Fall 2016 Optimization Theory and Its Applications
Project #5 (Feedback Gain Optimization)

Project Title: Feedback Gain Optimization

A system (such as an industrial plant) is characterized by $y = Gu + v$ and $y \in \mathbf{R}^n$ is the output, $u \in \mathbf{R}^n$ is the input, and $v \in \mathbf{R}^n$ is a disturbance signal. The matrix $G \in \mathbf{R}^{n\times n}$, which is known, is called the system input-output matrix. The input signal u is found using a linear feedback (control) policy: $u = Fy$ where $F \in \mathbf{R}^{n\times n}$ is the feedback (gain) matrix, which is what we need to determine. From the equations given above, we have

$$y = (I - GF)^{-1}v, \quad u = F(I - GF)^{-1}v.$$

(You can simply assume that $I - GF$ will be invertible and you can also assume G is invertible if needed.)

The disturbance v is random with $\mathbb{E}[v] = 0$ and $\mathbb{E}[vv^T] = \sigma^2 I$, σ^2 is known. The objective is to minimize $\max_{i=1,\ldots,n} \mathbb{E}[y_i^2]$ (the maximum mean square value of the output components) subject to the constraint that $\mathbb{E}[u_i^2] \leq 1$, for $i = 1, 2, \ldots, n$, i.e., each input component has a mean square value not exceeding one. The variable to be chosen is the matrix $F \in \mathbf{R}^{n\times n}$. **Formulate a convex/quasi-convex optimization to find an optimal feedback gain matrix F under the constraints given in above to fulfill the optimization goal stated in above. Properly assign the values in G and the value of σ^2 while doing your numerical simulations**.