

Frame Assignments

1 FRAME DIAGRAM

The Quanser Arm's frame diagram is attached in Figure 1. This was developed using the Standard Denavit Hartenberg (DH) parameters [1].

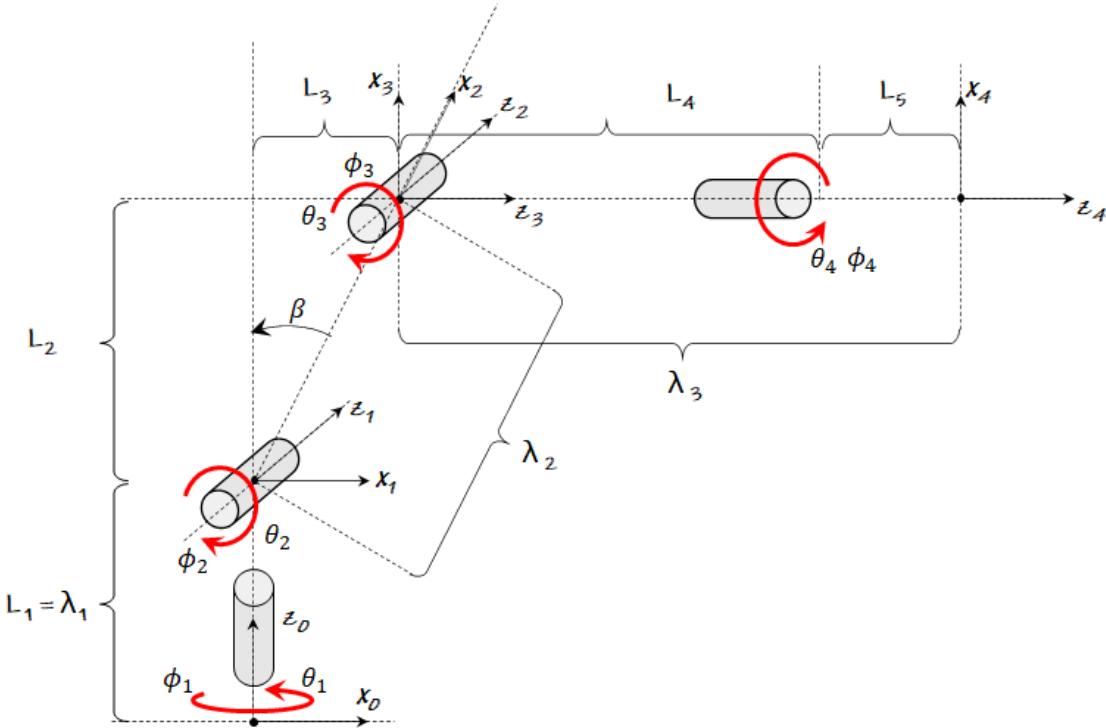


Figure 1. Frame diagram for the Quanser Arm manipulator

Note that the manipulator shown in Figure 1 is currently in its home position. In this state, the joint space vector $\vec{\theta}$ is,

$$\vec{\theta} = \begin{bmatrix} 0 \\ \beta - \frac{\pi}{2} \\ -\beta \\ 0 \end{bmatrix} \quad (3)$$

The manipulator's encoders and actuators though, are calibrated at this position. Thus, an actuator command of $[0 \ 0 \ 0 \ 0]^T$ would move the manipulator to the home position, where it's encoders would read a joint position of $[0 \ 0 \ 0 \ 0]^T$ as well. We can represent this alternate joint space as $\vec{\phi}$. A mapping summarized in Table 1 will allow us to describe the manipulator in $\vec{\phi}$ space, while we carry out the mathematics in $\vec{\theta}$ space without having to carry around the offset in equation 3. For example, a command of $\phi_2 = 0$ will imply $\theta_2 = \beta - \frac{\pi}{2}$ which corresponds to joint 2 in the home position. Algebraically, a reference to θ_2 in the mathematics already includes this offset.

New parameter	Original Parameter	New parameter	Original Parameter
λ_1	L_1	ϕ_1	θ_1
λ_2	$\sqrt{L_2^2 + L_3^2}$	ϕ_2	$\theta_2 + \frac{\pi}{2} - \beta$
λ_3	$L_4 + L_5$	ϕ_3	$\theta_3 + \beta$
β	$\tan^{-1}\left(\frac{L_3}{L_2}\right)$	ϕ_4	θ_4

Table 1. Linear mapping to simplify the mathematical formulations

2 DH TABLE

The DH table corresponding to the frame diagram in Figure 1 is presented in Table 2.

i	a_i	α_i	d_i	θ_i
1	0	$-\pi/2$	λ_1	θ_1
2	λ_2	0	0	θ_2
3	0	$-\pi/2$	0	θ_3
4	0	0	λ_3	θ_4

Table 2. DH Table