

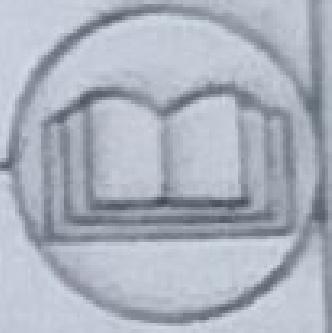
Assignment- III

I. For the data given below use least-squares regression to find

X	6	7	11	15	17	21	23	29	29	37	39
Y	29	21	29	14	21	15	7	7	13	6	3

- (i) the slope (a_1)
- (ii) intercept (a_0)
- (iii) fit a straight line
- (iv) compute the Standard error of the estimate
- (v) the Correlation Coefficient

Ans:-	X	Y	x^2	xy	$(y - \bar{y})^2$	$(y - a_0 - a_1 x_i)^2$
	6	29	36	174	211.7025	6.93268
	7	21	49	147	42.9025	21.03590
	11	29	121	319	211.7025	42.71276
	15	14	225	210	0.2025	28.54231
	17	21	289	357	42.9025	10.35874
	21	15	441	315	0.3025	0.11594
	23	7	529	161	55.5025	37.19170
	29	7	841	203	55.5025	2.00364
	29	13	841	377	2.1025	21.01764
	37	0	1361	0	208.8025	4.71541
	39	3	1521	117	131.1025	5.70941
			6262			
Total	234	159	6262	2380	962.7275	180.3364



$$- \bar{x} = \frac{\sum x_i}{n} = \frac{234}{11} = 21.27$$

$$- \bar{y} = \frac{\sum y_i}{n} = \frac{159}{11} = 14.45$$

$$(i) a_1 = \frac{n \sum xy - \sum x_i \sum y_i}{n \sum x_i^2 - (\sum x_i)^2}$$

$$= \frac{11(2386) - (234)(159)}{11(6262) - (234)^2}$$

$$= \frac{2610 - 37206}{68882 - 54756}$$

$$= \frac{-11026}{14126}$$

$$= -0.7805$$

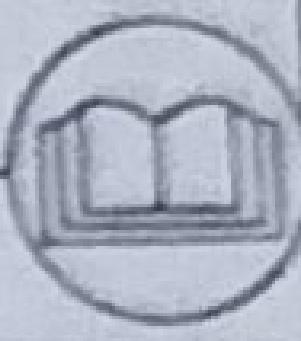
$$(ii) a_0 = \bar{y} - a_1 \bar{x}$$

$$= 14.45 - (-0.7805)(21.27)$$

$$= 14.45 + 16.60$$

$$= 31.05$$

$$(iii) y = 31.05 - 0.7805x$$



$$(iv) S_{y/x} = \sqrt{\frac{S_{yy}}{n-2}}$$

$$= \sqrt{\frac{180.3364}{11-2}}$$

$$= \sqrt{\frac{180.3364}{9}}$$

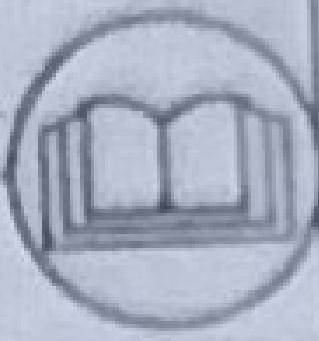
$$= 4.47$$

$$(v) S_{T^2} = \frac{S_{T^2} - S_{yy}}{S_{T^2}}$$

$$= \frac{962.7275 - 180.3364}{962.7275}$$

$$= \frac{782.3911}{962.7275}$$

$$= 0.81$$



2 Fit the data with model ($y = ab^x$), $x = 9$

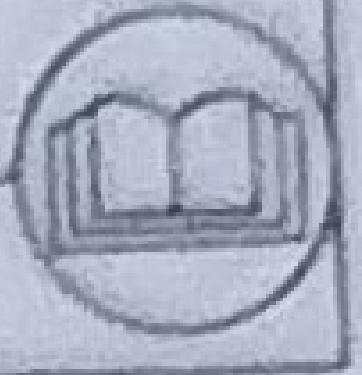
x	2.5	3.5	5	6	7.5	10	12.5	15	17.5
y	13	11	8.5	8.2	7	6.2	5.2	4.8	4.6

Ans: $y = ab^x$

$$x = x$$

$$y = \ln y$$

x	y	$\ln y$	x^2	xy
2.5	13	2.5649	6.25	6.4123
3.5	11	2.3979	12.25	8.3927
5	8.5	2.1401	25	10.7005
6	8.2	2.1041	36	12.6246
7.5	7	1.9459	56.25	14.5943
10	6.2	1.8245	100	18.2450
12.5	5.2	1.6486	156.25	20.6075
15	4.8	1.5686	225	23.5290
17.5	4.6	1.5261	306.25	26.7063
20	4.3	1.4586	400	29.1720
Total	99.5	19.1793	1329.25	170.9842



$$- \bar{x} = \frac{\sum x_i}{n} = \frac{99.5}{10} = 9.95$$

$$- \bar{y} = \frac{\sum y_i}{n} = \frac{19.1793}{10} = 1.9179$$

$$- b = \frac{n \sum xy - \sum x \sum y}{n \sum x^2 - (\sum x)^2}$$

$$\frac{10(170.9842) - (99.5)(19.1793)}{10(1323.25) - (99.5)^2}$$

$$= \frac{1709.842 - 1908.4035}{13232.5 - 9900.25}$$

$$= \frac{-198.4983}{3332.25}$$

$$= -0.0595$$

$$- a = \bar{y} - b(\bar{x})$$

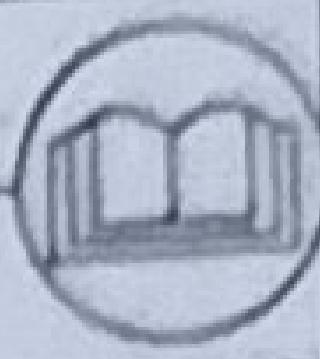
$$= 1.9179 + 0.0595(9.95)$$

$$= 2.5094$$

$$- a = e^{2.5094} = 12.31$$

$$b = e^{-0.0595} = 0.9422$$

$$- y = 12.31(0.9422)^x$$



3 Evaluate the following integral

$$\int_0^6 \frac{dx}{1+x^2} \quad n = 6$$

- (i) Trapezoidal rule
 (ii) Simpson's one-third rule
 (iii) Use Simpson's 3/8 rule $h = 1$

i	x_i	$f(x_i) = \frac{1}{1+x^2}$
x_0	0	1
x_1	1	0.5
x_2	2	0.25
x_3	3	0.111
x_4	4	0.0588
x_5	5	0.0385
x_6	6	0.0270

(i) Trapezoidal rule:-

$$= \frac{h}{2} \left[f(x_0) + 2 \left[f(x_1) + f(x_2) + f(x_3) + f(x_4) + f(x_5) \right] + f(x_6) \right]$$



$$\begin{aligned}
 &= \frac{1}{2} \left[1 + 2 \left[(0.5) + (0.2) + (0.1) + (0.588) + (0.3) \right] + 0.0270 \right] \\
 &= \frac{1}{2} \left[1 + 2 (0.8973) + 0.0270 \right] \\
 &= 1.4108
 \end{aligned}$$

(ii) Simpson's 1/3 Rule :-

$$\begin{aligned}
 &= \frac{h}{3} \left[f(x_0) + 4 \left[f(x_1) + f(x_3) + f(x_5) \right] + 2 \left[f(x_2) + f(x_4) \right] + f(x_6) \right] \\
 &= \frac{1}{3} \left[1 + 4 \left[(0.5) + (0.1) + (0.0385) \right] + 2 \left[(0.2) + 0.05 \right] + 0.0270 \right]
 \end{aligned}$$

$$= \frac{1}{3} (4.0986)$$

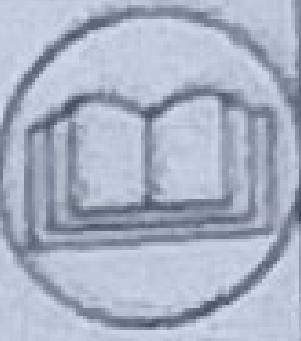
$$= 1.3662$$



(iii) Simpson's 3/8 Rule:-

$$\begin{aligned}
 & - \frac{3h}{8} \left[f(x_0) + 3[f(x_1) + f(x_2) + f(x_4) + f(x_5)] + \right. \\
 & \quad \left. 2[f(x_3) + f(x_6)] \right] \\
 & - \frac{3}{8} \left[1 + 3[(0.5) + 0.2 + 0.0588 + 0.0385] + 2(0.1) \right. \\
 & \quad \left. + 0.0270 \right] \\
 & = \frac{3}{8} (3.6189)
 \end{aligned}$$

$$1 = 1.3571$$



4 Evaluate the following integral by taking
 (i) $n=2$ (ii) $n=3$ and using Gauss quadrature
 formula

$$\int_{-1}^1 \frac{dx}{1+x^2}$$

Ans:-

$$n=2$$

$$x_1 = -\frac{1}{\sqrt{3}}, \quad x_2 = \frac{1}{\sqrt{3}}$$

$$f(x) = \frac{1}{1+x^2}$$

$$I = w_1 f(x_1) + w_2 f(x_2)$$

$$= 1 \left[\frac{1}{1 + \left(-\frac{1}{\sqrt{3}} \right)^2} \right] + 1 \left[\frac{1}{1 + \left(\frac{1}{\sqrt{3}} \right)^2} \right]$$

$$= \frac{1}{1 + \frac{1}{3}} + \frac{1}{1 + \frac{1}{3}}$$

$$= 2 \left[\frac{3}{4} \right]$$



$$= \frac{3}{2}$$

$$\boxed{= 1.5}$$

$$\rightarrow n = 3$$

$$x_1 = \sqrt{-\frac{3}{5}}, \quad x_2 = 0, \quad x_3 = \sqrt{\frac{3}{5}}$$

$$I = w_1 \frac{1}{1+x_1^2} + w_2 \frac{1}{1+x_2^2} + w_3 \frac{1}{1+x_3^2}$$

$$= \frac{5}{9} \left(\frac{1}{1+3/5} \right) + \frac{8}{9} (1) + \frac{5}{9} \left(\frac{1}{1+3/5} \right)$$

$$= \frac{25}{72} + \frac{8}{9} + \frac{25}{72}$$

$$= \frac{114}{72}$$

$$= \frac{19}{12}$$

$$\boxed{= 1.5833}$$



5 Explain the Trapezoidal Rule.

Ans:- The Trapezoidal rule is a numerical method for approximating the definite integral of a function.

→ Given a $f(x)$ over an interval $[a, b]$
Formula is:

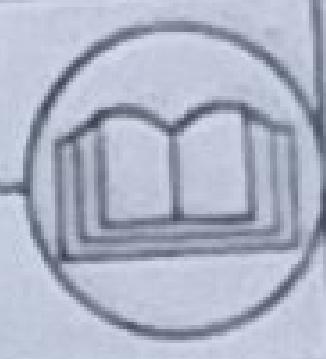
$$\int_a^b f(x) dx = \frac{b-a}{2} (f(a) + f(b))$$

$$\int_a^b f(x) dx = \frac{h}{2} [f(x_0) + 2f(x_i) + f(x_n)]$$

$$h = \frac{b-a}{n}$$

6. Explain graphical interpretation of Simpson 1/3 rule.

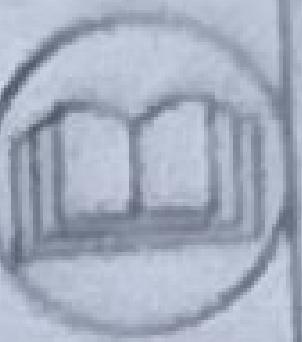
Ans:- The Simpson's 1/3 rule is another numerical integration method, which approximates the area under a curve by fitting Parabolas curve.



- Graphically, it is based on the idea that the curve can be better approximated by functions than by lines.
- For Simpson's $\frac{1}{3}$ rule, the interval $[a, b]$ is divided into an even number of sub intervals.

Formula is:-

$$\int_a^b f(x) dx \approx \frac{h}{3} \left[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) \right]$$



7 Use Euler's method to solve $y' = 1 + y^2$ and Compute $y(0.8)$ taking $h = 0.2$

Ans!- $y' = 1 + y^2, h = 0.2$

Step 1!- $x_0 = 0, y_0 = 0$

$$\begin{aligned} y_1 &= y_0 + h(1 + y_0^2) \\ &= 0 + 0.2(1 + 0^2) \\ &= 0.2 \end{aligned}$$

Step 2 - $x_1 = 0.2, y_1 = 0.2$

$$\begin{aligned} y_2 &= y_1 + h(1 + y_1^2) \\ &= 0.2 + 0.2(1 + (0.2)^2) \\ &= 0.2 + 0.208 \\ &= 0.408 \end{aligned}$$

Step 3 $x_2 = 0.4, y_2 = 0.408$

$$\begin{aligned} y_3 &= y_2 + h(1 + y_2^2) \\ &= 0.408 + 0.2(1 + (0.408)^2) \\ &= 0.408 + 0.2332 \\ &= 0.6412 \end{aligned}$$

Step 4:- $x_3 = 0.6, y_3 = 0.6412$

$$\begin{aligned}
 y_4 &= y_3 + h(1 + (y_3)^2) \\
 &= 0.6412 + 0.2(1 + (0.6412)^2) \\
 &= 0.6412 + 0.2822 \\
 &= 0.9235
 \end{aligned}$$

Step 5:- $x_4 = 0.8, y_4 = 0.9235$

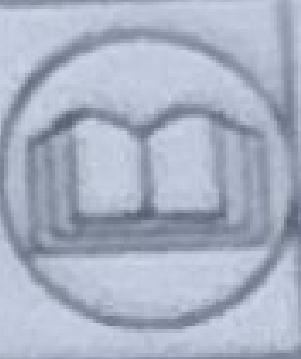
$$\begin{aligned}
 y_5 &= y_4 + h(1 + (y_4)^2) \\
 &= 0.9235 + 0.2(1 + (0.9235)^2) \\
 &= 0.9235 + 0.3705 \\
 &= 1.294
 \end{aligned}$$

8 Use 4th order RK method to solve
 $y' = x+y$ from $x=0$ to 0.4 taking $h=0.1$

Ans:- $y' = x+y$
 $h=0.1$

Step 1:- $n=0, x_0=0, y_0=1$

$$\begin{aligned}
 k_1 &= h \cdot f(x_n, y_n) \\
 &= 0.1(0+1) \\
 &= 0.1
 \end{aligned}$$



$$\begin{aligned}
 - K_2 &= h \cdot f(x_n + h/2, y_n + k_1/2) \\
 &= 0.1 \left(0 + \frac{0.1}{2}; 1 + \frac{0.1}{2} \right) \\
 &= 0.1 (0.05 + 1.05) \\
 &= 0.1 (1.1) \\
 &= 0.11
 \end{aligned}$$

$$\begin{aligned}
 - K_3 &= h F(x_n + h/2, y_n + k_2/2) \\
 &= 0.1 \left(0 + \frac{0.1}{2}, 1 + \frac{0.11}{2} \right) \\
 &= 0.1 (0.05 + 1.055) \\
 &= 0.1 (1.105) \\
 &= 0.1105
 \end{aligned}$$

$$\begin{aligned}
 - K_4 &= h f(x_n + 2h/2; y_n + k_3) \\
 &= 0.1 (0 + 0.1, 1 + 0.1105) \\
 &= 0.1 (1.2105) \\
 &= 0.12105
 \end{aligned}$$

$$\begin{aligned}
 - y_1 &= y_0 + \frac{1}{6} (K_1 + 2K_2 + 2K_3 + K_4) \\
 &= 1 + \frac{1}{6} (0.1 + 0.22 + 0.221 + 0.12105) \\
 &= 1 + \frac{1}{6} (0.66205) \\
 &= 1 + 0.110342 \\
 &= 1.110342
 \end{aligned}$$



Step 2 :- $n=1, x_1=0.1, y_1= 1.11034$

$$\begin{aligned} - k_1 &= 0.1 (0.1 + 1.11034) \\ &= 0.1 (1.21034) \\ &= 0.121034 \end{aligned}$$

$$\begin{aligned} - k_2 &= 0.1 \left(0.1 + \frac{0.1}{2}, 1.11034 + \frac{0.121034}{2} \right) \\ &= 0.1 (0.15 + 1.17085) \\ &= 0.13208 \end{aligned}$$

$$\begin{aligned} - k_3 &= 0.1 \left[0.1 + \frac{0.1}{2}, 1.11034 + \frac{0.13208}{2} \right] \\ &= 0.1 (0.15 + 1.17638) \\ &= 0.13263 \end{aligned}$$

$$\begin{aligned} - k_4 &= 0.1 (0.1 + 0.1, 1.11034 + 1.13263) \\ &= 0.1 (0.2 + 1.24297) \\ &= 0.14429 \end{aligned}$$

$$\begin{aligned} - y_2 &= y_1 + \frac{1}{6} (k_1 + 2k_2 + 2k_3 + k_4) \\ &= 1.11034 + \frac{1}{6} (0.121034 + 0.26416 + 0.2674 \\ &\quad + 0.14429) \\ &= 1.11034 + 0.13236 \\ &= 1.2428 \end{aligned}$$

Step 3 $n = 2$, $x = 0.2$, $y_2 = 1.2428$

- $K_1 = 0.1 (0.2 + 1.2428)$
 $\boxed{= 0.14428}$

- $K_2 = 0.1 \left(0.2 + \frac{0.1}{2}, 1.2428 + \frac{0.14428}{2} \right)$
 $= 0.1 (0.25 + 1.31494)$
 $\boxed{= 0.15649}$

- $K_3 = 0.1 \left(0.2 + \frac{0.1}{2}, 1.2428 + \frac{0.15649}{2} \right)$
 $= 0.1 (0.25 + 1.321045)$
 $\boxed{= 0.15710}$

- $K_4 = 0.1 (0.2 + 0.1, 1.2428 + 0.15710)$
 $= 0.1 (0.3 + 1.3999)$
 $\boxed{= 0.16999}$

- $y_3 = y_2 + \frac{1}{6} (K_1 + 2K_2 + 2K_3 + K_4)$
 $= 1.2428 + \frac{1}{6} (0.14428 + 0.321045 + 0.3142 + 0.16999)$

$\boxed{y_3 = 1.3997}$



9 The differential equation $y' = y - x^2$, is satisfied by $y(0) = 1$, $y(0.2) = 1.12186$, $y(0.4) = 1.46820$, $y(0.6) = 1.7359$.

Compute the value of $y(0.8)$ by milne's Predictor- Corrector formula.

Ans:- $y' = y - x^2$

→ Given values:- $y(0) = 1$, $y(0.2) = 1.12186$
 $y(0.4) = 1.46820$, $y(0.6) = 1.7359$

- $F(x, y) = y - x^2$

(i) $F(0.2, y(0.2)) = 1.12186 - (0.2)^2$
 $= 1.12186 - 0.04$
 $\boxed{= 1.08186}$

(ii) $F(0.4, y(0.4)) = 1.46820 - (0.4)^2$
 $= 1.46820 - 0.16$
 $\boxed{= 1.3082}$

(iii) $F(0.6, y(0.6)) = 1.7359 - (0.6)^2$
 $= 1.7359 - 0.36$
 $\boxed{= 1.3759}$



- Predictor Formule

$$- y(n)^P = y_{n-1} + \frac{4h}{3} [2f_i - f_{i-1} + 2f_{i-2}]$$

$$= y(0.6) + \frac{4(0.2)}{3} [2f(0.6) - f(0.4) + 2f(0.2)]$$

$$= 1.7359 + \frac{0.8}{3} [2(1.3759) - 1.3082 + 2(1.0818)]$$

$$= 1.7359 + \frac{0.8}{3} [2.7518 - 1.3082 + 2.1637]$$

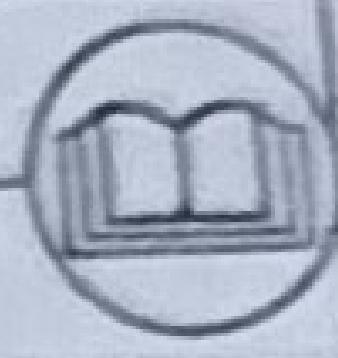
$$= 1.7359 + \frac{0.8}{3} (3.6073)$$

$$= 1.7359 + 0.9619$$

$$= 2.6978$$

$$- f(0.8, y(0.8)) = 2.6978 - (0.8)^2$$

$$= 2.0578$$



Correct

$$y(n)^c = y_{n-1} + \frac{h}{3} (f_{n+1} + 4f_n + f_{n-1})$$

$$= y(0.6) + \frac{0.2}{3} (f(0.8) + 4(0.6) + f(0.4))$$

$$= 1.7359 + \frac{0.2}{3} (2.0578 + 5.5036 + 1.3082)$$

$$= 1.7359 + \frac{0.2}{3} (8.8696)$$

$$= 1.7359 + 0.5913$$

$$= 2.3272$$



10

Solve $y' = 1-y$ with initial condition $x=0, y=0$ using Euler's algorithm and tabulate the solutions at $x=0.1, 0.2, 0.3, 0.4$ using this result find (0.5).

- using Adams- Bashforth method.

Ans! - $h=0.1, \quad y' = 1-y$

Step 1 $x=0.1, \quad y_0=0$

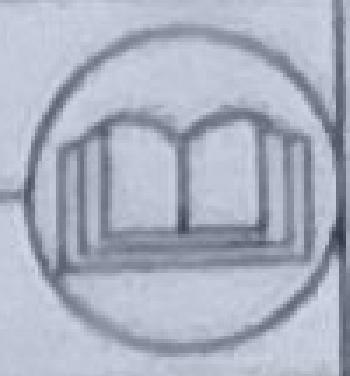
$$\begin{aligned}
 y_1 &= y_0 + h(1-y) \\
 &= 0 + 0.1(1-0) \\
 &= 0 + 0.1 \\
 &= 0.1
 \end{aligned}$$

Step 2 $x=0.2, \quad y_1=0.1$

$$\begin{aligned}
 y_2 &= y_1 + h(1-y_1) \\
 &= 0.1 + 0.1(1-0.1) \\
 &= 0.1 + 0.1(0.9) \\
 &= 0.19
 \end{aligned}$$

Step 3 $x=0.3, \quad y_2=0.19$

$$\begin{aligned}
 y_3 &= y_2 + h(1-y_2) \\
 &= 0.19 + 0.1(1-0.19) \\
 &= 0.19 + 0.081 \\
 &= 0.271
 \end{aligned}$$



Step-4

$$x = 0.4, y_3 = 0.271$$

$$\begin{aligned}
 y_4 &= y_3 + h(1 - y_3) \\
 &= 0.271 + 0.1(1 - 0.271) \\
 &= 0.271 + 0.0729 \\
 &= 0.3439
 \end{aligned}$$

- $f(0.1, y(0.1)) = 1 - 0.1 = 0.9$
- $f(0.2, y(0.2)) = 1 - 0.19 = 0.81$
- $f(0.3, y(0.3)) = 1 - 0.271 = 0.729$
- $f(0.4, y(0.4)) = 1 - 0.3439 = 0.6561$

Predictor

- $y(0.5)^P = y(0.4) + \frac{0.1}{2} (3f(0.4) - f(0.3))$

$$\begin{aligned}
 &= 0.3439 + \frac{0.1}{2} (3(0.6561) - 0.729) \\
 &= 0.3439 + 0.05 (1.9683 - 0.729) \\
 &= 0.4058
 \end{aligned}$$

Correct

- $y(0.5)^C = y(0.4) + \frac{0.1}{2} (f(0.5) + f(0.4))$

$$\begin{aligned}
 &= 0.3439 + 0.5 (1.25023) \\
 &= 0.3439 + 0.0625 \\
 &= 0.4062
 \end{aligned}$$



11 What are Predictor-Connector methods for solving the initial value problem $y' = f(x, y)$, $y(x_0) = y_0$?

Ans: We are use Adams-Basforth - Method, Euler with Trapezoidal Connect and Milne's method use for this pattern.

12 Compute $f'(0)$ from the data

x	0	1	2	3	4
y	1	2.718	7.381	20.086	54.598

Ans: $f'(0) = \frac{f(x+h) - f(x)}{h}$

$$= \frac{f(1) - f(0)}{1}$$

$$= \frac{2.718 - 1}{1}$$

$$f'(0) = 1.718$$