E1 254
Game theory and
Mechanism Design
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# Evolutionary Games on Graphs

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## Introduction

**Evolution**: Change in *heritable* characteristics of populations over generations.
**Natural Selection**: Mutations randomly arise in individuals, conferring a change in *fitness*. Individuals with highest fitness pass on their traits. Gradual changes accumulate to bring change in population.
**Games on Graphs**: Representation of a traditional game using graph. Captures interactions between players too.

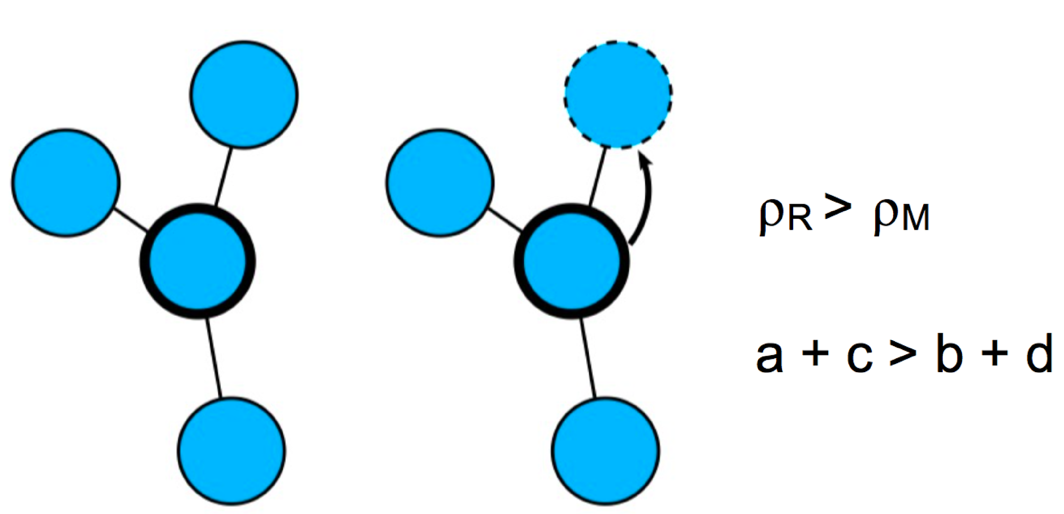
Graphs $\rightarrow$ Population
Games on Graphs $\rightarrow$ Evolution

## Objectives

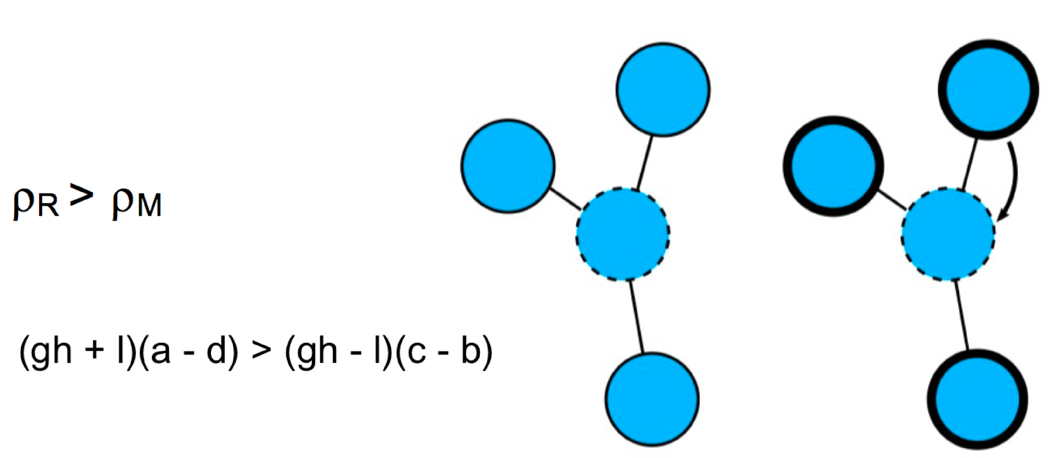
- To model evolution in a population
- To study the payoff matrix affects how the equilibrium population
- To explore applications and extensions of the existing model

## Work On Games on Graphs

- **Update Methods**:



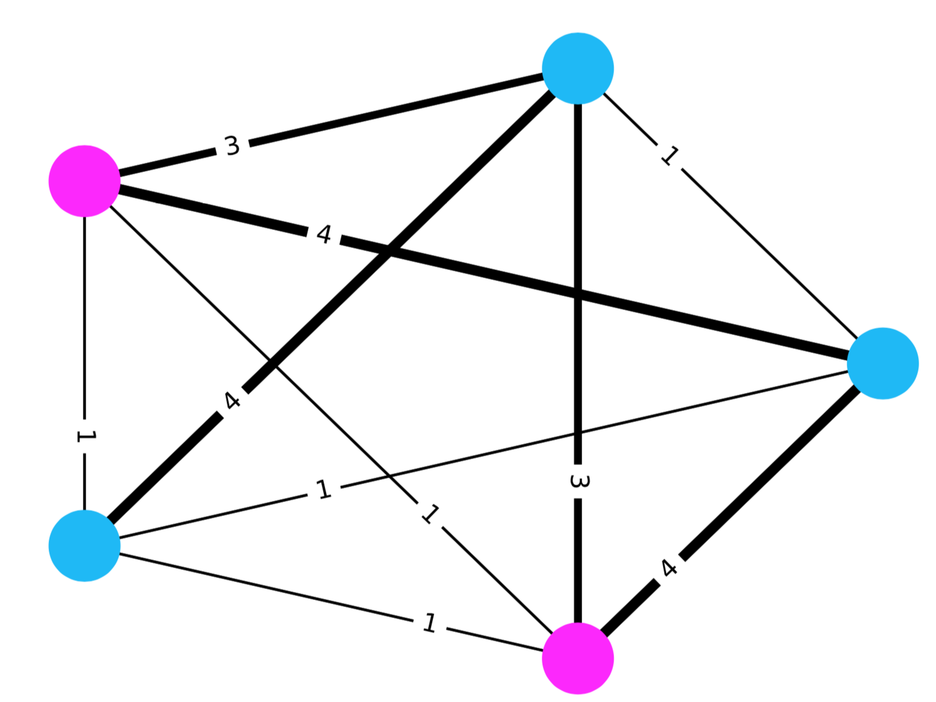

BD or BD-D




DB or DB-D

- **Moran Process**: Simple stochastic process describing finite populations with no interactions. It captures genetic drift and natural selection to an extent. Given $i$ mutants, fixation probability is $i/n$.

## Model




- Each vertex is an individual.
- 2 variants: Residents $\mathcal{R}$ and mutants $\mathcal{M}$.
- Edge weight $\implies$ Strength of connection
- Payoff matrix determines the dynamics

$$G = \begin{array}{c c} & \begin{array}{cc} \mathcal{R} & \mathcal{M} \end{array} \\ \begin{array}{c} \mathcal{R} \\ \mathcal{M} \end{array} & \begin{pmatrix} a & b \\ c & d \end{pmatrix} \end{array}$$

- Fixation probability of mutant $\rho_{\mathcal{M}} =$ $\Pr[(\mathcal{M}, \mathcal{R}, \mathcal{R}, \dots, \mathcal{R}) \rightarrow (\mathcal{M}, \mathcal{M}, \mathcal{M}, \dots, \mathcal{M})]$

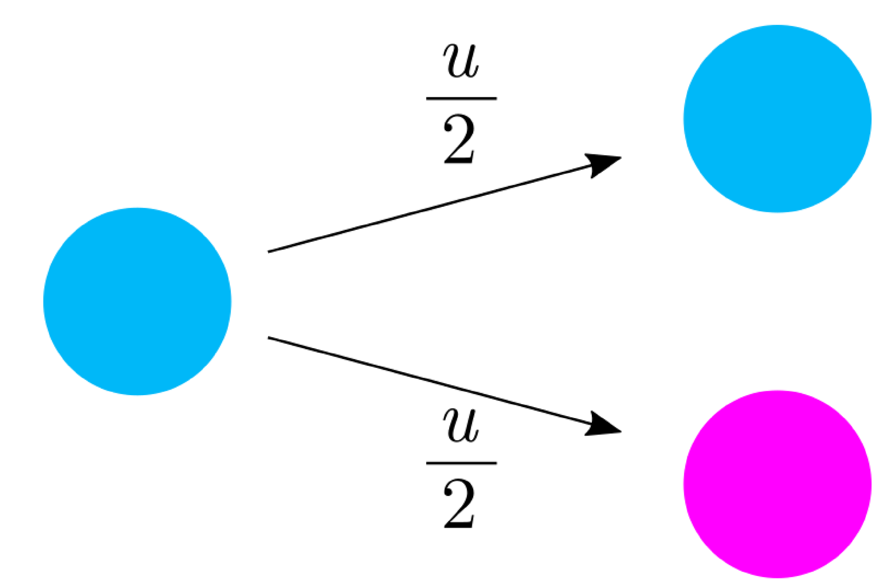
## Applications

Designing matrices from literature, the model was used to find MSE for these cases:

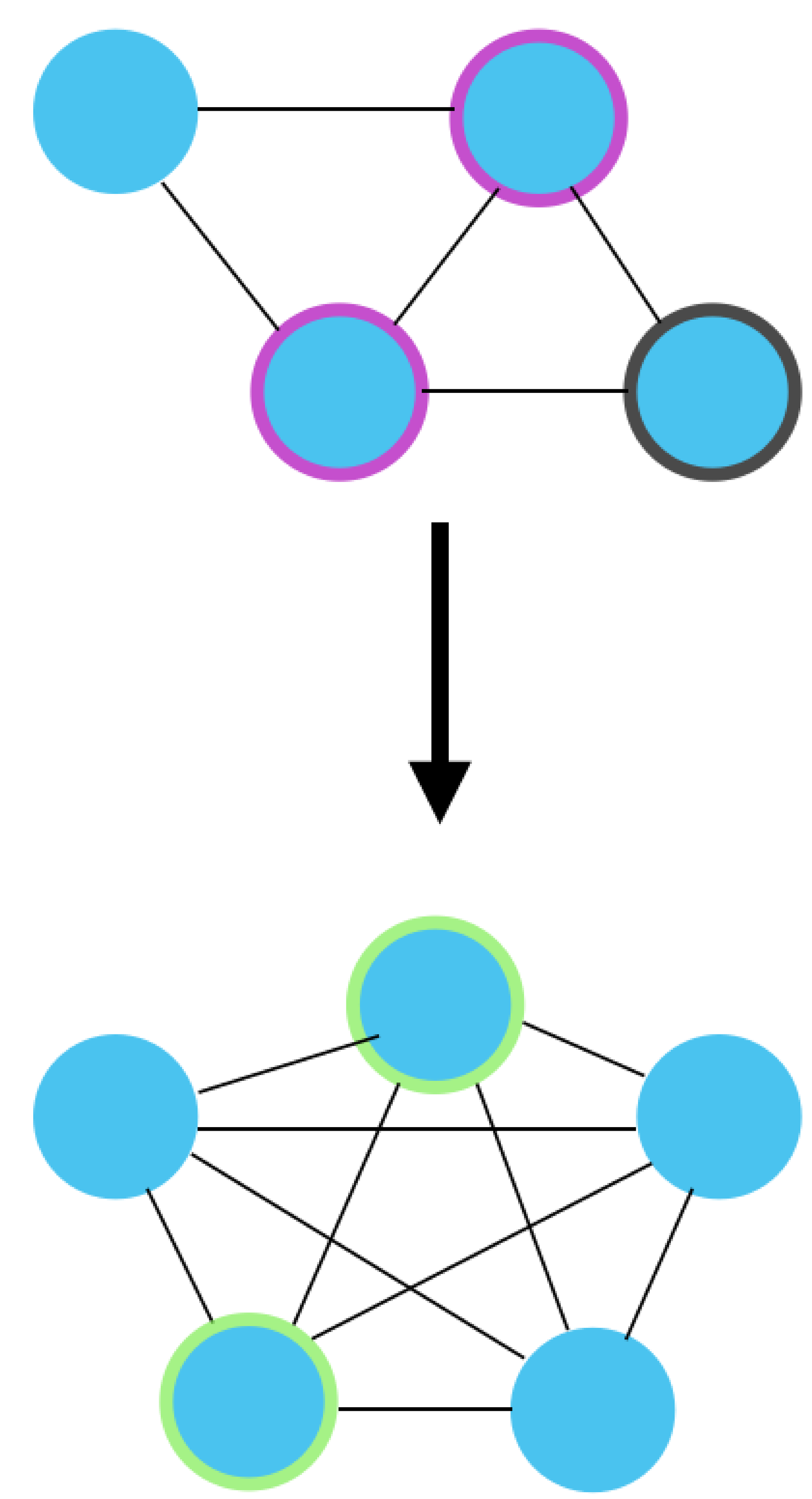
- **Cooperative breeding**: Females lay eggs in same nest and take care of offspring together. *Mutants* would lay eggs, but not take care of offspring
- **Vaccinations** Resident: Vaccinated.
  Fitness Proportional to network
  Mutant: Non-Vaccinated.
  More susceptible to infection

## Extensions

- **Mutations**
  Errors in reproduction can lead to residents giving birth to mutants and vice versa.




- **Sexual reproduction**
  - Two individuals mate and form two offsprings.
  - Offspring node connections are convex combinations of parents' node connections.
  - System has finite carrying capacity.



## Future Prospects

We hope to see the use of Evolutionary Games on Graphs for the following applications:

1. Sympatric Speciation
2. Designing networks such as organizational structures and scientific collaboration graphs
3. Studying Social Dilemma using games on graphs
4. Epidemiology

## Summary

- We characterized evolutionary games on graphs
- We applied to two natural systems, cooperative breeding and vaccinations in populations
- We extended the model for mutations and sexual reproduction

## References

[1] H. Ohtsuki, J. M. Pacheco, and M. A. Nowak, "Evolutionary graph theory: breaking the symmetry between interaction and replacement," *Journal of theoretical biology*, vol. 246, pp. 681–694, 06 2007.
[2] B. Allen, A. Traulsen, C. E. Tarnita, and M. A. Nowak, "How mutation affects evolutionary games on graphs," *Journal of Theoretical Biology*, vol. 299, pp. 97–105, 04 2012.
[3] B. Allen and M. A. Nowak, "Games on graphs," *EMS surveys in mathematical sciences*, vol. 1, no. 1, pp. 113–151, 2014.
[4] E. Lieberman, C. Hauert, and M. A. Nowak, "Evolutionary dynamics on graphs," *Nature*, vol. 433, pp. 312 EP –, 01 2005.
[5] M. A. Nowak, C. E. Tarnita, and T. Antal, "Evolutionary dynamics in structured populations," *Philosophical Transactions of the Royal Society B: Biological Sciences*, vol. 365, pp. 19–30, 01 2010.

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