Assignment - III

1. For the data given below use least-squares regression to find

x	6	7	11	15	17	21	23	29	29	37	39
y	29	21	29	14	21	15	7	7	13	0	3

(i) the slope (a_1)
(ii) intercept (a_0)
(iii) fit a straight line
(iv) Compute the standard error of the estimate
(v) the Correlation coefficient

Ans:-

x	y	x^2	xy	$(y-\bar{y})^2$	$(y-a_0-a_1x_i)^2$
6	29	36	174	211.7025	6.93268
7	21	49	147	42.9025	21.03590
11	29	121	319	211.7025	42.71276
15	14	225	210	0.2025	28.54231
17	21	289	357	42.9025	10.35874
21	15	441	315	0.3025	0.11594
23	7	529	161	55.5025	37.19170
29	7	841	203	55.5025	2.00364
29	13	841	377	2.1025	21.01764
37	0	1361	0	208.8025	4.71541
39	3	1521	117	131.1025	5.70971
Total 234	159	6262	2380	962.7275	180.3364

- $\bar{x} = \frac{\Sigma x_i}{n} = \frac{234}{11} = 21.27$

- $\bar{y} = \frac{\Sigma y_i}{n} = \frac{159}{11} = 14.45$

(i) $a_1 = \frac{n \Sigma xy - \Sigma x_i \Sigma y_i}{n \Sigma x_i^2 - (\Sigma x_i)^2}$

$= \frac{11(2380) - (234)(159)}{11(6262) - (234)^2}$

$= \frac{26180 - 37206}{68882 - 54756}$

$= \frac{-11026}{14126}$

$\boxed{= -0.7805}$

(ii) $a_0 = \bar{y} - a_1 \bar{x}$

$= 14.45 - (-0.7805)(21.27)$

$= 14.45 + 16.60$

$\boxed{= 31.05}$

(iii) $y = 31.05 - 0.7805x$

(iv) $S_{y/x} = \sqrt{\dfrac{S_r}{n-2}}$

$= \sqrt{\dfrac{180.3364}{11-2}}$

$= \sqrt{\dfrac{180.3364}{9}}$

$\boxed{= 4.47}$

(v) $r^2 = \dfrac{S_t - S_r}{S_t}$

$= \dfrac{962.7275 - 180.3364}{962.7275}$

$= \dfrac{782.3911}{962.7275}$

$\boxed{= 0.81}$

2 Fit the data with model ($y = ab^x$). $x = 9$

x	2.5	3.5	5	6	7.5	10	12.5	15	17.5
y	13	11	8.5	8.2	7	6.2	5.2	4.8	4.6

Ans:- $y = ab^x$

$X = x$
$Y = \ln y$

x	y	$Y = \ln(y)$	x^2	xY
2.5	13	2.5649	6.25	6.4123
3.5	11	2.3979	12.25	8.3927
5	8.5	2.1401	25	10.7005
6	8.2	2.1041	36	12.6246
7.5	7	1.9459	56.25	14.5943
10	6.2	1.8245	100	18.2450
12.5	5.2	1.6486	156.25	20.6075
15	4.8	1.5686	225	23.5290
17.5	4.6	1.5261	306.25	26.7063
20	4.3	1.4586	400	29.1720
Total 99.5		19.1793	1323.25	170.9842

- $\bar{x} = \frac{\Sigma x_i}{n} = \frac{99.5}{10} = 9.95$

- $\bar{y} = \frac{\Sigma y_i}{n} = \frac{19.1793}{10} = 1.9179$

- $b = \frac{n\Sigma xy - \Sigma x \Sigma y}{n\Sigma x^2 - (\Sigma x)^2}$

$= \frac{10(170.9842) - (99.5)(19.1793)}{10(1323.25) - (99.5)^2}$

$= \frac{1709.842 - 1908.4035}{13232.5 - 9900.25}$

$= \frac{-198.4983}{3332.25}$

$\boxed{= -0.0595}$

- $a = \bar{y} - b(\bar{x})$

$= 1.9179 + 0.0595(9.95)$

$\boxed{= 2.5094}$

- $a = e^{2.5094} = 12.31$

$b = e^{-0.0595} = 0.9422$

- $\boxed{y = 12.31(0.9422)^x}$

3 Evaluate the following integral

$$\int_0^6 \frac{dx}{1+x^2} \qquad n=6$$

(i) Trapezoidal rule
(ii) Simpson's one-third rule.
(iii) Use Simpson's 3/8 rule $\quad h = 1$

i	x_i	$f(x_i) = \frac{1}{1+x^2}$
x_0	0	1
x_1	1	0.5
x_2	2	0.2
x_3	3	0.1
x_4	4	0.0588
x_5	5	0.0385
x_6	6	0.0270

(i) Trapezoidal rule:-

$$= \frac{h}{2}\left[f(x_0) + 2\left[f(x_1) + f(x_2) + f(x_3) \dots f(x_5)\right] + f(x_6)\right]$$

$= \frac{1}{2}\left[1 + 2\left[(0.5) + (0.2) + (0.1) + (0.588) + (0.3)\right] + 0.0270\right]$

$= \frac{1}{2}\left[1 + 2(0.8973) + 00270\right]$

$\boxed{I = 1.4108}$

(ii) Simpson's 1/3 Rule:-

$= \frac{h}{3}\left[F(x_0) + 4\left[F(x_1) + F(x_3) + F(x_5)\right] + 2\left[F(x_2) + F(x_4)\right] + F(x_6)\right]$

$= \frac{1}{3}\left[1 + 4\left[(0.5) + (0.1) + (0.0385)\right] + 2\left[(0.2) + (0.05\ldots)\right] + 0.0270\right]$

$= \frac{1}{3}(4.0986)$

$\boxed{I = 1.3662}$

(iii) Simpson's 3/8 Rule :-

$$= \frac{3h}{8}\left[f(x_0) + 3\left[f(x_1) + f(x_2) + f(x_4) + f(x_5)\right] + 2\left[f(x_3)\right] + f(x_6)\right]$$

$$= \frac{3}{8}\left[1 + 3\left[(0.5) + 0.2 + 0.0588 + 0.0385\right) + 2(0.1) + 0.0270\right]$$

$$= \frac{3}{8}(3.6189)$$

$$\boxed{= 1.3571}$$

4 Evaluate the following integral by taking (i) $n=2$ (ii) $n=3$ and using Gauss quadrature Formula

$$\int_{-1}^{1} \frac{dx}{1+x^2}$$

Ans :-

$n=2$

$$x_1 = -\frac{1}{\sqrt{3}}, \quad x_2 = \frac{1}{\sqrt{3}}$$

$$f(x) = \frac{1}{1+x^2}$$

$$I = w_1 f(x_1) + w_2 f(x_2)$$

$$= 1\left[\frac{1}{1+\left(-\frac{1}{\sqrt{3}}\right)^2}\right] + 1\left[\frac{1}{1+\left(\frac{1}{\sqrt{3}}\right)^2}\right]$$

$$= \frac{1}{1+\frac{1}{3}} + \frac{1}{1+\frac{1}{3}}$$

$$= 2\left[\frac{3}{4}\right]$$

$$= \frac{3}{2}$$

$$\boxed{= 1.5}$$

$\rightarrow n = 3$

$$x_1 = \sqrt{-\frac{3}{5}}, \quad x_2 = 0, \quad x_3 = \sqrt{\frac{3}{5}}$$

$$- I = w_1 \frac{1}{1+x^2} + w_2 \frac{1}{1+x^2} + w_3 \frac{1}{1+x^2}$$

$$= \frac{5}{9}\left(\frac{1}{1+3/5}\right) + \frac{8}{9}(1) + \frac{5}{9}\left(\frac{1}{1+3/5}\right)$$

$$= \frac{25}{72} + \frac{8}{9} + \frac{25}{72}$$

$$= \frac{114}{72}$$

$$= \frac{19}{12}$$

$$\boxed{= 1.5833}$$

5 Explain the trapezoidal Rule.

Ans:- The Trapezoidal rule is a numerical method for approximating the definite integral of a Function.

→ Given a f(x) over an interval [a,b] Formula is:

$$\int_a^b f(x)\,dx = \frac{b-a}{2}\left(f(a)+f(b)\right)$$

$$\int_a^b f(x)\,dx = \frac{h}{2}\left[f(x_0)+2f(x_i)+f(x_n)\right]$$

$$h = \frac{b-a}{n}$$

6. Explain graphical interpretation of Simpson 1/3 rule.

Ans:- The Simpson's 1/3 rule is another numerical integration method, which approximates the area under a curve by fitting Parabolas curve.

-▷ Graphically, it is based on the idea that the curve can be better approximated by functions than by lines.

-▷ For Simpson's 1/3 rule, the interval [a,b] is divided into an even number of sub intervals.

Formula is :-

$$\int_a^b f(x)\,dx = \frac{h}{3}\left[f(x_0)+4f(x_1)+2f(x_2)+4f(x_3)\right]$$

7 Use Euler's method to solve $y' = 1 + y^2$ and Compute $y(0.8)$ taking $h = 0.2$

Ans:- $y' = 1 + y^2$, $h = 0.2$

Step 1:- $x_0 = 0$, $y_0 = 0$

$$y_1 = y_0 + h(1 + y_0^2)$$
$$= 0 + 0.2(1 + 0^2)$$
$$\boxed{= 0.2}$$

Step 2 :- $x_1 = 0.2$, $y_1 = 0.2$

$$y_2 = y_1 + h(1 + y_1^2)$$
$$= 0.2 + 0.2(1 + (0.2)^2)$$
$$= 0.2 + 0.208$$
$$\boxed{= 0.408}$$

Step 3 $x_2 = 0.4$, $y_2 = 0.408$

$$y_3 = y_2 + h(1 + y_2^2)$$
$$= 0.408 + 0.2(1 + (0.408)^2)$$
$$= 0.408 + 0.2332$$
$$\boxed{= 0.6412}$$

Step 4:- $x_3 = 0.6,\ y_3 = 0.6412$

$y_4 = y_3 + h(1 + (y_3)^2)$
$= 0.6412 + 0.2(1 + (0.6412)^2)$
$= 0.6412 + 0.2822$
$\boxed{= 0.9235}$

Step 5:- $x_4 = 0.8,\ y_4 = 0.9235$

$y_5 = y_4 + h(1 + (y_4)^2)$
$= 0.9235 + 0.2(1 + (0.9235)^2)$
$= 0.9235 + 0.3705$
$\boxed{= 1.7112}$

8 Use 4th order RK method to slove $y' = x + y$ From $x = 0$ to 0.4 taking $h = 0$

Ans:- $y' = x + y$
$h = 0.1$

Step 1:- $n = 0,\ x_0 = 0,\ y_0 = 1$

$k_1 = h \cdot f(x_n, y_n)$
$= 0.1(0 + 1)$
$\boxed{= 0.1}$

- $k_2 = h \cdot f(x_n + h/2, \; y_n + k_1/2)$

  $= 0.1 \left(0 + \frac{0.1}{2} \; ; \; 1 + \frac{0.1}{2}\right)$

  $= 0.1 \, (0.05 + 1.05)$

  $= 0.1 \, (1.1)$

  $\boxed{= 0.11}$

- $k_3 = h f(x_n + h/2, \; y_n + k_2/2)$

  $= 0.1 \left(0 + \frac{0.1}{2}, \; 1 + \frac{0.11}{2}\right)$

  $= 0.1 \, (0.05 + 1.055)$

  $= 0.1 \, (1.105)$

  $\boxed{= 0.1105}$

- $k_4 = h f(x_n + h \; ; \; y_n + k_3)$

  $= 0.1 \, (0 + 0.1, \; 1 + 0.1105)$

  $= 0.1 \, (1.2105)$

  $\boxed{= 0.12105}$

- $y_1 = y_0 + 1/6 \, (k_1 + 2k_2 + 2k_3 + k_4)$

  $= 1 + 1/6 \, (0.1 + 0.22 + 0.221 + 0.12105$

  $= 1 + 1/6 \, (0.66205)$

  $= 1 + 0.110342$

  $\boxed{= 1.110342}$

Step 2 :- $n = 1, \quad x_1 = 0.1, \quad y_1 = 1.11034$

- $k_1 = 0.1\,(0.1 + 1.11034)$

$= 0.1\,(1.21034)$

$\boxed{= 0.121034}$

- $k_2 = 0.1\left(0.1 + \frac{0.1}{2},\ 1.11034 + \frac{0.121034}{2}\right)$

$= 0.1\,(0.15 + 1.17085)$

$\boxed{= 0.13208}$

- $k_3 = 0.1\left[0.1 + \frac{0.1}{2},\ 1.11034 + \frac{0.13208}{2}\right]$

$= 0.1\,(0.15 + 1.17638)$

$\boxed{= 0.13263}$

- $k_4 = 0.1\,(0.1 + 0.1,\ 1.11034 + 1.13263)$

$= 0.1\,(0.2 + 1.24297)$

$\boxed{= 0.14429}$

- $y_2 = y_1 + 1/6\,(k_1 + 2k_2 + 2k_3 + k_4)$

$= 1.11034 + 1/6\,(0.121034 + 0.26416 + 0.2674$

$+ 0.1442$

$= 1.11034 + 0.13236$

$\boxed{= 1.2428}$

Step 3 $n = 2$, $x = 0.2$, $y_2 = 1.2428$

- $K_1 = 0.1\,(0.2 + 1.2428)$
  $\boxed{= 0.14428}$

- $K_2 = 0.1\left(0.2 + \frac{0.1}{2},\ 1.2428 + \frac{0.14428}{2}\right)$
  $= 0.1\,(0.25 + 1.31494)$
  $\boxed{= 0.15649}$

- $K_3 = 0.1\left(0.2 + \frac{0.1}{2},\ 1.2428 + \frac{0.15649}{2}\right)$
  $= 0.1\,(0.25 + 1.321045)$
  $\boxed{= 0.15710}$

- $K_4 = 0.1\,(0.2 + 0.1,\ 1.2428 + 0.15710)$
  $= 0.1\,(0.3 + 1.3999)$
  $\boxed{= 0.16999}$

- $y_3 = y_2 + \frac{1}{6}\,(K_1 + 2K_2 + 2K_3 + K_4)$
  $= 1.2428 + \frac{1}{6}\,(0.14428 + 0.321298 +$
  $0.3142 + 0.16999)$

$\boxed{y_3 = 1.3997}$

9 The differential equation $y' = y - x^2$, is Satisfied by $y(0) = 1$, $y(0.2) = 1.12186$, $y(0.4) = 1.46820$, $y(0.6) = 1.7359$.
Compute the value of $y(0.8)$ by milne's Predictor - Corrector Formula.

Ans:- $y' = y - x^2$

→ Given values:- $y(0) = 1$, $y(0.2) = 1.12186$
$y(0.4) = 1.46820$, $y(0.6) = 1.7359$

- $f(x, y) = y - x^2$

(i) $f(0.2, y(0.2)) = 1.12186 - (0.2)^2$
$= 1.12186 - 0.04$
$\boxed{= 1.08186}$

(ii) $f(0.4, y(0.4)) = 1.46820 - (0.4)^2$
$= 1.46820 - 0.16$
$\boxed{= 1.3082}$

(iii) $f(0.6, y(0.6)) = 1.7359 - (0.6)^2$
$= 1.7359 - 0.36$
$\boxed{= 1.3759}$

- Predictor Formula

- $y(n)^P = y_{n-1} + \frac{4h}{3}\left[2f_i - f_{i-1} + 2f_{i-2}\right)$

$= y(0.6) + \frac{4(0.2)}{3}\left[2f(0.6) - f(0.4) + 2f(0.2\right.$

$= 1.7359 + \frac{0.8}{3}\left[2(1.3759) - 1.30820 + 2(1.0818\right.$

$= 1.7359 + \frac{0.8}{3}\left[2.7518 - 1.3082 + 2.16372\right]$

$= 1.7359 + \frac{0.8}{3}(3.6073)$

$= 1.7359 + 0.9619$

$\boxed{= 2.6978}$

- $f(0.8, y(0.8)) = 2.6978 - (0.8)^2$
$\boxed{= 2.0578}$

- Corrrect

- $y(n)^c = y_{n-1} + \frac{h}{3}\left(f_{n+1} + 4f_n + f_{n-1}\right)$

$$= y(0.6) + \frac{0.2}{3}\left(f(0.8) + 4(0.6) + f(0.4)\right)$$

$$= 1.7359 + \frac{0.2}{3}\left[2.0578 + 5.5036 + 1.30820\right]$$

$$= 1.7359 + \frac{0.2}{3}(8.8696)$$

$$= 1.7359 + 0.5913$$

$$\boxed{= 2.3272}$$

10 Solve $y' = 1-y$ with inital condition $x=0, y=0$ using Euler's algorithm and tabulate the solutions at $x=0.1, 0.2, 0.3, 0.4$ using this result find (0.5).
- using Admas-Bashforth method.

Ans:- $h = 0.1, \quad y' = 1-y$

Step 1 $\quad x = 0.1, \quad y_0 = 0$

$$y_1 = y_0 + h(1-y)$$
$$= 0 + 0.1(1-0)$$
$$= 0 + 0.1$$
$$\boxed{= 0.1}$$

Step 2 $\quad x = 0.2, \quad y_1 = 0.1$

$$y_2 = y_1 + h(1-y_1)$$
$$= 0.1 + 0.1(1-0.1)$$
$$= 0.1 + 0.1(0.9)$$
$$\boxed{= 0.19}$$

Step 3 $\quad x = 0.3, \quad y_2 = 0.19$

$$y_3 = y_2 + h(1-y_2)$$
$$= 0.19 + 0.1(1-0.19)$$
$$= 0.19 + 0.081$$
$$\boxed{= 0.271}$$

Date :

Page : 66

Step. 4 $x = 0.4$, $y_3 = 0.271$

$$y_4 = y_3 + h(1 - y_3)$$
$$= 0.271 + 0.1(1 - 0.271)$$
$$= 0.271 + 0.0729$$
$$\boxed{= 0.3439}$$

- $F(0.1, y(0.1)) = 1 - 0.1 = 0.9$
  $F(0.2, y(0.2)) = 1 - 0.19 = 0.81$
  $F(0.3, y(0.3)) = 1 - 0.271 = 0.729$
  $F(0.4, y(0.4)) = 1 - 0.3439 = 0.6561$

- Poredictar

- $y(0.5)^P = y(0.4) + \frac{0.1}{2}(3F(0.4) - F(0.3))$

$$= 0.3439 + \frac{0.1}{2}(3(0.6561) - 0.729$$
$$= 0.3439 + 0.05(1.9683 - 0.729)$$
$$\boxed{= 0.4058}$$

- Corrrect

- $y(0.5)^c = y(0.4) + \frac{0.1}{2}(F(0.5) + F(0.4))$

$$= 0.3439 + 0.5(1.25023)$$
$$= 0.3439 + 0.0625$$
$$\boxed{= 0.4062}$$

11 What are Predictor - Corrector methods For sloving the initial value Problem $y' = f(x,y)$, $y(x_0) = y_0$?

Ans:- We are use Admas - Bashfourth - Method, Euler with Trapezoidal Correct and Mine's method use For this pattern.

12 Compute $f'(0)$ from the data

X	0	1	2	3	4
y	1	2.718	7.381	20.086	54.598

Ans:- $f'(x) = \dfrac{f(x+h) - f(x)}{h}$

$= \dfrac{f(1) - f(0)}{1}$

$= \dfrac{2.178 - 1}{1}$

$\boxed{f'(0) = 1.718}$