Aston University

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Module: EE4108 – Machine Learning

Program: MSc SmartNet (EMJMD)

Assignment # 3

Ans 1 i)

Conditional probability distribution $\boldsymbol{P(y|x)} = E\{P(y|x)(L(y_{hat}, y)\}$

$= P_0.L(y_{hat}, 0) + P_1.L(y_{hat}, 1)$

$Since, P_1 = 1 - P_0$

$= P_0.L(y_{hat}, 0) + (1 - P_0).L(y_{hat}, 1)$

$= P_0.L(y_{hat}, 0) - P_0.L(y_{hat}, 1) + L(y_{hat}, 1)$

$= L(y_{hat}, 1) + P_0\big(L(y_{hat}, 0) - L(y_{hat}, 1)\big)$

Therefore, if we plug in values of y_{hat}:

$P(0|x) = \lambda_{01} - P_0.\lambda_{01}$

$P(1|x) = P_0.\lambda_{10}$

P(0|x) and P(1|x) are linear functions of P0, the unique optimal threshold is reached when:

$$P(0|x) = P(1|x)$$

For example:

$\lambda_{01} - P_0.\lambda_{01} = P_0.\lambda_{10}$

$\lambda_{01} = P_0.\lambda_{10} + P_0.\lambda_{01}$

$\lambda_{01} = P_0(\lambda_{10} + \lambda_{01})$

$P_1 = 1 - P_0$

$$\boldsymbol{P_0 = \frac{\lambda_{01}}{\lambda_{01} + \lambda_{10}} = 1 - \theta}$$

$$\boldsymbol{P_1 = \frac{\lambda_{10}}{\lambda_{01} + \lambda_{10}} = \theta}$$

Ans 1 ii)

We need to derive the loss matrix where the threshold is 0.1

Let, $\lambda_{10} = 1,\ \ \lambda_{01} = 9$

We can execute then:

$$P_1 = 0.1 = \frac{\lambda_{10}}{\lambda_{01} + \lambda_{10}} = \theta$$

$$0.1\lambda_{01} + 0.1\lambda_{10} = \lambda_{10} \dots\dots\dots\ eq(1)$$

$$\boldsymbol{\lambda_{01} = 9\lambda_{10}}$$

$$P_0 = \frac{\lambda_{01}}{\lambda_{01} + \lambda_{10}} = 0.9$$

$$0.9\lambda_{01} + 0.9\lambda_{10} = \lambda_{01} \dots\dots\dots\ eq(2)$$

$$\lambda_{10} = \frac{\lambda_{01}}{9}$$

$$\boldsymbol{9\lambda_{10} = \lambda_{01}}$$

Loss Matrix:

predicted label ŷ	***true label y***	
	0	***1***
0	***0***	***9***
1	***1***	***0***