^2, zp^2\right) \;-\; \frac{1}{2}\; F_g(r;x,p_\perp^2,p^2)\right]\, \gamma_{g\to gg}\,(z,\epsilon)\;\; a_g(z,p^2)$$

$$\hat{A}_g^{q\bar{q}}\, F_g \;=\; \lambda_\chi\, n_f(p^2)\; F_g(r;x,p_\perp^2,p^2) \int_0^1 dz\, \gamma_{g\to q\bar{q}}\,(z,\epsilon)\;\; a_g(z,p^2)$$

$$\hat{A}_q^{gq}\, F_{q+\bar{q}} \;=\; \lambda_\chi \int_0^1 \frac{dz}{z}\, F_{q+\bar{q}}\left(r;\frac{x}{z}, zp_\perp^2, zp^2\right)\, \gamma_{q\to gq}\,(z,\epsilon)\;\; a_q(z,p^2)\;. \tag{51}$$

Here $\lambda_\chi \equiv \lambda_\chi(\chi(r)) = 1-(\chi/\chi_0)^4 + O[(\chi/\chi_0)^6]$, and the function $a(z,p^2)$ is given by

$$a_{q,g}(z,p^2) \;:=\; \frac{1}{2\pi}\, T_{q,g}(p^2)\; \alpha_s\left(\,(1-z)p^2\right)\;, \tag{52}$$

with a "life-time" factor $T_{q,g}(p^2)$ that expresses the probability for a parton of virtuality $p^2$ to decay (branch) within a time interval $t$ in the laboratory frame,

$$T_{q,g}(p^2) \;=\; 1 \;-\; \exp\left[-\frac{t}{\tau_{q,g}(p^2)}\right]\;, \tag{53}$$

where $\tau(p^2) \propto E/p^2$ (explicit expressions can be found in Ref. [34]). Furthermore, $\alpha_s$ is the one-loop QCD coupling

$$\alpha_s(k^2) \;=\; \frac{12\pi}{(\,33-2n_f(k^2))\;\ln\left[(k^2+k_0^2)\,L_c^2\right]}\;, \tag{54}$$

and $n_f(k^2)$ is the effective number of quark flavors at $k^2$,

$$n_f(k^2) \;:=\; \sum_f^{N_f} \sqrt{1-\frac{4m_f^2}{k^2}}\;\theta\left(1-\frac{4m_f^2}{k^2}\right)\;. \tag{55}$$

In (54) we have assumed the correspondence $L_c \simeq \Lambda_{QCD}^{-1}$ to the intrinsic perturbative QCD scale, and $k_0$ is a parameter that prevents a divergence when $k^2 \to L_c^{-2}$, and defines a maximum value $\alpha_s(0)$. We will determine $k_0$ in Sec. 5 from the total parton multiplicity. The functions $\gamma(z,\epsilon)$ are are analogous to the standard branching kernels in the MLLA [29]. Note that the 4-gluon vertex vertex does not contribute in the MLLA in the gauge (42), because it is kinematically suppressed. As a consequence, the effect of the couplings $\kappa_L(\chi)$ and $\mu(\chi)$, eqs. (16) and (17), on the parton evolution reduces to 2-parton recombinations into color-singlet clusters − the terms proportional to $\hat{B},\hat{C}$ which will be given below.

The branching kernels $\gamma_{a\to bc}(z)$ are the familiar energy distributions for the branching $a\to bc$ with $z = x_b/x_a$ and $1-z = x_c/x_a$ the energy fractions of of daughter partons:

$$\gamma_{q\to qg}(z,\epsilon) \;=\; C_F\;\left(\frac{1+z^2}{1-z+\epsilon}\right)$$