STATISTICAL INFERENCE

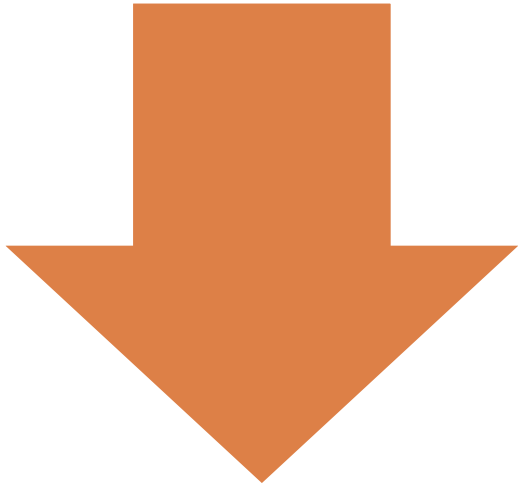
ESTIMATION

HYPOTHESIS TESTING

STATISTICAL INFERENCE

- What it is? :
  - the process of generalizing information obtained from a sample to a population.
- Areas of inferential statistics?:
  - Estimation – sample data are used to estimate the value of unknown parameters such as μ and *p*.
  - Hypothesis Testing – statements regarding a characteristics of one or more populations are tested using sample data.

INFERENTIAL STATISTICS



ESTIMATION

- "guessing" the value of the parameter
- provide the measure of the quality (reliability) of the guess

HYPOTHESIS TESTING

- making a "yes-no" decision regarding the parameter
- understand the chances of making incorrect decision

ESTIMATION

ESTIMATION THE SAMPLE SIZE FOR POPULATION MEAN AND POPULATION PROPORTION

ESTIMATION ON CONFIDENCE INTERVAL WITH TWO POPULATION MEAN

ESTIMATION ON CONFIDENCE INTERVAL WITH POPULATION PROPORTION

ESTIMATION

- Estimation from sample are only guesses (of the parameter)
- Every estimate has a standard error, and it is measure of the variation in the estimates.
  - Standard error – A value that measures the spread of the sample means ($\bar{x}$) around the population mean (μ). The standard error is reduced when the sample size is increased.
- Regardless of the population being estimated, we always expect sampling error:
  - Sampling error -the difference between a measure (statistics) computed from a sample and the corresponding measure (parameter) computed from the population.

POINT ESTIMATE

- **Definition**: is the value of a statistic that estimate the value of a parameter.
- For example:
  - the sample mean, $\bar{x}$ is a point estimate of the population mean, μ
  - the sample proportion $\hat{p}$ is a point estimate of the population proportion, p.
- It may subject to sampling error

INTERVAL ESTIMATE

- **Definition**: Is defined by two numbers, between which parameter is said to lie
- For example:

  $a < x < b$ is an interval estimate of the population mean, μ. It indicates that the population mean is greater than a but less than b
- A common procedure to calculate an interval estimate is know as confidence interval.

Proportion: individual in the sample does or does not have certain characteristics.
Population proportion = p
Sample proportion = $\hat{p}$

CONFIDENCE INTERVAL (CI)

- A **Confidence Interval** for an unknown parameter consists of an interval of numbers based on a point estimate.

- Confidence Interval estimates are of the form:

  point estimate ± margin of error

- **Level of confidence** (or confidence level) is a frequency (i.e., the proportion) of possible confidence intervals that contain the true value of their corresponding parameter.

  Denoted as: **$(1 - \alpha) \cdot 100\%$**

MARGIN ERROR

- A measure of how close we expect the point estimate to be to the population parameter with the specified level of confidence.

Margin of Error for Estimating μ, σ Known

$$e = z\frac{\sigma}{\sqrt{n}}$$

where:

e = Margin of error
z = Critical value
$\frac{\sigma}{\sqrt{n}}$ = Standard error of the sample distibution

CRITICAL VALUE

- Critical Value
  - Represent the number of standard deviations the sample statistics can be from the parameter and still result in an interval that includes the parameter.
  - Indicate as the value of $z_{\frac{\alpha}{2}}$

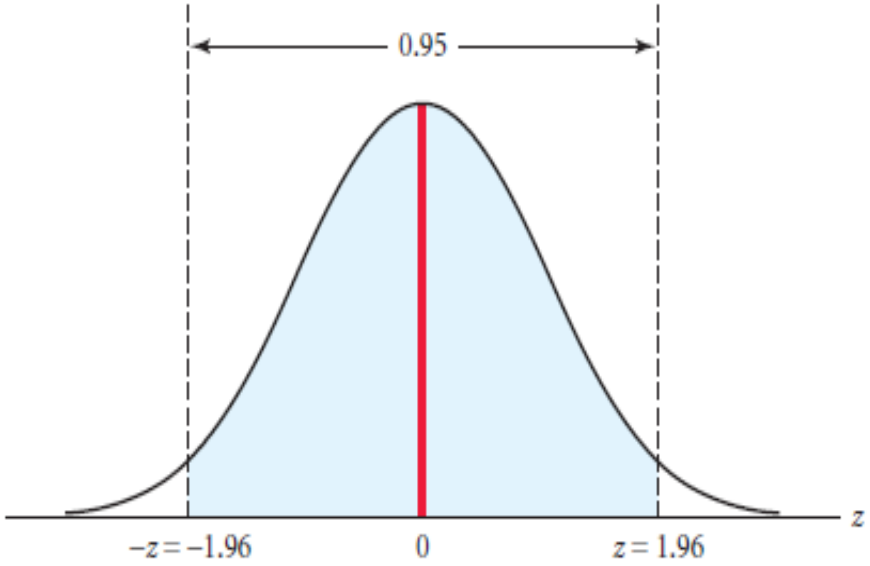
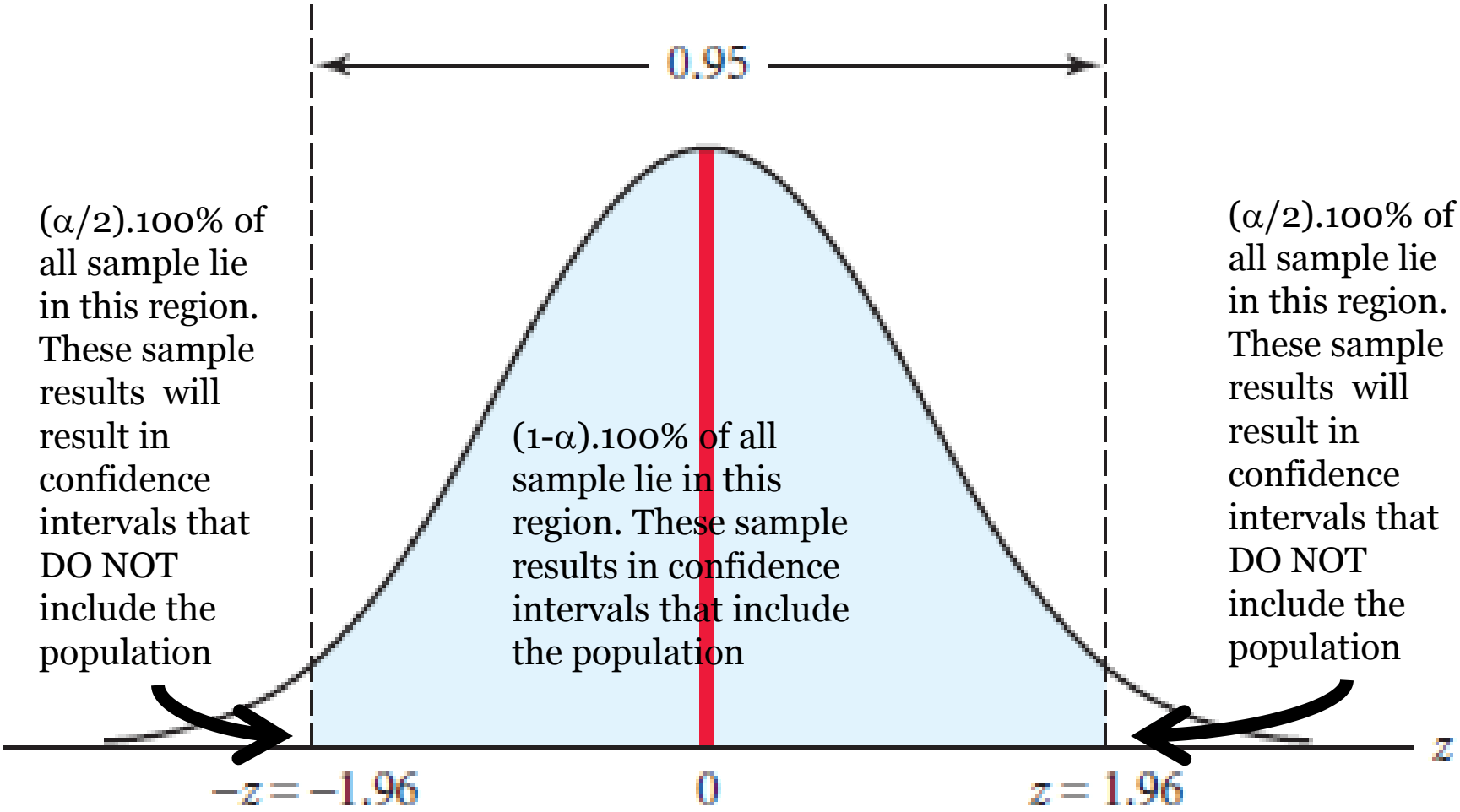


Figure: critical value for a 95% confidence interval

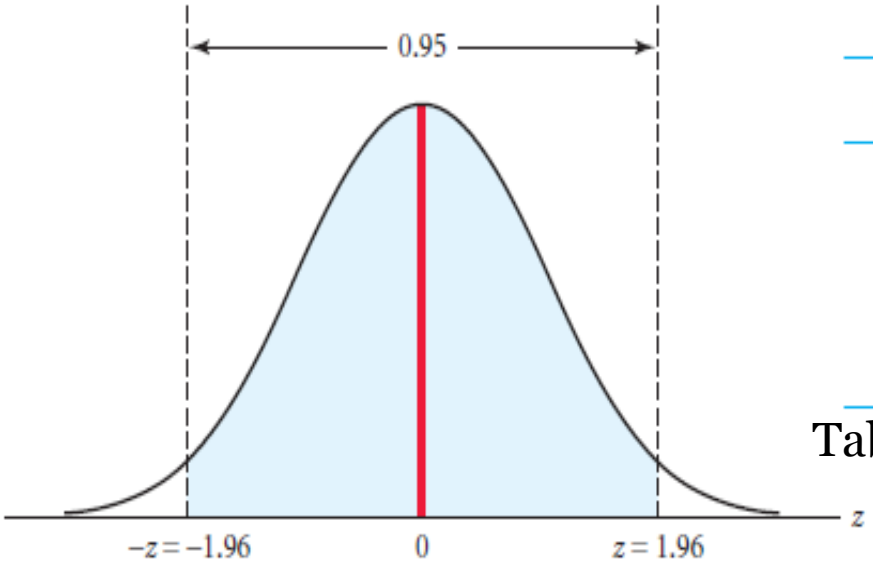
CRITICAL VALUE



How to find z- value (critical value)?

You need to refer z-table (standard normal table):
For example:

- 95% CI, the level of significance = 5% or =0.05
  since it is 2 sided (lower and upper), thus, 0.05/2 = 0.025
- From table locate the value of 0.025
- The critical value = 1.96



Critical Value	Confidence Level
$z = 1.28$	80%
$z = 1.645$	90%
$z = 1.96$	95%
$z = 2.575$	99%

Table: Most commonly use confidence level

Figure: critical value for a 95% confidence interval

IMPACT OF CHANGING THE CONFIDENCE LEVEL

PROBLEM:

National Recycling operates a garbage hauling company in a southern Marine city, Australia. Each year, the company must apply for a new contract with the state. The contract is in part based on the pounds of recycled materials collected. Part of the analysis that goes into contract development is an estimate of the mean pounds of recycled material submitted by each customer in the city on a quarterly basis. The city has asked for both 99% and 90% confidence interval estimates for the mean. The simple random sample of n = 100 customers is selected for the analysis. The sample mean, $\bar{x}$= 40.78 pounds and the population standard deviation, $\sigma_{\bar{x}}$ = 1.26 pounds. If, after the contract has been signed, the actual mean pounds deviates from the estimate over time, an adjustment will be made (up or down) in the amount National Recycling receives. Identify the margin error if both confidence level estimates for the mean is applied.

IMPACT OF CHANGING THE CONFIDENCE LEVEL?

SOLUTION:

For 99% confidence level:

$\bar{x} \pm z\frac{\sigma}{\sqrt{n}} = 40.78 \pm 2.575\frac{1.26}{\sqrt{100}} = 40.78 \pm 3.24$ = [40.456 pounds, 41.10 pounds]

The margin error for 99% is ± 0.64 pounds

For 90% confidence level:

$\bar{x} \pm z\frac{\sigma}{\sqrt{n}} = 40.78 \pm 1.645\frac{1.26}{\sqrt{100}} = 40.78 \pm 0.207$ = [40.57 pounds, 40.98 pounds]

The margin error for 90% is ± 0.41 pounds

SUMMARY:

The margin error is only 2.07 pounds when the confidence level is reduced from 99% to 90%.

It shows that the margin error will be smaller when the confidence level is smaller.

IMPACT OF CHANGING THE CONFIDENCE LEVEL?

Values that affect the margin error to be smaller:

- The confidence level:
  - the margin error will be smaller when the confidence level is smaller.
- Population standard deviation (σ):
  - the more σ can be reduced, the smaller the margin error will be.
  - However, reducing the population standard deviation (σ) is not possible
- Sample size:
  - An increase sample size reduced the standard error of the sampling distribution. Thus as the direct way of reducing margin error

ESTIMATION

ESTIMATING THE SAMPLE SIZE FOR POPULATION MEAN AND POPULATION PROPORTION

ESTIMATING THE SAMPLE SIZE

- To estimate the sample size, the decision maker need to specify their confidence level and their desired margin error, *e*
- 3 Scenario to consider:
  1) For population mean, when population standard deviation, σ known
  2) For population mean, when population standard deviation, σ unknown
     - 3 approaches to estimate unknown σ:
       - to use a value for *s* that is considered to be at least as large as the true *s*.
       - to select a **pilot sample**
       - to use the range of the population to estimate the population's standard deviation. ($\sigma \approx \frac{R}{6}$)
  3) For population proportion

ESTIMATING THE SAMPLE SIZE FOR POPULATION MEAN.

1) When population standard deviation, σ known

Sample Size Requirement for Estimating μ, σ Known

$$n = \left(\frac{z\sigma}{e}\right)^2 = \frac{z^2\sigma^2}{e^2}$$

where:

z = Critical value for the specified confidence level
e = Desired margin of error
σ = Population standard deviation

When population standard deviation, σ known

Problem:

Consider the Mission Valley Power Company (MVP) in northwest Michigan, which has more than 6,000 residential customers. In response to a request by the Michigan Public Utility Commission, MVP needs to estimate the average kilowatts of electricity used by customers on February 1. The only way to get this number is to select a random sample of customers and take a meter reading after 5:00 P.M. on January 31 and again after 5:00 P.M. on February 1. The commission has specified that any estimate presented in the utility's report must be based on a 95% confidence level. Further, the margin of error must not exceed 30 kilowatts. Given these requirements, what size sample is needed?

Solution:

Equation: $n = \frac{z^2\sigma^2}{e^2}$, the z value for 95% is 1.96 (refer to Standard Normal Distribution table). For known σ (MPV has conduct in previous study), the value is 200.

Therefore, $n = \frac{1.96^2 200^2}{30^2} = 170.73 \sim 171$ customers

What if the confidence level changes? What is the new sample size?

ESTIMATING THE SAMPLE SIZE FOR POPULATION MEAN.

2) When population standard deviation, σ unknown (with the 3 option as in slide #16, now σ is unknown)

Sample Size Requirement for Estimating μ, σ Known

$$n = \left(\frac{z\sigma}{e}\right)^2 = \frac{z^2\sigma^2}{e^2}$$

where:

z = Critical value for the specified confidence level
e = Desired margin of error
σ = Population standard deviation

When population standard deviation, σ unknown

Problem:

Consider a situation in which the regional manager for XYZ Fuel Station in Penang wishes to know the average gallons of petrol purchased by customers each time they fill up their car. Not only does he not know μ, he also does not know the population standard deviation, σ. He wants a 90% confidence level and is willing to have a margin of error of 0.50 gallons in estimating the true mean gallons purchased.

Steps to solution:

1. **Specify the desired margin of error.**

The manager wants the estimate to be within ±0.50 gallons of the true mean. Thus, $e = 0.50$

2. **Determine an estimate for the population standard deviation.**

The manager will select a pilot sample of $n = 20$ fill-ups and record the number of gallons for each. These values are:

18.9	22.4	24.6	25.7	26.3	28.4	21.7	31.0	19.0	31.7
17.4	25.5	20.1	34.3	25.9	20.3	21.6	25.8	31.6	28.8

The estimate for the population standard deviation is the sample standard deviation for the pilot sample. This is computed using,

$$s = \sqrt{\frac{\sum(x-\bar{x})^2}{n-1}} = \sqrt{\frac{(18.9-25.05)^2+(22.4-25.05)^2+(24.6-25.05)^2+\cdots+(28.8-25.05)^2}{20-1}} = 4.85$$

Therefore the $\sigma \approx 4.85$

3. Determine the critical value for the desired level of confidence.

The critical value will be a *z*-value from the standard normal table. The 90% confidence level gives $z = 1.645$

4. Calculate the required sample size using pilot sample's standard deviation.

$$n = \frac{z^2\sigma^2}{e^2} = \frac{(1.645^2)(4.85^2)}{0.50^2} = 254.61 \approx 255$$

Summary:

The required sample size is 255 fill-ups, but we can use the pilot sample as part of this total. Thus, the net required sample size in this case is 255 - 20 = 235

ESTIMATING THE SAMPLE SIZE FOR POPULATION PROPORTION

- Margin of error for estimating population proportion, p

$$e = z\sqrt{\frac{p(1-p}{n}}$$

where:

p = population proportion
n = sample size
z = critical value from standard normal distribution table for desired confidence interval

- Sample size for estimating population proportion, p

$$n = \frac{z^2 p(1-p)}{e^2}$$

where:
e = desired margin error
p = population proportion
n = sample size
z = critical value from standard normal distribution table for desired confidence interval

Problem:

The customer account manager for Neumann Research, a marketing research company located in Cincinnati, Ohio, is interested in estimating the proportion of a client's customers who like a new television commercial. She wishes to develop a 90% confidence interval estimate and would like to have the estimate be within 0.05 of the true population proportion. Can you help her to determine the required sample size?

Steps to solution:

1. **Specify the desired level of confidence and the critical value.**

The desired confidence level is 90%, thus the critical value, $z = 1.645$

2. **Determine the desired margin error.**

The account manager whishes the margin of error to be 0.05

3. **Obtain at a value to use for population proportion, *p***

Two options can be used to obtain a value for p:

1. Use a pilot sample and compute $\bar{p}$, the sample proportion. Use $\bar{p}$ to approximate p.
2. Select a value for p that is closer to 0.50 than you actually believe the value to be. If you have no idea what p might be, use $p = 0.50$, which will give the largest possible sample size for the stated confidence level and margin of error.

Steps to solution:

In this case, suppose the account manager has no idea what p is but wants to make sure that her sample is sufficiently large to meet her estimation requirements. Then she will use $p = 0.50$.

4. Determine the sample size.

$$n = \frac{z^2 p(1-p)}{e^2} = \frac{1.645^2(0.50)(1-0.50)}{0.05^2} = 270.0625 \approx 271$$

The account manager should randomly survey 271 customers.

Problem:

An economist wants to know if the proportion of the US population who commutes to work via carpooling is on the rise. What size sample should be obtained if the economist wants an estimate within 2 percentage points of the true proportion with 90% confidence if

a) the economist uses the 2009 estimate of 10% obtained from the American Community Survey
b) the economist does not use any prior estimates?
c) What you conclude by having (a) and (b)?

Steps to solution:

1. **Specify the desired level of confidence and the critical value.**

The desired confidence level is 90%, thus the critical value, $z = 1.645$

2. **Determine the desired margin error.**

The account manager whishes the margin of error to be 2% or 0.02

To answer (a):

$$n = \frac{z^2 p(1-p)}{e^2} = \frac{1.645^2(0.10)(1-0.10)}{0.02^2} = 608.9 \approx 609$$

The economist must survey 609 randomly selected residents of the United States

Steps to solution:

To answer (b):
In this case, the economist does not use any prior estimates but wants to make sure that her sample is sufficiently large to meet her estimation requirements. Then she will use $p = 0.50$.

$$n = \frac{z^2 p(1-p)}{e^2} = \frac{1.645^2(0.50)(1-0.50)}{0.02^2} = 1691.3 \approx 1692$$

The economist must survey 1692 randomly selected residents of the United States.

To answer (c):
The effect of not having prior estimate of population proportion, p is that the sample size more than a double!

ESTIMATION

ESTIMATING CONFIDENCE INTERVAL FOR POPULATION PROPORTION

ESTIMATING A POPULATION PROPORTION

- Why study proportion? : some may interested in situations in which the value of interest is the proportion of items in the population that possess a particular attribute.
  - Example: You may wish to estimate the proportion of customers who are satisfied with the service provided by your company

- Notation: p = population proportion

  $\bar{p}$= sample proportion

CONFIDENCE INTERVAL ESTIMATE FOR POP. PROPORTION

- Equation for estimating confidence interval for population proportion:

$$\bar{p} \pm z_{\frac{\alpha}{2}} \sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

where:

$\bar{p}$ = sample proportion

n = sample size

z = critical value from standard normal distribution table for desired confidence interval

Problem:

The Swiss Hotel, Residence and Resort is thinking of starting a new promotion. When a customer checks out of the resort after spending 5 or more days, the customer would be given a voucher that is good for 2 free nights on the next stay of 5 more or more nights at the resort. The marketing manager is interested in estimating proportion of customers who return after getting a voucher. From a simple random sample of 100 customers, 62 returned within 1 year after receiving the voucher. Help the marketing manager in estimating the confidence interval for the returning customer after getting the voucher.

Steps to solution:

1. **Specify the desired level of confidence and the critical value.**

Assuming the marketing manager is desired for 95% confidence level, thus the critical value, $z = 1.96$

2. **Compute the point estimate based on the sample data.**

$$\bar{p} = \frac{x}{n} = \frac{62}{100} = 0.62$$

where x is number of items in the sample with the attribute of interest

3. Compute the confidence interval.

The 95% confidence interval estimate is:

$$\bar{p} \pm z_{\frac{\alpha}{2}} \sqrt{\frac{\bar{p}(1-\bar{p}}{n}}$$

$$0.62 \pm 1.96 \sqrt{\frac{0.62(1-0.62}{100}}$$

$$0.62 \pm 0.095$$

$$[0.525 \_\_\_\_\_ 0.715]$$

The confidence interval for the returning customer after getting the voucher at 95% confidence level is between [0.525,0.715]

Problem:

In the Parent-Teen Cell Phone survey conducted by Princeton Survey Research Associates International, 800 randomly sampled 16-17 year old living in the United States were asked whether they have ever used their cell phone to text while driving. Of the 800 teenagers surveyed, 272 indicated that they text while driving. Obtain a 95% confidence interval for the proportion 16-17 year old who text while driving.

Steps to solution:

1. Specify the desired level of confidence and the critical value.

The desired confidence level is 95% confidence level, thus the critical value, $z_{\frac{\alpha}{2}}$ $=z_{0.025} = 1.96$

2. Compute the point estimate based on the sample data.

$\hat{p} = \frac{x}{n} = \frac{272}{800} = 0.34$

where x is number of items in the sample with the attribute of interest

Note:

point estimate for population proportion is the sample proportion $\hat{p}$

3. Compute the confidence interval.

The 95% confidence interval estimate is:

$$\hat{p} \pm z_{\frac{\alpha}{2}} \sqrt{\frac{\bar{p}(1-\bar{p}}{n}}$$

$$0.34 \pm 1.96 \sqrt{\frac{0.34(1-0.34}{800}}$$

$$0.34 \pm 0.033$$

$$[0.307 \_\_\_\_\_ 0.373]$$

We are 95% confidence that the proportion of 16-17 year old who text while driving is between [0.307,0.373]

ESTIMATION

ESTIMATING CONFIDENCE INTERVAL FOR SINGLE POPULATION MEANS

- Point Estimate: the sample mean, $\bar{x}$ is a point estimate of the population mean, μ
- Equation for estimating confidence interval for single population mean:

$$\bar{x} \pm t_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}}$$

where:

$\bar{x}$ = point estimate of sample mean
n = sample size
s = sample standard deviation
t = critical value from Student's t-distribution table for desired confidence interval

- Equation for estimating margin error for single population mean:

$$e = t_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}}$$

- Steps in constructing confidence interval for single population mean.
  1. Compute the value of sample mean, $\bar{x}$ and sample standard deviation, s

     Sample standard deviation, $s = \sqrt{\frac{\sum(x-\bar{x})^2}{n-1}}$ where x = data element, n = sample size
  2. Determine the critical value from student's t-distribution $t_{\frac{\alpha}{2}}$ with n-1 degree of freedom.
  3. Use the formula to determine the lower and upper bounds of the confidence interval.
  4. Interpret the result.

Problem:

The website fueleconomy.gov allows drivers to report the miles per gallon of their vehicle. The data is illustrate in the Table below. Construct a 95% confidence interval for the mean mules per gallon for the respective data.

35.7	37.2	34.1	38.9
32.0	41.3	32.5	37.1
37.3	38.3	38.2	39.6
32.2	40.9	37.0	36.0

Steps to solution:

1. Compute the value of sample mean, $\bar{x}$ and sample standard deviation, s

$\bar{x}$ = 36.8 mpg and the sample standard deviation, s = 2.92 mpg

2. Determine the critical value from student's t-distribution $t_{\frac{\alpha}{2}}$ with n-1 degree of freedom.

Since we want 95% confidence level, the α = 0.05. With the sample size, n = 16, therefore with n-1 degree of freedom is 15

Thus, the critical value for $t_{\frac{\alpha}{2}}$ with n-1 degree of freedom according to students t- distribution table is 2.131

Steps to solution:

3. Use the formula to determine the lower and upper bounds of the confidence interval.

$$\bar{x} \pm t_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}}$$

The lower bound: $36.8 - 2.131 \frac{2.92}{\sqrt{16}} = 35.24$

The upper bound: $36.8 + 2.131 \frac{2.92}{\sqrt{16}} = 38.36$

The 95% confidence interval are: [35.24 _____38.36]

We are 95% confidence that the mean miles per gallon for the mentioned data is between [35.24, 38.36]

ESTIMATION

ESTIMATING CONFIDENCE INTERVAL FOR TWO POPULATION MEANS:

- Independent Samples
- Paired Samples

WHY NEED TO KNOW?

- Previously, we have discuss estimation involving a single population parameter.
- In many business decision-making situations, managers must decide between two or more alternatives.
  - For example, farmers must decide which of several brands and types of wheat to plant.

- There are statistical procedures that can help decision makers use sample information to compare different populations.
- The samples from the two populations will be either:
  - Independent Samples
  - Paired Samples

ESTIMATION FOR TWO POPULATION MEANS USING INDEPENDENT SAMPLES

- Independent Samples:
  - Samples selected from two or more populations in such a way that the occurrence of values in one sample has no influence on the probability of the occurrence of values in the other sample(s).

- The estimation between the means will cover:
  1. The population standard deviation are known and the samples are independent
  2. The population standard deviation are unknown and the samples are independent

ESTIMATING THE DIFFERENCE BETWEEN TWO POPULATION MEANS WHEN σ_1 AND σ_2 ARE KNOWN, USING INDEPENDENT SAMPLES

- In business application, sometimes it may be interested in estimating the difference between two population means.
  - For instance:
    - Estimating the difference in mean starting salaries between males and females
    - Estimating the difference in mean service times at two different fast-food businesses.

- Point estimate = $\bar{x}_1 - \bar{x}_2$

ESTIMATING THE DIFFERENCE BETWEEN TWO POPULATION MEANS WHEN σ_1 AND σ_2 ARE KNOWN, USING INDEPENDENT SAMPLES

Standard Error of $\bar{x}_1 - \bar{x}_2$ When σ_1 and σ_2 Are Known

$$\sigma_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

where:

σ_1^2 = Variance of population 1

σ_2^2 = Variance of population 2

n_1 and n_2 = Sample sizes from populations 1 and 2

Confidence Interval of μ_1-μ_2 When σ_1 and σ_2 Are Known, Independent Samples

$$(\bar{x}_1 - \bar{x}_2) \pm z\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

Critical Value	Confidence Level
$z = 1.28$	80%
$z = 1.645$	90%
$z = 1.96$	95%
$z = 2.575$	99%

Table: The *z* values for several of the most commonly used confidence levels

ESTIMATING THE DIFFERENCE BETWEEN TWO POPULATION MEANS WHEN σ_1 AND σ_2 ARE KNOWN, USING INDEPENDENT SAMPLES

Problem:

Healthcare Associates operate 33 medical clinics in Minnesota and Wisconsin. As part of the company's ongoing efforts to examine its customer service, Healthcare worked with one of the Minnesota universities on a project in which a team of students observed patients at the clinics to estimate the difference in mean time spent per visit for men and women patients. The student team has selected simple random samples of 100 males, the $\bar{x}_{males} = 34.5$ minutes and 100 females, the $\bar{x}_{females} = 42.4$ minutes at different times in different clinics owned by the company across Minnesota and Wisconsin. Previous studies indicate that the standard deviation is 11 minutes for males and 16 minutes for females. Determine the 95% confidence interval estimate for the difference in mean times for men and women patients.

Steps to solution:

1. **Define the population parameter of interest and select independent samples from the two populations.**

The healthcare association is interested in estimating the difference in mean time spent in the clinic between males and females. The measure of interest is $\boldsymbol{\mu_1 - \mu_2}$.

Steps to solution:

2. Specify the desired confidence level.

To have a 95% confidence interval estimate.

3. Compute the point estimate.

Point estimate = $\bar{x}_{males} - \bar{x}_{females} = 34.5 - 42.4 = -7.9$ minutes

Women in the sample spent an average of 7.9 minutes longer in the clinic.

4. Determine the standard error of the sampling distribution

standard error, $\sigma_{\bar{x}_1,\bar{x}_2} = \sqrt{\frac{\sigma 2_1}{n_1} + \frac{\sigma 2_2}{n_2}} = \sqrt{\frac{11^2}{100} + \frac{16^2}{100}} = 1.9416$

5. Determine the critical value, *z*, from the standard normal table.

The critical value *z* at 95% confidence level are $z = 1.96$

6. Develop the confidence interval estimate

confidence interval, $(\bar{x}_1 - \bar{x}_2) \pm z\left(\sigma_{\bar{x}_1,\bar{x}_2}\right) = (-7.9) \pm 1.96(1.9416)$

$= -7.9 \pm 3.8056$

7. Identify the summary for the problem studied

The 95% confidence interval estimate for the difference in mean time spent in the medical clinics between men and women is -11.706 minutes ≤ (μ_1-μ_2) ≤ -4.094 minutes. Thus, based on the sample data and the specified confidence level, women spend on average between 4.09 and 11.71 minutes longer at the Healthcare Associates clinics.

ESTIMATING THE DIFFERENCE BETWEEN TWO POPULATION MEANS WHEN σ_1 AND σ_2 ARE UNKNOWN, USING INDEPENDENT SAMPLES

- When estimating two population mean when the population standard deviation is unknown and the sample sizes are small, the critical value is a *t*-value from the *t*-distribution.

- The following assumptions hold:
  - The populations are normally distributed.
  - The populations have equal variances.
  - The samples are independent.

ESTIMATING THE DIFFERENCE BETWEEN TWO POPULATION MEANS WHEN σ_1 AND σ_2 ARE UNKNOWN, USING INDEPENDENT SAMPLES

Problem:

The head of research and development at Sneva Pharmaceutical Research is interested in estimating the difference between individuals age 50 and under and those over 50 years old with respect to the mean time from when a patient takes a new medication until the medication can be detected in the blood. Once she estimates the difference, if a difference does exist, the company can use this information to guide doctors in how to advise their patients to use this medication. A simple random sample of six people age 50 or younger and eight people over 50 participated in the study. The research manager wishes to have a 95% confidence interval estimate. The resulting sample means and sample standard deviations for the two groups are:

Age ≤ 50 years	**Age > 50 years**
$\bar{x}_1 = 13.6$ minutes	$\bar{x}_1 = 11.2$ minutes
$S_1 = 3.1$ minutes	$S_2 = 5.0$ minutes

Steps to solution:

1. **Define the population parameter of interest and select independent samples from the two populations.**

The objective here is to estimate the difference in mean time between the two age groups with respect to the speed at which the medication reaches the blood. The research lab has selected simple random samples of six "younger" and eight "older" people. Because the impact of the medication in one person does not influence the impact in another person, the samples are independent.

2. **Specify the desired confidence level.**

To have a 95% confidence interval estimate.

3. **Compute the point estimate.**

Point estimate = $\bar{x}_1 - \bar{x}_2 = 13.6 - 11.2 = 2.4$ minutes

4. **Determine the standard error of the sampling distribution**

The pooled standard deviation is computed using: $s_p = \sqrt{\frac{(n_1-1)s^2{}_1+(n_2-1)s^2{}_2}{n_1+n_2-2}}$

$$s_p = \sqrt{\frac{(6-1)3.1^2 + (8-1)5.0^2}{6+8-2}} = 4.31$$

then standard error is calculated as, $s_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}} = 4.31\sqrt{\frac{1}{6}+\frac{1}{8}} = 2.3277$

Steps to solution:

5. Determine the critical value, *t*, from the student's t table.

Because the population standard deviations are unknown, the critical value will be a *t*-value from the *t*-distribution as long as the population variances are equal and the populations are assumed to be normally distributed.

The critical *t* for 95% confidence, $t = (n_1+n_2-2)$ *degree of freedom*, thus t = 6 + 8 - 2 = 12 degrees of freedom is $t = 2.1788$

6. Develop the confidence interval estimate

confidence interval, $(\bar{x}_1 - \bar{x}_2) \pm t(s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}) = (2.4) \pm 2.1788(2.3277)$

$=2.4 \pm 5.0715 = -2.6715 \leq (\mu_1-\mu_2) \leq 7.4715$

7. Identify the summary for the problem studied

Because the interval crosses zero, the research manager cannot conclude that a difference exists between the age groups with respect to the mean time needed for the medication to be detected in the blood. Thus, with respect to this factor, it does not seem to matter whether the patient is 50 or younger or over 50, so no advice for doctors is warranted.

STATISTICAL INFERENCE

ESTIMATION FOR TWO POPULATION MEANS:

Paired Samples

ESTIMATION FOR TWO POPULATION MEAN WITH PAIRED SAMPLES

- Previously, we have look into estimating the two population mean when the samples are independent.

- There are many situations in business where using paired samples should be considered.
  - What is paired samples? : Samples that are selected in such a way that values in one sample are matched with the values in the second sample for the purpose of controlling for extraneous factors.
  - Another term for paired samples is dependent samples.
  - Why paired samples? – to control for any variation in a sample. Example: different cars (and drivers), different painter and etc

ESTIMATION FOR TWO POPULATION MEAN WITH PAIRED SAMPLES

Example of business application:
A major oil company wanted to estimate the difference in average mileage for cars using a regular engine oil compared with cars using a synthetic-oil product. A random sample of 10 motorists (and their cars) was selected. Each car was filled with gasoline, the oil was drained, and new, regular oil was added. The car was driven 200 miles on a specified route. The car then was filled with gasoline and the miles per gallon were computed. After the 10 cars completed this process, the same steps were performed using synthetic oil. Because the same cars and drivers tested both types of oil, the miles-per-gallon measurements for synthetic oil and regular engine oil will most likely be related.

$$d = x_1 - x_2$$

where:

d = Paired difference

x_1 and x_2 = Values from samples 1 and 2, respectively

Paired Difference

$$\bar{d} = \frac{\sum_{i=1}^{n} d_i}{n}$$

where:

d_i = ith paired difference value

n = Number of paired differences

Point Estimate for the Population Mean Paired Difference, μ_d

$$s_d = \sqrt{\frac{\sum_{i=1}^{n} (d_i - \bar{d})^2}{n-1}}$$

where:

d_i = ith paired difference

$\bar{d}$ = Mean paired difference

Sample Standard Deviation for Paired Differences

Confidence Interval Estimate for Population Mean Paired Difference, μ_d

$$\bar{d} \pm t\frac{s_d}{\sqrt{n}}$$

where:

t = Critical t value from t-distribution with $n-1$ degrees of freedom

$\bar{d}$ = Sample mean paired difference

s_d = Sample standard deviation of paired differences

n = Number of paired differences (sample size)

ESTIMATION FOR TWO POPULATION MEAN WITH PAIRED SAMPLES

Problem:

Technology has done more to change golf than possibly any other sport in recent years. Titanium woods, hybrid irons and new golf ball designs have impacted professional and amateur golfers alike. PGA of America is the association that only professional golfers can belong to. The association provides many services for golf professionals, including operating an equipment training center in Florida. Recently, a maker of golf balls developed a new ball technology, and PGA of America is interested in estimating the mean difference in driving distance for this new ball versus the existing best-seller. To conduct the test, the PGA of America staff selected six professional golfers and had each golfer hit each ball one time. The sample data is as follows:

Calculate the 95% confidence interval estimate for the difference in population means for paired samples.

Golfer	Existing Ball	New Ball
1	280	276
2	299	301
3	278	285
4	301	299
5	268	273
6	295	300

Steps to solution:

1. **Define the population parameter of interest**

Because the same golfers hit each golf ball, the company is controlling for the variation in the golfers' ability to hit a golf ball. The samples are paired, and the population value of interest is μ_d, the mean paired difference in distance. We assume that the population of paired differences is normally distributed.

2. **From the sample data collected, compute the point estimate, $\bar{d}$**

Golfer	Existing Ball	New Ball	$\bar{d}$
1	280	276	4
2	299	301	-2
3	278	285	-7
4	301	299	2
5	268	273	-5
6	295	300	-5

The Point estimate is, $\bar{d} = \frac{\sum d}{n} = -2.17$ yards

Steps to solution:

3. Calculate the standard deviation, s_d

Standard deviation is computed using: $s_d = \sqrt{\frac{\sum(d-\bar{d})^2}{n-1}} = 4.36$ yard

4. Determine the critical value, *t*, from the *t* distribution table.

The critical *t* for 95% confidence, *t = (n-1) degree of freedom*, thus *t* = 6 - 1 = 5 degrees of freedom is *t* = 2.5706

5. Develop the confidence interval estimate

confidence interval, $\bar{d} \pm t\left(\frac{s_d}{\sqrt{n}}\right)$

$$(-2.17) \pm 2.5706\left(\frac{4.36}{\sqrt{6}}\right)$$

=-2.17 ± 4.58 = -6.75 yards _____ 2.41 yards

6. Identify the summary for the problem studied

Based on these sample data and the confidence interval estimate, which contains zero, the PGA of America must conclude that the new ball's average distance may not be any longer than that of the existing best-seller. This may affect whether the company that developed the new ball will continue to make it.

STATISTICAL INFERENCE

- end for ESTIMATION
- next: HYPOTHESIS TESTING