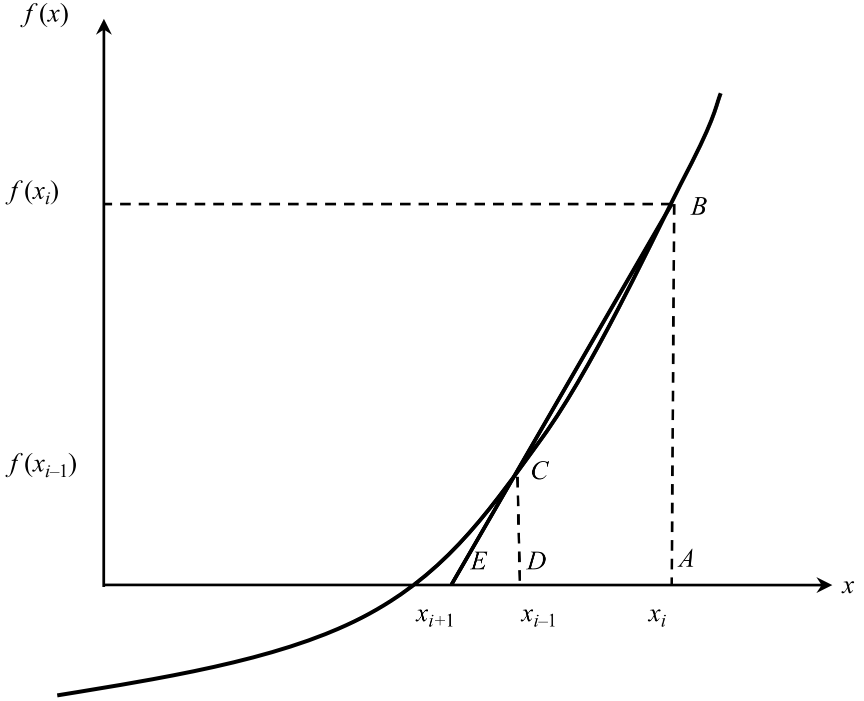
Secant Method

Secant Method

- Secant method is an open method. That is, no more interval under consideration needs to bracket the root, though it still requires two initial guesses. Let us denote these initial guesses by x_{-1} and x_0 . The formula for generating the sequence of approximations is

$$x_{i+1} = \frac{x_{i-1} f(x_i) - x_i f(x_{i-1})}{f(x_i) - f(x_{i-1})}$$



Secant Method

- Firstly, initial choices of x_{-1} and x_0 need not be bracketing the root. So, that is, root need not lie within the endpoints x_{-1} and x_0.That is f(x_{-1})and f(x_0)can be of same sign. No more, one needs to ensure that f(x_{-1}).f(x_0) < 0.

- Though, in practice one usually chooses x_{-1} and x_0 as ones which bracket the root, there is no such compulsion.

Secant Method

- Secondly, in computation of next approximation, older approximation is discarded. Recent most two approximations are used for iteration, irrespective of function values at these end points. So graphically, straight line joining (x_{i-1}, f (x_{i-1})) and (x_i , f (x_i)) is drawn to generate x_{i+1} , next approximation. No more, function values sign are checked to ensure that root lies between (x_{i-1}, f (x_{i-1})) and (x_i , f (x_i)) . Because secant is drawn joining (x_{i-1}, f (x_{i-1})) and (x_i , f (x_i)), it is called Secant Method.

Algorithm

The steps of the Secant method to find the root of an equation f(x) = 0 are

1. Let k =1, Evaluate $f(x_i), f(x_{i-1})$
2. Use an initial guess of the root, x_i , to estimate the new value of the root, x_{i+1} , as

$$x_{i+1} = \frac{x_{i-1} f(x_i) - x_i f(x_{i-1})}{f(x_i) - f(x_{i-1})}$$

3. Compare the absolute approximate error $|c_k - c_{k-1}|$ or absolute relative approximate error $\frac{|c_k - c_{k-1}|}{|c_k|}$ with the pre-specified relative error tolerance ϵ.

where

c_k = estimated root from present iteration

c_{k-1}= estimated root from previous iteration

$If\ |c_k - c_{k-1}| < \epsilon\ or\ \frac{|c_k - c_{k-1}|}{|c_k|} < \epsilon$ then exit

Else

k = k + 1

go to Step 2

Note one should also check whether the number of iterations is more than the maximum number of iterations allowed. If so, one needs to terminate the algorithm and notify the user about it.

Advantages and Disadvantages

Advantages:

1. No constraint of end points of interval to converge
2. Very rapid convergence

Disadvantages:

1. If initial guess is not sufficiently near the root, it may diverge.

Example

This polynomial obviously has roots at x = 2 and at x = 4; one is a double root, the other a triple root:

$f(x) = (x-2)^3(x-4)^2$

$= x^5 - 14x^4 + 76x^3 - 200x^2 + 256x - 128$

a. Which root can you get with bisection? Which root can't you get?

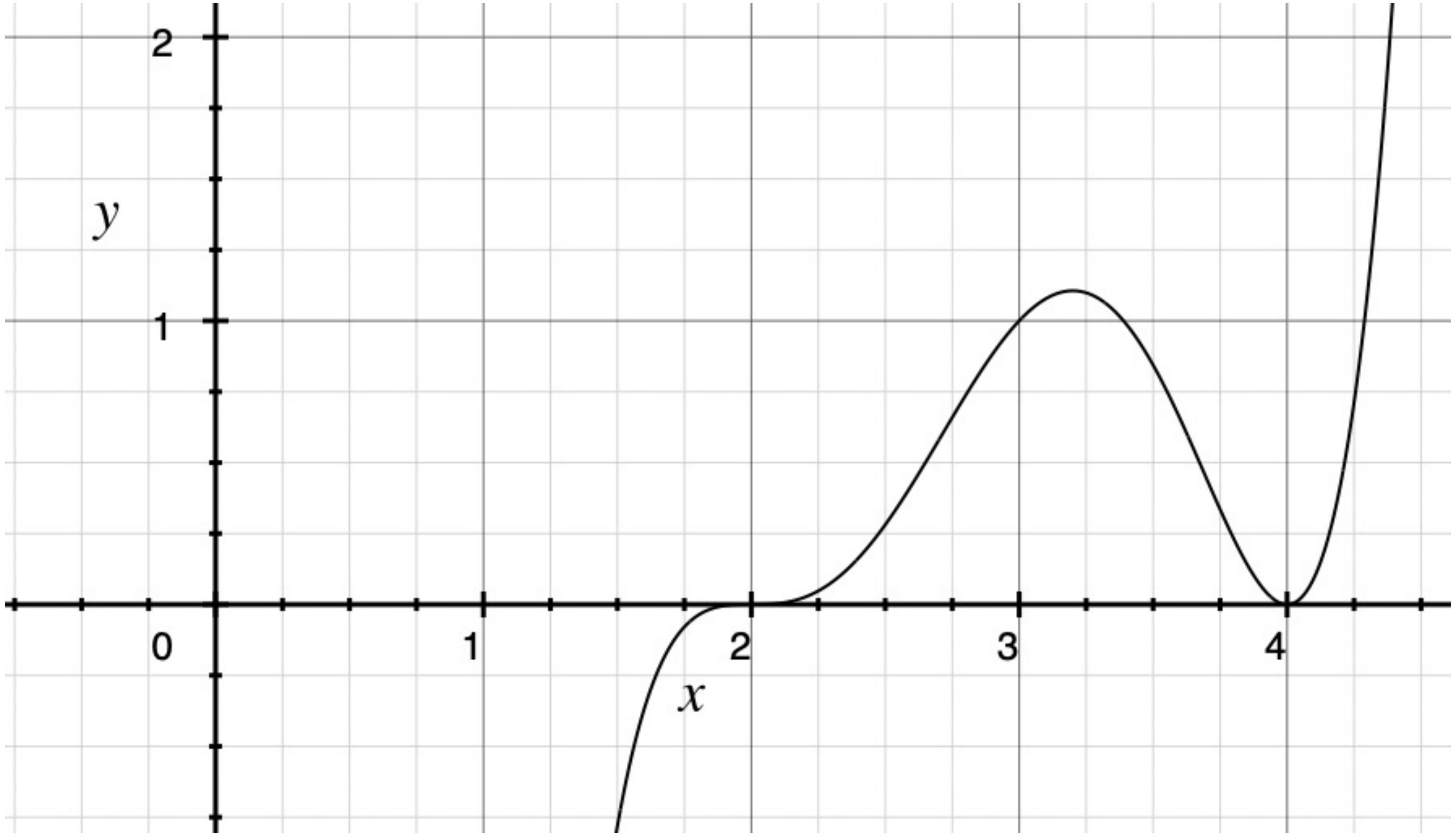
b. Repeat part (a) with the secant method.

c. If you begin with the interval [1,5], which root will you get with

 (1) bisection, (2) the secant method, (3) false position?

d. Use Newton's method with x_0 = 3. Does it converge? To which root?

Example



Example

This polynomial obviously has roots at x = 2 and at x = 4; one is a double root, the other a triple root:

$$f(x) = (x-2)^3(x-4)^2$$
$$= x^5 - 14x^4 + 76x^3 - 200x^2 + 256x - 128$$

a.

b. Both roots can be reached with secant method

c. If you begin with the interval [1,5], which root will you get with

(2) the secant method: It will converge to 2

d.

Example

Eg 1. *Find the real root of the equation* $x^3 - 4x - 9 = 0$ *by* Secant Method *correct*.

Take x_0 = *2,* x_1 = *3,* ϵ = 0.01

Eg 2. *Find the real root of the equation* $x^3 - x - 1 = 0$ *by* Secant Method *correct. Take* x_0 = *1,* x_1 = *1.5,* ϵ = 0.001

Eg 3. $f(x) = 3x - e^x$ starting from 0,1 and ϵ = 0.01

Secant Method

Eg 1. *Find the real root of the equation* $x^3 - 4x - 9 = 0$ *by* Secant Method *correct*.

x	0	1	2	3
f(x)	-9	-12	-9	6

Take $x_0 = 2$, $x_1 = 3$, $\epsilon = 0.01$

Secant Method

Eg 1. *Find the real root of the equation* $x^3 - 4x - 9 = 0$ *by* Secant Method *correct. Take* $x_0 = 2$, $x_1 = 3$, $\epsilon = 0.001$

$f(x) = x^3 - 4x - 9$

$f(2) = -9$

and

$f(3) = 6$

x_0	f(x_0)	x_1	f(x_1)
2	-9	3	6

Secant Method

First approximation to the root is

$$x_2 = \frac{(2)(6) - (3)(-9)}{6 - (-9)}$$

x_2= 2.6

Now $f(x_2)$ = – 1.824

x_0	f(x_0)	x_1	f(x_1)	x_2	f(x_2)	$\|x_k - x_{k-1}\|$
2	-9	3	6	2.6	– 1.824	-

Secant Method

x_0 = *3*, x_1 = 2.6

Second approximation to the root is

$$x_2 = \frac{(3)(-1.824) - (2.6)6}{(-1.824) - 6}$$

= 2.6933

Now $f(x_2)$ = -0.2372

x_0	f(x_0)	x_1	f(x_1)	x_2	f(x_2)	$\lvert x_k - x_{k-1} \rvert$
2	-9	3	6	2.6	– 1.824	-
3	6	2.6	– 1.824	2.6933	-0.2372	0.0933

Secant Method

x_0 = 2.6, x_1 = 2.6933

Third approximation to the root is

$$x_2 = \frac{(2.6)(-0.2372) - (2.6933)(-1.824\ )}{(-0.2372) - (2.6933)}$$

= 2.7072

Now $f(x_2\ )$ = 0.012

x_0	f(x_0)	x_1	f(x_1)	x_2	f(x_2)	$\lvert x_k - x_{k-1} \rvert$
2	-9	3	6	2.6	–1.824	-
3	6	2.6	–1.824	2.6933	-0.2372	0.0933
2.6	–1.824	2.6933	-0.2372	2.7072	0.012	0.0139

Secant Method

x_0 = 2.6933, x_1 = 2.7072

Fourth approximation to the root is

$$x_2 = \frac{(2.6933)(0.012)-(2.7072)(-0.2372)}{0.012-(-0.2372)} = 2.7065$$

ϵ = 0.001, $|x_k - x_{k-1}|$ = 0.0007

Hence, the root is 2.7065, correct to three decimal places.

x_0	f(x_0)	x_1	f(x_1)	x_2	f(x_2)	$\lvert x_k - x_{k-1} \rvert$
2	-9	3	6	2.6	– 1.824	-
3	6	2.6	– 1.824	2.6933	-0.2372	0.0933
2.6	– 1.824	2.6933	-0.2372	2.7072	0.012	0.0139
2.6933	-0.2372	2.7072	0.012	2.7065	0	0.0007

Secant Method

Eg 2. *Find the real root of the equation $x^3 - x - 1 = 0$ by* Secant Method *correct. Take* x_0 = *1*, x_1 = *1.5*, ϵ = 0.001

$f(x) = x^3 - x - 1$

$f(1) = -1$

and

$f(1.5) = 0.875$

x_0	f(x_0)	x_1	f(x_1)
1	-1	1.5	0.875

Secant Method

First approximation to the root is

$$x_2 = \frac{(1)(0.875) - (1.5)(-1)}{0.875 - (-1)}$$

x_2= 1.2667

Now $f(x_2)$ = – 0.2344

x_0	f(x_0)	x_1	f(x_1)	x_2	f(x_2)	$\|x_k - x_{k-1}\|$
1	-1	1.5	0.875	1.2667	– 0.2344	-

Secant Method

x_0 = *1.5*, x_1 = 1.2667

Second approximation to the root is

$$x_2 = \frac{(1.5)(-0.2344) - (1.2667)0.875}{(-0.2344) - 0.875}$$

= 1.316

Now $f(x_2)$ = -0.037

x_0	f(x_0)	x_1	f(x_1)	x_2	f(x_2)	$\lvert x_k - x_{k-1} \rvert$
1	-1	1.5	0.875	1.2667	– 0.2344	-
1.5	0.875	1.2667	– 0.2344	1.316	-0.037	0.0493

Secant Method

x_0 = 1.2667, x_1 = 1.316

Third approximation to the root is

$$x_2 = \frac{(1.2667)(-0.037) - (1.316)(-0.2344)}{(-0.037) - (-0.2344)}$$

=1.3252

Now $f(x_2)$ = 0.0021

x_0	f(x_0)	x_1	f(x_1)	x_2	f(x_2)	$\lvert x_k - x_{k-1} \rvert$
1	-1	1.5	0.875	1.2667	– 0.2344	-
1.5	0.875	1.2667	– 0.2344	1.316	-0.037	0.0493
1.2667	– 0.2344	1.316	-0.037	1.3252	0.0021	0.0092

Secant Method

x_0 = 1.316, x_1 = 1.3252

Fourth approximation to the root is

$$x_2 = \frac{(1.316)(0.0021)-(1.3252)(-0.037)}{0.0021-(-0.037)} = 1.3247$$

ϵ = 0.001, $|x_k - x_{k-1}|$ = 0.0005

Hence, the root is 1.3247, correct to three decimal places.

x_0	f(x_0)	x_1	f(x_1)	x_2	f(x_2)	$\lvert x_k - x_{k-1} \rvert$
1	-1	1.5	0.875	1.2667	– 0.2344	-
1.5	0.875	1.2667	– 0.2344	1.316	-0.037	0.0493
1.2667	– 0.2344	1.316	-0.037	1.3252	0.0021	0.0092
1.316	-0.037	1.3252	0.0021	1.3247	0	0.0005

Example

- f(x) = 3x - e^x starting from 0,1 and ϵ = 0.01

x_0	f(x_0)	x_1	f(x_1)	x_2	f(x_2)	$\lvert x_k - x_{k-1} \rvert$
0	-1	1	0.2817	0.7802	0.1587	-
1	0.2817	0.7802	0.1587	0.4967	-0.1532	0.2835
0.7802	0.1587	0.4967	-0.1532	0.636	0.019	0.1393
0.4967	-0.1532	0.636	0.019	0.6206	0.0017	0.0154
0.636	0.019	0.6206	0.0017	0.619	0	0.0016