Examples

Eg 4. *Find the real root of the equation* $x^3 - x - 1$ *by Bisection method correct to two decimal places*.

ϵ = 0.01, *Take a = 1.25, b = 1.5*

Eg 5. *Find the real root of the equation*f(x) = 3x - e^x *by Bisection method correct to two decimal places*.

ϵ = 0.01, *Take a = 0, b = 1*

Examples

Eg 4. *Find the real root of the equation* $x^3 - x - 1$ *by Bisection method correct to two decimal places.*

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Eg 5. *Find the real root of the equation*f(x) = 3x - e^x *by Bisection method correct to two decimal places.*

ϵ = 0.01, *Take a = 0, b = 1*

Example 4

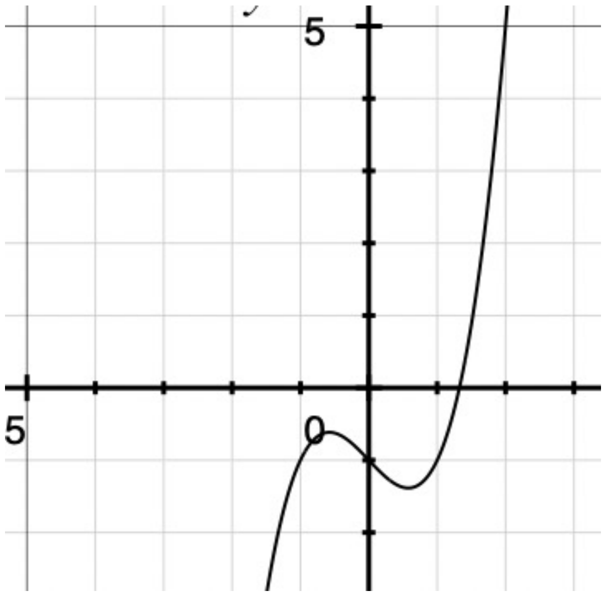
Eg 4. *Find the real root of the equation* $x^3 - x - 1$ *by Bisection method correct to two decimal places.*

$\epsilon = 0.01$, *Take a = 1.25, b = 1.5*

x	0	1	2
f(x)	-1	-1	5

Solution 4

Eg 4. *Find the real root of the equation* $x^3 - x - 1$ *by Bisection method correct to two decimal places. Take a = 1.25, b = 1.5,*



Solution 4

Eg 4. *Find the real root of the equation* $x^3 - x - 1$ *by Bisection method correct to two decimal places. Take a = 1.25, b = 1.5,*

ϵ = 0.01

f(*x*) = $x^3 - x - 1$

f(1.25) = – 0.2969 (–)ve

f(1.5) = 0.875 (+)ve
Hence, the root lies between 1.25 and 1.5.

a	f(a)	b	f(b)
1.25	– 0.2969	1.5	0.875

Solution 4

∴ First approximation to the root is

$$c = \frac{(1.25 + 1.5)}{2}$$

c = 1.375

a	f(a)	b	f(b)	c = (a+b)/2	f(c)
1.25	– 0.2969	1.5	0.875	1.375	0.2246

Now f (c) = f(1.375) = 0.2246 (+)ve

and f(a) = – 0.2969 (–)ve

Hence, the root lies between 1.25 and 1.375

Solution 4

Second approximation to the root is

$$c = \frac{(1.25 + 1.375)}{2}$$

= 1.3125

a	f(a)	b	f(b)	c = (a+b)/2	f(c)	$\lvert c_k - c_{k-1} \rvert$
1.25	– 0.2969	1.5	0.875	1.375	0.2246	-
1.25	– 0.2969	1.375	0.2246	1.3125	-0.0515	0.0625

Now $f(c)$ = f(1.3125) = -0.0515 (-)ve and f(b) = 0.2246 (+)ve

Hence, the root lies between 1.3125 and 1.375.

Solution 4

Third approximation to the root is

$c = \frac{(1.3125+1.375)}{2}$ = 1.3438

a	f(a)	b	f(b)	c = (a+b)/2	f(c)	$\lvert c_k - c_{k-1} \rvert$
1.25	– 0.2969	1.5	0.875	1.375	0.2246	-
1.25	– 0.2969	1.375	0.2246	1.3125	-0.0515	0.0625
1.3125	-0.0515	1.375	0.2246	1.3438	0.0826	0.0313

Now $f(c)$ = f(1.3438) = 0.0826 *i.e.,* (+)ve

and f(a) = -0.0515 (–)ve

Hence, the root lies between 1.3125 and 1.3438 .

Solution 4

Fourth approximation to the root is

$c = \frac{(1.3125+1.3438)}{2}$ = 1.3281

a	f(a)	b	f(b)	c = (a+b)/2	f(c)	$\lvert c_k - c_{k-1} \rvert$
1.25	– 0.2969	1.5	0.875	1.375	0.2246	-
1.25	– 0.2969	1.375	0.2246	1.3125	-0.0515	0.0625
1.3125	-0.0515	1.375	0.2246	1.3438	0.0826	0.0313
1.3125	-0.0515	1.3438	0.0826	1.3281	0.0146	0.0157

Now *f*(c) = *f*(1.3281) = 0.0146 *i.e.,* (+)ve and

f(b) = *f*(1.3125) = -0.0515 *i.e.,* (-)ve

Hence, the root lies between 1.3125 and 1.3281

Solution 4

Fifth approximation to the root is

c = $\frac{(1.3125+1.3281)}{2}$ = 1.3203

ϵ = 0.01 $|c_k - c_{k-1}|$ = 0.0078 < ϵ

The approximate real root is 1.3203

a	f(a)	b	f(b)	c = (a+b)/2	f(c)	$\lvert c_k - c_{k-1} \rvert$
1.25	– 0.2969	1.5	0.875	1.375	0.2246	-
1.25	– 0.2969	1.375	0.2246	1.3125	-0.0515	0.0625
1.3125	-0.0515	1.375	0.2246	1.3438	0.0826	0.0313
1.3125	-0.0515	1.3438	0.0826	1.3281	0.0146	0.0157
1.3125	-0.0515	1.3281	0.0146	1.3203		0.0078

Solution 5

- f(x) = 3x - e^x

a	$f(a)$	b	$f(b)$	$c = \frac{a+b}{2}$	$f(c)$
0	-1	1	0.2817	0.5	-0.1487
0.5	-0.1487	1	0.2817	0.75	0.133
0.5	-0.1487	0.75	0.133	0.625	0.0068
0.5	-0.1487	0.625	0.0068	0.5625	-0.0676
0.5625	-0.0676	0.625	0.0068	0.5938	-0.0295
0.5938	-0.0295	0.625	0.0068	0.6094	-0.0112
0.6094	-0.0112	0.625	0.0068	0.6172	-0.0021
0.6172	-0.0021	0.625	0.0068	0.6211	0.0023
0.6172	-0.0021	0.6211	0.0023	0.6191	0.0001

Algorithm for the bisection method

1. Choose a and b as two guesses for the root such that f(a).f(b) < 0 Let k =1
2. Estimate the root, c, of the equation as the mid-point between a and b as $c_k = \frac{a+b}{2}$
3. Now check the following

   If f(a).f(c) < 0 , then the root lies between a and c; then b = c.

   If f(a).f(c) > 0 , then the root lies between c and b; then a = c

   If f(c) = 0; then the root is c. Stop the algorithm if this is true.
4. Find the new estimate of the root $c_k = \frac{a+b}{2}$
5. Compare the absolute approximate error $|c_k - c_{k-1}|$ or absolute relative approximate error $\frac{|c_k - c_{k-1}|}{|c_k|}$

with the pre-specified relative error tolerance ϵ, where c_k = estimated root from present iteration, c_{k-1} = estimated root from previous iteration

If $|c_k - c_{k-1}| < \epsilon$ or $\frac{|c_k - c_{k-1}|}{|c_k|} < \epsilon$ then exit

Else

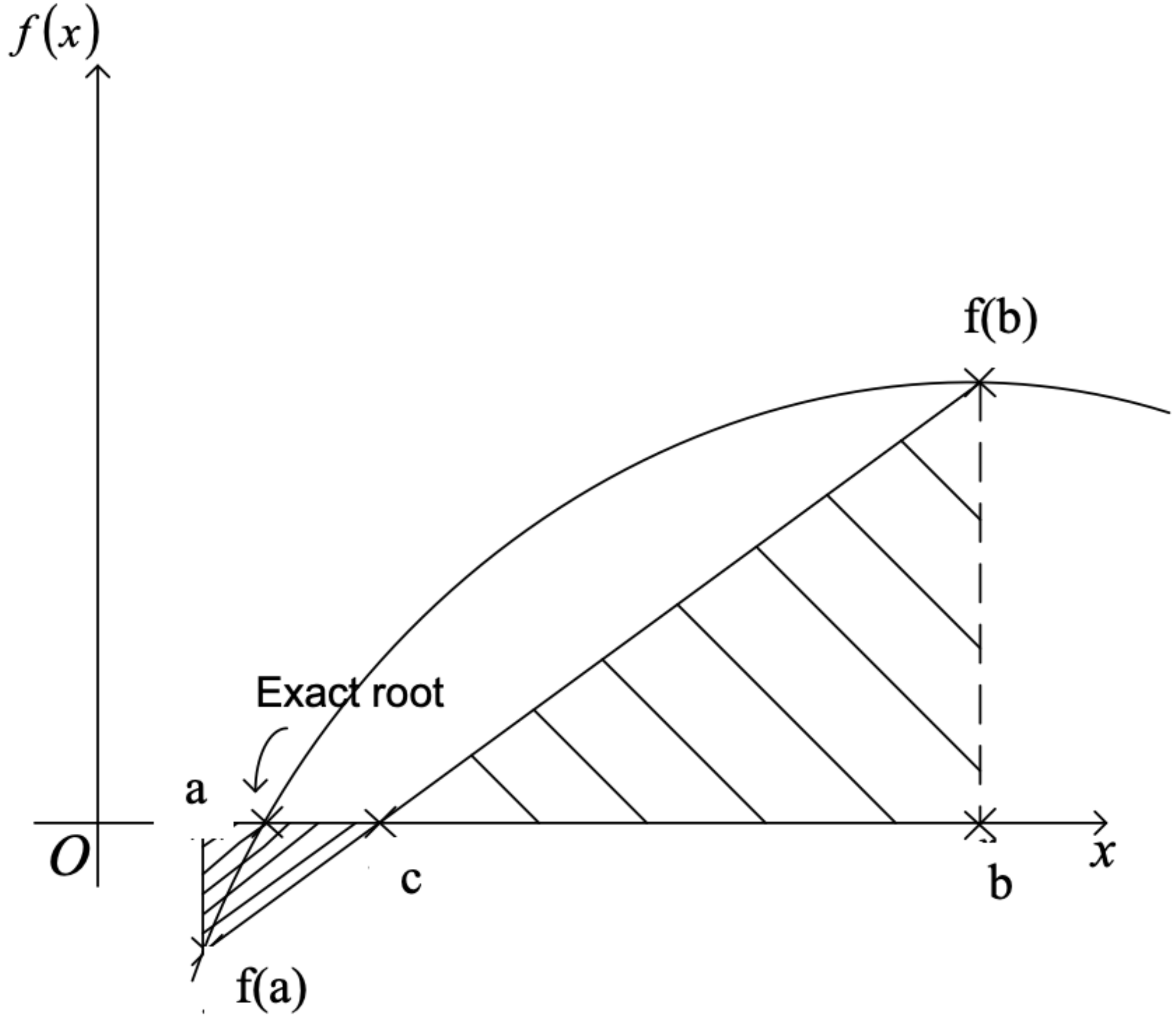
k = k + 1

go to Step 3

Note one should also check whether the number of iterations is more than the maximum number of iterations allowed.

False Position

False position method



False position method

- In the bisection method, we identify proper values of a (lower bound value) and b (upper bound value) for the current bracket.
- However, in the example shown in Figure , the bisection method may not be efficient because it does not take into consideration that f(a) is much closer to the zero of the function f(x) as compared to f(b). In other words, the next predicted root c would be closer to a, than the mid-point between a and b. The false-position method takes advantage of this observation mathematically by drawing a straight line from the function value at a to the function value at b, and estimates the root as where it crosses the *x*-axis.

False position method

So, in regular falsi method, to take into consideration the function values at a and b, straight line is drawn, joining (a,f(a))and(b,f(b)).

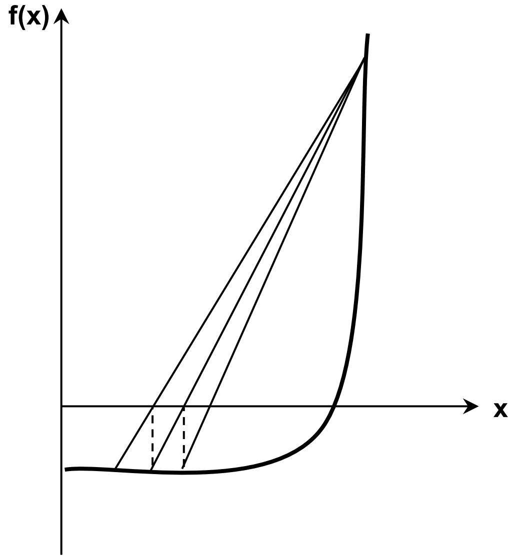
The point, where it cuts X axis is, the new estimate of the root. Point of intersection of straight line with X – axis being on X-axis is (c , 0).

$$c = \frac{af(b) - bf(a)}{f(b) - f(a)}$$

Advantages and Disadvantages

Disadvantages:

1. Not self starting. One needs two initial guesses a and b such that $f(a) \cdot f(b) < 0$.
2. Though, faster than Bisection Method, still regarded as slow.
3. In rare cases, it may become slower than Bisection Method.



Slow convergence of the false-position method for $f(x) = x^{10} - 1 = 0$

Advantages and Disadvantages

Advantages:

1. It is faster than Bisection Method.
2. It is also simple.
3. It guarantees convergence.
4. Only one function evaluation per iteration is required.

Example

This polynomial obviously has roots at x = 2 and at x = 4; one is a double root, the other a triple root:

$f(x) = (x-2)^3(x-4)^2$

$= x^5 - 14x^4 + 76x^3 - 200x^2 + 256x - 128$

a. Which root can you get with bisection? Which root can't you get?

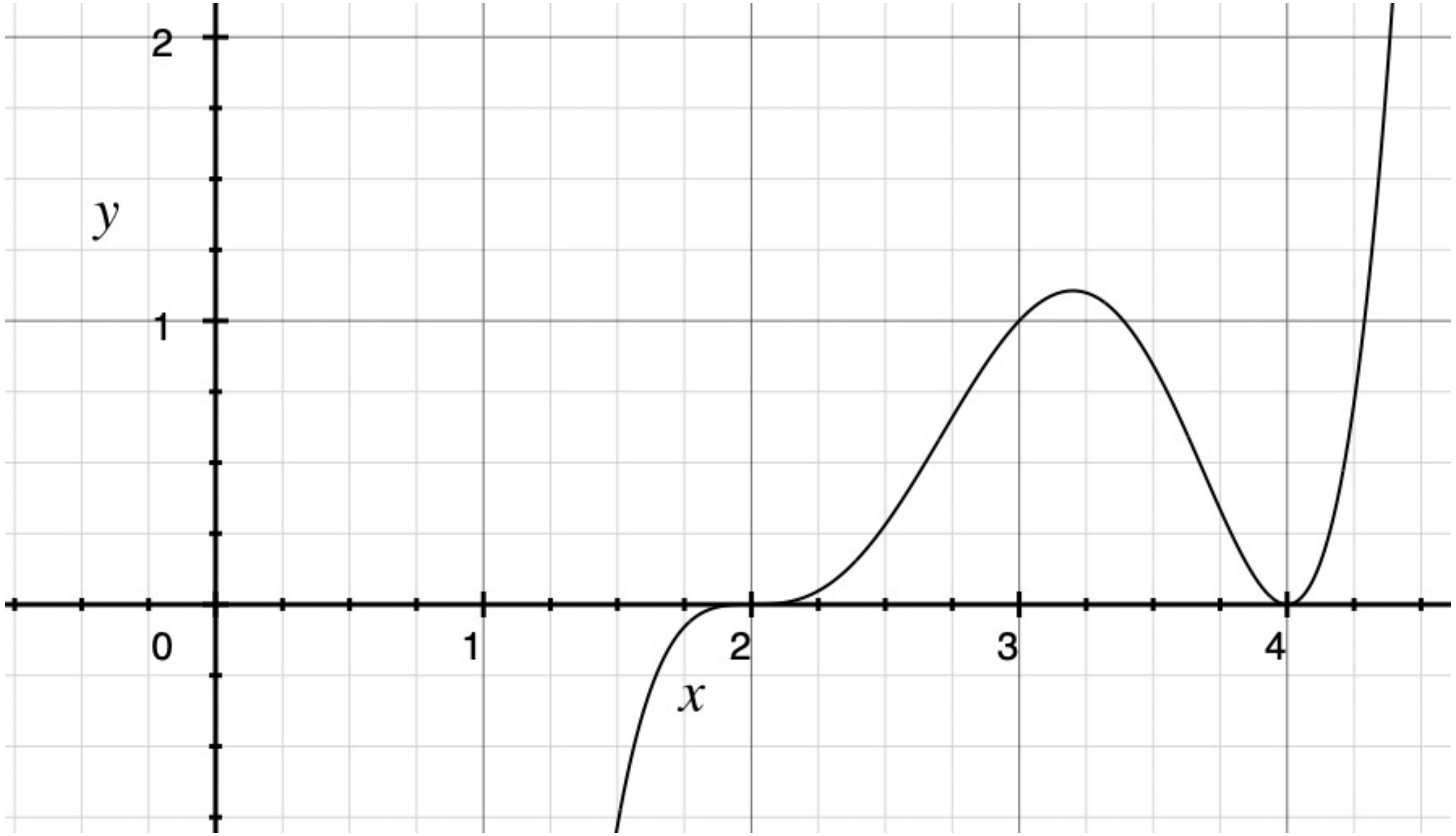
b. Repeat part (a) with the secant method.

c. If you begin with the interval [1,5], which root will you get with

(1) bisection, (2) the secant method, (3) false position?

d. Use Newton's method with x_0 = 3. Does it converge? To which root?

Solution



Solution

This polynomial obviously has roots at x = 2 and at x = 4; one is a double root, the other a triple root:

$f(x) = (x-2)^3(x-4)^2$

$= x^5 - 14x^4 + 76x^3 - 200x^2 + 256x - 128$

a.

b.

c. If you begin with the interval [1,5],

(3) False Position: Converge to 2

d.

Examples

Eg 1. *Find the real root of the equation* $x^3 - 4x - 9 = 0$ *by* False position method *correct*.

Take $a = 2$, $b = 3$, $\epsilon = 0.01$

Eg 2. *Find the real root of the equation* $x^3 - x - 1$ *by* False position method *correct to three decimal places*. $\epsilon = 0.001$

Take $a = 1$, $b = 1.5$

Solution 1

Eg 1. *Find the real root of the equation* $x^3 - 4x - 9 = 0$ *by* False position method *correct. Take* $a = 2$, $b = 3$, $\epsilon = 0.01$

$f(x) = x^3 - 4x - 9$

x	0	1	2	3
f(x)	-9	-12	-9	6

$f(2) = -9$ *i.e.,* (–)ve

and

$f(3) = 6$ *i.e.,* (+)ve

Hence, the root lies between 2 and 3.

a	f(a)	b	f(b)
2	-9	3	6

Solution 1

First approximation to the root is

$$c = \frac{(2)(6) - (3)(-9)}{6 - 9}$$

c = 2.6

a	f(a)	b	f(b)	c	f(c)	$\lvert c_k - c_{k-1} \rvert$
2	-9	3	6	2.6	– 1.824	-

Now *f*(c) = – 1.824 *i.e.*, (–)ve and

f(b) = 6 *i.e.*, (+)ve

Hence, the root lies between 2. 6 and 3.

Solution 1

Second approximation to the root is

$$c = \frac{(2.6)(6) - (3)(-1.824)}{6 - (-1.824)}$$

= 2.6933

Now $f(c)$ = -0.2372 *i.e.,* (-)ve and $f(b)$ = 6 *i.e.,* (+)ve

Hence, the root lies between 2.6933 and 3

a	f(a)	b	f(b)	c	f(c)	$\lvert c_k - c_{k-1} \rvert$
2	-9	3	6	2.6	– 1.824	-
2.6	– 1.824	3	6	2.6933	-0.2372	0.0933

Solution 1

Third approximation to the root is

$$c = \frac{(2.6933)(6) - (3)(-0.2372)}{6 - (-0.2372)}$$

= 2.7049

Now *f*(c) = -0.0289 *i.e.,* (-)ve and *f*(b) = 6 *i.e.,* (+)ve

Hence, the root lies between 2.7049 and 3.

a	f(a)	b	f(b)	c	f(c)	$\lvert c_k - c_{k-1} \rvert$
2	-9	3	6	2.6	– 1.824	-
2.6	– 1.824	3	6	2.6933	-0.2372	0.0933
2.6933	-0.2372	3	6	2.7049	-0.0289	0.0116

Solution 1

Fourth approximation to the root is

$$c = \frac{(2.7049)(6)-(3)(-0.0289)}{6-(-0.0289)} = 2.7063$$

$\epsilon = 0.01$, $|c_k - c_{k-1}| = 0.0014$

Hence, the root is 2.7063, correct to two decimal places.

a	f(a)	b	f(b)	c	f(c)	$\lvert c_k - c_{k-1} \rvert$
2	-9	3	6	2.6	−1.824	-
2.6	−1.824	3	6	2.6933	-0.2372	0.0933
2.6933	-0.2372	3	6	2.7049	-0.0289	0.0116
2.7049	-0.0289	3	6	2.7063	-0.0035	0.0014

Example 2

Eg 2. *Find the real root of the equation* $x^3 - x - 1$ *by* False position method *correct to three decimal places.* ϵ = 0.001

x	0	1	2
f(x)	-1	-1	5

Take a = 1, b = 1.5

Solution 2

Eg 2. *Find the real root of the equation* $x^3 - x - 1$ *by* False position method *correct to three decimal places*. *Take a = 1, b = 1.5,*

ϵ = 0.001

x	0	1	2
f(x)	-1	-1	5

f(*x*) = $x^3 - x - 1$

f(1) = – 1 (–)ve

f(1.5) = 0.875 (+)ve

Hence, the root lies between 1 and 1.5.

a	f(a)	b	f(b)
1	– 1	1.5	0.875

Solution 2

∴ First approximation to the root is

$$c = \frac{1(0.875) - (1.5)(-1)}{0.875 - (-1)}$$

c = 1.2667

a	f(a)	b	f(b)	c	f(c)
1	– 1	1.5	0.875	1.2667	-0.2344

Now *f* (c) = *f*(1.2667) = -0.2344 (-)ve

and *f*(b) = 0.875 (+)ve

Hence, the root lies between 1.2667 and 1.5

Solution 2

Second approximation to the root is

$$c = \frac{(1.2667)(0.875) - (1.5)(-0.2344)}{0.875 - (-0.2344)}$$

= 1.316

a	f(a)	b	f(b)	c	f(c)	$\lvert c_k - c_{k-1} \rvert$
1	– 1	1.5	0.875	1.2667	-0.2344	-
1.2667	-0.2344	1.5	0.875	1.316	-0.037	0.0493

Now *f*(c) = *f*(1.316) = -0.037(-)ve and *f*(b) = 0.875 (+)ve

Hence, the root lies between 1.316 and 1.5.

Solution 2

Third approximation to the root is

$$c = \frac{(1.316)(0.875) - (1.5)(-0.037)}{0.875 - (-0.037)} = 1.3234$$

a	f(a)	b	f(b)	c	f(c)	$\lvert c_k - c_{k-1} \rvert$
1	– 1	1.5	0.875	1.2667	-0.2344	-
1.2667	-0.2344	1.5	0.875	1.316	-0.037	0.0493
1.316	-0.037	1.5	0.875	1.3234	-0.0055	0.0074

Now $f(c)$ = f(1.3234) = -0.0055 *i.e.*, (-)ve

and f(b) = 0.875 (+)ve

Hence, the root lies between 1.3234 and 1.5 .

Solution 2

Fourth approximation to the root is

$$c = \frac{(1.3234)(0.875) - (1.5)(-0.0055)}{0.875 - (-0.0055)} = 1.3245$$

a	f(a)	b	f(b)	c	f(c)	$\lvert c_k - c_{k-1} \rvert$
1	-1	1.5	0.875	1.2667	-0.2344	-
1.2667	-0.2344	1.5	0.875	1.316	-0.037	0.0493
1.316	-0.037	1.5	0.875	1.3234	-0.0055	0.0074
1.3234	-0.0055	1.5	0.875	1.3245	-0.0008	0.0011

Now *f*(c) = *f*(1.3245) = -0.0008 *i.e.*, (-)ve and

and f(b) = 0.875 (+)ve

Hence, the root lies between 1.3245 and 1.5

Solution 2

Fifth approximation to the root is

$$c = \frac{(1.3234)(0.875) - (1.5)(-0.0055)}{0.875 - (-0.0055)} = 1.3247$$

$\epsilon = 0.001$ $|c_k - c_{k-1}| = 0.0078 < \epsilon$

The approximate real root is 1.3247

a	f(a)	b	f(b)	c	f(c)	$\lvert c_k - c_{k-1} \rvert$
1	– 1	1.5	0.875	1.2667	-0.2344	-
1.2667	-0.2344	1.5	0.875	1.316	-0.037	0.0493
1.316	-0.037	1.5	0.875	1.3234	-0.0055	0.0074
1.3234	-0.0055	1.5	0.875	1.3245	-0.0008	0.0011
1.3245	-0.0008	1.5	0.875	1.3247	-0.0001	0.0002

Algorithm for the False Position method

1. Choose a and b as two guesses for the root such that f(a).f(b) < 0 Let k =1

2. Estimate the root, c, of the equation as a point between a and b as $c_k = \frac{af(b) - bf(a)}{f(b) - f(a)}$

3. Now check the following

    If f(a).f(c) < 0 , then the root lies between a and c; then b = c.

    If f(a).f(c) > 0 , then the root lies between c and b; then a = c

    If f(c) = 0; then the root is c. Stop the algorithm if this is true.

4. Find the new estimate of the root $c_k = \frac{af(b) - bf(a)}{f(b) - f(a)}$

5. Compare the absolute approximate error $|c_k - c_{k-1}|$ or absolute relative approximate error $\frac{|c_k - c_{k-1}|}{|c_k|}$ with the pre-specified relative error tolerance ϵ, where c_k = estimated root from present iteration, c_{k-1} = estimated root from previous iteration

If $|c_k - c_{k-1}| < \epsilon$ or $\frac{|c_k - c_{k-1}|}{|c_k|} < \epsilon$ then exit

Else

k = k + 1

go to Step 3

Note one should also check whether the number of iterations is more than the maximum number of iterations allowed.

Example 4

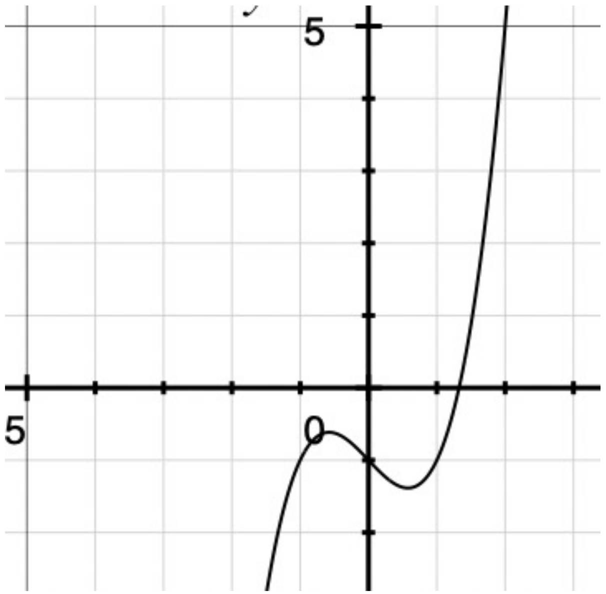
Eg 4. *Find the real root of the equation* $x^3 - x - 1$ *by Bisection method correct to two decimal places*.

$\epsilon = 0.01$, *Take a = 1.25, b = 1.5*

x	0	1	2
f(x)	-1	-1	5

Solution 4

Eg 4. *Find the real root of the equation* $x^3 - x - 1$ *by Bisection method correct to two decimal places. Take a = 1.25, b = 1.5,*



Solution 4

Eg 4. *Find the real root of the equation* $x^3 - x - 1$ *by Bisection method correct to two decimal places. Take a = 1.25, b = 1.5,*

ϵ = 0.01

f(*x*) = $x^3 - x - 1$

f(1.25) = – 0.2969 (–)ve

f(1.5) = 0.875 (+)ve

Hence, the root lies between 1.25 and 1.5.

a	f(a)	b	f(b)
1.25	– 0.2969	1.5	0.875

Solution 4

∴ First approximation to the root is

$$c = \frac{(1.25 + 1.5)}{2}$$

c = 1.375

a	f(a)	b	f(b)	c = (a+b)/2	f(c)
1.25	– 0.2969	1.5	0.875	1.375	0.2246

Now $f(c) = f(1.375)$ = 0.2246 (+)ve

and f(a) = – 0.2969 (–)ve

Hence, the root lies between 1.25 and 1.375

Solution 4

Second approximation to the root is

$$c = \frac{(1.25 + 1.375)}{2}$$

= 1.3125

a	f(a)	b	f(b)	c = (a+b)/2	f(c)	$\lvert c_k - c_{k-1} \rvert$
1.25	– 0.2969	1.5	0.875	1.375	0.2246	-
1.25	– 0.2969	1.375	0.2246	1.3125	-0.0515	0.0625

Now $f(c)$ = f(1.3125) = -0.0515 (-)ve and f(b) = 0.2246 (+)ve

Hence, the root lies between 1.3125 and 1.375.

Solution 4

Third approximation to the root is

$c = \frac{(1.3125+1.375)}{2}$ = 1.3438

a	f(a)	b	f(b)	c = (a+b)/2	f(c)	$\lvert c_k - c_{k-1} \rvert$
1.25	– 0.2969	1.5	0.875	1.375	0.2246	-
1.25	– 0.2969	1.375	0.2246	1.3125	-0.0515	0.0625
1.3125	-0.0515	1.375	0.2246	1.3438	0.0826	0.0313

Now $f(c)$ = f(1.3438) = 0.0826 *i.e.,* (+)ve

and f(a) = -0.0515 (–)ve

Hence, the root lies between 1.3125 and 1.3438 .

Solution 4

Fourth approximation to the root is

$$c = \frac{(1.3125+1.3438)}{2} = 1.3281$$

a	f(a)	b	f(b)	c = (a+b)/2	f(c)	$\lvert c_k - c_{k-1} \rvert$
1.25	– 0.2969	1.5	0.875	1.375	0.2246	-
1.25	– 0.2969	1.375	0.2246	1.3125	-0.0515	0.0625
1.3125	-0.0515	1.375	0.2246	1.3438	0.0826	0.0313
1.3125	-0.0515	1.3438	0.0826	1.3281	0.0146	0.0157

Now $f(c)$ = f(1.3281) = 0.0146 *i.e.,* (+)ve and

$f(b)$ = f(1.3125) = -0.0515 *i.e.,* (-)ve

Hence, the root lies between 1.3125 and 1.3281

Solution 4

Fifth approximation to the root is

c = $\frac{(1.3125+1.3281)}{2}$ = 1.3203

ϵ = 0.01 $|c_k - c_{k-1}|$ = 0.0078 < ϵ

The approximate real root is 1.3203

a	f(a)	b	f(b)	c = (a+b)/2	f(c)	$|c_k - c_{k-1}|$
1.25	– 0.2969	1.5	0.875	1.375	0.2246	-
1.25	– 0.2969	1.375	0.2246	1.3125	-0.0515	0.0625
1.3125	-0.0515	1.375	0.2246	1.3438	0.0826	0.0313
1.3125	-0.0515	1.3438	0.0826	1.3281	0.0146	0.0157
1.3125	-0.0515	1.3281	0.0146	1.3203		0.0078

Solution 5

- f(x) = 3x - e^x

a	$f(a)$	b	$f(b)$	$c = \frac{a+b}{2}$	$f(c)$
0	-1	1	0.2817	0.5	-0.1487
0.5	-0.1487	1	0.2817	0.75	0.133
0.5	-0.1487	0.75	0.133	0.625	0.0068
0.5	-0.1487	0.625	0.0068	0.5625	-0.0676
0.5625	-0.0676	0.625	0.0068	0.5938	-0.0295
0.5938	-0.0295	0.625	0.0068	0.6094	-0.0112
0.6094	-0.0112	0.625	0.0068	0.6172	-0.0021
0.6172	-0.0021	0.625	0.0068	0.6211	0.0023
0.6172	-0.0021	0.6211	0.0023	0.6191	0.0001

Algorithm for the bisection method

1. Choose a and b as two guesses for the root such that f(a).f(b) < 0 Let k =1

2. Estimate the root, c, of the equation as the mid-point between a and b as $c_k = \frac{a+b}{2}$

3. Now check the following

    If f(a).f(c) < 0 , then the root lies between a and c; then b = c.

    If f(a).f(c) > 0 , then the root lies between c and b; then a = c

    If f(c) = 0; then the root is c. Stop the algorithm if this is true.

4. Find the new estimate of the root $c_k = \frac{a+b}{2}$

5. Compare the absolute approximate error $|c_k - c_{k-1}|$ or absolute relative approximate error $\frac{|c_k - c_{k-1}|}{|c_k|}$ with the pre-specified relative error tolerance ϵ, where c_k = estimated root from present iteration, c_{k-1} = estimated root from previous iteration

If $|c_k - c_{k-1}| < \epsilon$ or $\frac{|c_k - c_{k-1}|}{|c_k|} < \epsilon$ then exit

Else

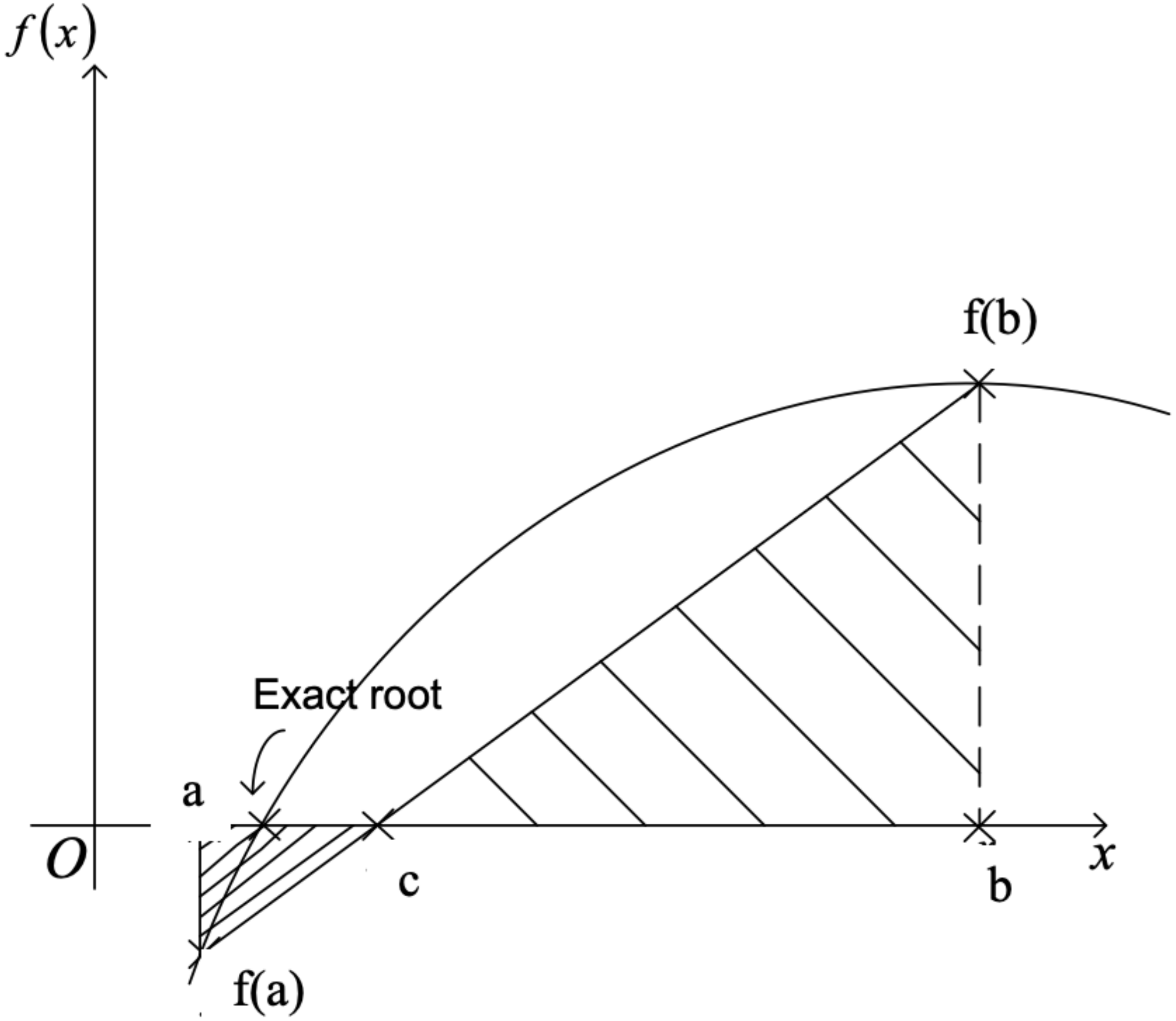
k = k + 1

go to Step 3

Note one should also check whether the number of iterations is more than the maximum number of iterations allowed.

False Position

False position method



False position method

- In the bisection method, we identify proper values of a (lower bound value) and b (upper bound value) for the current bracket.
- However, in the example shown in Figure , the bisection method may not be efficient because it does not take into consideration that f(a) is much closer to the zero of the function f(x) as compared to f(b). In other words, the next predicted root c would be closer to a, than the mid-point between a and b. The false-position method takes advantage of this observation mathematically by drawing a straight line from the function value at a to the function value at b, and estimates the root as where it crosses the *x*-axis.

False position method

So, in regular falsi method, to take into consideration the function values at a and b, straight line is drawn, joining (a,f(a))and(b,f(b)).

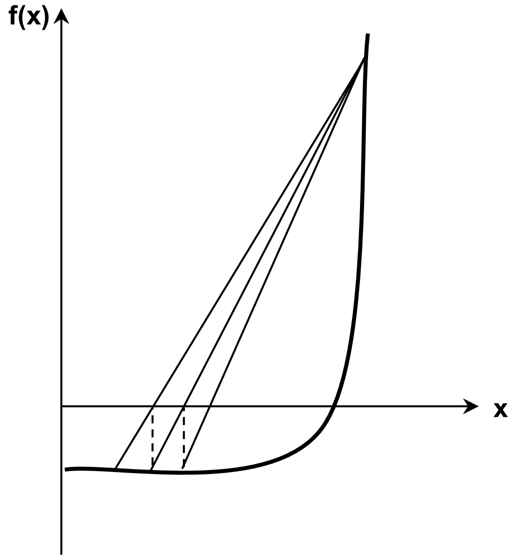
The point, where it cuts X axis is, the new estimate of the root. Point of intersection of straight line with X – axis being on X-axis is (c , 0).

$$c = \frac{af(b) - bf(a)}{f(b) - f(a)}$$

Advantages and Disadvantages

Disadvantages:

1. Not self starting. One needs two initial guesses a and b such that $f(a) \cdot f(b) < 0$.
2. Though, faster than Bisection Method, still regarded as slow.
3. In rare cases, it may become slower than Bisection Method.



Slow convergence of the false-position method for $f(x) = x^{10} - 1 = 0$

Advantages and Disadvantages

Advantages:

1. It is faster than Bisection Method.
2. It is also simple.
3. It guarantees convergence.
4. Only one function evaluation per iteration is required.

Example

This polynomial obviously has roots at x = 2 and at x = 4; one is a double root, the other a triple root:

$f(x) = (x-2)^3(x-4)^2$
$= x^5 - 14x^4 + 76x^3 - 200x^2 + 256x - 128$

a. Which root can you get with bisection? Which root can't you get?

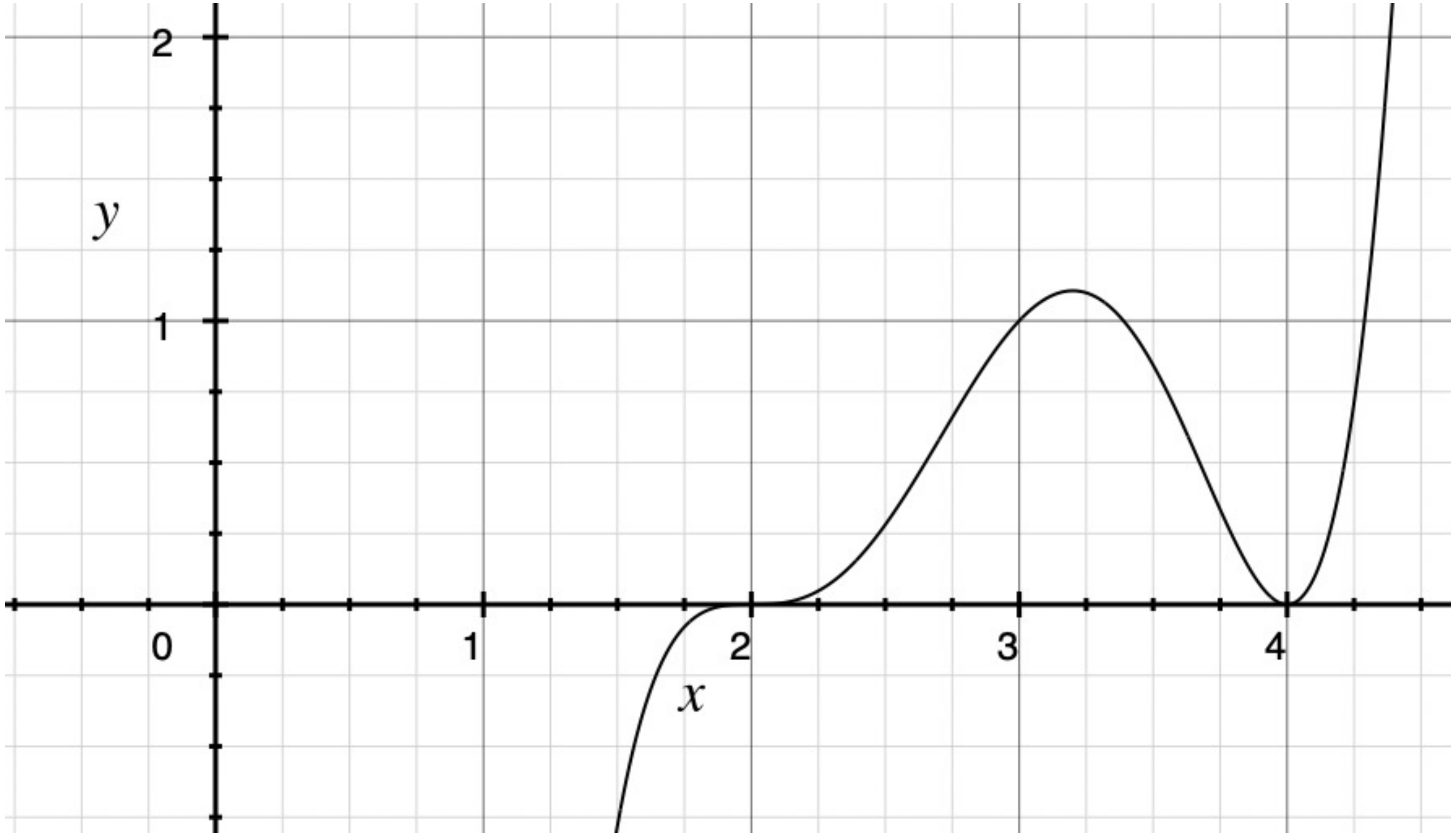
b. Repeat part (a) with the secant method.

c. If you begin with the interval [1,5], which root will you get with

(1) bisection, (2) the secant method, (3) false position?

d. Use Newton's method with x_0 = 3. Does it converge? To which root?

Solution



Solution

This polynomial obviously has roots at x = 2 and at x = 4; one is a double root, the other a triple root:

$$f(x) = (x-2)^3(x-4)^2$$
$$= x^5 - 14x^4 + 76x^3 - 200x^2 + 256x - 128$$

a.

b.

c. If you begin with the interval [1,5],

(3) False Position: Converge to 2

d.

Examples

Eg 1. *Find the real root of the equation* $x^3 - 4x - 9 = 0$ *by* False position method *correct*.

Take a = 2, b = 3, $\epsilon = 0.01$

Eg 2. *Find the real root of the equation* $x^3 - x - 1$ *by* False position method *correct to three decimal places.* $\epsilon = 0.001$

Take a = 1, b = 1.5

Solution 1

Eg 1. *Find the real root of the equation* $x^3 - 4x - 9 = 0$ *by* False position method *correct. Take* $a = 2$, $b = 3$, $\epsilon = 0.01$

$f(x) = x^3 - 4x - 9$

x	0	1	2	3
f(x)	-9	-12	-9	6

$f(2) = -9$ *i.e.*, (–)ve

and

$f(3) = 6$ *i.e.*, (+)ve

Hence, the root lies between 2 and 3.

a	f(a)	b	f(b)
2	-9	3	6

Solution 1

First approximation to the root is

$$c = \frac{(2)(6) - (3)(-9)}{6 - 9}$$

c = 2.6

a	f(a)	b	f(b)	c	f(c)	$\lvert c_k - c_{k-1} \rvert$
2	-9	3	6	2.6	– 1.824	-

Now $f(c)$ = – 1.824 *i.e.*, (–)ve and

f(b) = 6 *i.e.*, (+)ve

Hence, the root lies between 2. 6 and 3.

Solution 1

Second approximation to the root is

$$c = \frac{(2.6)(6) - (3)(-1.824)}{6 - (-1.824)}$$

= 2.6933

Now *f*(c) = -0.2372 *i.e.*, (-)ve and *f*(b) = 6 *i.e.*, (+)ve

Hence, the root lies between 2.6933 and 3

a	f(a)	b	f(b)	c	f(c)	$\lvert c_k - c_{k-1} \rvert$
2	-9	3	6	2.6	– 1.824	-
2.6	– 1.824	3	6	2.6933	-0.2372	0.0933

Solution 1

Third approximation to the root is

$$c = \frac{(2.6933)(6) - (3)(-0.2372)}{6 - (-0.2372)}$$

= 2.7049

Now $f(c)$ = -0.0289 *i.e.,* (-)ve and $f(b)$ = 6 *i.e.,* (+)ve

Hence, the root lies between 2.7049 and 3.

a	f(a)	b	f(b)	c	f(c)	$\lvert c_k - c_{k-1} \rvert$
2	-9	3	6	2.6	– 1.824	-
2.6	– 1.824	3	6	2.6933	-0.2372	0.0933
2.6933	-0.2372	3	6	2.7049	-0.0289	0.0116

Solution 1

Fourth approximation to the root is

$$c = \frac{(2.7049)(6)-(3)(-0.0289)}{6-(-0.0289)} = 2.7063$$

$\epsilon = 0.01$, $|c_k - c_{k-1}| = 0.0014$

Hence, the root is 2.7063, correct to two decimal places.

a	f(a)	b	f(b)	c	f(c)	$\lvert c_k - c_{k-1} \rvert$
2	-9	3	6	2.6	– 1.824	-
2.6	– 1.824	3	6	2.6933	-0.2372	0.0933
2.6933	-0.2372	3	6	2.7049	-0.0289	0.0116
2.7049	-0.0289	3	6	2.7063	-0.0035	0.0014

Example 2

Eg 2. *Find the real root of the equation* $x^3 - x - 1$ *by* False position method *correct to three decimal places.* ϵ = 0.001

x	0	1	2
f(x)	-1	-1	5

Take a = 1, b = 1.5

Solution 2

Eg 2. *Find the real root of the equation* $x^3 - x - 1$ *by* False position method *correct to three decimal places*. *Take a = 1, b = 1.5,*

$\epsilon = 0.001$

x	0	1	2
f(x)	-1	-1	5

f(*x*) = $x^3 - x - 1$

f(1) = – 1 (–)ve

f(1.5) = 0.875 (+)ve

Hence, the root lies between 1 and 1.5.

a	f(a)	b	f(b)
1	– 1	1.5	0.875

Solution 2

∴ First approximation to the root is

$$c = \frac{1(0.875) - (1.5)(-1)}{0.875 - (-1)}$$

c = 1.2667

a	f(a)	b	f(b)	c	f(c)
1	– 1	1.5	0.875	1.2667	-0.2344

Now f (c) = f(1.2667) = -0.2344 (-)ve

and f(b) = 0.875 (+)ve

Hence, the root lies between 1.2667 and 1.5

Solution 2

Second approximation to the root is

$$c = \frac{(1.2667)(0.875) - (1.5)(-0.2344)}{0.875 - (-0.2344)}$$

= 1.316

a	f(a)	b	f(b)	c	f(c)	$\lvert c_k - c_{k-1} \rvert$
1	– 1	1.5	0.875	1.2667	-0.2344	-
1.2667	-0.2344	1.5	0.875	1.316	-0.037	0.0493

Now *f*(c) = *f*(1.316) = -0.037(-)ve and *f*(b) = 0.875 (+)ve

Hence, the root lies between 1.316 and 1.5.

Solution 2

Third approximation to the root is

$$c = \frac{(1.316)(0.875) - (1.5)(-0.037)}{0.875 - (-0.037)} = 1.3234$$

a	f(a)	b	f(b)	c	f(c)	$\|c_k - c_{k-1}\|$
1	– 1	1.5	0.875	1.2667	-0.2344	-
1.2667	-0.2344	1.5	0.875	1.316	-0.037	0.0493
1.316	-0.037	1.5	0.875	1.3234	-0.0055	0.0074

Now *f*(c) = *f*(1.3234) = -0.0055 *i.e.,* (-)ve

and f(b) = 0.875 (+)ve

Hence, the root lies between 1.3234 and 1.5 .

Solution 2

Fourth approximation to the root is

$$c = \frac{(1.3234)(0.875) - (1.5)(-0.0055)}{0.875 - (-0.0055)} = 1.3245$$

a	f(a)	b	f(b)	c	f(c)	$\lvert c_k - c_{k-1} \rvert$
1	– 1	1.5	0.875	1.2667	-0.2344	-
1.2667	-0.2344	1.5	0.875	1.316	-0.037	0.0493
1.316	-0.037	1.5	0.875	1.3234	-0.0055	0.0074
1.3234	-0.0055	1.5	0.875	1.3245	-0.0008	0.0011

Now $f(c)$ = f(1.3245) = -0.0008 *i.e.*, (-)ve and

and f(b) = 0.875 (+)ve

Hence, the root lies between 1.3245 and 1.5

Solution 2

Fifth approximation to the root is

$$c = \frac{(1.3234)(0.875) - (1.5)(-0.0055)}{0.875 - (-0.0055)} = 1.3247$$

$\epsilon = 0.001$ $|c_k - c_{k-1}| = 0.0078 < \epsilon$

The approximate real root is 1.3247

a	f(a)	b	f(b)	c	f(c)	$\lvert c_k - c_{k-1} \rvert$
1	– 1	1.5	0.875	1.2667	-0.2344	-
1.2667	-0.2344	1.5	0.875	1.316	-0.037	0.0493
1.316	-0.037	1.5	0.875	1.3234	-0.0055	0.0074
1.3234	-0.0055	1.5	0.875	1.3245	-0.0008	0.0011
1.3245	-0.0008	1.5	0.875	1.3247	-0.0001	0.0002

Algorithm for the False Position method

1. Choose a and b as two guesses for the root such that f(a).f(b) < 0 Let k =1
2. Estimate the root, c, of the equation as a point between a and b as $c_k = \frac{af(b) - bf(a)}{f(b) - f(a)}$
3. Now check the following

   If f(a).f(c) < 0 , then the root lies between a and c; then b = c.

   If f(a).f(c) > 0 , then the root lies between c and b; then a = c

   If f(c) = 0; then the root is c. Stop the algorithm if this is true.
4. Find the new estimate of the root $c_k = \frac{af(b) - bf(a)}{f(b) - f(a)}$
5. Compare the absolute approximate error $|c_k - c_{k-1}|$ or absolute relative approximate error $\frac{|c_k - c_{k-1}|}{|c_k|}$

with the pre-specified relative error tolerance ϵ, where c_k = estimated root from present iteration, c_{k-1} = estimated root from previous iteration

If $|c_k - c_{k-1}| < \epsilon$ or $\frac{|c_k - c_{k-1}|}{|c_k|} < \epsilon$ then exit

Else

k = k + 1

go to Step 3

Note one should also check whether the number of iterations is more than the maximum number of iterations allowed.