Assignment 3 Task 2

August 6, 2021

[36]:

```python
import numpy as np
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d.axes3d import Axes3D
```

[79]:

```python
def plotting(M,x,y,theta_overfit,theta_underfit):
    fig  = plt.figure(figsize = (10,7))
    axes = fig.add_axes([0.1, 0.1, 0.8, 0.8])
    Y_overfit=np.zeros((M, 1))
    Y_underfit=np.zeros((M, 1))

    for It in range (0,M):
        np.random.seed()
        eta = sigma*np.random.randn(M, 1)
        a=theta_overfit.shape[0]
        b=theta_underfit.shape[0]
        Y_overfit_temp= (np.dot(np.array([x**0, x,  x**2, x**3, x**4,x**5, ␣
↪x**6,
                                          x**7,x**8,  x**9, x**10]).
↪T,theta_overfit[:,It].reshape(a, 1)))
        Y_overfit = np.concatenate((Y_overfit,Y_overfit_temp),axis=1)
        Y_underfit_temp= np.dot(np.array([x**0, x,  x**2]).T,theta_underfit[:
↪,It].reshape(b, 1))
        Y_underfit = np.concatenate((Y_underfit,Y_underfit_temp),axis=1)
        Y_overfit=Y_overfit[:,1:]
        Y_underfit=Y_underfit[:,1:]
        if It % 100 == 0:
            axes.plot(x,Y_overfit_temp, "b-", linewidth = 2)
            axes.plot(x, Y_underfit_temp, "g-", linewidth = 2)
            axes.set_xlabel("x", fontsize = 18)
            axes.set_ylabel("y", rotation = 0, fontsize = 18)

    axes.plot(x,Y_overfit_temp, "b-", linewidth = 2, label = 'over fit')
    axes.plot(x, Y_underfit_temp, "g-", linewidth = 2, label = 'under fit')
    plt.title('10 graphs chosen randomly')
    plt.legend()
    plt.show()
```

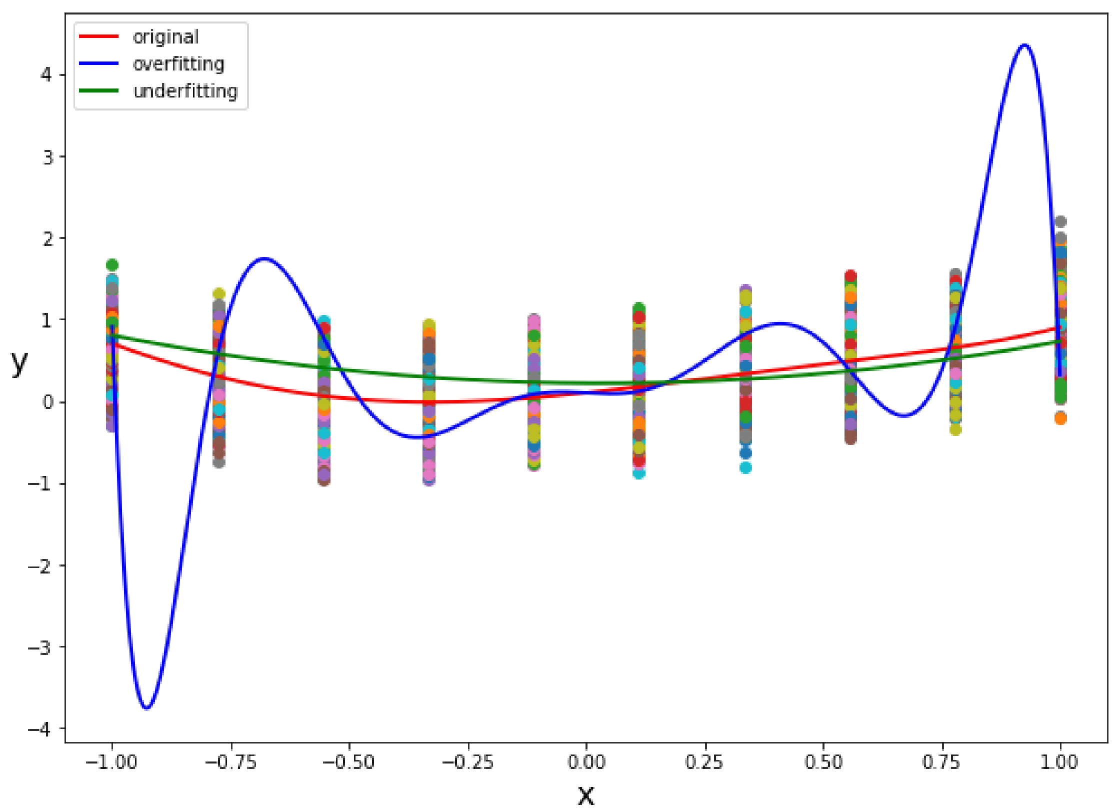
```
    fig  = plt.figure(figsize = (10,7))
    axes = fig.add_axes([0.1, 0.1, 0.8, 0.8])

    Y_overfit = np.array([np.mean(Y_overfit, axis=1)]).reshape(M,1)
    Y_underfit = np.array([np.mean(Y_underfit, axis=1)]).reshape(M,1)
    axes.plot(x, Y_overfit, "y-", linewidth = 4, label='overfit average')
    axes.plot(x, Y_underfit, "r-", linewidth = 4,label='underfit average')
    axes.plot(x, y, "g-", linewidth = 2, label = "original")
    axes.set_xlabel("x", fontsize = 18)
    axes.set_ylabel("y", rotation = 0, fontsize = 18)
    plt.title('Average of 1000 graphs')
    axes.legend()
    plt.show()
```

1 Part i

[80]:

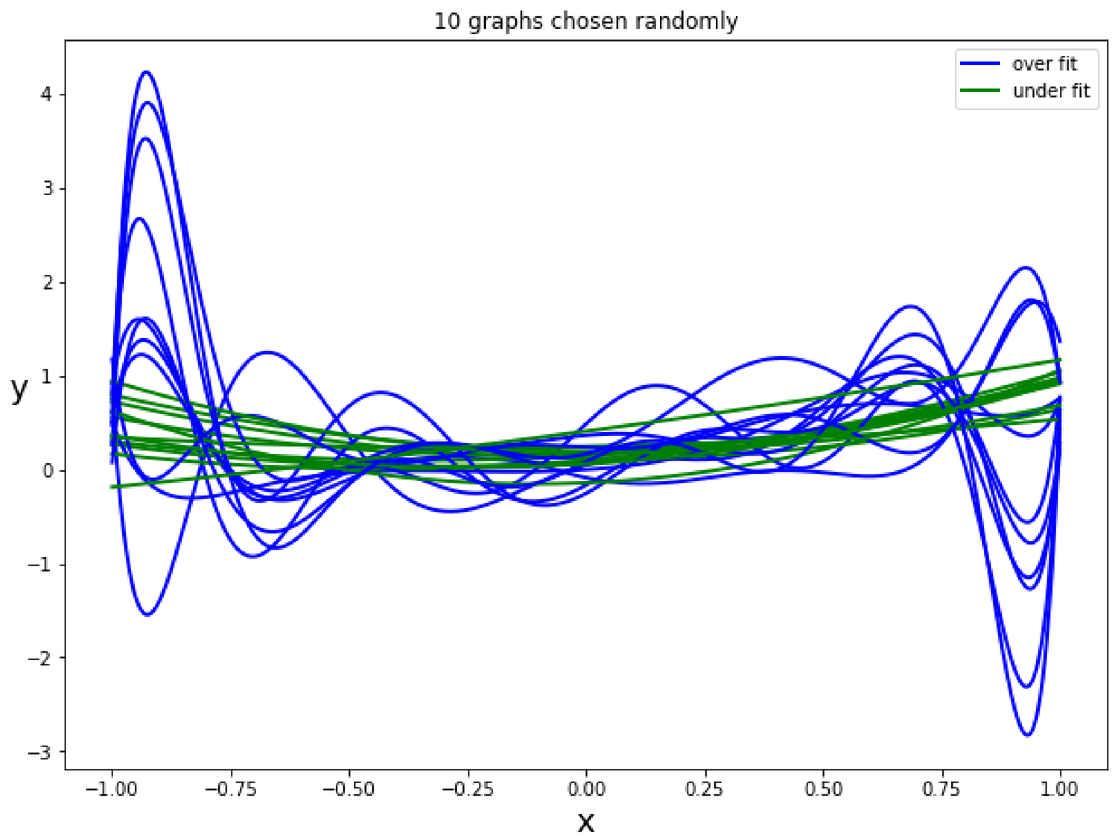
```
M = 1000
N = 10
sigma = np.sqrt(0.1)
theta = np.array([[0.1, 0.6, 0.5, -0.8, 0.2, 0.3]]).T
X0 = np.ones((N, 1))
X1 = np.array([np.linspace(-1,1,10)]).T
X2 = X1**2; X3 = X1**3; X4 = X1**4; X5 = X1**5; X6 = X1**6; X7 = X1**7;
X8 = X1**8; X9 = X1**9; X10 = X1**10
X_original = np.hstack((X0, X1, X2, X3, X4, X5))
X_overfit = np.hstack((X0, X1, X2, X3, X4, X5, X6, X7, X8, X9, X10))
X_underfit = np.hstack((X0, X1, X2))

Y=np.zeros((N, 1))

fig  = plt.figure(figsize = (10, 7))
axes = fig.add_axes([0.1, 0.1, 0.8, 0.8])

for It in range(0, M):
        np.random.seed()
        eta = sigma*np.random.randn(N, 1)
        Y_temp = np.dot(X_original, theta) + eta
        Y = np.concatenate((Y,Y_temp),axis=1)
        axes.scatter(X1, Y_temp)

Y = Y[:,1:]
theta_underfit = np.linalg.inv(X_underfit.T.dot(X_underfit)).dot(X_underfit.T).
 ↪dot(Y)
theta_overfit = (X_overfit.T).dot(np.linalg.inv(X_overfit.dot(X_overfit.T)).
 ↪dot(Y))
```

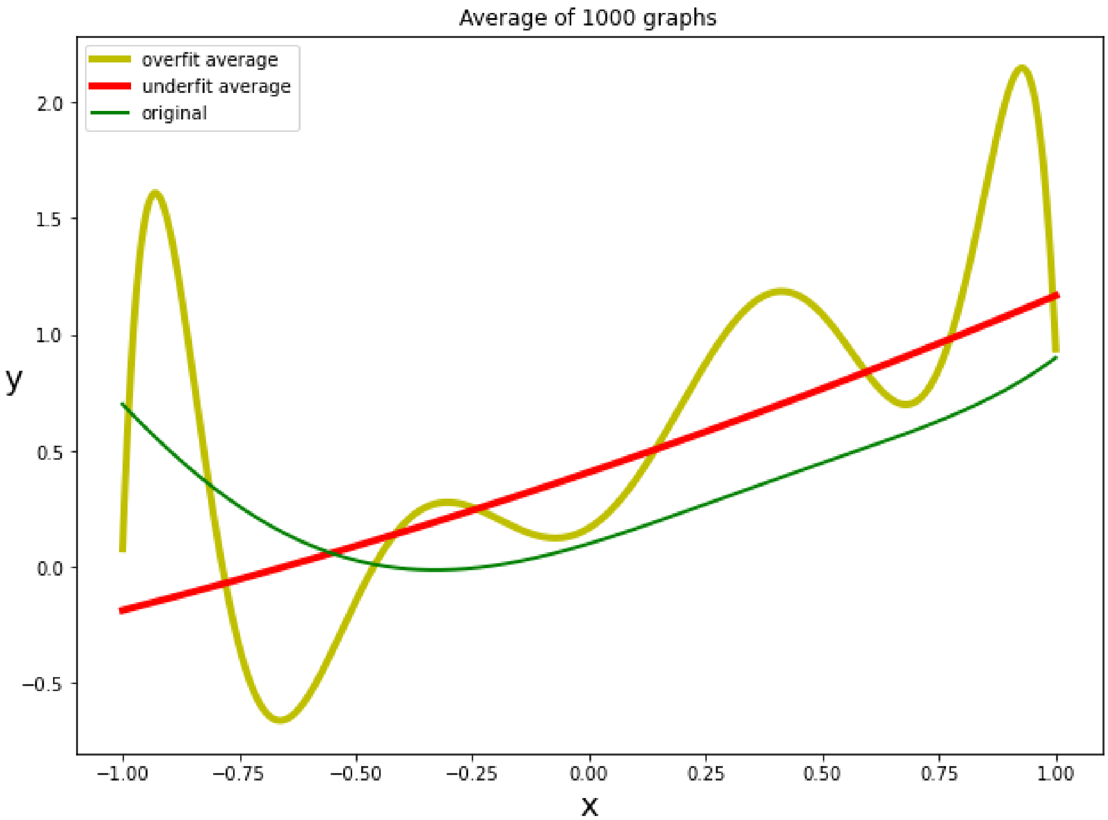
```
x = np.linspace(-1, 1, M)
y = 0.1 + 0.6*x + 0.5*x**2 - 0.8*x**3 + 0.2*x**4 + 0.3*x**5
axes.plot(x, y, "r-", linewidth = 2, label = "original")

theta_matrix = np.random.randint(M)
y_overfit= np.dot(np.array([x**0, x,  x**2, x**3, x**4,x**5,  x**6, x**7,x**8,
                                x**9, x**10]).T,theta_overfit[:,theta_matrix].
↪reshape(11, 1))
axes.plot(x, y_overfit, "b-", linewidth = 2, label = "overfitting")

y_underfit= np.dot(np.array([x**0, x,  x**2]).T,theta_underfit[:,theta_matrix].
↪reshape(3, 1))
axes.plot(x, y_underfit, "g-", linewidth = 2, label = "underfitting")

axes.set_xlabel("x", fontsize = 18)
axes.set_ylabel("y", rotation = 0, fontsize = 18)
axes.legend()
plt.show()
```



2 Part ii and iii

[81]:
```
plotting(M,x,y,theta_overfit,theta_underfit)
```

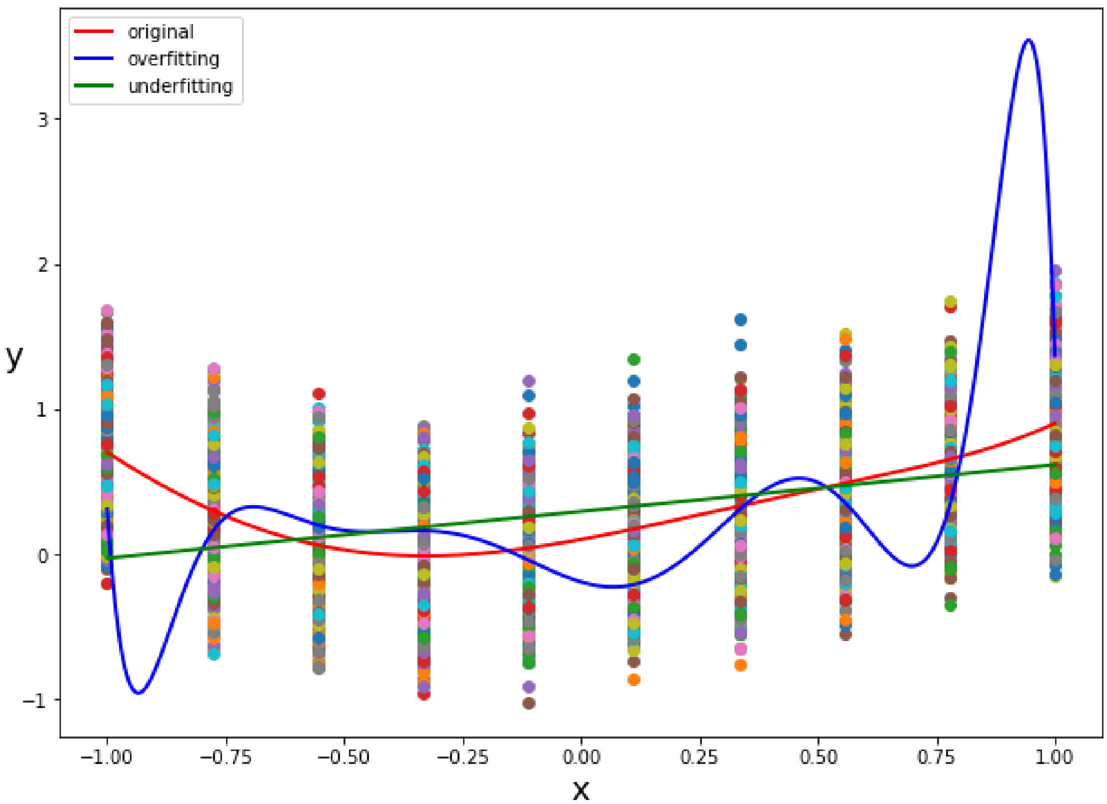




3 Part iv

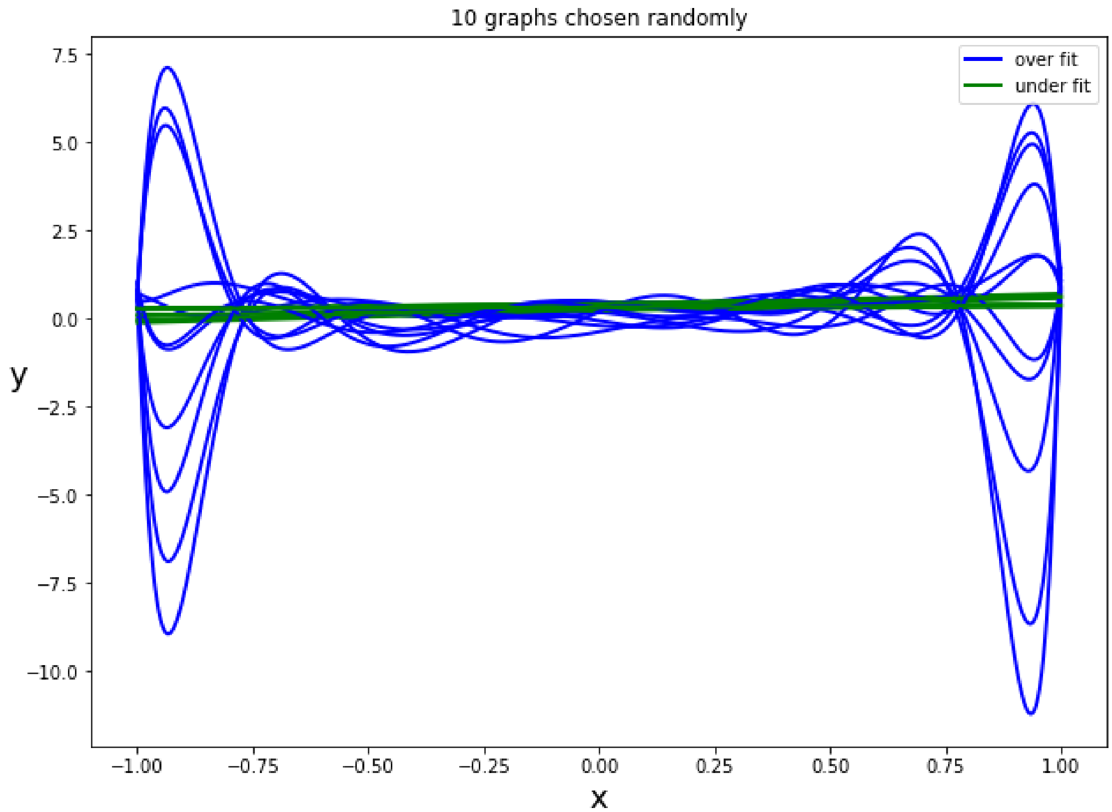
[82]:
```
M = 1000
N = 10
sigma = np.sqrt(0.1)
theta = np.array([[0.1, 0.6, 0.5, -0.8, 0.2, 0.3]]).T
X0 = np.ones((N, 1))
X1 = np.array([np.linspace(-1,1,10)]).T
X2 = X1**2; X3 = X1**3; X4 = X1**4; X5 = X1**5; X6 = X1**6; X7 = X1**7; X8 =␣
↪X1**8;
X9 = X1**9; X10 = X1**10; X11 = X1**11; X12 = X1**12; X13 = X1**13; X14 = X1**14
X_original = np.hstack((X0, X1, X2, X3, X4, X5))
X_overfit = np.hstack((X0, X1, X2, X3, X4, X5, X6, X7, X8, X9, X10, X11,X12,␣
↪X13, X14))
X_underfit = np.hstack((X0, X1))



Y=np.zeros((N, 1))
```

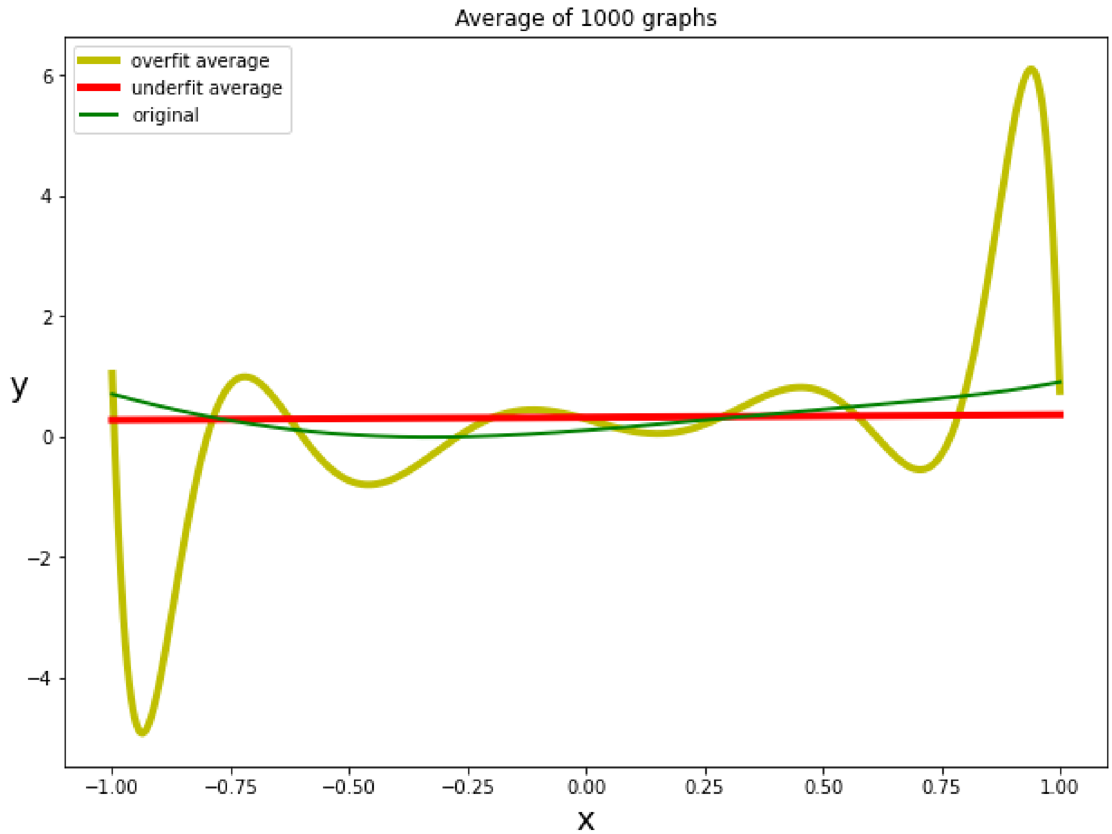
```python
fig  = plt.figure(figsize = (10, 7))
axes = fig.add_axes([0.1, 0.1, 0.8, 0.8])

for It in range(0, M):  # It=1:IterNo
        np.random.seed()
        eta = sigma*np.random.randn(N, 1)
        Y_temp = np.dot(X_original, theta) + eta # output matrix  : Y = X * theta + eta
        Y = np.concatenate((Y,Y_temp),axis=1)
        axes.scatter(X1, Y_temp)

Y = Y[:,1:]

theta_underfit = np.linalg.inv(X_underfit.T.dot(X_underfit)).dot(X_underfit.T).dot(Y)
theta_overfit = (X_overfit.T).dot(np.linalg.inv(X_overfit.dot(X_overfit.T)).dot(Y))
a=theta_underfit.shape[0]
b=theta_overfit.shape[0]

x = np.linspace(-1, 1, M)
y = 0.1 + 0.6*x + 0.5*x**2 - 0.8*x**3 + 0.2*x**4 + 0.3*x**5

theta_index = np.random.randint(M)

y_overfit= np.dot(np.array([x**0, x,  x**2, x**3, x**4,x**5,  x**6, x**7,x**8, x**9, x**10, x**11,
                            x**12,  x**13, x**14]).T,theta_overfit[:,theta_index].reshape(b, 1))

y_underfit= np.dot(np.array([x**0, x]).T,theta_underfit[:,theta_index].reshape(a, 1))

axes.plot(x, y, "r-", linewidth = 2, label = "original")
axes.plot(x, y_overfit, "b-", linewidth = 2, label = "overfitting")
axes.plot(x, y_underfit, "g-", linewidth = 2, label = "underfitting")
axes.set_xlabel("x", fontsize = 18)
axes.set_ylabel("y", rotation = 0, fontsize = 18)
axes.legend()
plt.show()
```



[83]:
```
fig  = plt.figure(figsize = (10,7))
axes = fig.add_axes([0.1, 0.1, 0.8, 0.8])
Y_overfit=np.zeros((M, 1))
Y_underfit=np.zeros((M, 1))

for It in range (0,M):
    np.random.seed()
    eta = sigma*np.random.randn(M, 1)
    a=theta_overfit.shape[0]
    b=theta_underfit.shape[0]

    # 14th order polynomial:
    Y_overfit_temp= (np.dot(np.array([x**0, x,  x**2, x**3, x**4,x**5,  x**6,␣
 ↪x**7,x**8,  x**9, x**10, x**11,
                                                     x**12,  x**13, x**14]).
 ↪T,theta_overfit[:,It].reshape(a, 1)))
    Y_overfit = np.concatenate((Y_overfit,Y_overfit_temp),axis=1)

    # linear equation:
    Y_underfit_temp= np.dot(np.array([x**0, x]).T,theta_underfit[:,It].
 ↪reshape(b, 1))
```

```
    Y_underfit = np.concatenate((Y_underfit,Y_underfit_temp),axis=1)

    Y_overfit=Y_overfit[:,1:]
    Y_underfit=Y_underfit[:,1:]

    if It % 100 == 0:
        axes.plot(x,Y_overfit_temp, "b-", linewidth = 2)
        axes.plot(x, Y_underfit_temp, "g-", linewidth = 2)
        axes.set_xlabel("x", fontsize = 18)
        axes.set_ylabel("y", rotation = 0, fontsize = 18)

axes.plot(x,Y_overfit_temp, "b-", linewidth = 2, label = 'over fit')
axes.plot(x,Y_underfit_temp, "g-", linewidth = 2, label = 'under fit')
plt.title('10 graphs chosen randomly')
plt.legend()
plt.show()


fig  = plt.figure(figsize = (10,7))
axes = fig.add_axes([0.1, 0.1, 0.8, 0.8])

Y_overfit = np.array([np.mean(Y_overfit, axis=1)]).reshape(M,1)
Y_underfit = np.array([np.mean(Y_underfit, axis=1)]).reshape(M,1)

axes.plot(x, Y_overfit, "y-", linewidth = 4, label='overfit average')
axes.plot(x, Y_underfit, "r-", linewidth = 4,label='underfit average')
axes.plot(x, y, "g-", linewidth = 2, label = "original")
axes.set_xlabel("x", fontsize = 18)
axes.set_ylabel("y", rotation = 0, fontsize = 18)
plt.title('Average of 1000 graphs')
axes.legend()
plt.show()
```


10 graphs chosen randomly
over fit
under fit
7.5
5.0
2.5
0.0
−2.5
−5.0
−7.5
−10.0
y
−1.00
−0.75
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00
x



4 Comments:

1. Choosing a higher order polynomial (10th or 14th order) overfits the original data and choosing a lower order polynomial (2nd or 1st order) polynomial underfits the data, this is because the actual data is generated using 5th order polynomial.

2. Choosing higher noise variance results in wider spread of data points which inturn results in larger variation of polynomial and creates a greater deviation between actual curve and predicted curves using either of the polynomials.