

Assignment 3 Regression, Integration, Differential Equations, Differentiation

1. For the data given below use least-squares regression to find

x	6	7	11	15	17	21	23	29	29	37	39
y	29	21	29	14	21	15	7	7	13	0	3

- (i) the slope (a_1)
- (ii) intercept (a_0)
- (iii) fit a straight line
- (iv) compute the standard error of the estimate
- (v) the correlation coefficient.

2. Fit the data with the model ($y = ab^x$). Use the resulting equation to predict y at $x = 9$:

x	2.5	3.5	5	6	7.5	10	12.5	15	17.5	20
y	13	11	8.5	8.2	7	6.2	5.2	4.8	4.6	4.3

3. Evaluate the following integral:

$$\int_0^6 \frac{dx}{1+x^2}$$

by taking $n = 6$ and using the following rules:

- (i) Trapezoidal rule and (ii) Simpson's one-third rule (iii) Use Simpson's 3/8 rule

4. Evaluate the following integral by taking (i) $n = 2$ (ii) $n = 3$ and using Gauss quadrature formula

$$\int_{-1}^1 \frac{dx}{1+x^2}$$

5. Explain the trapezoidal rule.

6. Explain graphical interpretation of Simpson's 1/3 rule.

7. Use Euler's method to solve $y' = 1 + y^2$ and compute $y(0.8)$ taking $h = 0.2$

8. Use 4th order RK method to solve $y' = x + y$ from $x = 0$ to 0.4 taking $h = 0.1$

9. The differential equation $y' = y - x^2$, is satisfied by $y(0) = 1$, $y(0.2) = 1.12186$, $y(0.4) = 1.46820$, $y(0.6) = 1.7359$. Compute the value of $y(0.8)$ by Milne's predictor-corrector formula.

10. Solve $y' = 1 - y$ with the initial condition $x = 0$, $y = 0$, using Euler's algorithm and tabulate the solutions at $x = 0.1, 0.2, 0.3, 0.4$. Using these results find $y(0.5)$, using Adams-Bashforth predictor-corrector method.

11. What are predictor-corrector methods for solving the initial value problem $y' = f(x, y)$, $y(x_0) = y_0$?

12. Compute $f'(0)$ from the data

x	0	1	2	3	4
y	1	2.718	7.381	20.086	54.598

Solution

1. i) -0.424 ii) 23.869 iii) $y = 23.869 - 0.424x$ iv) 5.602 v) 0.8041
2. $12.313(0.9422)^x$
3. i) 1.4108 ii) 1.3662 iii) 1.3571
4. i) 1.5 ii) 1.5833
7. $0.2, 0.48, 0.73$
8. $1.11034, 1.2428, 1.399711$
9. $y(0.8) = 2.013683$.
10. $y(0.1) = 0.1, y(0.2) = 0.19, y(0.3) = 0.271, y(0.4) = 0.3439, y(0.5) = 0.406293$.
12. -0.2225