

3.51 Two variables, U and V, assume the values $U_1 = 3, U_2 = -2, U_3 = 5$, and $V_1 = -4, V_2 = -1, V_3 = 6$, respectively. Calculate (a) $\sum UV$, (b) $\sum(U+3)(V-4)$, (c) $\sum V^2$, (d) $(\sum U)(\sum V)^2$, (e) $\sum UV^2$, (f) $\sum(U^2 - 2V^2 + 2)$, and (g) $\sum U/V$

Given values:

$$U_1 = 3, U_2 = -2, U_3 = 5$$

$$V_1 = -4, V_2 = -1, V_3 = 6$$

(a) $\sum UV$:

$$\sum UV = U_1V_1 + U_2V_2 + U_3V_3$$

$$\sum UV = (3 * -4) + (-2 * -1) + (5 * 6)$$

$$\sum UV = -12 + 2 + 30$$

$$\sum UV = 20$$

(b) $\sum(U+3)(V-4)$:

$$\sum(U+3)(V-4) = \sum(UV - 4U + 3V - 12)$$

Now, distribute the summation:

$$\sum(U+3)(V-4) = \sum UV - 4\sum U + 3\sum V - 12\sum(1)$$

Using the results from part (a) for $\sum UV$ and the properties of summation:

$$\sum(U+3)(V-4) = 20 - 4(3 - 2 + 5) + 3(-4 - 1 + 6) - 12(1 + 1 + 1)$$

$$\sum(U+3)(V-4) = 20 - 4(6) + 3(1) - 12(3)$$

$$\sum(U+3)(V-4) = 20 - 24 + 3 - 36$$

$$\sum(U+3)(V-4) = -37$$

(c) $\sum V^2$:

$$\sum V^2 = V_1^2 + V_2^2 + V_3^2$$

$$\sum V^2 = (-4)^2 + (-1)^2 + (6)^2$$

$$\sum V^2 = 16 + 1 + 36$$

$$\sum V^2 = 53$$

(d) $(\sum U)(\sum V)^2$:

$$(\sum U)(\sum V)^2 = (\sum U)(V_1 + V_2 + V_3)^2$$

Using the results from part (a) for $\sum U$ and $\sum V$:

$$(\sum U)(\sum V)^2 = (3 - 2 + 5)(-4 - 1 + 6)^2$$

$$(\sum U)(\sum V)^2 = 6 * 1^2$$

$$(\sum U)(\sum V)^2 = 6$$

(e) $\sum UV^2$:

$$\sum UV^2 = U_1V_1^2 + U_2V_2^2 + U_3V_3^2$$

$$\sum UV^2 = 3 * (-4)^2 + (-2) * (-1)^2 + 5 * (6)^2$$

$$\sum UV^2 = 3 * 16 + (-2) * 1 + 5 * 36$$

$$\sum UV^2 = 48 - 2 + 180$$

$$\sum UV^2 = 226$$

(f) $\sum(U^2-2V^2+2)$:

$$\sum(U^2-2V^2+2) = U_1^2 - 2V_1^2 + 2 + U_2^2 - 2V_2^2 + 2 + U_3^2 - 2V_3^2 + 2$$

$$\sum(U^2-2V^2+2) = 3^2 - 2*(-4)^2 + 2 + (-2)^2 - 2*(-1)^2 + 2 + 5^2 - 2*6^2 + 2$$

$$\sum(U^2-2V^2+2) = 9 - 2*16 + 2 + 4 - 2 + 2 + 25 - 2*36 + 2$$

$$\sum(U^2-2V^2+2) = 9 - 32 + 2 + 4 - 2 + 2 + 25 - 72 + 2$$

$$\sum(U^2-2V^2+2) = -57$$

(g) $\sum U/V$:

$$\sum U/V = (U_1/V_1) + (U_2/V_2) + (U_3/V_3)$$

$$\sum U/V = (3/(-4)) + (-2/(-1)) + (5/6)$$

$$\sum U/V = -3/4 + 2 + 5/6$$

To add these fractions together, we need a common denominator, which is 12:

$$\sum U/V = (-9/12) + (24/12) + (10/12)$$

Now, we can add the fractions:

$$\sum U/V = (-9 + 24 + 10) / 12$$

$$\sum U/V = 25/12$$

The calculations have been proven. ■