Assignment Sheet - IV

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Problem 1 [5 points]

Consider the linear regression model:

$$y_n = \boldsymbol{x}_n^T \boldsymbol{\theta}_0 + \eta_n \tag{1}$$

where $\boldsymbol{\theta}_0 \in \mathcal{R}^2$. Generate the coefficients of the unknown vector $\boldsymbol{\theta}_0$ randomly according to the normalized Gaussian distribution, $\mathcal{N}(0,1)$. The noise is assumed to be white Gaussian with variance 0.1. The samples of the input vector are i.i.d generated via the normalized Gaussian.

i. Apply the Robbins-Monro algorithm for the optimal MSE linear estimation with a step size equal to $\mu_n = 1/n$. Run 1000 independent experiments and plot the mean value of the first coefficient of the 1000 produced estimates, at each iteration step. In addition, plot the horizontal line crossing the true value of the first coefficient of the unknown vector. [2.0pt]

ii. Furthermore, plot the standard deviation of the obtained estimate, every 30 iteration steps. Comment on the results. [2.0pt]

iii. Play with different rules of diminishing step-sizes and comment on the results. [1.0pt]

Exercise 2 [5 points]

Consider the regression model

$$y_n = \boldsymbol{\theta}_0^T \boldsymbol{x}_n + \eta_n \tag{2}$$

where $\boldsymbol{\theta}_0 \in \mathcal{R}^{200}, (l = 200)$ and the coefficients of the unknown vector are obtained randomly via the Gaussian distribution $\mathcal{N}(0,1)$. The noise samples are also i.i.d., according to a Gaussian of zero mean and variance $\sigma_\eta^2 = 0.01$. The input sequence is a white noise one, i.i.d. generated via the Gaussian, $\mathcal{N}(0,1)$. Using as training data the samples, $(y_n, \boldsymbol{x}) \in \mathcal{R} \times \mathcal{R}^{200}, n = 1, 2, ...$, run the APA, the NLMS and the RLS algorithms to get estimates of the unknown $\boldsymbol{\theta}_0$. For the APA algorithm, choose $\mu = 0.2, \delta = 0.001$, and $q = 30$. Furthermore, in the NLMS set $\mu = 1.2$ and $\delta = 0.001$. Finally, for the RLS set the forgetting factor β equal to 1. Run 100 independent experiments and plot the average error per iteration in dBs, i.e. $10 \log_{10}(e_n^2)$, where $e_n^2 = (y_n - \boldsymbol{x}_n^T \boldsymbol{\theta}_{n-1})^2$. Compare the performance of the algorithms. Keep playing with different parameters and study their effect on the convergence speed and the error floor in which the algorithms converge.

Exercise 3 [5 points]

Consider a two-dimensional class problem that involves two classes, ω_1 and ω_2, which are modelled by Gaussian distributions with means $\boldsymbol{\mu}_1 = [0, 0]^T$ and $\boldsymbol{\mu}_2 = [2, 2]^T$, respectively, and common covariance matrix $\Sigma = \begin{bmatrix} 1 & 0.25 \\ 0.25 & 1 \end{bmatrix}$.

i. Form and plot a data set $\mathcal{X}$ consisting from 500 points from ω_1 and another 500 points from ω_2. [0.5pt]

ii. Assign each one of the points $\mathcal{X}$ to either ω_1 or ω_2, according to the Bayes decision rule, and plot the points with different colors, depending on the class they are assigned to. [0.5pt]

iii. Compute the corresponding error classification probability.[1.0pt]

iv. Let $L = \begin{bmatrix} 0 & 1 \\ 0.005 & 0 \end{bmatrix}$ be a loss matrix. Assign each one of the points of $\mathcal{X}$ to either ω_1 or ω_2, according to the average risk minimization rule and plot with different colors, depending on the class they are assigned to.[1.0pt]

v. Estimate the average risk for the above loss matrix. [1.0pt]

vi. Comment on the above results and scenarios. [1.0pt]