Module 7 - Recursion

Example 1

$$T(n) = \begin{cases} 1 & \text{if } n == 0 \\ T(n-1) + 2 & \text{if } n > 0 \end{cases}$$

$T(n) = T(n-1) + 2$

$T(n-1) = T(n-2) + 2$

$T(n-2) = T(n-3) + 2$

$\vdots$

$$\begin{aligned}
T(n) &= T(n-1) + 2 = T(n-1) + 1 \times 2 && \text{step 1} \\
&= T(n-2) + 2 + 2 = T(n-2) + 2 \times 2 && \text{step 2} \\
&= T(n-3) + 2 + 2 + 2 = T(n-3) + 3 \times 2 && \text{step 3} \\
&= \cdots && \vdots \\
&= T(n-k) + k \cdot 2 && \text{step } K \\
&= \cdots && \vdots \\
&= \underbrace{T(n-n)}_{T(0)=1} + n \cdot 2 && \text{step } n
\end{aligned}$$

$T(n) = T(0) + n \cdot 2 = 1 + 2n$

$\downarrow$

$T(n) = 2n + 1 \Rightarrow T(n) = O(n)$

Example 2:

$$T(n) = \begin{cases} 1 & \text{if } n==1 \\ 2T(\frac{n}{2}) + 2n & \text{if } n>1 \end{cases}$$

$T(2^k) = 2T(2^{k-1}) + 2\cdot 2^k$

$T(2^{k-1}) = 2T(2^{k-2}) + 2\cdot 2^{k-1}$

$T(2^{k-2}) = 2T(2^{k-3}) + 2\cdot 2^{k-2}$

$\vdots$

Let $n = 2^k$, then $\frac{n}{2} = 2^{k-1}$

$T(n) = 2T(2^{k-1}) + 2\cdot 2^k = 2^1 T(2^{k-1}) + 1\cdot 2\cdot 2^k$ step 1

$= 2(2T(2^{k-2}) + 2\cdot 2^{k-1}) + 2\cdot 2^k = 2^2 T(2^{k-2}) + 2\cdot 2\cdot 2^k$ step2

$= 2^2(2T(2^{k-3}) + 2\cdot 2^{k-2}) + 2\cdot 2\cdot 2^k = 2^3 T(2^{k-3}) + 3\cdot 2\cdot 2^k$ step3

$= \cdots$

$= \underbrace{2^k}_{n}\, \underbrace{T(2^{k-k})}_{1} + \underbrace{k}_{\log n} \cdot 2 \cdot \underbrace{2^k}_{n}$ step K

if $n = 2^k$

then $k = \log n$

$T(n) = 2n\log n + n \Rightarrow T(n) = O(n\log n)$