

What is  
LCS?

hello  
↓ ↓ ↓ ↓  
helo

4 "helo"

abcd  
↓ ↓ ↓  
abce

3 "abc"

abcd  
↓ ↓ ↓ ↓  
acbd

3 "acd"  
"abd"

Sequence alignment of DNA

G A C G T  
| | | |  
G A T C G G

Top down algorithm

def LCS(i, j):

if  $s1[i] == s2[j]$ :

return  $1 + \text{LCS}(i+1, j+1)$

else:

return  $\max(\text{LCS}(i+1, j), \text{LCS}(i, j+1))$

if  $dp[i][j] > 0$   
return  $dp[i][j]$

$dp[i][j] =$   
 $\text{res}$

without memoization  
Space

$O(\max(m, n))$

Time

$O(2^{m+n})$

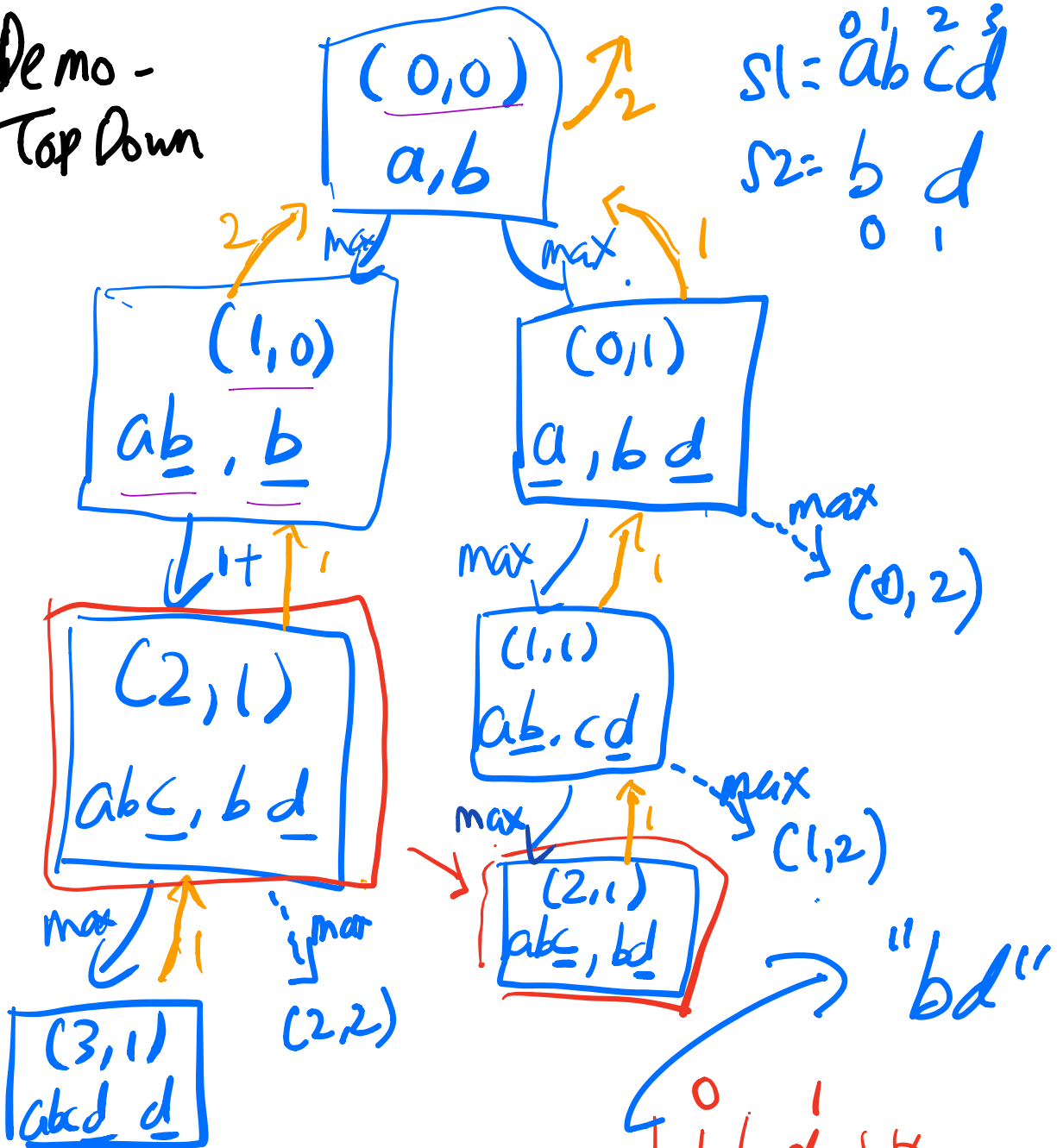
with memoization  
Space

$O(m \times n)$

Time

$O(m \times n)$

Demo -  
Top Down



	0	1	2	3
0 a	0	2	1	0
1 b	0	2	1	0
2 c	0	0	1	0
3 d	0	0	1	0

## Top Down

- start from Top - recurse the way down to the bottom.

```
def LCS(i, j):
```

```
    if S1[i] == S2[j]:
```

```
        return 1 + LCS(i+1, j+1)
```

```
    else
```

```
        return max(
            LCS(i+1, j),
            LCS(i, j+1))
```

if  $dp[i][j] \neq 0$

return  $dp[i][j]$

(with memoization)

	b	d	$\phi$
a	2	1	0
b	2	1	0
c	0	1	0
d	0	1	0
$\phi$	0	0	0

## Bottom up

- Start from Bottom build it up.

```
def LCS(i, j):
```

```
    for i in [1, |S1|]:
```

```
        for j in [1, |S2|]:
```

```
            if S1[i-1] == S2[j-1]:
```

```
                 $dp[i][j] = dp[i-1][j-1] + 1$ 
```

```
            else
```

```
                 $dp[i][j] = \max$ 
```

```
                    ( $dp[i-1][j]$ ,
```

```
                     $dp[i][j-1]$ )
```

```
        return  $dp[|S1|][|S2|]$ 
```

	$\phi$	b	d
$\phi$	0	0	0
a	0	1	1
b	0	2	1
c	0	1	2
d	0	1	2

Space &

Time

$O(m \times n)$

Bottom  
up with String

def lcs( $s_1, s_2$ ):

for  $i \in [1, |s_1|]$

for  $j \in [1, |s_2|]$

if  $s_1[i-1] == s_2[j-1]$

$dp[i][j] = dp[i-1][j-1] + 1$

else:

$dp[i][j] = \max(dp[i-1][j], dp[i][j-1])$

return  $dp[|s_1|][|s_2|]$

	$\phi$	b	d
$\phi$	$\phi$	$\phi$	$\phi$
a	$\phi$	$\phi$	$\phi$
b	$\phi$	b	b
c	$\phi$	b	b
d	$\phi$	b	bd

purpose of extra  $\phi$  col and row:

- ① easier calculation as explained
- ② when  $s_1 = \phi$  or  $s_2 = \phi$ ,  $dp[|s_1|][|s_2|]$  still work as expected.