(35), and account for the uncertainty in spatial localization of a quantum particle due to its momentum that is determined by the particle's self interaction. The self-consistent solution of the transport and constraint equations (34) and (35) therefore corresponds to summing over all possible quantum paths r in space-time with fluctuations in energy-momentum p, constrained by the uncertainty principle [19]. This simultanous evolution in r and p of the Wigner functions F_α is illustrated in Fig. 4.

3.3 Macroscopic quantities

The functions F_α, eq. (38), contain the microscopic information that is required for the statistical description of the time evolution of a many-particle system in complete phase space [24]. Depending on the physical situation under consideration, one starts from specified initial distributions at $t = t_0$ and follows the time evolution of the phase-space densities $F_\alpha(r,p)$ according to the master equations (34) and (35). At any time $t > t_0$, $F_\alpha(r,p)$ reflects the state of the system around $\vec{r}$ and $\vec{p}$. One can then calculate, in a Lorentz-invariant manner, directly from the microscopic densities F_α, the relevant macroscopic quantities that are related to observables. Relativistic transport theory [25] relates physical quantities to phase-space integrals over products of combinations of four-momenta or tensors and the particle distributions. Specifically, using (37), performing the traces over color and spin indices where necessary, and taking the ensemble average, one obtains the local space-time-dependent particle currents n_α and the corresponding energy-momentum tensors $T^{\mu\nu}_\alpha$ for the different particle species $\alpha = q, \bar{q}, g, \chi, U$, which are given by [24]

$$n^\mu_\alpha(r) \;=\; \int d\Omega_\alpha\, p^\mu F_\alpha(p,r)\;, \qquad T^{\mu\nu}_\alpha(r) \;=\; \int d\Omega_\alpha p^\mu p^\nu\, F_\alpha(r,p)\;, \tag{40}$$

where $d\Omega_\alpha = \gamma_\alpha dM^2 d^3p/(16\pi^3 p^0)$, the γ_α are degeneracy factors for the internal degrees of freedom (color, spin, etc.), M_α measures the amount by which a particle α is off mass shell as a result of the self energy terms in (34), (35), and $p^0 \equiv E = +\sqrt{\vec{p}^{\,2} + M_\alpha^2}$. These macroscopic quantities can be written in Lorentz-invariant form by introducing for each species α the associated matter flow velocity $u^\mu_\alpha(r)$, defined as a unit-norm time-like vector at each space-time point, $(u_\mu u^\mu)_\alpha = 1$. A natural choice is e.g. $u^\mu_\alpha = n^\mu_\alpha/\sqrt{n_{\nu\,\alpha} n^\nu_\alpha}$. By contracting the quantities (40) with the local flow velocities u^μ_α, one can now obtain corresponding invariant scalars of particle density, pressure, and energy density, for each particle