Assignment 3 Task 4

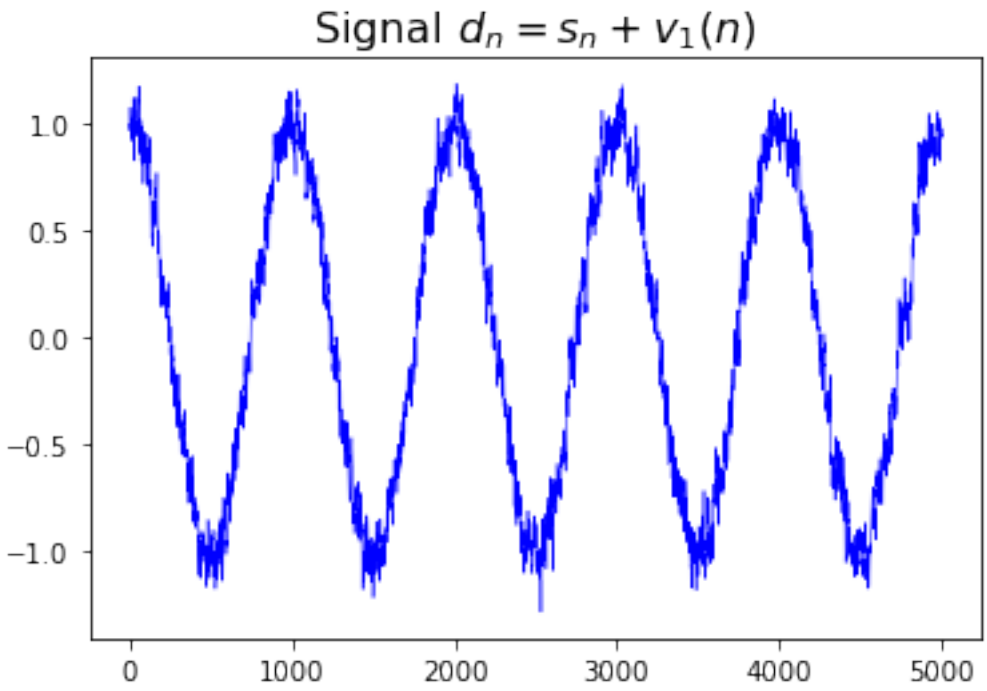
August 6, 2021

[34]:
```
import numpy as np
import matplotlib.pyplot as plt
import warnings
warnings.filterwarnings('ignore')
```

[35]:
```
def circ_shift(a): # Function defined for circular shifting and concatenation
    X = np.array([np.roll(a,1)]).T
    a = np.array([a]).T
    V2_modified = np.concatenate((a,X),axis=1)
    return V2_modified
```

1 Part i, ii and iii

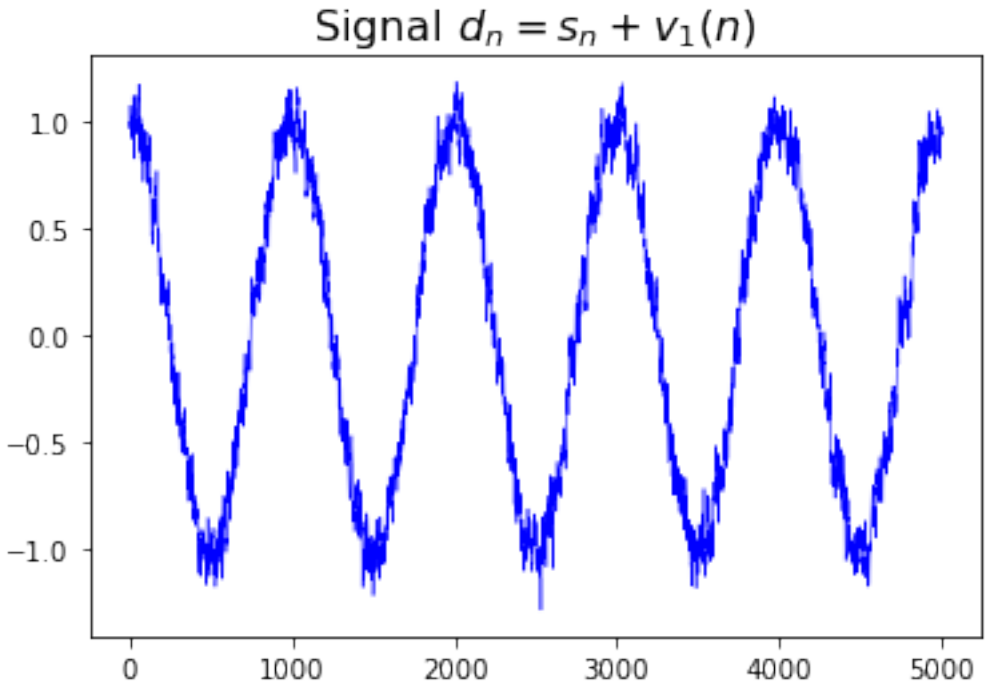
[36]:
```
np.random.seed()
N = 5000
omega = 2 * 10**(-3) * np.pi
sigma = np.sqrt(0.0025)
a1 = 0.8
a2 = 0.75
noise = sigma * np.random.randn(N,1)
S = np.cos(omega * np.arange(N)).reshape(-1,1)

# Part (b)
v1 = np.zeros((N,1))
v2 = np.zeros((N,1))

for i in range(1,N):
    v1[i] = a1 * v1[i-1] + noise[i]

# Part(c)
d_n= S + v1
x_axis = np.arange(0,N,1)
plt.figure(1)
plt.plot(x_axis,d_n,'b',lw=0.5)
plt.title(r'Signal $d_n=s_n+v_1(n)$', fontsize=16)
plt.show()
```

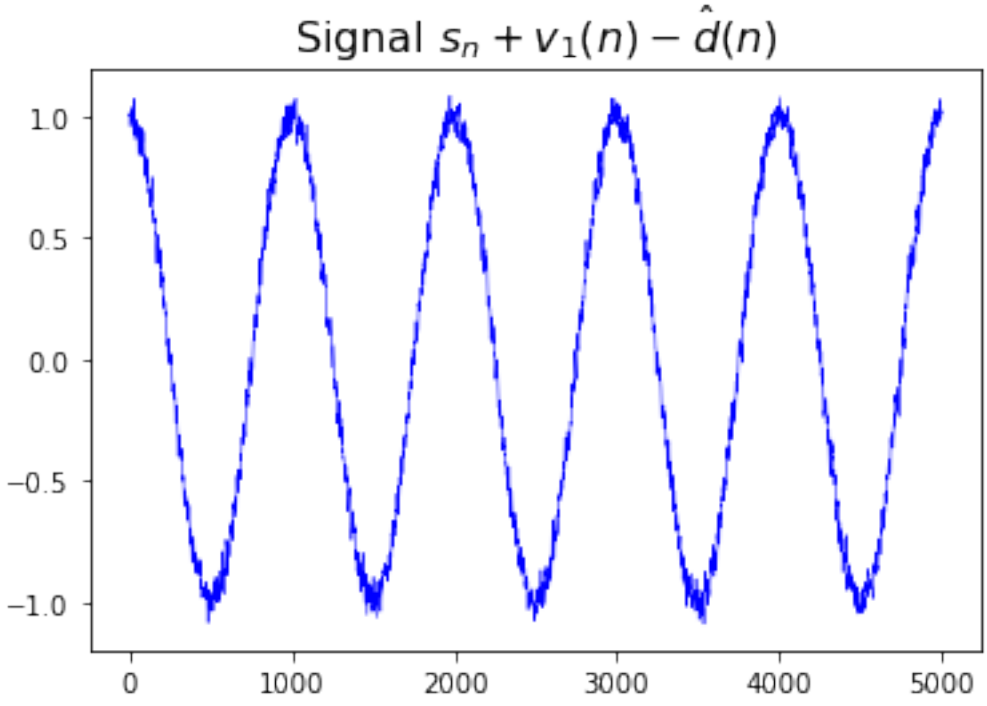


2 Part iv and v

[37]:
```
for i in range(1,N):
    v2[i] = a2 * v2[i-1] + noise[i]


v2_modified = circ_shift(v2.ravel())
W = np.linalg.inv(np.dot(v2_modified.T,v2_modified)).dot(v2_modified.T).dot(d_n)
d_est = v2_modified.dot(W)
S_hat_n = d_n - d_est
```

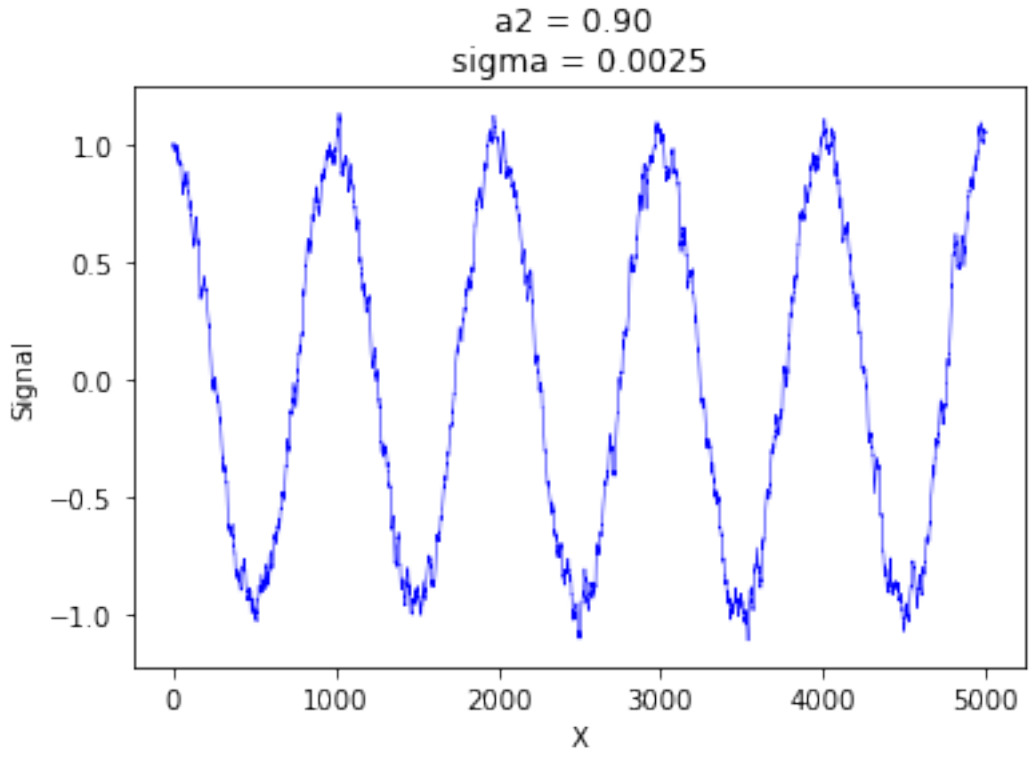
[38]:
```
x_axis = np.arange(0,N,1)
plt.figure(2)
plt.plot(x_axis,d_n, 'b',lw=0.5)
plt.title(r'Signal $d_n=s_n+v_1(n)$', fontsize=16)

plt.figure(1)
plt.plot(x_axis,d_n-d_est, 'b',lw=0.5)
plt.title(r'Signal $s_n+v_1(n)-\hat d(n)$', fontsize=16)

plt.show()
```

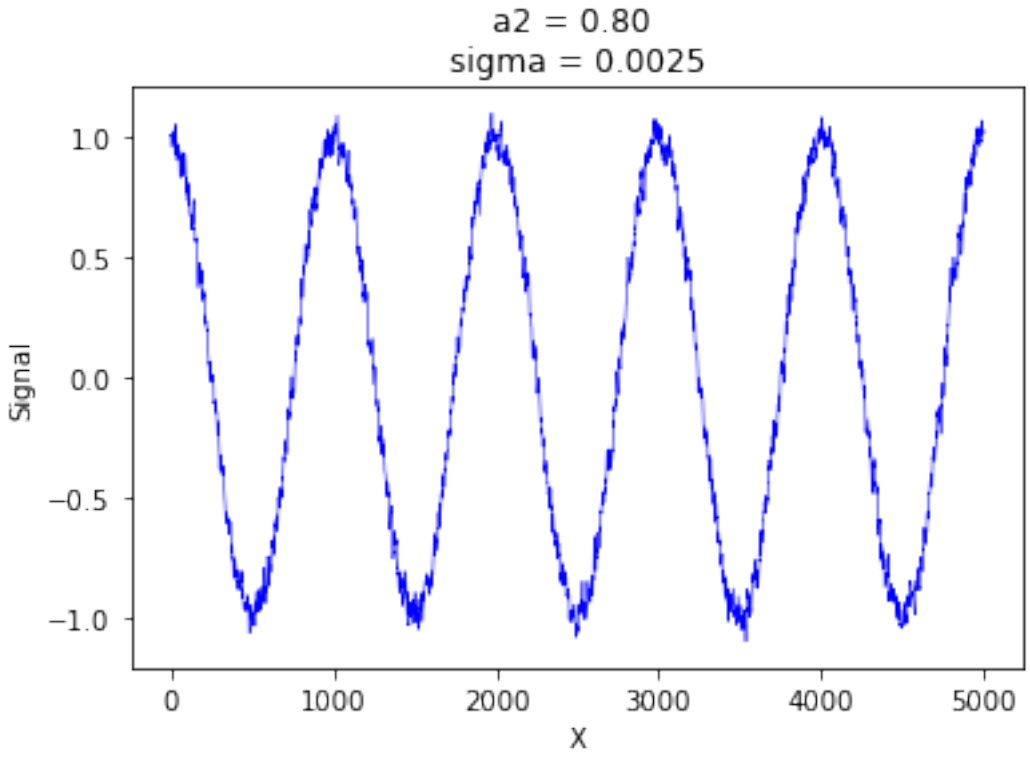

Signal $d_n = s_n + v_1(n)$
1.0
0.5
0.0
−0.5
−1.0
0
1000
2000
3000
4000
5000

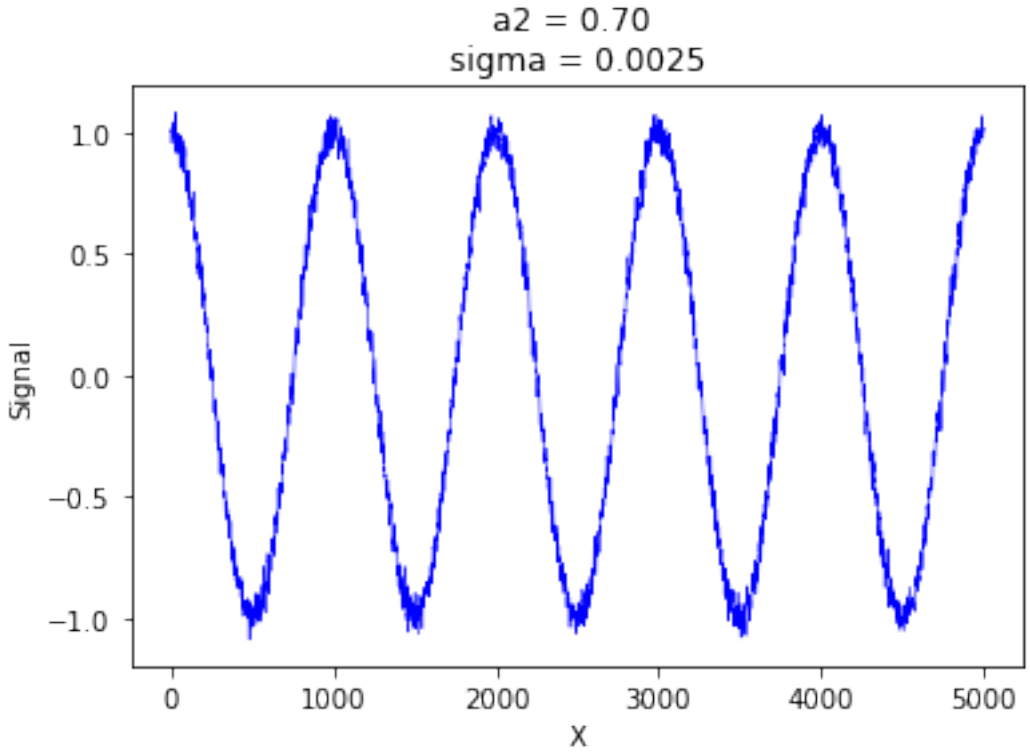

Signal $s_n + v_1(n) - \hat{d}(n)$
1.0
0.5
0.0
−0.5
−1.0
0
1000
2000
3000
4000
5000

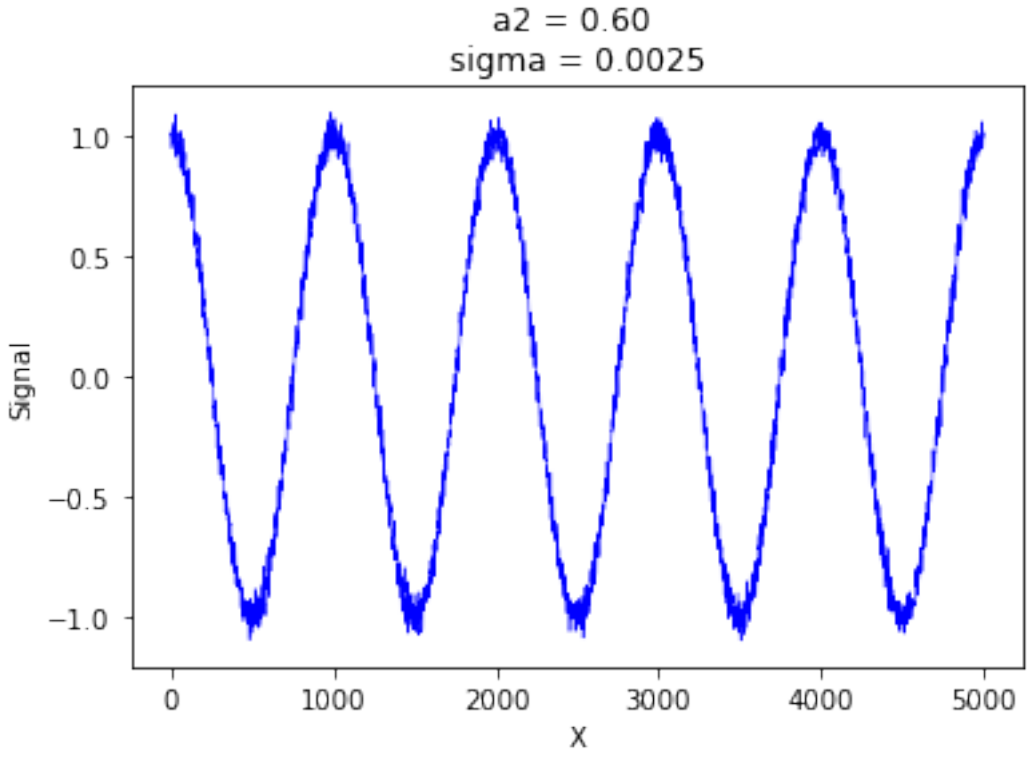
3 Part vi

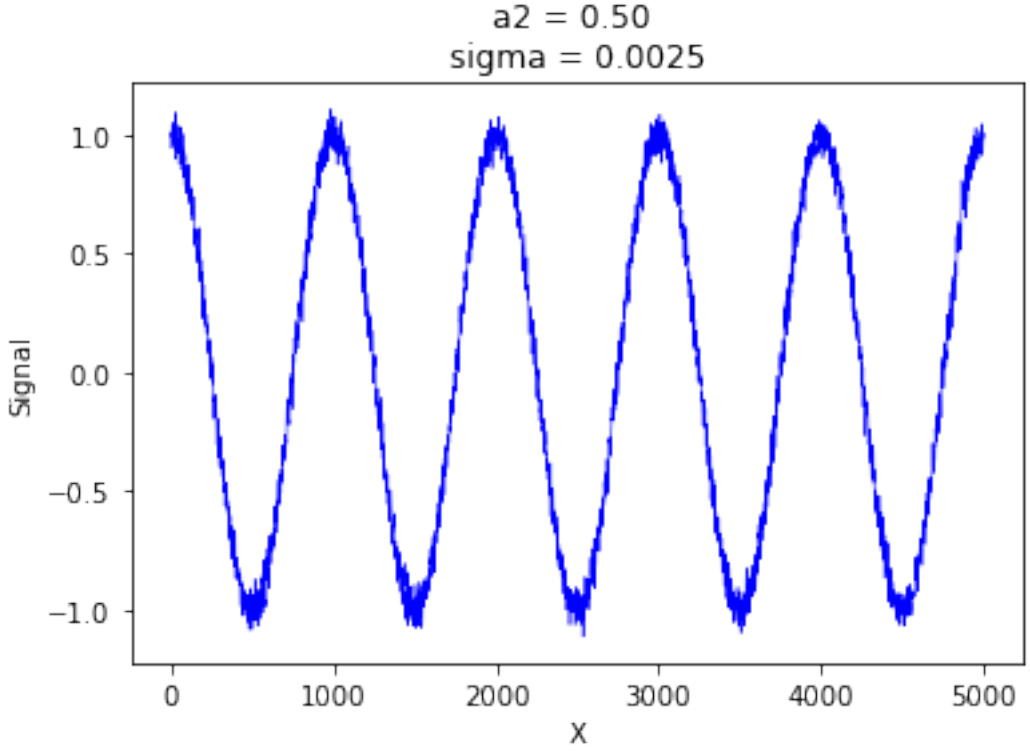
[39]:
```
a2 = np.array([0.9,0.8,0.7,0.6,0.5,0.3])
for j in range (len(a2)):
    for i in range(1,N):
        v2[i] = a2[j] * v2[i-1] + noise[i]
    v2_modified = circ_shift(v2.ravel())
    W = np.linalg.inv(np.dot(v2_modified.T,v2_modified)).dot(v2_modified.T).
↪dot(d_n)
    d_est = v2_modified.dot(W)
    S_hat_n = d_n - d_est
    x_axis = np.arange(0,N,1)
    plt.figure()
    plt.plot(x_axis,d_n-d_est, 'b',lw=0.5)
    plt.title('a2 = {:.2f} \nsigma = {:.4f}'.format(a2[j],sigma**2))
    plt.xlabel('X')
    plt.ylabel('Signal')
    plt.show()
    # Optimum value of a2 is between 0.8 and 0.7
```

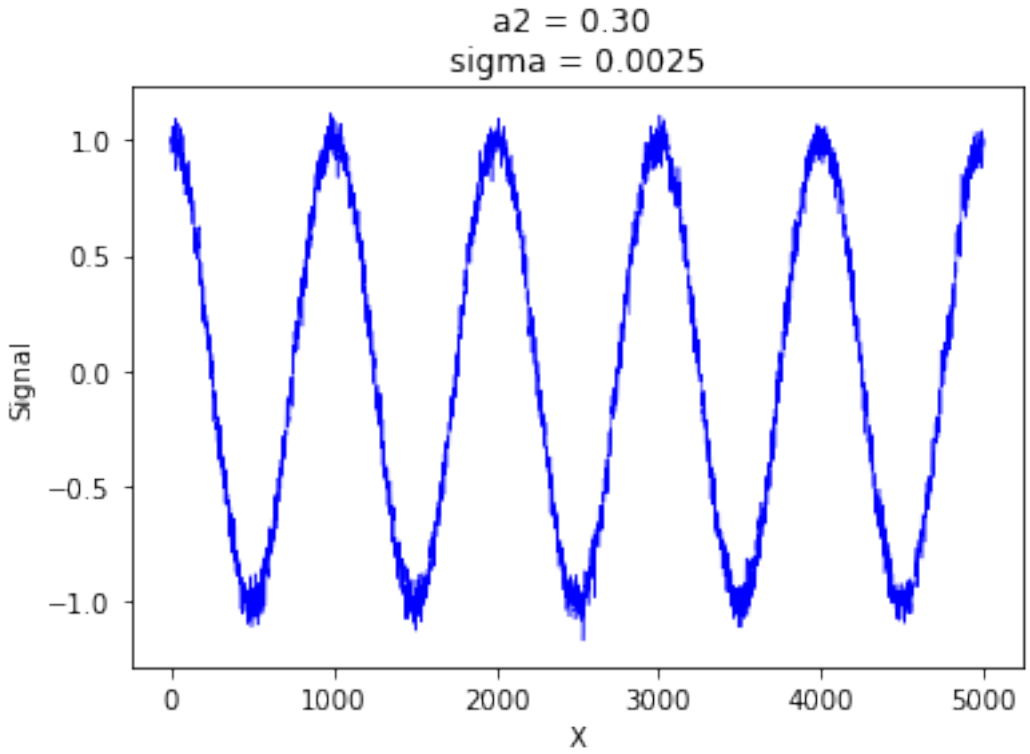



a2 = 0.80
sigma = 0.0025
Signal
X
1.0
0.5
0.0
−0.5
−1.0
0
1000
2000
3000
4000
5000


a2 = 0.70
sigma = 0.0025
Signal
X
1.0
0.5
0.0
−0.5
−1.0
0
1000
2000
3000
4000
5000


a2 = 0.60
sigma = 0.0025
Signal
X
1.0
0.5
0.0
−0.5
−1.0
0
1000
2000
3000
4000
5000

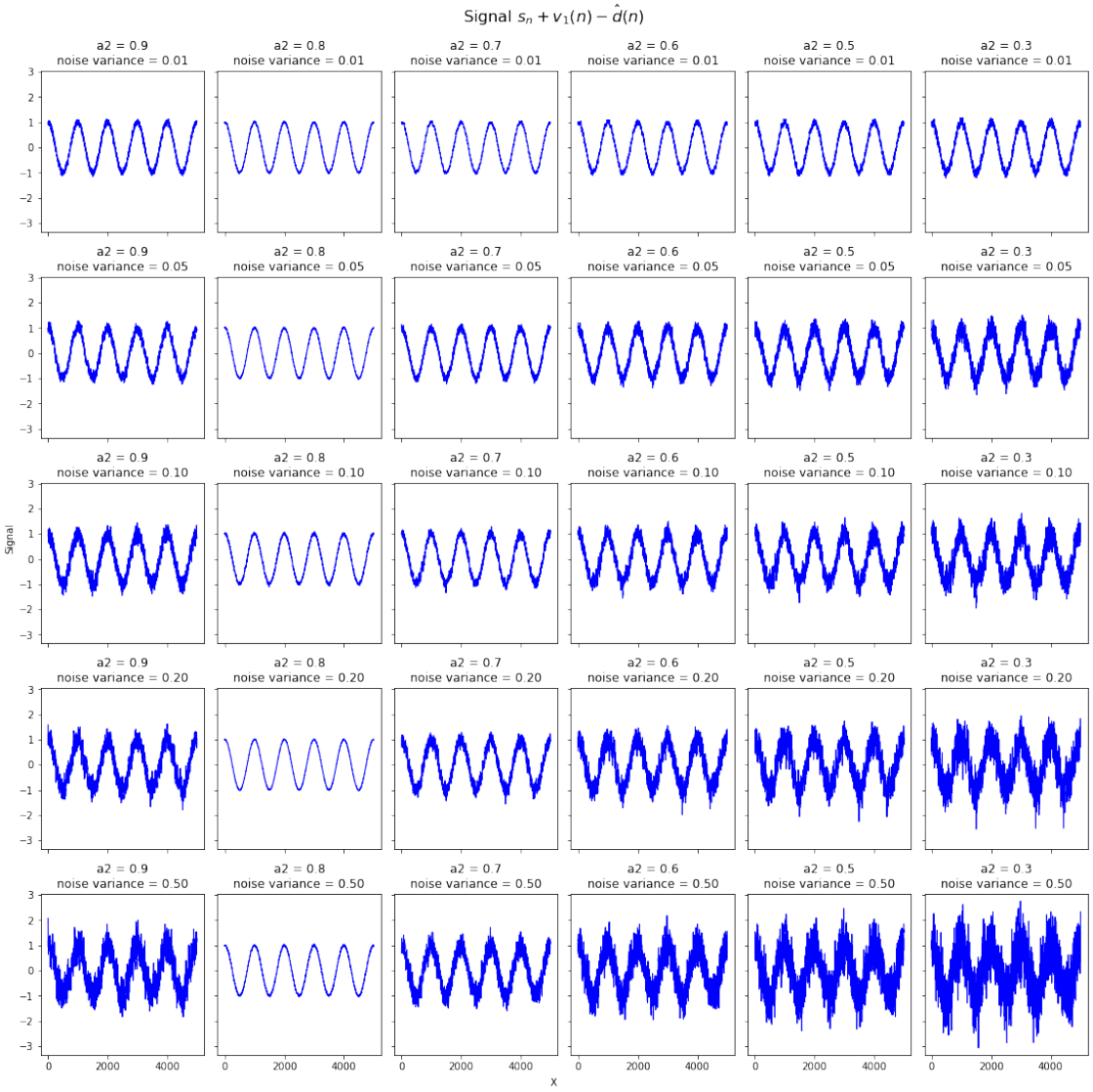

a2 = 0.50
sigma = 0.0025
Signal
X
1.0
0.5
0.0
−0.5
−1.0
0
1000
2000
3000
4000
5000



4 Part vii

[40]:
```
sigma_vec = np.sqrt(np.array([0.01,0.05,0.1,0.2,0.5]))
a2_vec = np.array([0.9,0.8,0.7,0.6,0.5,0.3])
fig, ax = plt.subplots(len(sigma_vec), len(a2_vec), sharex = True, sharey =␣
↪True, figsize=(15,15), constrained_layout=True)
fig.suptitle(r'Signal $s_n+v_1(n)-\hat d(n)$', fontsize=16)
for k in range (len(sigma_vec)):
    noise = sigma_vec[k] * np.random.randn(N,1)
    for j in range (len(a2)):
        for i in range(1,N):
            v1[i] = a1 * v1[i-1] + noise[i]
            v2[i] = a2_vec[j] * v2[i-1] + noise[i]
        d_n= S + v1
        v2_modified = circ_shift(v2.ravel())
        W = np.linalg.inv(np.dot(v2_modified.T,v2_modified)).dot(v2_modified.T).
↪dot(d_n)
        d_est = v2_modified.dot(W)
        S_hat_n = (d_n) - (d_est)
```

```
        x_axis = np.arange(0,N,1)
        ax[k,j].plot(x_axis,d_n-d_est, 'b',lw=1)
        ax[k,j].set_title("a2 = {}\nnoise variance = {:3.2f}".
 ↪format(a2_vec[j],sigma_vec[k]**2))
fig.text(0.5, -0.01, 'X', ha='center')
fig.text(-0.01, 0.5, 'Signal', va='center', rotation='vertical')
plt.show()
```



5 Comments:

1. It can be clearly seen that optimum value of a2 is between 0.8 and 0.7, it cancels most of the noise. If a2 is more or less than this value then V2 removes more noise

than required (virtually adds more noise) or it removes less noise, in either of these cases, the final output remains noisy

2. When noise variance is increased it results in noisy data even after noise cancellation.

3. The best case scenario is when noise variance is equal to 0.20 and a2 is 0.8.