QCD Spectroscopy

An Introduction

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OLD DOMINION UNIVERSITY

Jefferson Lab
Thomas Jefferson National Accelerator Facility

Outline

Introduction

QCD Spectroscopy, Scattering & Bound States, Case Study — *the Deuteron*

Kinematics

Energy & Momentum, Phase Space, Binary Reactions

Scattering Theory

Scattering Amplitudes, Unitarity, Phase Shifts

Spectroscopy

Bound States, Poles & Couplings, Binding Energy & Momentum

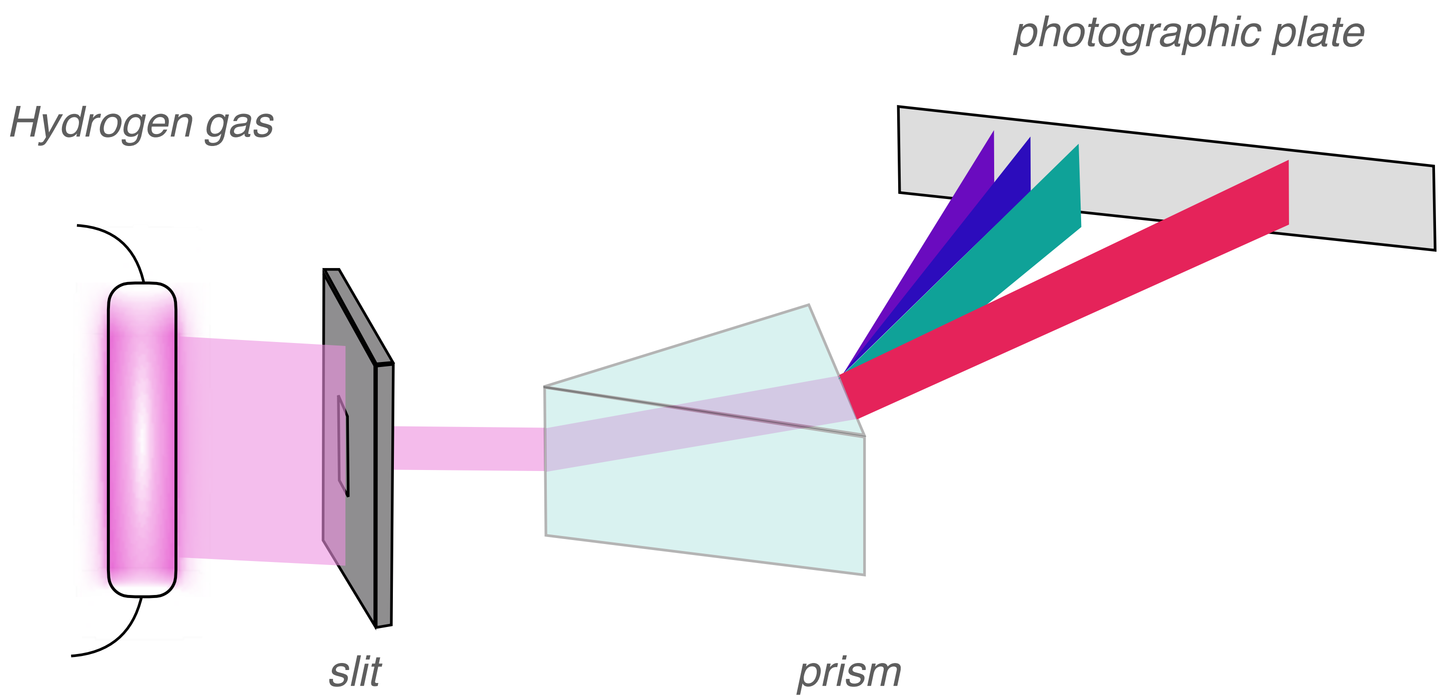
Neutron-Proton Scattering

Spin, Cross-Section, Effective Range Parameters, the Deuteron

Spectroscopy

Studying the ***spectrum*** is key to understanding physical phenomena

- e.g., the Hydrogen atom led to the discovery of Quantum Mechanics



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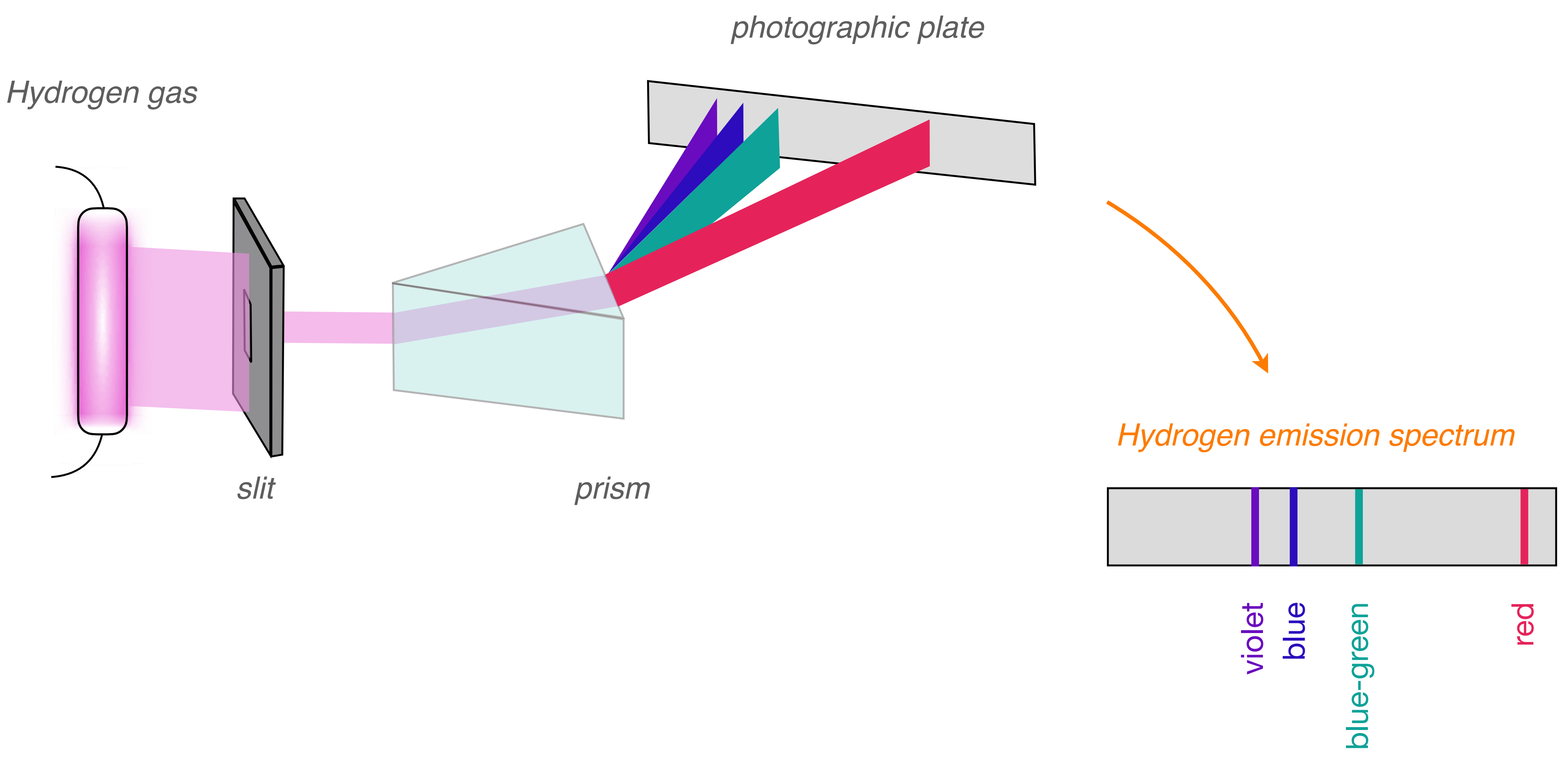

Quantized spectrum

Spectroscopy

$\hbar = c = 1$

Studying the ***spectrum*** is key to understanding physical phenomena

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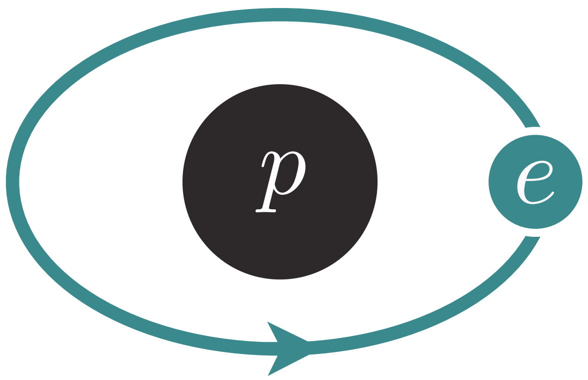
- Precision spectroscopy paved the way for modern particle physics



$$\widehat{\mathrm{H}} \left|\psi\right\rangle = E_n \left|\psi\right\rangle$$ *Schrödinger equation*

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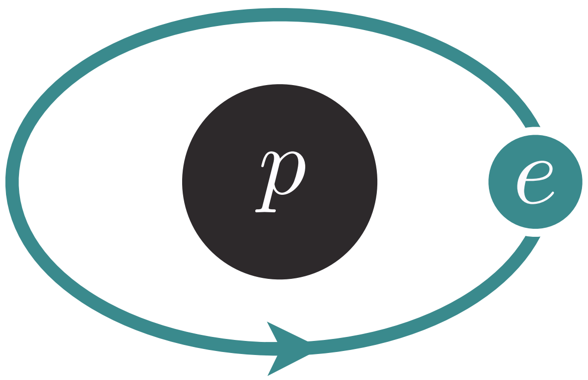
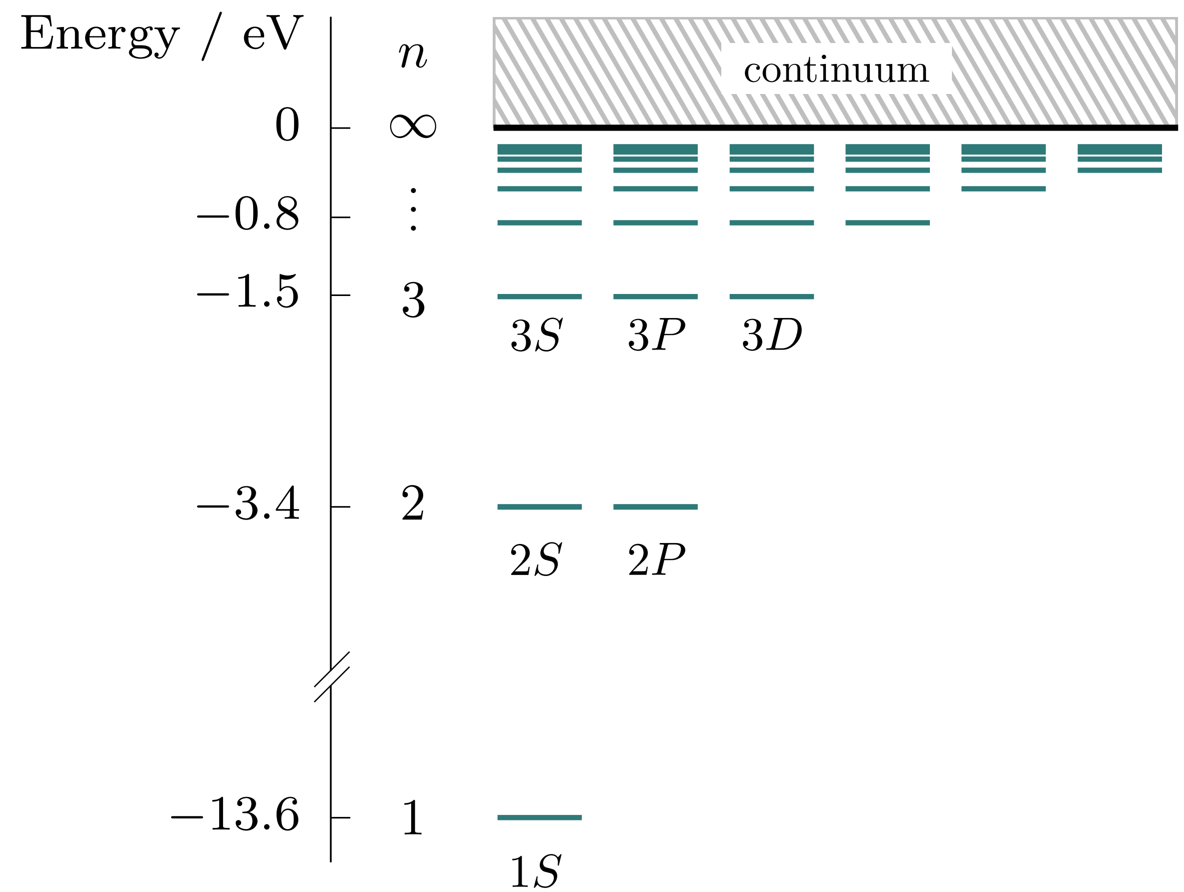


$$\widehat{\mathrm{H}}\,|\psi\rangle = E_n\,|\psi\rangle$$

Schrödinger equation

spectrum

$$E_n = -\frac{m\alpha^2}{2n}$$



$\alpha = \frac{e^2}{4\pi}$

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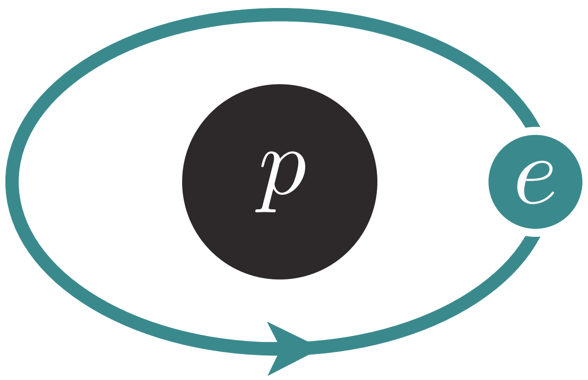
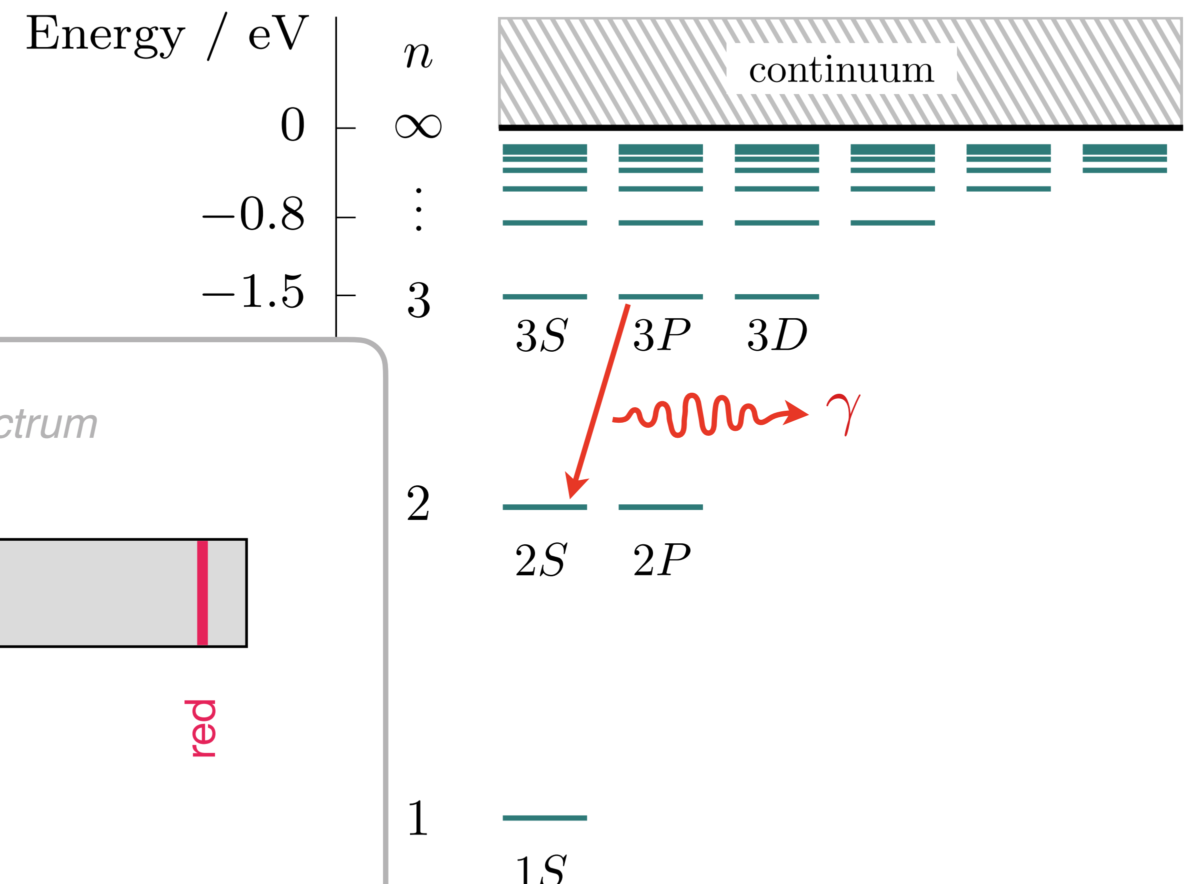
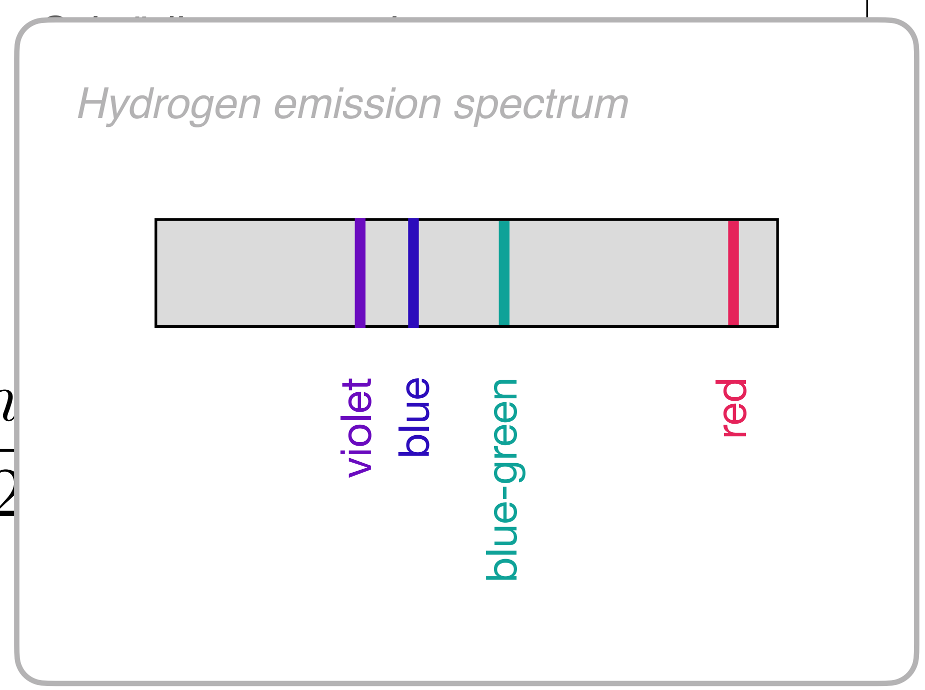




$$\widehat{\mathrm{H}}\,|\psi\rangle = E_n\,|\psi\rangle$$

spectrum

$$E_n = -\frac{m}{2}$$



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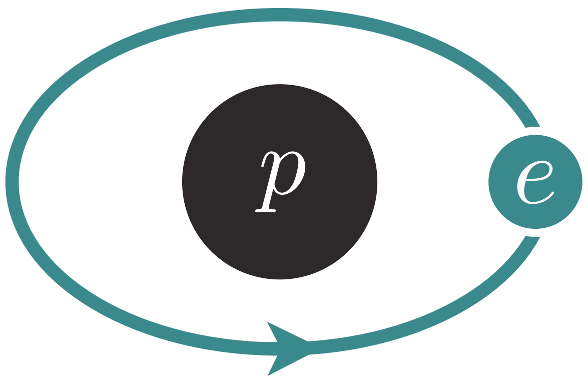


$$\widehat{\mathrm{H}}\,|\psi\rangle = E_n\,|\psi\rangle$$

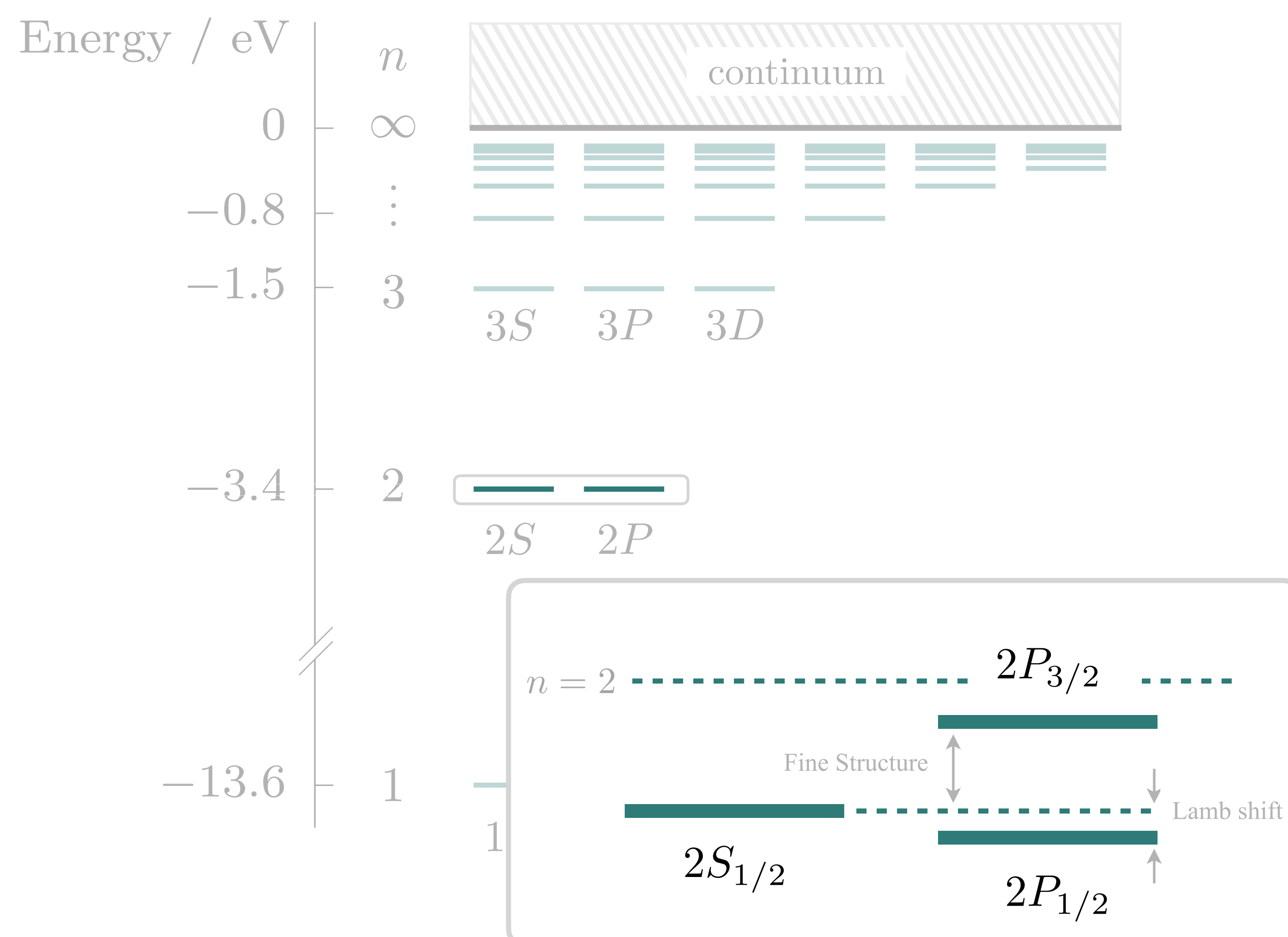
Schrödinger equation

spectrum

$$E_n = -\frac{m\alpha^2}{2n} + \mathcal{O}\left(\alpha^4\right)$$

$$\alpha = \frac{e^2}{4\pi}$$

Relativistic effects, QED, …



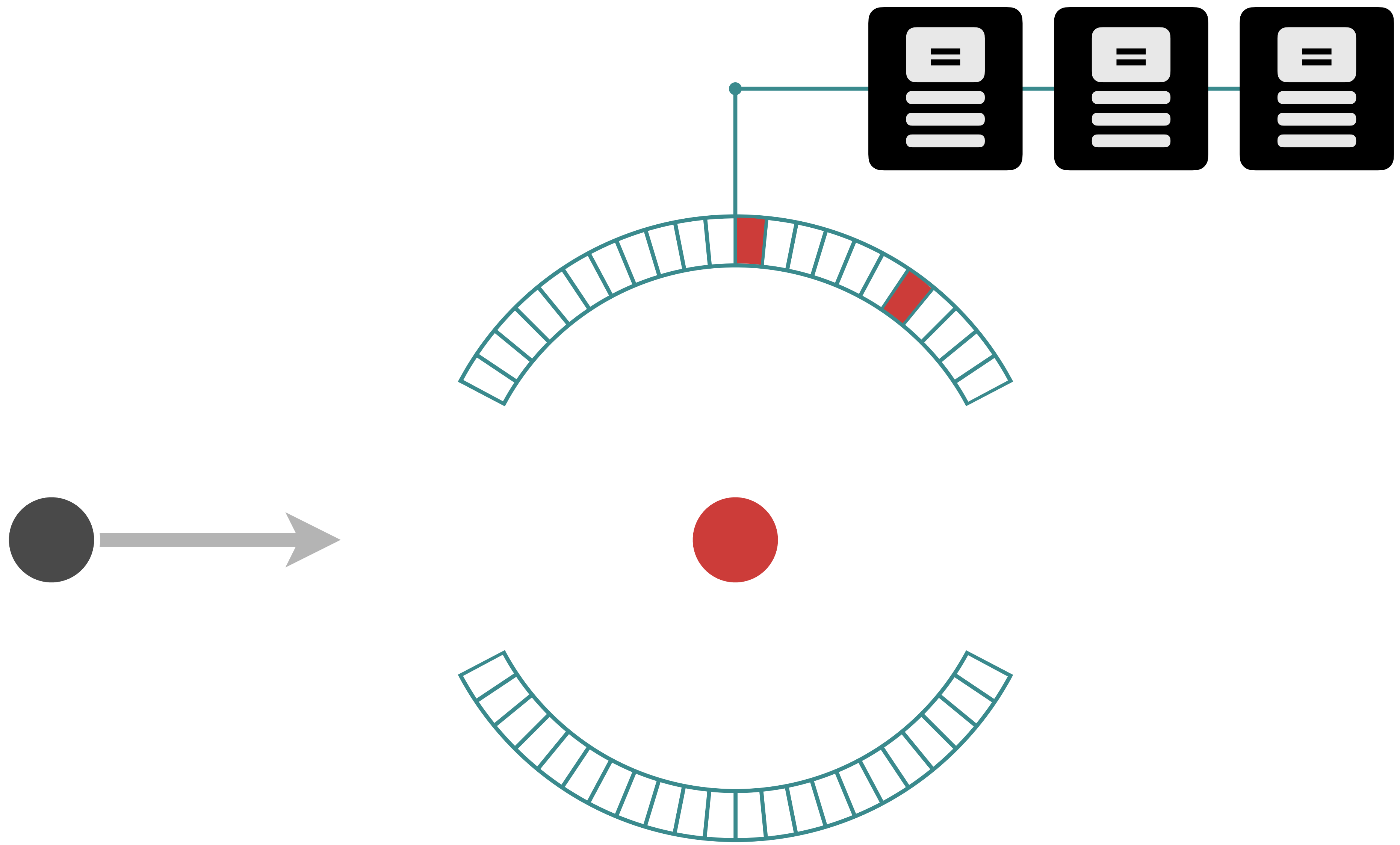
QCD Spectroscopy

The strong interaction spectrum, composed of ***hadrons***, contains many interesting features

$$N \propto \sigma \propto |\mathcal{M}|^2$$

Experimentally measured

Contains physical information



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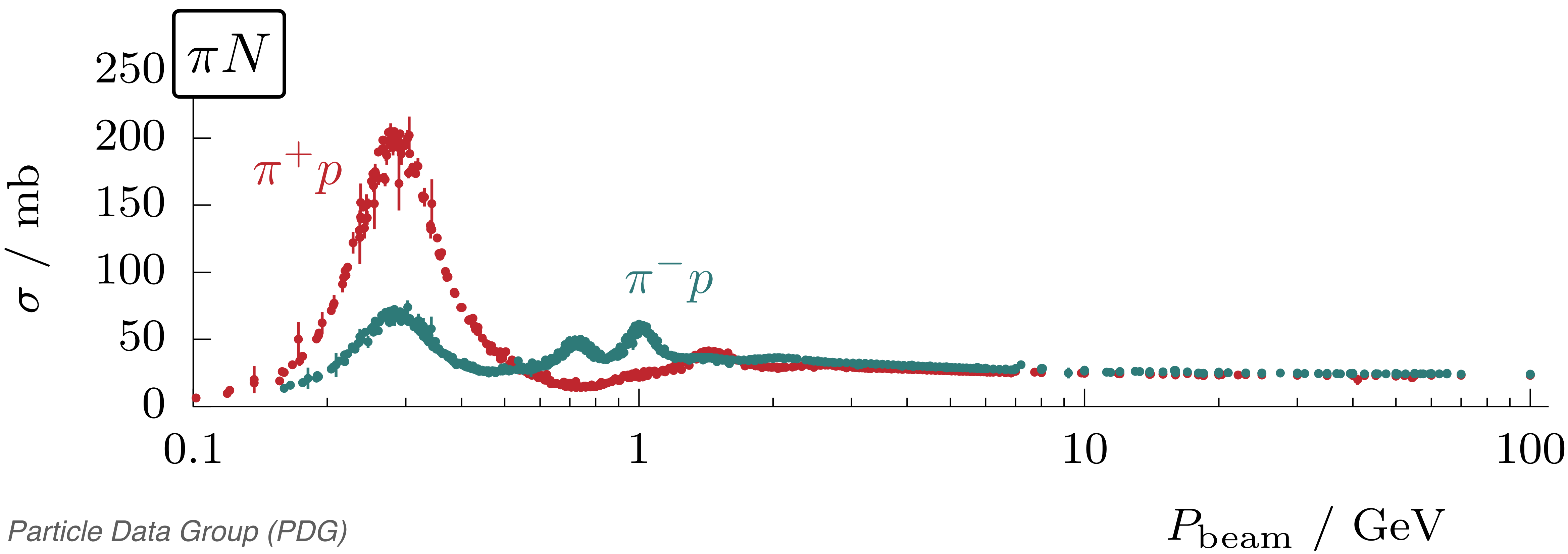
e.g. πN spectrum



Particle Data Group (PDG)
The Review of Particle Physics (RPP)

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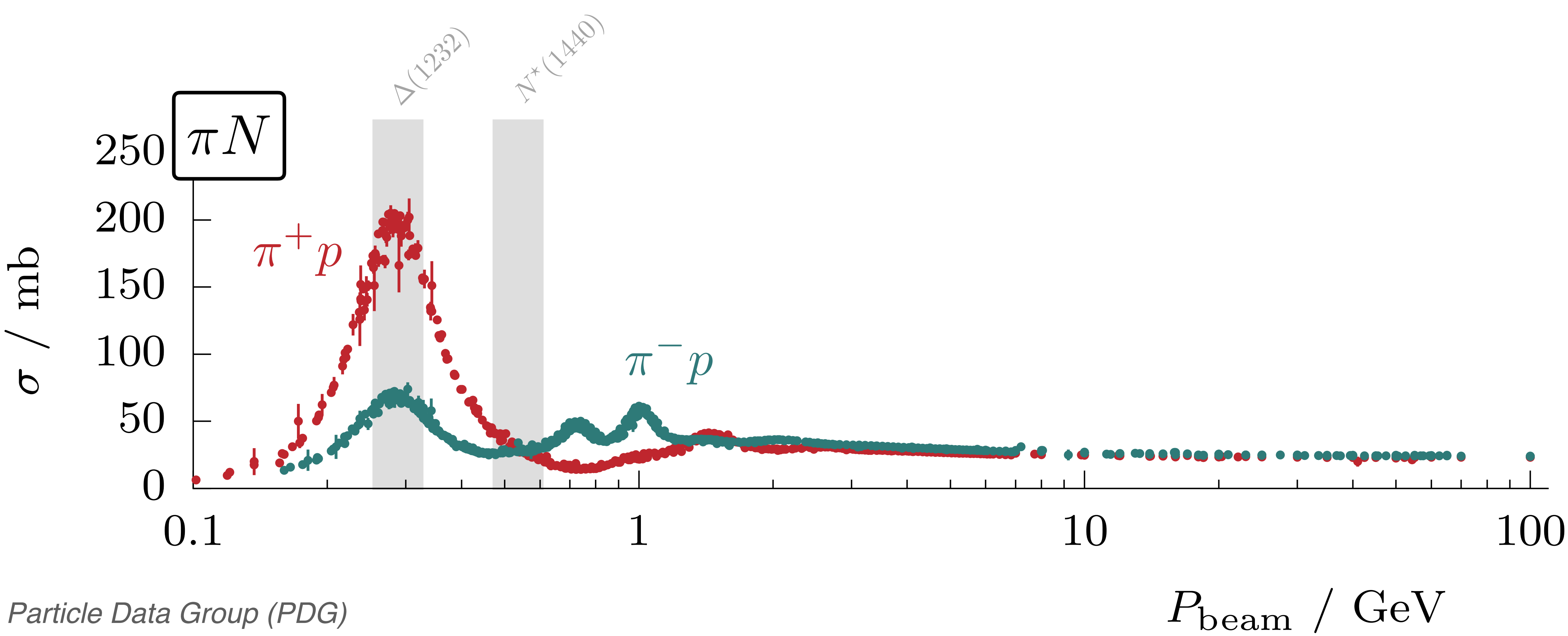

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Scattering ⟺ Bound States

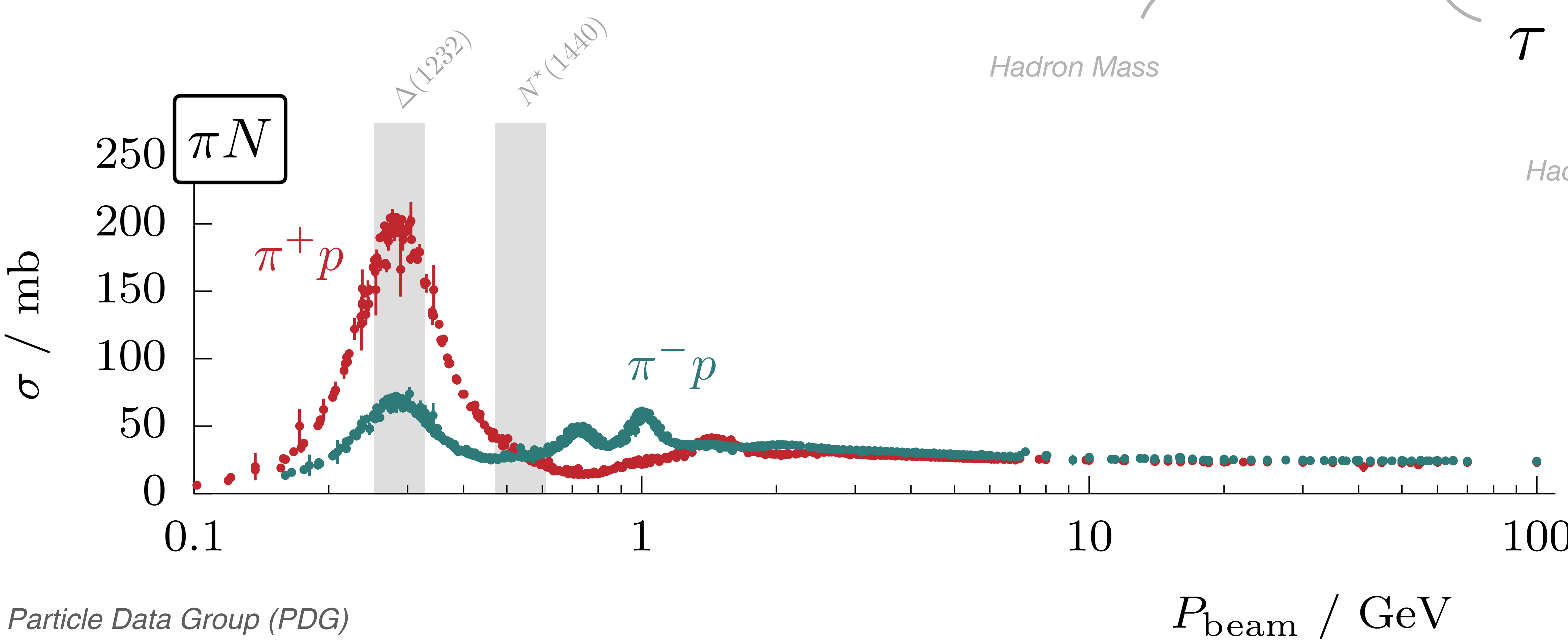
$$\mathcal{M} \propto \frac{1}{E - M - i\Gamma/2}$$

Hadron Mass

$$\tau = \frac{1}{\Gamma}$$

Hadron Lifetime

e.g. πN spectrum



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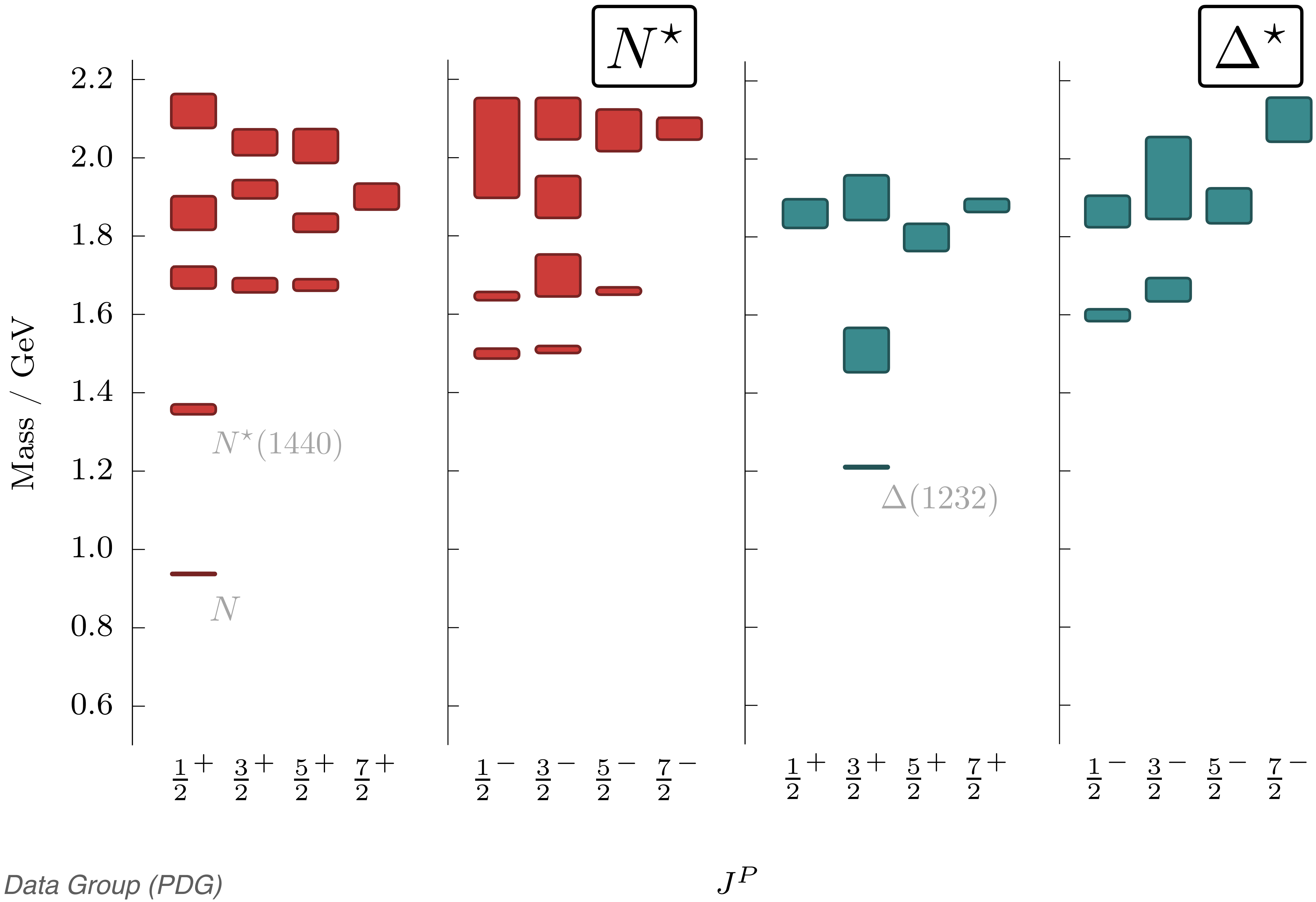

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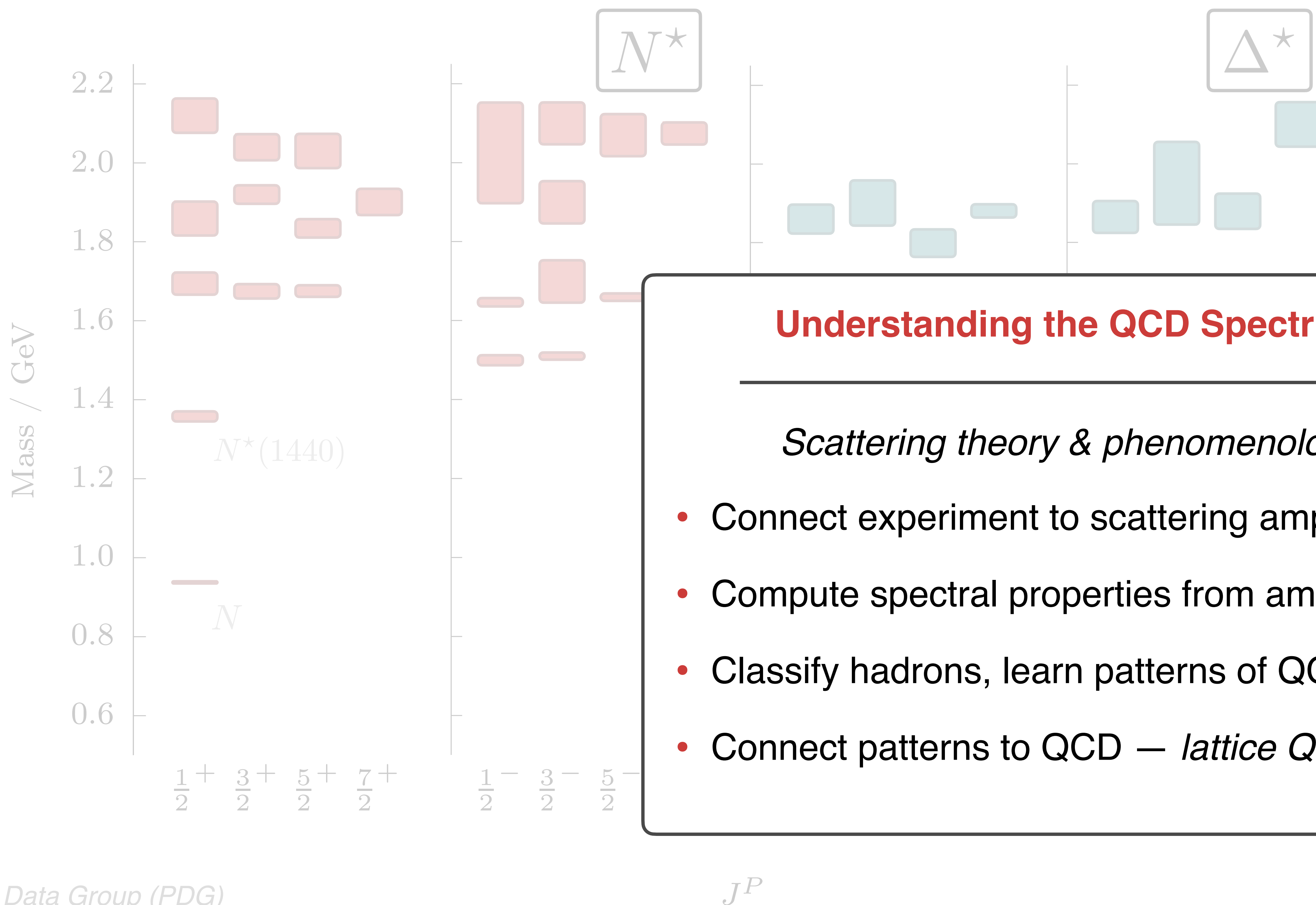

Understanding the QCD Spectrum

Scattering theory & phenomenology

- Connect experiment to scattering amplitudes
- Compute spectral properties from amplitudes
- Classify hadrons, learn patterns of QCD
- Connect patterns to QCD — *lattice QCD*

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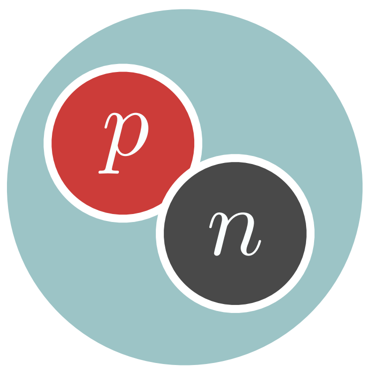
Case study — *the deuteron*

As an example of connecting scattering physics to hadron properties, consider ***the deuteron***

- Simplest nucleus, bound state of ***proton*** and ***neutron***
- “Hydrogen atom of nuclear physics” — study nature of two-body nuclear forces



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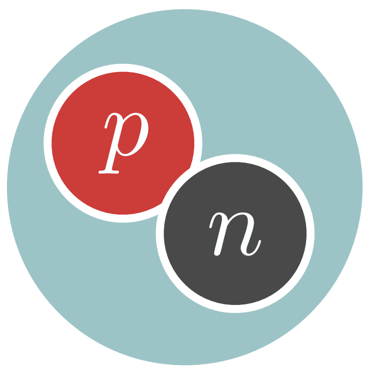

Deuteron

Spectral properties checklist

- ☐ Mass
- ☐ Lifetime
- ☐ Spin
- ☐ Parity
- ☐ Radius
- ☐ Charge

…

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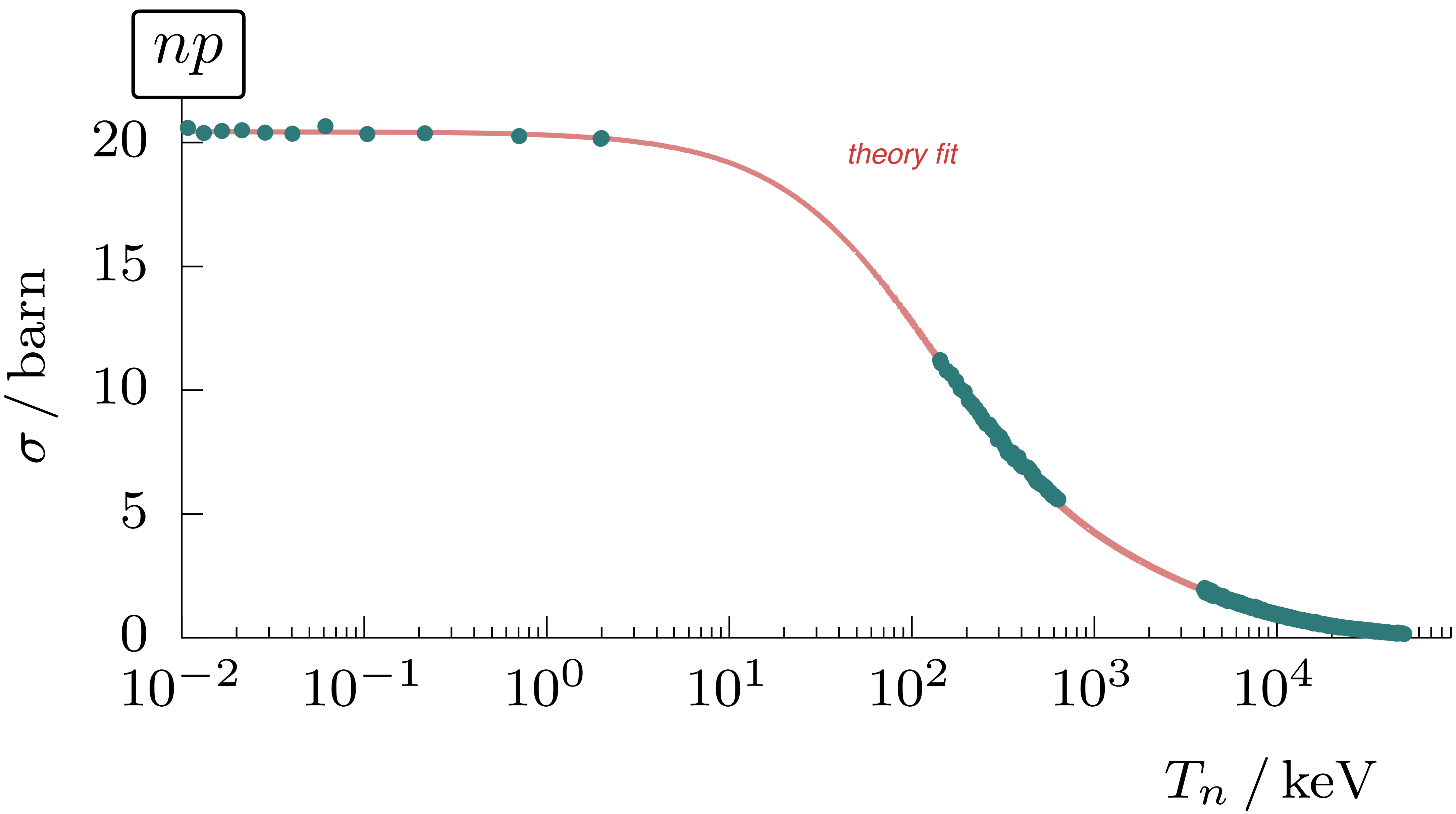
- Examine low-energy *neutron-proton elastic scattering*



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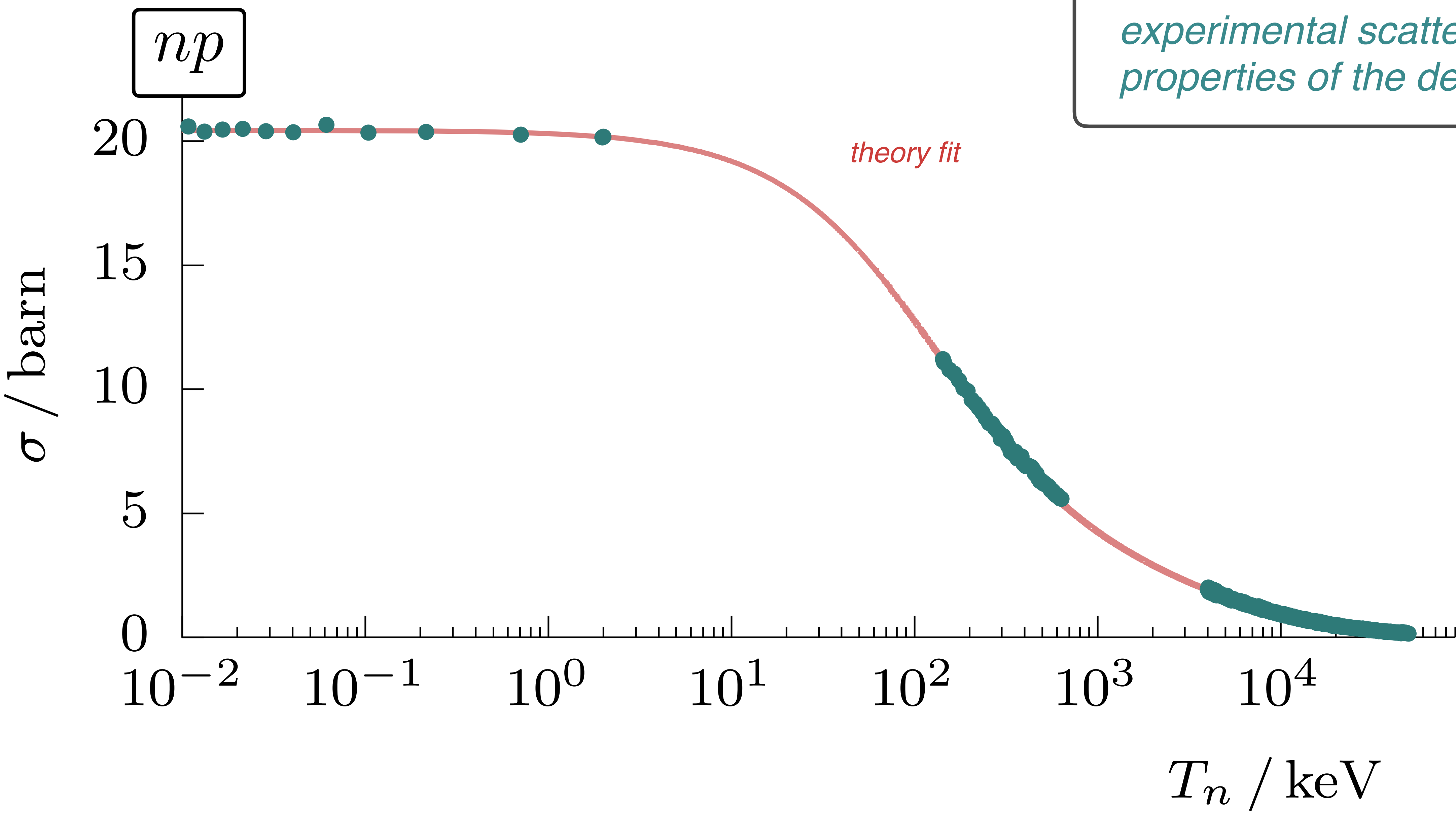

How do we convert experimental scattering data to properties of the deuteron?

Kinematics — Energy & Momentum

$\hbar = c = 1$

Let us define the kinematics of protons and neutrons

- proton and neutron masses are nearly equal — different isospin projections of Nucleon

$$N = \begin{pmatrix} p \\ n \end{pmatrix}$$

Nucleon

$$m_p \approx m_n$$

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Nucleon

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First approximation — Isospin limit

Nucleon mass

$$m_p = m_n \equiv m$$

$$\approx 940 \text{ MeV}$$

Kinematics — Energy & Momentum

Let us define the kinematics of protons and neutrons

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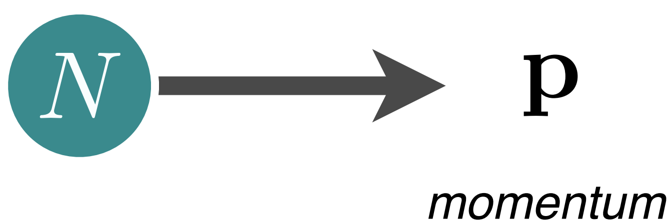
- Relativistic nucleon kinematics



energy

$$\omega_{\mathbf{p}} = \sqrt{m^2 + \mathbf{p}^2}$$

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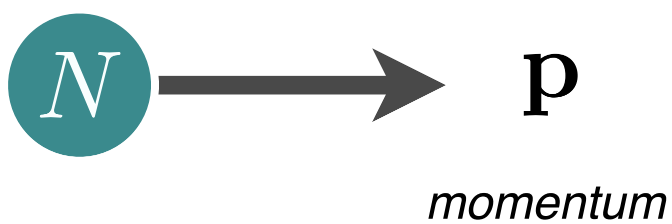

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$$\omega_{\mathbf{p}} = \sqrt{m^2 + \mathbf{p}^2}$$

$$= m + \frac{\mathbf{p}^2}{2m} + \mathcal{O}\left(\mathbf{p}^4\right)$$

non-relativistic limit, $p/m \ll 1$

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non-relativistic limit, $p/m \ll 1$

kinetic energy

$$T_{\mathbf{p}} = \omega_{\mathbf{p}} - m$$

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Exercise

Prove the non-relativistic expansion

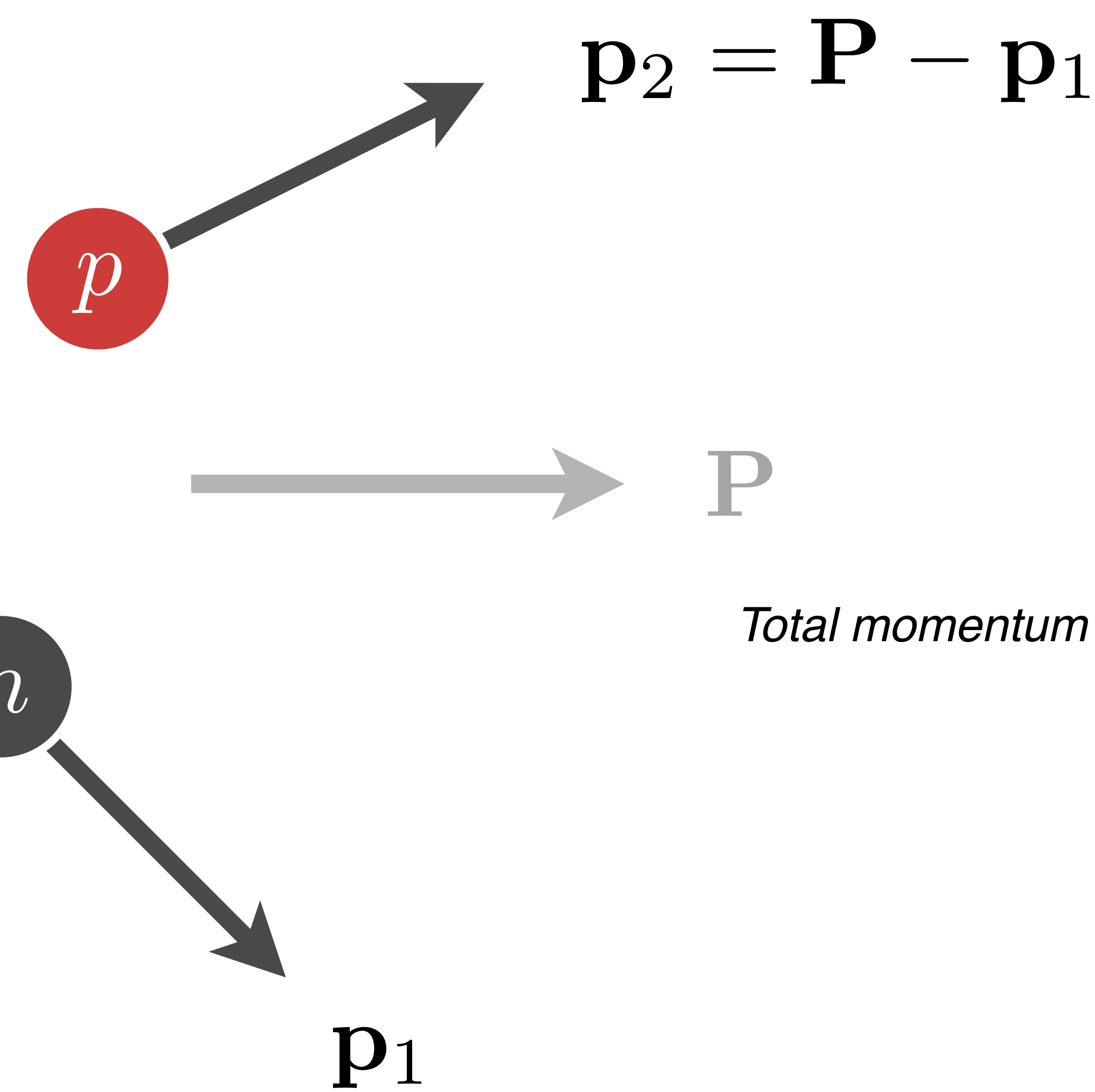
Kinematics — Energy & Momentum

Let us define the kinematics of protons and neutrons

- proton and neutron masses are nearly equal — different isospin projections of Nucleon
- Relativistic nucleon kinematics
- Two-nucleon kinematics

Total energy

$$E = \sqrt{m^2 + \mathbf{p}_1^2} + \sqrt{m^2 + \mathbf{p}_2^2}$$



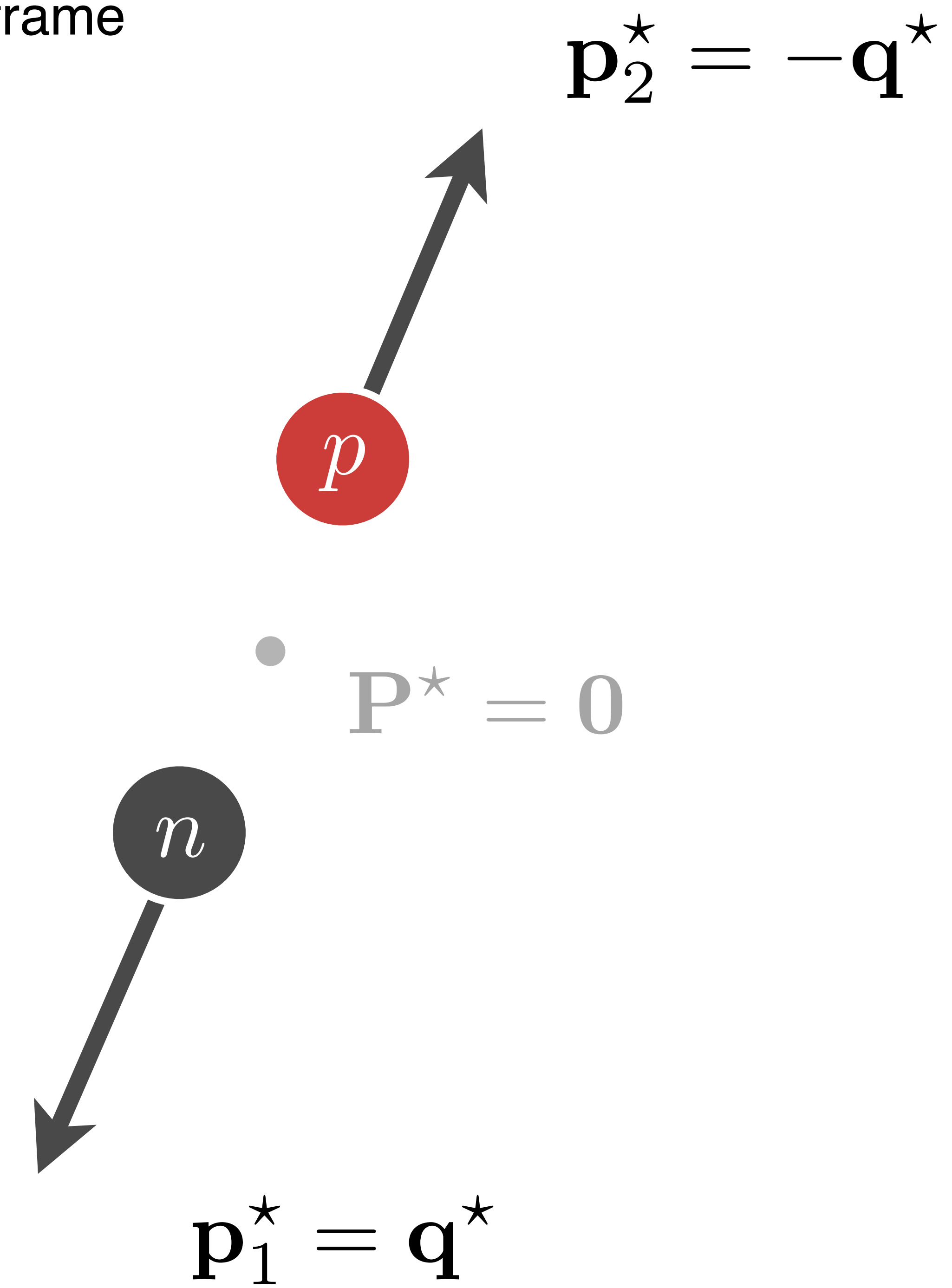
Kinematics — Energy & Momentum

Let us define the kinematics of protons and neutrons

- proton and neutron masses are nearly equal — different isospin projections of Nucleon
- Relativistic nucleon kinematics
- Two-nucleon kinematics — choose *center-of-momentum* frame

Total CM energy

$$E^\star = 2\sqrt{m^2 + q^{\star\,2}}$$



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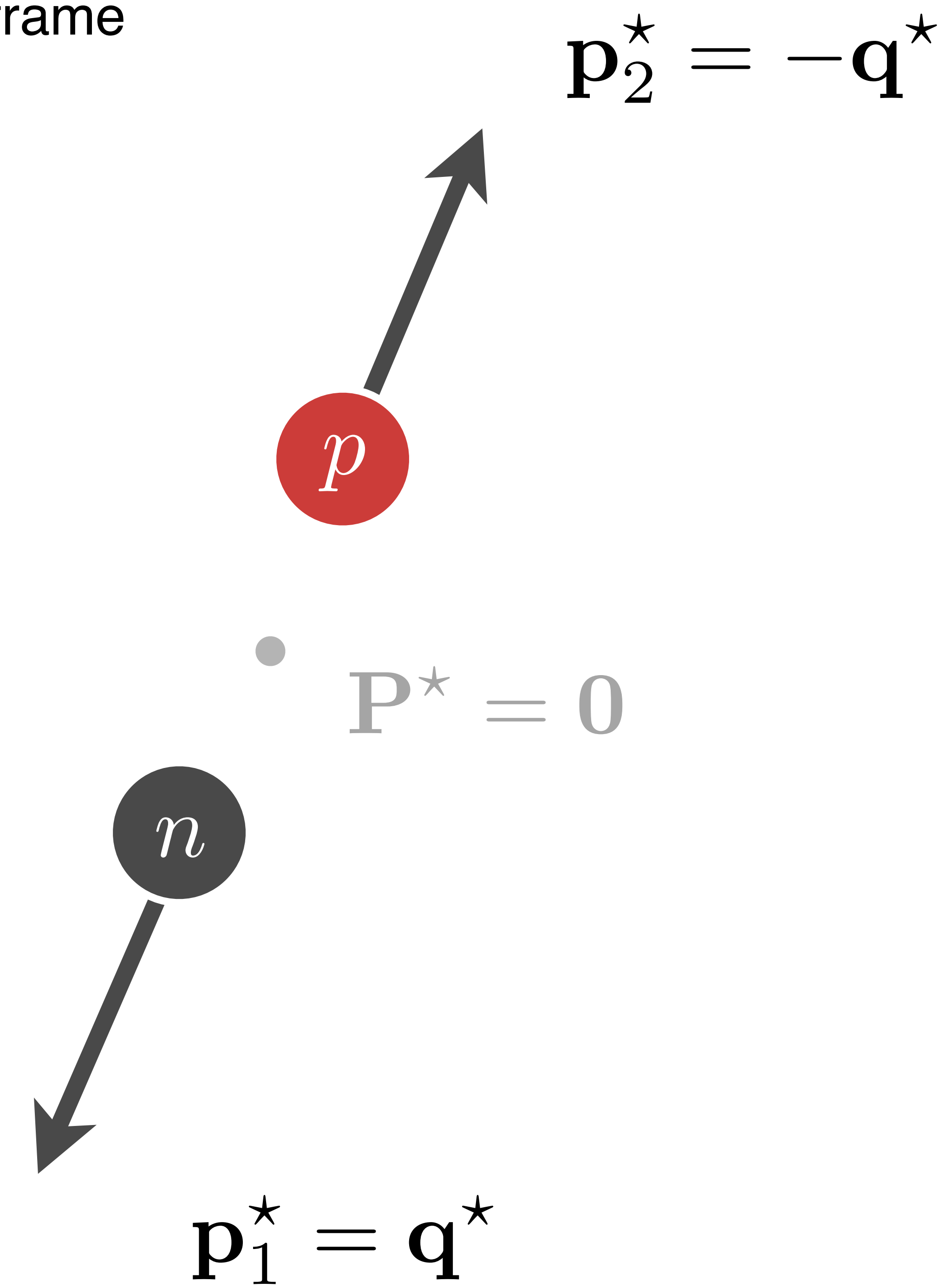
$$\Longrightarrow \qquad q^\star = \frac{1}{2}\sqrt{E^{\star\,2} - 4m^2}$$



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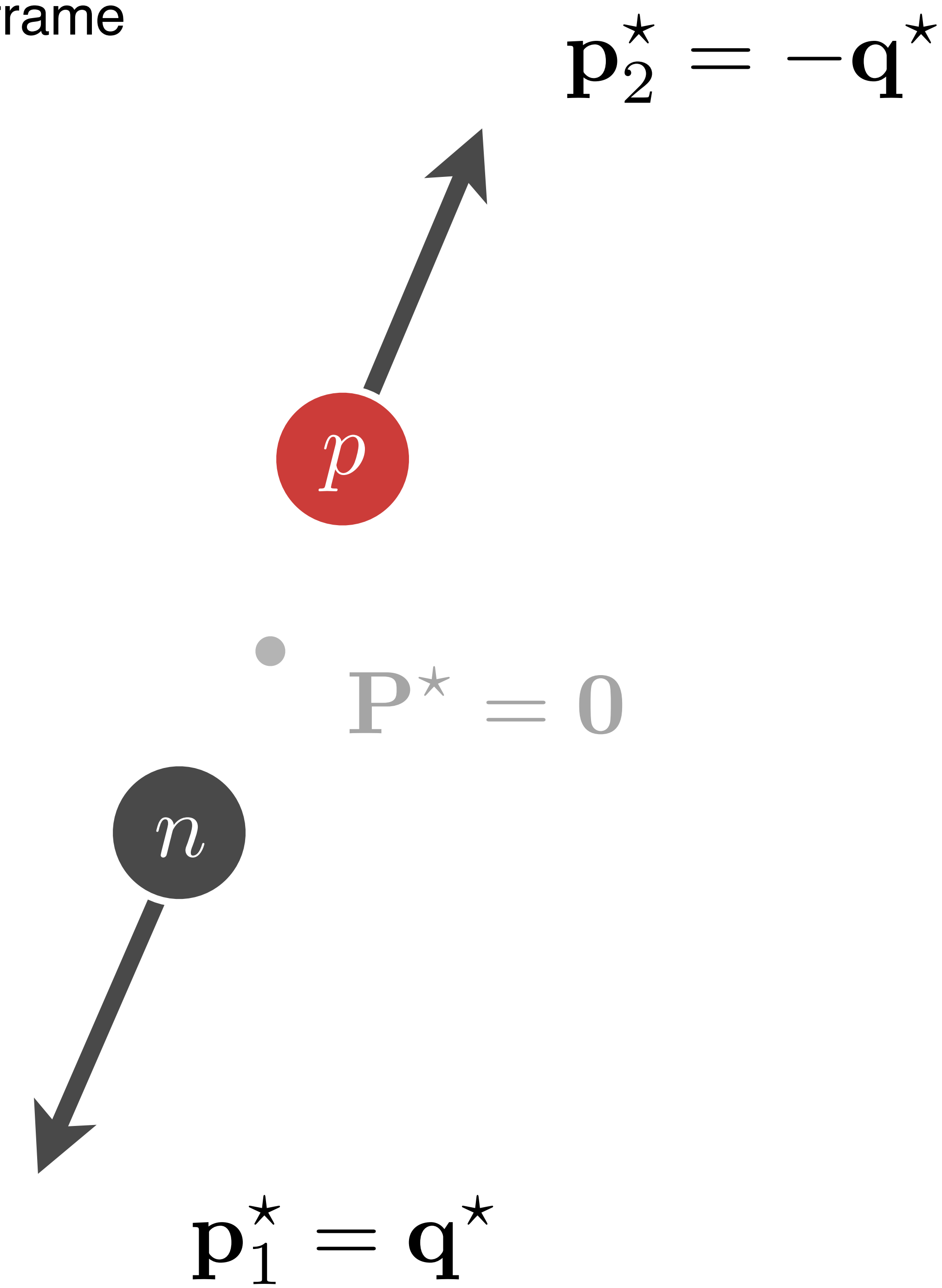

Total CM energy

$$E^\star = 2\sqrt{m^2 + q^{\star\,2}}$$

$$\implies \qquad q^\star = \frac{1}{2}\sqrt{E^{\star\,2} - 4m^2}$$

Exercise

(a) *Show that* $q^\star = \sqrt{E^{\star\,2}/4 - m^2}$

(b) *Plot* $q^\star$ *as a function of* $E^\star$

Two-nucleon energy is restricted by energy- and momentum-conservation

- Kinematic *phase space* — available space of kinematic configurations

Warning!

Some fancy mathematics ahead

Kinematics — Phase Space

Two-nucleon energy is restricted by energy- and momentum-conservation

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Particle 2 distribution

Lorentz invariant phase space

$$\mathrm{d}\Phi_2(P \to p_1 + p_2) = (2\pi)^4 \delta(E - \omega_{\mathbf{p}_1} - \omega_{\mathbf{p}_1})\, \delta^{(3)}(\mathbf{P} - \mathbf{p}_1 - \mathbf{p}_2)\, \frac{\mathrm{d}^3\mathbf{p}_1}{(2\pi)^3\, 2\omega_{\mathbf{p}_1}} \frac{\mathrm{d}^3\mathbf{p}_2}{(2\pi)^3\, 2\omega_{\mathbf{p}_2}}$$

Energy conservation
$E = \omega_{\mathbf{p}_1} + \omega_{\mathbf{p}_2}$

Momentum conservation
$\mathbf{P} = \mathbf{p}_1 + \mathbf{p}_2$

Particle 1 distribution

Kinematics — Phase Space

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$$= \cdots$$

$$= 2\rho\, \Theta(E^\star - 2m) \frac{\mathrm{d}\Omega^\star_{\mathbf{q}}}{4\pi}$$

Orientation (not fixed)

Energy greater than threshold
$E^\star \geq 2m$

$$E^\star_{\mathrm{thr.}} = 2m \iff q^\star_{\mathrm{thr.}} = 0$$

$$m \approx 940\,\mathrm{MeV}$$

$$E^\star_{\mathrm{thr.}} \approx 1880\,\mathrm{MeV}$$

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Two-body kinematic phase space factor

$$\rho = \frac{q^\star}{8\pi E^\star}$$

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Exercise

Plot ρ *as a function of* $E^\star$

** Derive* ρ

Two-body kinematic phase space factor

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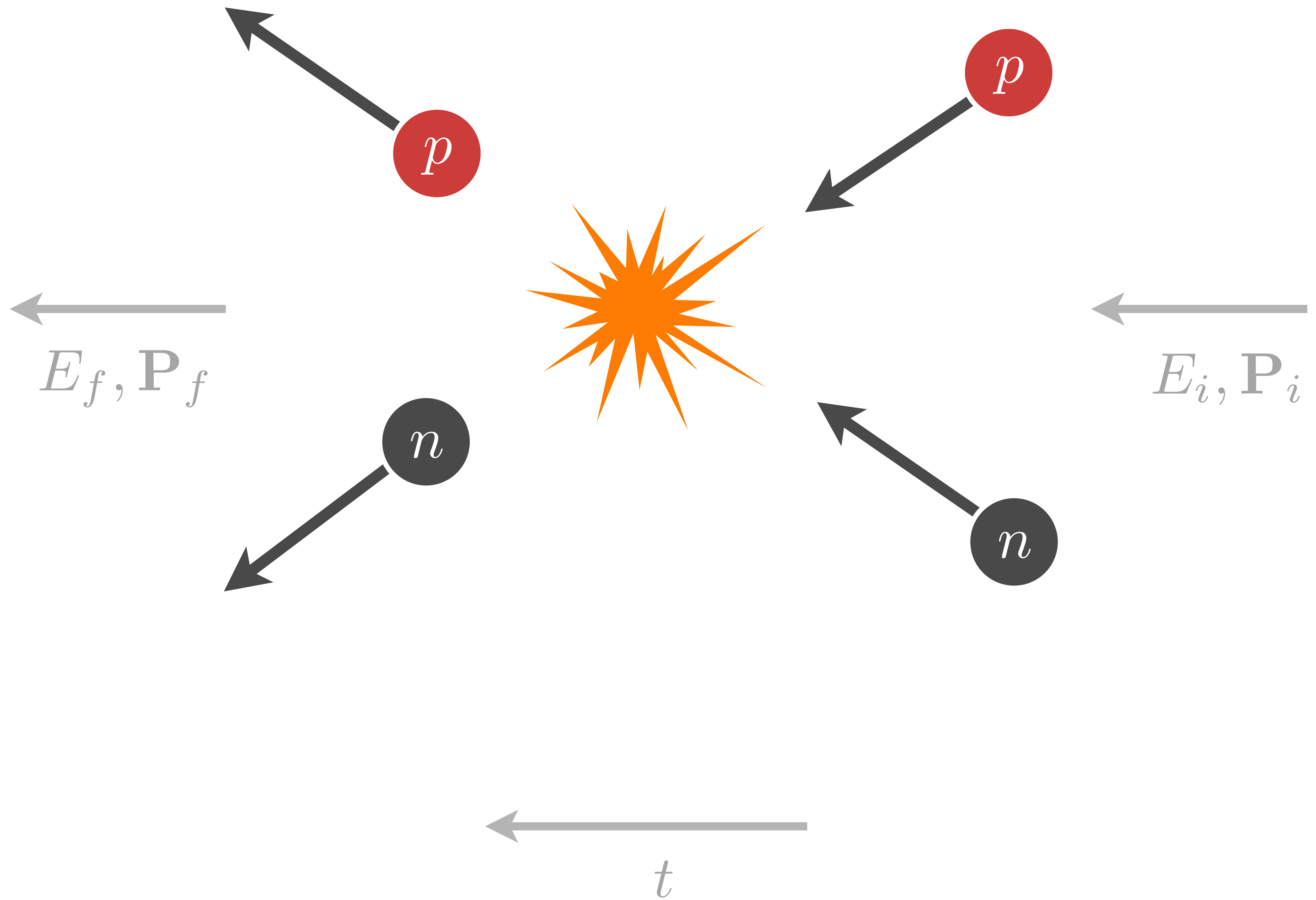
Kinematics — Binary Reactions

np-elastic scattering



Kinematics — Binary Reactions

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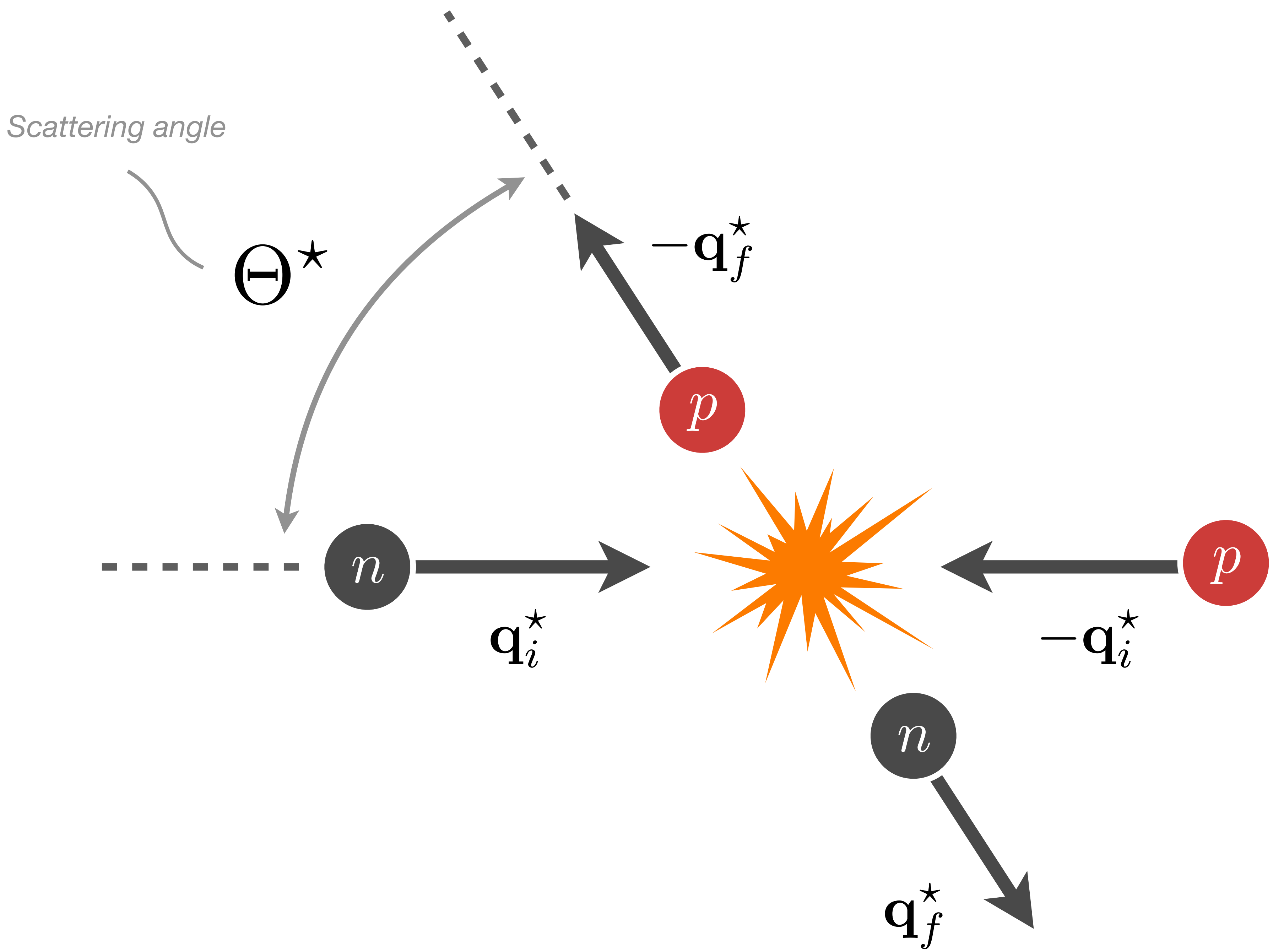
- Total energy-momentum is *conserved* — $E_i = E_f \equiv E\,,\ \mathbf{P}_i = \mathbf{P}_f \equiv \mathbf{P}$
- Analyze in *center-of-momentum* (CM) frame — $\mathbf{P}^\star = \mathbf{0}$
- Elastic, equal mass scattering — $q_i^\star = q_f^\star \equiv q^\star$



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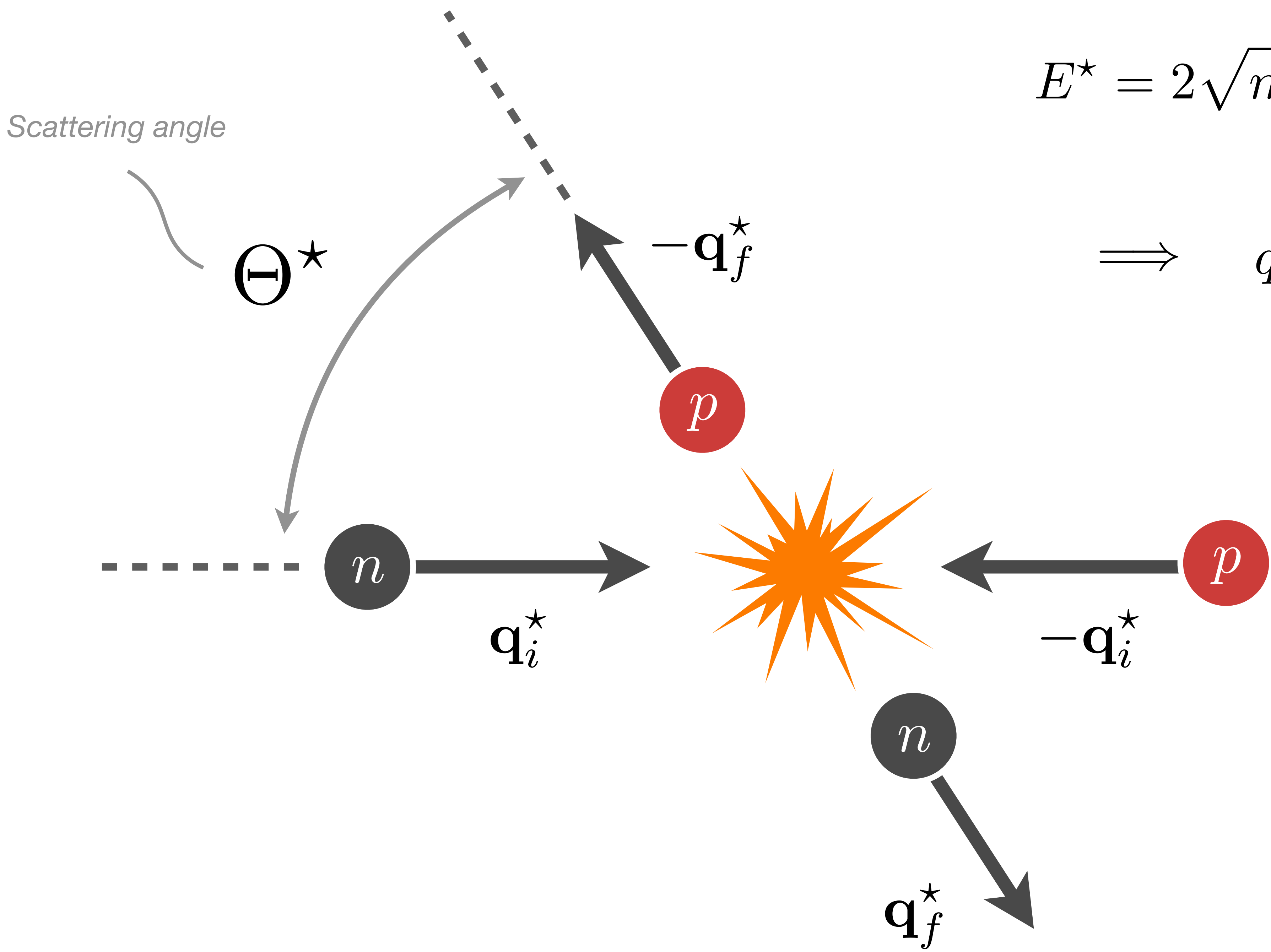

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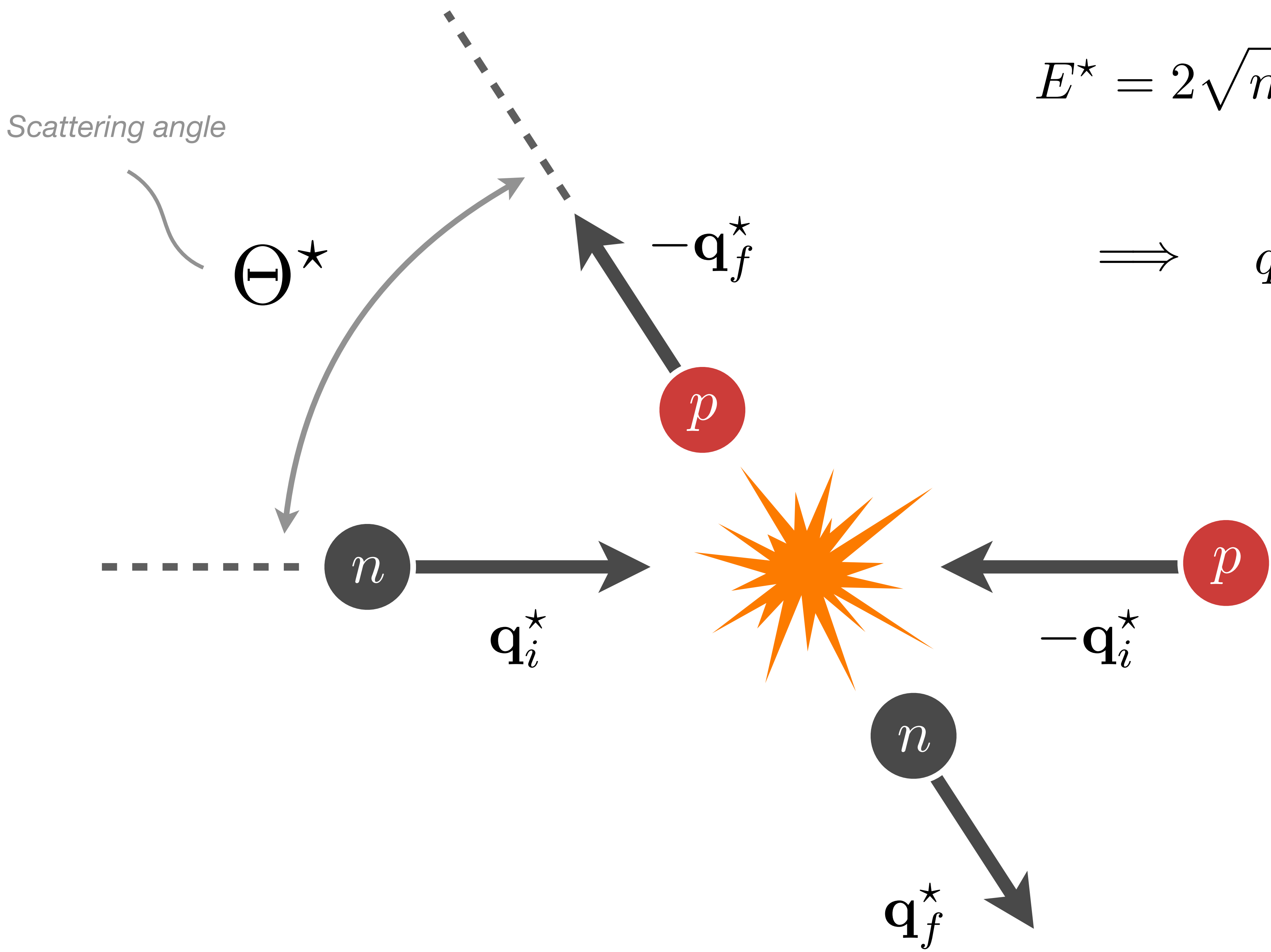

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Exercise

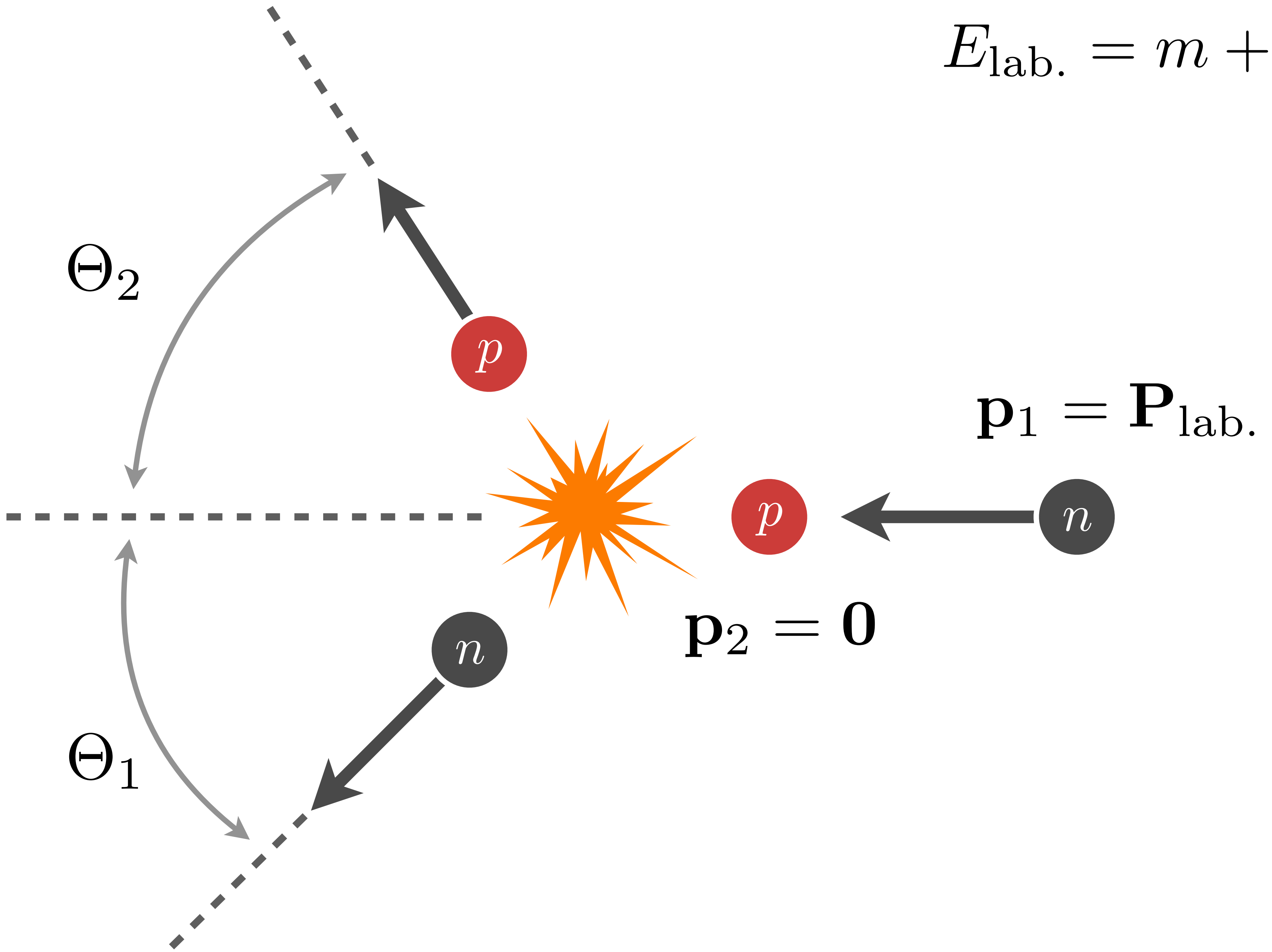
Show these results

Kinematics — Binary Reactions

np-elastic scattering

- Experiments often use *fixed-target* of *lab* frame

$$E_{\mathrm{lab.}} = m + \sqrt{m^2 + \mathbf{P}_{\mathrm{lab.}}^2}$$



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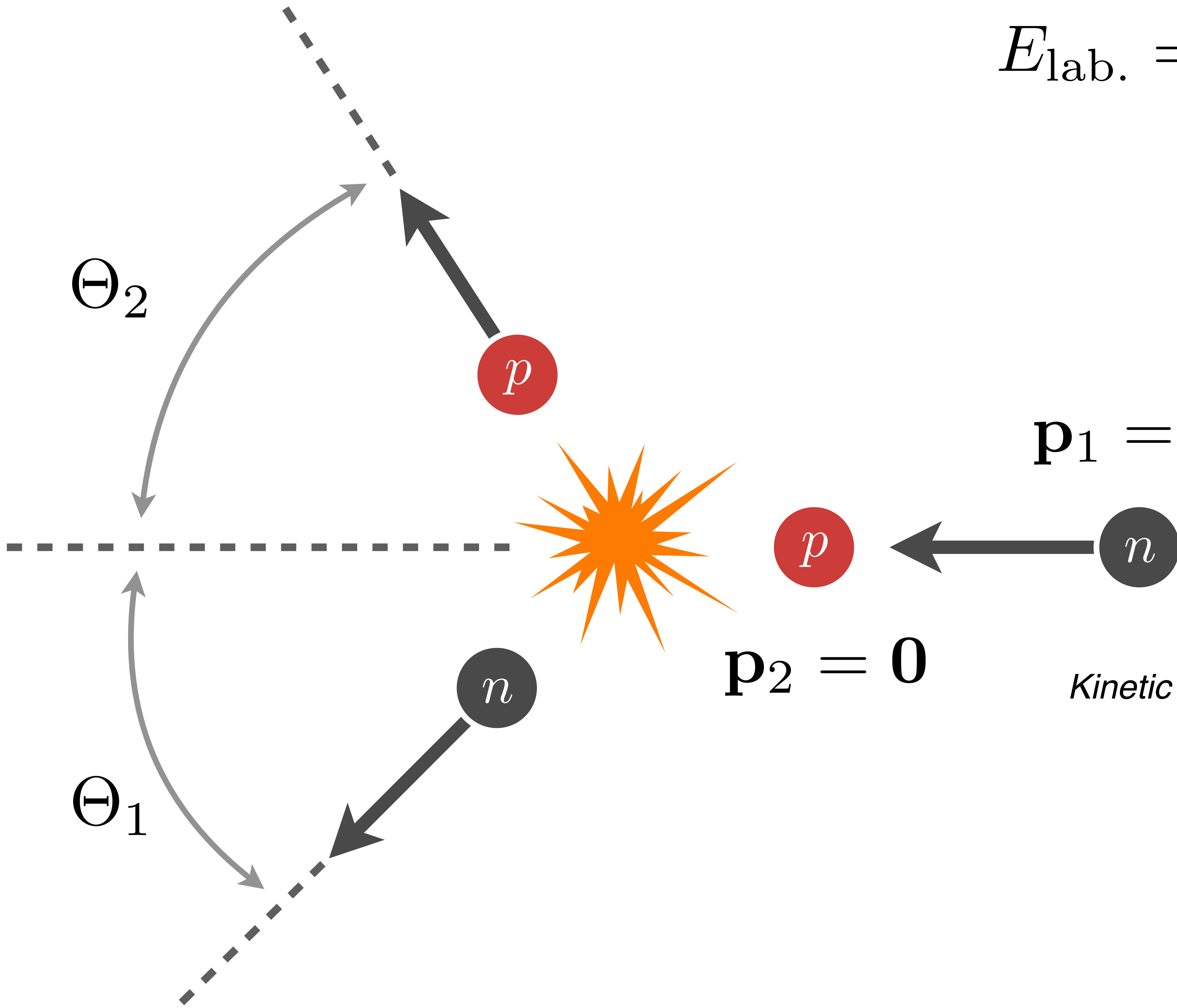


$$\mathbf{p}_1 = \mathbf{P}_{\text{lab.}}$$

$$\mathbf{p}_2 = \mathbf{0}$$

Kinetic energy

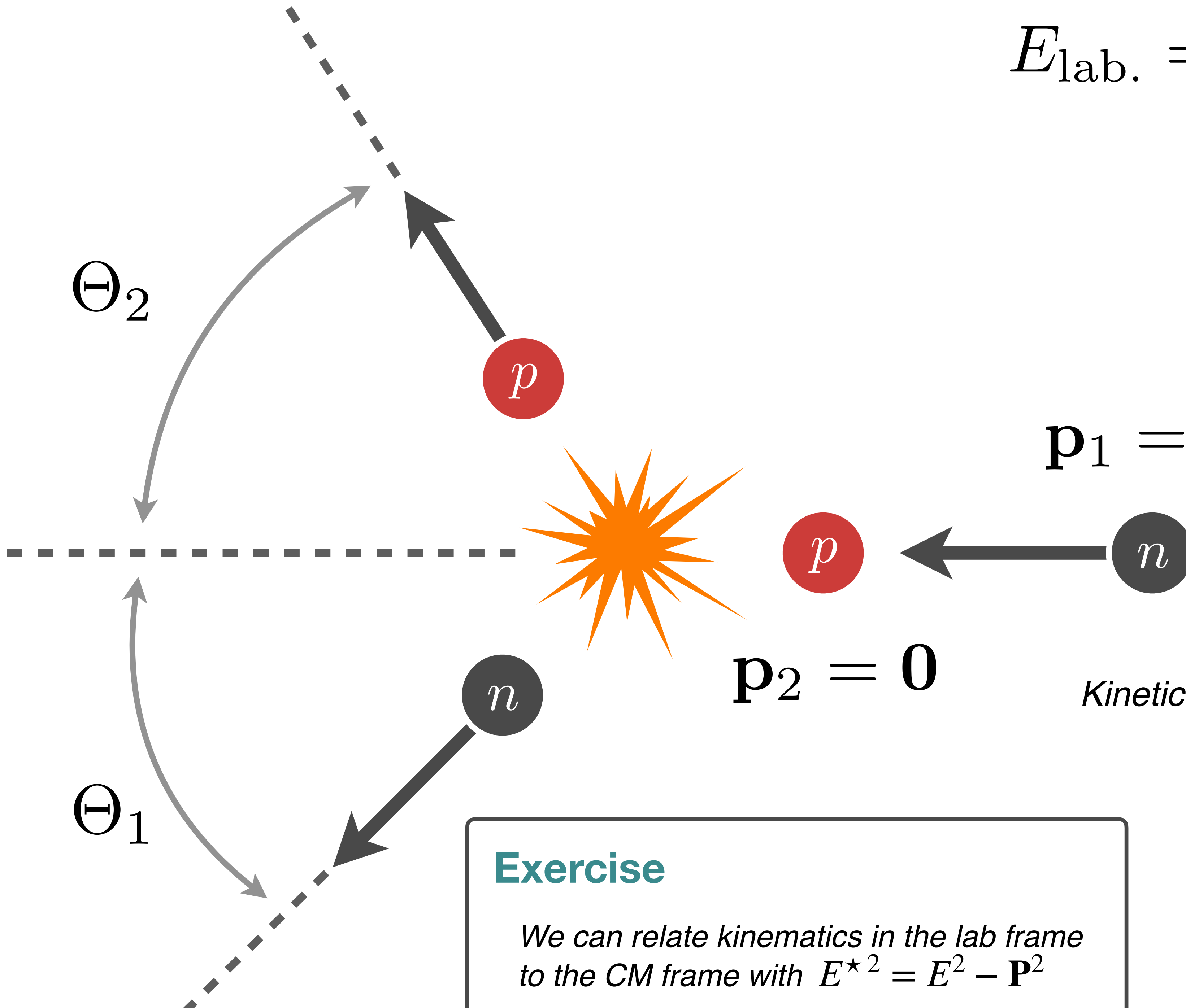
$$T_n = \sqrt{m^2 + \mathbf{P}^2_{\text{lab.}}} - m$$

$$q^{\star\,2} = \frac{1}{2} m T_n$$

np-elastic scattering

- Experiments often use *fixed-target* of *lab* frame

$$E_{\text{lab.}} = m + \sqrt{m^2 + \mathbf{P}^2_{\text{lab.}}}$$



Kinetic energy

$$T_n = \sqrt{m^2 + \mathbf{P}^2_{\text{lab.}}} - m$$

$$q^{\star\,2} = \frac{1}{2} m T_n$$

Exercise

We can relate kinematics in the lab frame to the CM frame with $E^{\star\,2} = E^2 - \mathbf{P}^2$

Show the relation between $q^\star$ *and* T_n

Scattering Theory — Amplitudes

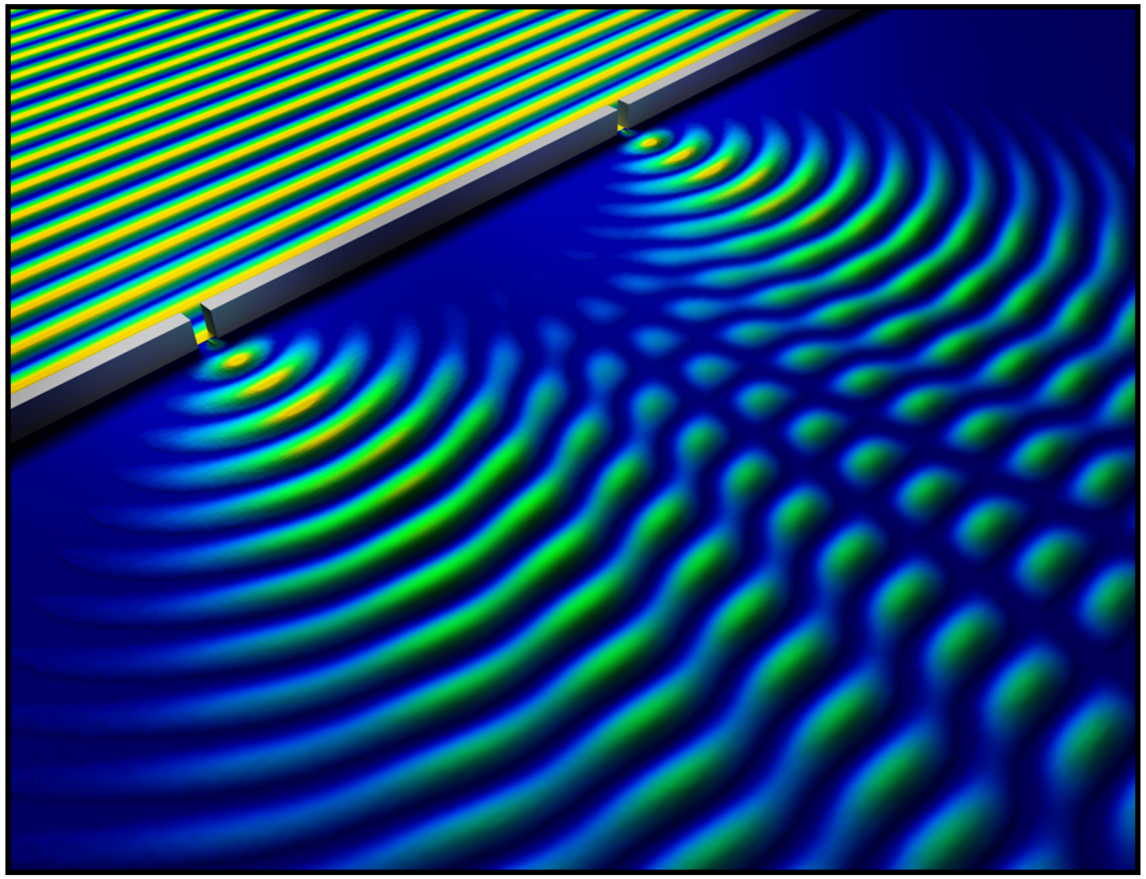
How do we dynamically describe reactions?

- Subatomic particles require quantum mechanics to properly describe physics
- Can only describe the probability for an event to occur
- Quantum mechanics describes *probability amplitudes*

$$\Psi(x,t) = Ae^{i(\omega t - kx)}$$

Quantum probability amplitude
"wavefunction"

$$\text{Prob} = \int_x \left|\Psi(x,t)\right|^2$$



$$\left|\Psi\right|^2 = \Psi^*\Psi$$

Mathematical Intermezzo — complex numbers

Complex numbers are *vital* in quantum mechanics

Cartesian representation

$$z = x + iy \qquad i \equiv \sqrt{-1} \qquad z \in \mathbb{C} \qquad x, y \in \mathbb{R}$$

Mathematical Intermezzo — complex numbers

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Complex conjugation — $z^* \equiv x - iy \qquad (z^*)^* = z$

Real and Imaginary parts — $\mathrm{Re}\, z = x = \dfrac{z + z^*}{2} \qquad \mathrm{Im}\, z = y = \dfrac{z - z^*}{2i}$

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Exercise

Show that

$$\mathrm{Re}\, z^* = \mathrm{Re}\, z$$

$$\mathrm{Im}\, z^* = -\,\mathrm{Im}\, z$$

Mathematical Intermezzo — complex numbers

Complex numbers are *vital* in quantum mechanics

Algebraic properties

$$
\begin{aligned}
z_1 \pm z_2 &= (x_1 + iy_1) \pm (x_2 + iy_2) \\
&= (x_1 \pm x_2) + i(y_1 \pm y_2)
\end{aligned}
$$

$$
\begin{aligned}
z_1 \cdot z_2 &= (x_1 + iy_1) \cdot (x_2 + iy_2) \\
&= (x_1 \cdot x_2 - y_1 \cdot y_2) + i(x_1 \cdot y_2 + x_2 \cdot y_1)
\end{aligned}
$$

$$
\begin{aligned}
\frac{1}{z} &= \frac{z^*}{z \cdot z^*} = \frac{z^*}{|z|^2} \\
&= \frac{x}{x^2 + y^2} - i\frac{y}{x^2 + y^2}
\end{aligned}
$$

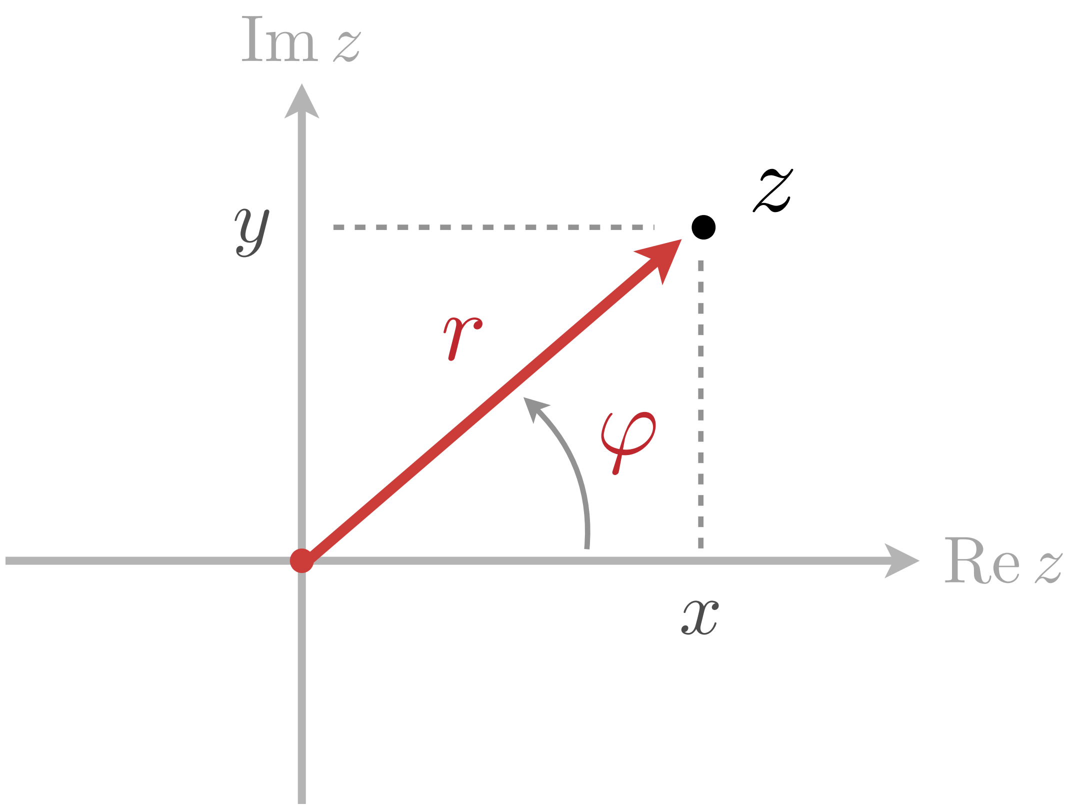
Mathematical Intermezzo — complex numbers

Complex numbers are *vital* in quantum mechanics

Polar representation

$$z = r\, e^{i\varphi} \qquad 0 \leq r < \infty \qquad \varphi \in (-\pi, \pi]$$

Components – $x = r\cos\varphi \qquad y = r\sin\varphi$



Euler Identity

$$e^{i\varphi} = \cos\varphi + i\sin\varphi$$

$$e^{i\pi} = -1$$

Mathematical Intermezzo — complex numbers

Complex numbers are *vital* in quantum mechanics

Polar representation

$$z = r\, e^{i\varphi} \qquad 0 \le r < \infty \qquad \varphi \in (-\pi, \pi]$$

Components — $x = r \cos\varphi \qquad y = r \sin\varphi$

Magnitude —
$$r = |z| \equiv \sqrt{z \cdot z^*} = \sqrt{x^2 + y^2} \qquad |z^*| = |z|$$

Phase —
$$\varphi = \arg(z) = \begin{cases} 2\arctan\left(\frac{y}{\sqrt{x^2+y^2}+x}\right) & \text{if } x > 0 \text{ or } y \neq 0\,, \\ \pi & \text{if } x < 0 \text{ and } y = 0\,, \\ \text{undefined} & \text{if } x = 0 \text{ and } y = 0 \end{cases}$$

Mathematical Intermezzo — complex numbers

Complex numbers are *vital* in quantum mechanics

Polar representation

$$z = r\,e^{i\varphi} \qquad 0 \leq r < \infty \qquad \varphi \in (-\pi, \pi]$$

Components — $x = r\cos\varphi \qquad y = r\sin\varphi$

Magnitude — $r = |z$

Phase — $\varphi = \mathrm{ar}$

Exercise

Let $z \in \mathbb{C}$

$$z = \frac{1-i}{1+i}\,,$$

1. *What are the real and imaginary components?*
2. *What is the magnitude* $|z|$*?*
3. *What is the phase of* z*?*
4. *Verify these numerically with the language of your choice (e.g. Python, C++, Fortran, …)*

Scattering Theory — Amplitudes

How do we dynamically describe reactions?

- In quantum mechanics, reaction dynamics encoded in *scattering probability amplitudes*

$$|\psi_f\rangle = \mathcal{S}\,|\psi_i\rangle$$

Initial state

Energy dependent amplitude $\mathcal{S} = \mathcal{S}(E^\star, \Theta^\star)$

Final state

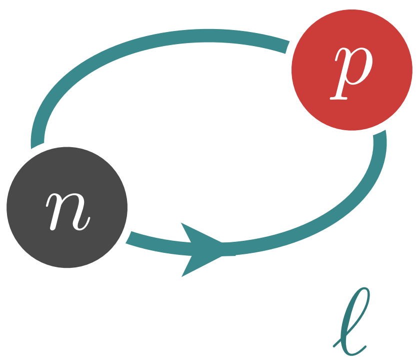
Probability for initial state to evolve to final state

$$\mathrm{Prob}(i \to f) = |\mathcal{S}|^2$$

Scattering Theory — Amplitudes

How do we dynamically describe reactions?

- In quantum mechanics, reaction dynamics encoded in *scattering probability amplitudes*
- Rotational symmetry — Break up interaction into angular momentum orbitals



Angular momentum quantized

$$\ell = 0, 1, 2, \dots$$

Scattering Theory — Amplitudes

How do we dynamically describe reactions?

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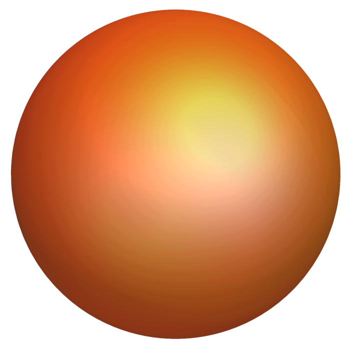
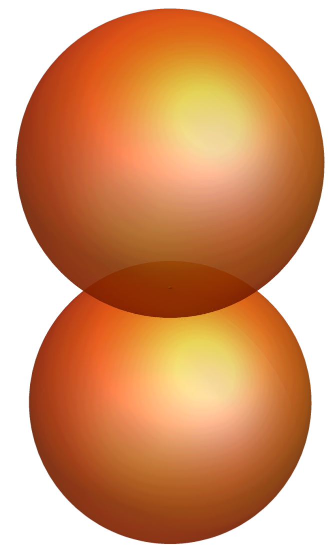
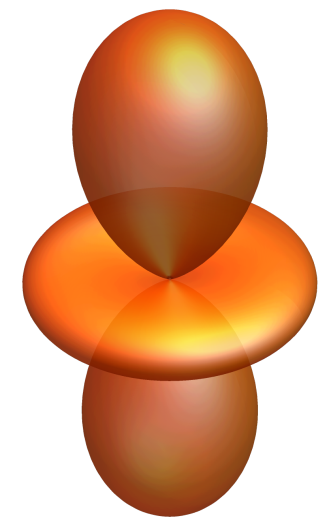
$$\mathcal{S}(E^\star, \Theta^\star) =$$



$+$



$+$



$+ \cdots$

$\mathcal{S}_0(E^\star)$ $\mathcal{S}_1(E^\star)$ $\mathcal{S}_2(E^\star)$

S-wave *P-wave* *D-wave*

$$\mathcal{S}(E^\star, \Theta^\star) = \sum_\ell (2\ell + 1)\, \mathcal{S}_\ell(E^\star) P_\ell(\cos \Theta^\star)$$

Scattering Theory — Amplitudes

How do we dynamically describe reactions?

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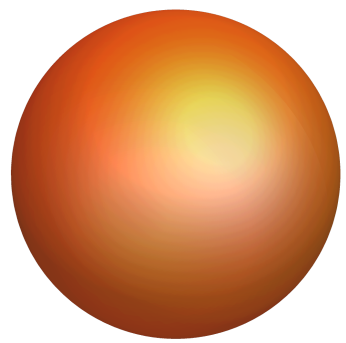
- Rotational symmetry — Break up interaction into angular momentum orbitals

$\mathcal{S}(E^\star, \Theta^\star) =$  $+$ $+$ $+ \cdots$

$\mathcal{S}_0(E^\star)$ $\mathcal{S}_1(E^\star)$ $\mathcal{S}_2(E^\star)$

S-wave *P-wave* *D-wave*

At low energies, isotropic interaction dominates

Ignore subscript $\mathcal{S}_0 \to \mathcal{S}$

$$\mathcal{S}(E^\star, \Theta^\star) = \sum_\ell (2\ell + 1)\, \mathcal{S}_\ell(E^\star) P_\ell(\cos \Theta^\star)$$

Scattering Theory — Amplitudes

How do we dynamically describe reactions?

- In quantum mechanics, reaction dynamics encoded in *scattering probability amplitudes*
- Rotational symmetry — Break up interaction into orbitals
- Remove trivial case where particles do not interact — define ***scattering amplitude***

$$\mathcal{S} = 1 + 2i\,\rho\,\mathcal{M}$$

No scattering

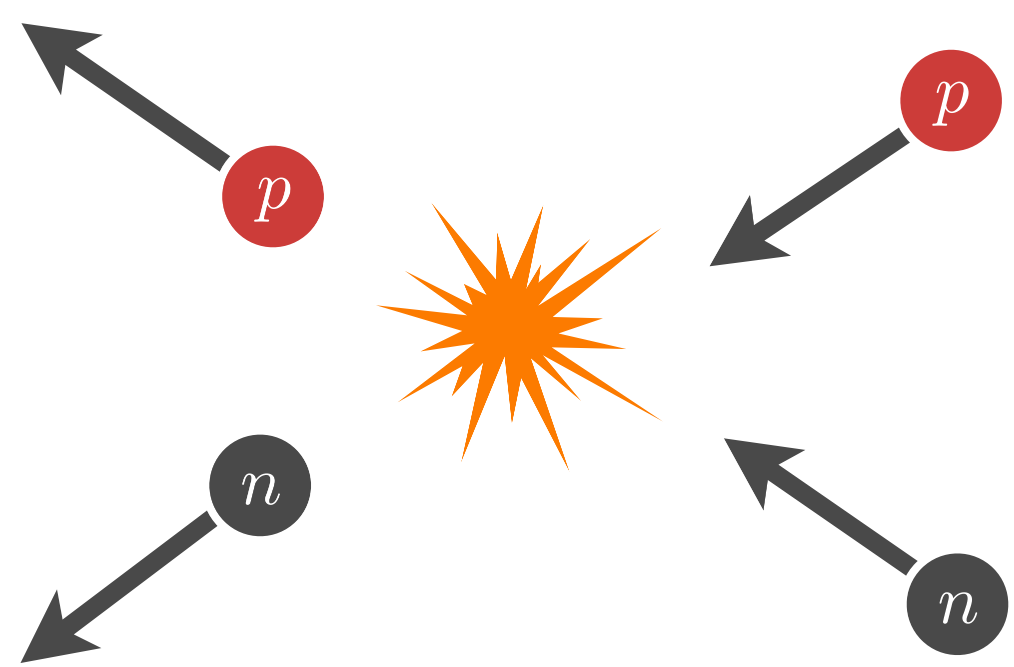
Available phase space

Scattering amplitude — encodes ALL dynamics

$$\rho = \frac{q^\star}{8\pi E^\star}$$

Relation to cross-section

$$\sigma \propto |\mathcal{M}|^2$$

$\mathcal{M} =$ 

Scattering Theory — Unitarity

Conservation of probability imposes constraints on the scattering amplitude

$$\sum_f \mathrm{Prob}(i \to f) = 1$$

Probability must be conserved!

Elastic scattering — only one final state

$$\implies \quad |\mathcal{S}|^2 = \mathcal{S}^*\mathcal{S} = 1$$

$$\mathrm{Prob}(i \to f) = |\mathcal{S}|^2$$

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Unitarity condition for scattering amplitude

$$\mathcal{S} = 1 + 2i\,\rho\mathcal{M}$$

$$\implies \quad \mathrm{Im}\,\mathcal{M} = \rho|\mathcal{M}|^2$$

$$\mathrm{Prob}(i \to f) = |\mathcal{S}|^2$$

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Unitarity condition for scattering amplitude

$$\mathcal{S} = 1 + 2i\,\rho\mathcal{M}$$

$$\implies \quad \mathrm{Im}\,\mathcal{M} = \rho|\mathcal{M}|^2$$

Exercise

Derive the unitarity condition for the scattering amplitude

$$\mathrm{Prob}(i \to f) = |\mathcal{S}|^2$$

Scattering Theory — Unitarity

Conservation of probability imposes constraints on the scattering amplitude

- At a fixed energy, amplitude needs *two* real numbers

$$\mathcal{M} = \mathrm{Re}\,\mathcal{M} + i\,\mathrm{Im}\,\mathcal{M}$$

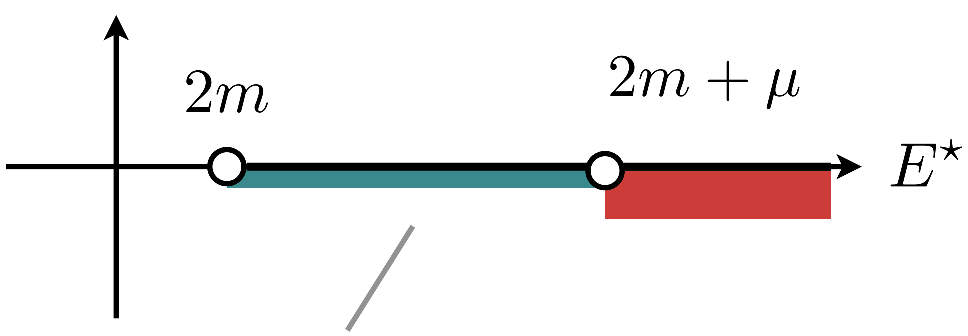
Unitarity relates real and imaginary parts

$$\mathrm{Im}\,\mathcal{M} = \rho|\mathcal{M}|^2$$

For $2m \leq E^\star < 2m + \mu$

Available phase space

$$\rho = \frac{q^\star}{8\pi E^\star}$$



Elastic scattering region

Scattering Theory — Unitarity

Conservation of probability imposes constraints on the scattering amplitude

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$$\mathcal{M} = \mathrm{Re}\,\mathcal{M} + i\,\mathrm{Im}\,\mathcal{M}$$

Unitarity relates real and imaginary parts

$$\mathrm{Im}\,\mathcal{M} = \rho|\mathcal{M}|^2$$

Imaginary part fixed by kinematics

Can rewrite as

$$\mathrm{Im}\,\mathcal{M}^{-1} = -\rho$$

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Only need one real number to completely specify scattering

$$\implies \mathcal{M}^{-1} = \mathcal{K}^{-1} - i\rho$$

Real function — encodes ALL dynamics!

Scattering Theory — Unitarity

Conservation of probability imposes constraints on the scattering amplitude

- At a fixed energy, amplitude needs *two* real numbers

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Imaginary part fixed by kinematics

Can rewrite as

$$\mathrm{Im}\,\mathcal{M}^{-1} = -\rho$$

Exercise

Show that $\mathrm{Im}\,\mathcal{M}^{-1} = -\rho$

Only need one real number to completely specify scattering

$$\Longrightarrow \quad \mathcal{M}^{-1} = \mathcal{K}^{-1} - i\rho$$

Real function — encodes ALL dynamics!

Scattering Theory — Unitarity

Unitarity implies only one real dynamical function needed to describe scattering

- Theoretically, want to compute *K* matrix
- Can infer *K* matrix by fitting experimental data

$$\mathcal{M}^{-1} = \mathcal{K}^{-1} - i\rho$$

Know this function

Want to determine this function

Scattering Theory — Phase shifts

Alternatively, can cast amplitude in terms of phase shifts

- Unitarity is starting point

$$\mathrm{Im}\,\mathcal{M} = \rho|\mathcal{M}|^2$$

Use polar representation

$$\mathcal{M} = |\mathcal{M}|\,e^{i\delta}$$

Scattering Theory — Phase shifts

Alternatively, can cast amplitude in terms of phase shifts

- Unitarity is starting point

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Use polar representation

$$\mathcal{M} = |\mathcal{M}|\, e^{i\delta}$$

Impose unitarity, can show

$$\implies \qquad \mathcal{M} = \frac{1}{\rho}\, e^{i\delta}\, \sin\delta$$

Phase shift, a real function of energy

Scattering Theory — Phase shifts

Alternatively, can cast amplitude in terms of phase shifts

- Unitarity is starting point

$$\mathrm{Im}\,\mathcal{M} = \rho|\mathcal{M}|^2$$

Use polar representation

$$\mathcal{M} = |\mathcal{M}|\,e^{i\delta}$$

Impose unitarity, can show

$$\implies \quad \mathcal{M} = \frac{1}{\rho}\,e^{i\delta}\,\sin\delta$$

Phase shift, a real function of energy

Exercise

(a) Derive the phase shift representation

(b) Show that $\mathcal{K}^{-1} = \rho\cot\delta$

Scattering Theory — Effective Range Expansion

Can use either K matrix or phase shifts to describe reaction (often use both)

- They are real functions, can Taylor expand in real variables — assign physical meaning

$$\mathcal{K}^{-1} = \rho \cot \delta$$

$$\propto q^\star \cot \delta = -\frac{1}{a} + \frac{1}{2} r {q^\star}^2 + \mathcal{O}({q^\star}^4)$$

Effective range expansion

Effective range — "range of interaction"

Scattering length — "strength of interaction"

$$\rho = \frac{q^\star}{8\pi E^\star}$$

Scattering Theory — Effective Range Expansion

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$$
\begin{aligned}
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\end{aligned}
$$

Effective range expansion

Scattering length — "strength of interaction"

Effective range — "range of interaction"

Amplitude at threshold is constant, fixed by strength of interaction

$$\mathcal{M}_{\text{thr.}} \equiv \mathcal{M}(E^{\star}_{\text{thr.}}) = -16\pi\, m a$$

$$\rho = \frac{q^{\star}}{8\pi E^{\star}}$$

Scattering Theory — Effective Range Expansion

Can use either *K* matrix or phase shifts to describe reaction (often use both)

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Effective range expansion

Scattering length — "strength of interaction"

Effective range — "range of interaction"

Amplitude at threshold is constant, fixed by strength of interaction

$\mathcal{M}_{\mathrm{thr.}} \equiv$

Exercise

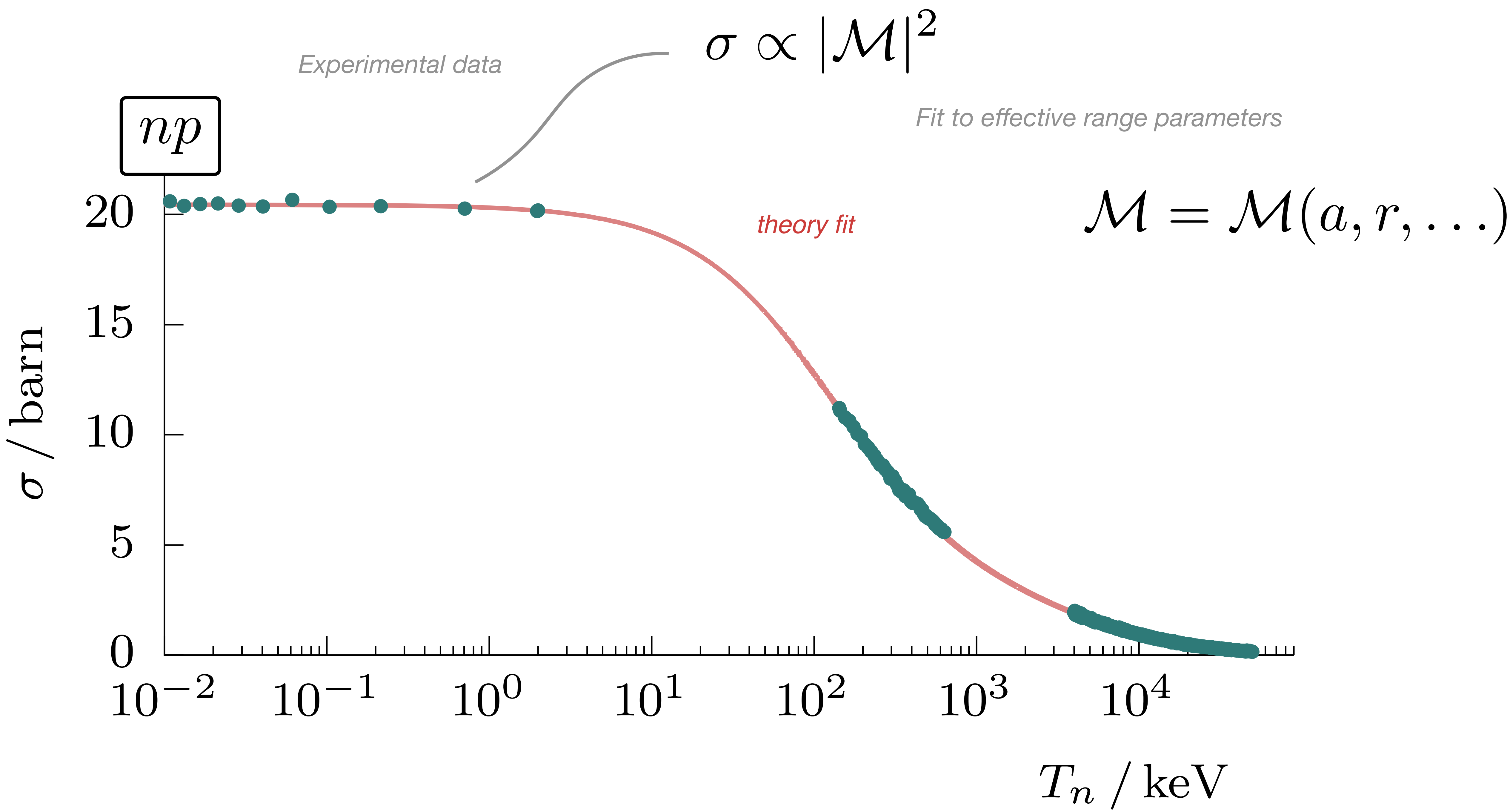
Show that at threshold the scattering amplitude is constant and fixed by the scattering length

$$\rho = \frac{q^\star}{8\pi E^\star}$$

Scattering Theory — Effective Range Expansion

Can use either K matrix or phase shifts to describe reaction (often use both)

- They are real functions, can Taylor expand in real variables — assign physical meaning
- These are fit parameters — can learn about the interaction



Spectroscopy — Bound states

Once we have the amplitude, we can now search for the ***spectrum***

- Bound states are enhancements in the amplitude — they are pole singularities

Scattering $\Longleftrightarrow$ Bound States

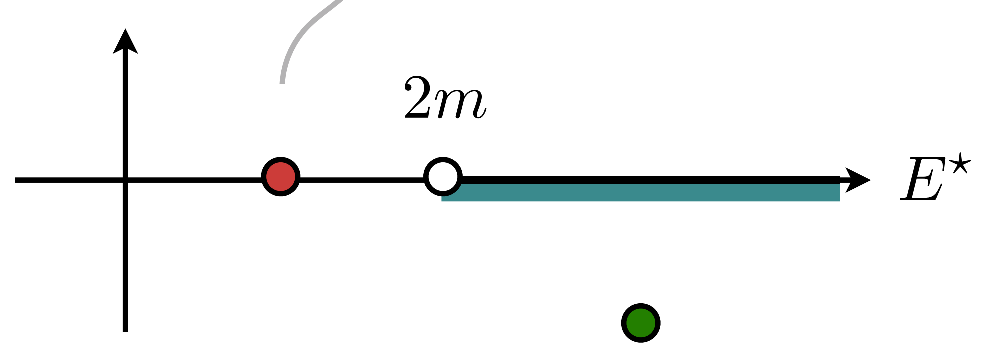
$$\mathcal{M} \propto \frac{1}{E^\star - M - i\Gamma/2}$$

Hadron Mass

$$\tau = \frac{1}{\Gamma}$$

Hadron Lifetime

Stable bound states ($\tau \to \infty$)



Unstable states (resonances)

Spectroscopy — Bound states

Once we have the amplitude, we can now search for the ***spectrum***

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Scattering $\Longleftrightarrow$ Bound States

$$\mathcal{M} \propto \frac{1}{E^\star - M - i\Gamma/2}$$

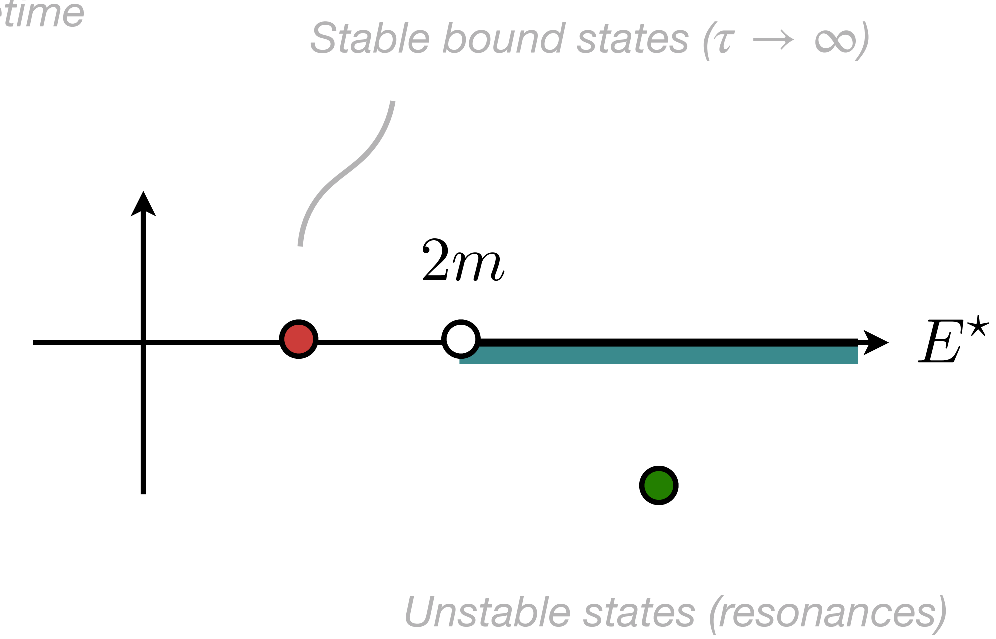
Hadron Mass

$$\tau = \frac{1}{\Gamma}$$

Hadron Lifetime

Search for zeroes of denominator

$$\mathcal{M}^{-1}(E_b^\star) = 0$$



Spectroscopy — Bound states

Once we have the amplitude, we can now search for the ***spectrum***

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$$\mathcal{M} = \frac{8\pi E^\star}{q^\star \cot\delta - iq^\star}$$

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$$\mathcal{M} = \frac{8\pi E^\star}{q^\star \cot\delta - iq^\star}$$

$$\mathcal{M}^{-1} = 0 \implies \quad q^\star \cot\delta = iq^\star$$

Spectroscopy — Bound states

Once we have the amplitude, we can now search for the ***spectrum***

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$$\mathcal{M} = \frac{8\pi E^\star}{q^\star \cot\delta - iq^\star}$$

$$\mathcal{M}^{-1} = 0 \implies q^\star \cot\delta = iq^\star$$

Assume leading order effective range, $a > 0$

$$q^\star \cot\delta = -\frac{1}{a} \qquad \implies \qquad q^\star = \frac{i}{a} \equiv i\kappa$$

Binding momentum

$$\begin{aligned} E^\star &= 2\sqrt{m^2 + q^{\star\,2}} \\ &= 2\sqrt{m^2 - \kappa^2} \end{aligned}$$

Spectroscopy — Bound states

Once we have the amplitude, we can now search for the ***spectrum***

- Bound states are enhancements in the amplitude — they are pole singularities

$$\mathcal{M} = \frac{8\pi E^\star}{q^\star \cot\delta - iq^\star}$$

$$\mathcal{M}^{-1} = 0 \implies \quad q^\star \cot\delta = iq^\star$$

Compare to

$$\mathcal{M} \propto \frac{1}{E^\star - M - i\Gamma/2}$$

$$\equiv i\kappa$$

Binding momentum

$$\implies \quad M = 2\sqrt{m^2 - \frac{1}{a^2}} \qquad \Gamma = 0$$

Spectroscopy — Bound states

Once we have the amplitude, we can now search for the ***spectrum***

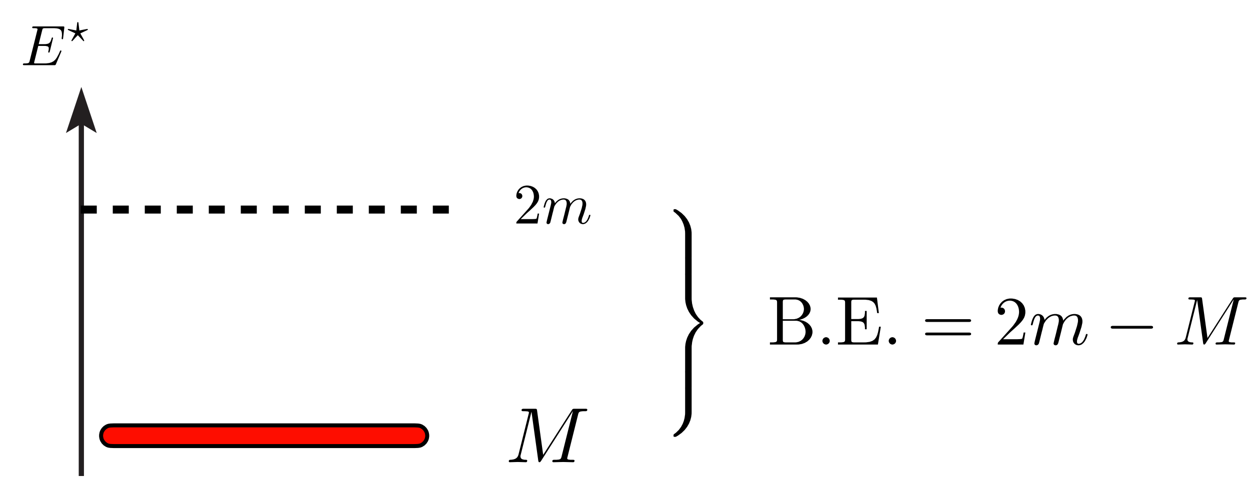
- Bound states are enhancements in the amplitude — they are pole singularities

For $a > 0$, find $M < 2m$ — Stable bound state

$$M = 2\sqrt{m^2 - \frac{1}{a^2}}$$

$$\Gamma = 0$$

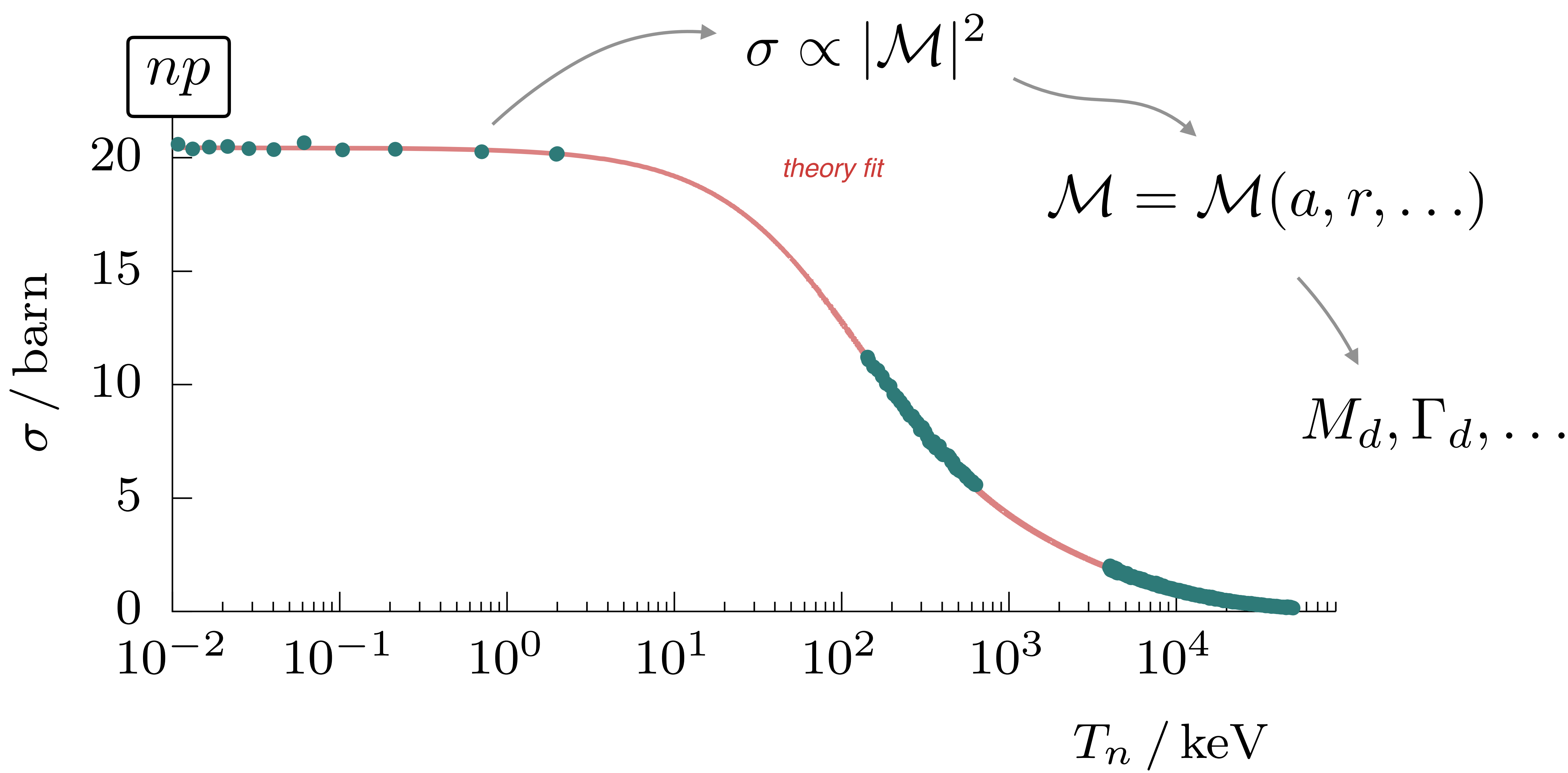
Binding energy



Neutron-Proton Scattering — Spin Effects

Goal — analyze low-energy neutron-proton scattering data

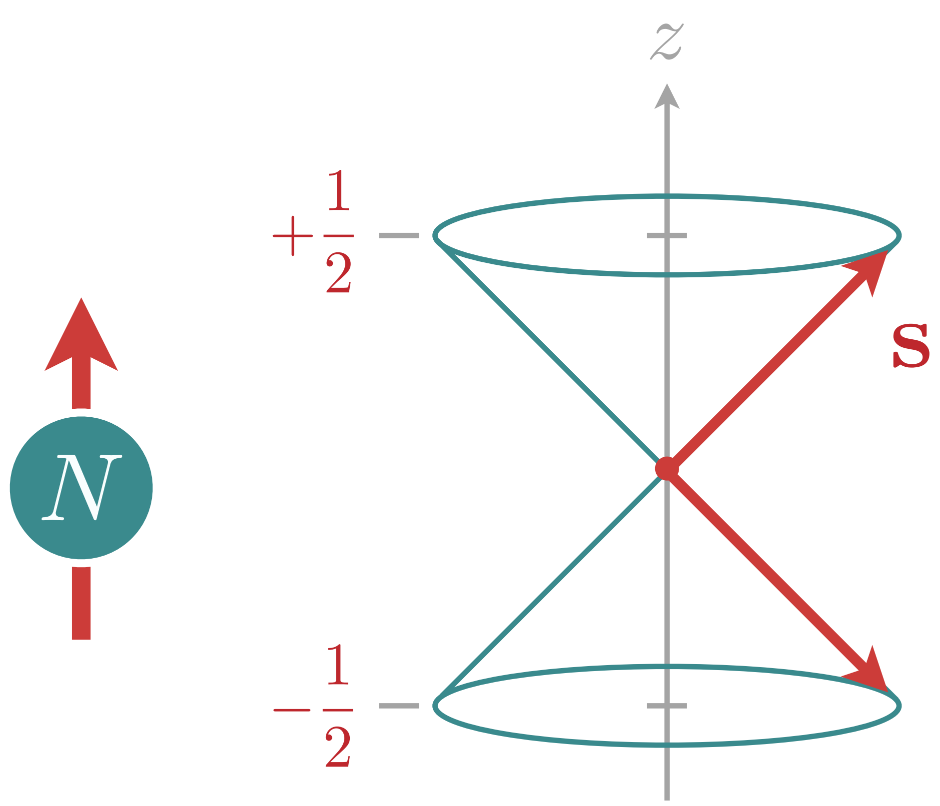
- Have a kinematic description of reaction
- Know how to parameterize amplitude in terms of physical parameters
- Can fit amplitude with unknown parameters to data
- Once we have scattering parameters, can determine bound state spectrum



Neutron-Proton Scattering — Spin Effects

So far, assumed nucleon was structure-less

- Nucleons have intrinsic *spin* angular momentum — spin-1/2 fermions
- The strong interactions can (and does) depend on the spin degrees-of-freedom
- Assume still in low-energy regime (no angular dependence)



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- The strong interactions can (and does) depend on the spin degrees-of-freedom
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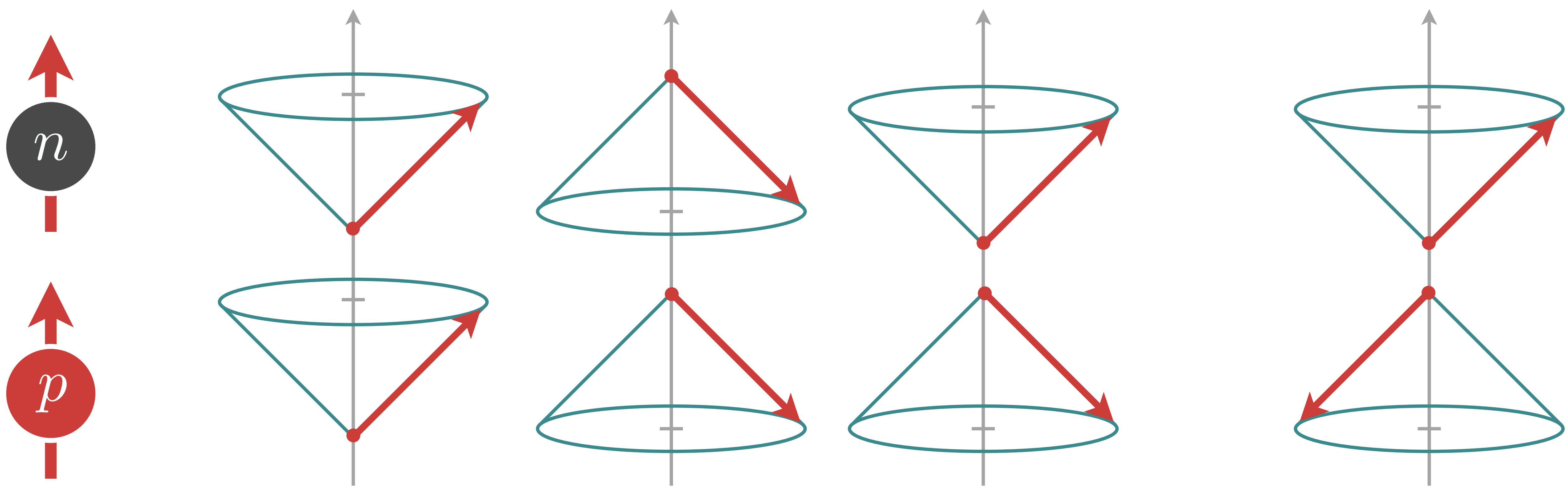
Total spin of neutron-proton system $$m_s = s_n + s_p$$

$$\Longrightarrow \quad s = 0 \text{ or } 1$$



$m_s = +1$ $m_s = -1$ $m_s = 0$ $m_s = 0$

Neutron-Proton Scattering — Spin Effects

So far, assumed nucleon was structure-less

- Nucleons have intrinsic *spin* angular momentum — spin-1/2 fermions
- The strong interactions can (and does) depend on the spin degrees-of-freedom
- Assume still in low-energy regime (no angular dependence)
- np scattering can occur in the singlet or triplet state
- Total cross section is weighted sum of singlet and triplet cross sections

$$\sigma = \frac{1}{4}\sigma_s + \frac{3}{4}\sigma_t$$

Triplet $(s = 1)$

Singlet $(s = 0)$

Total number of spin states = 4

No spin-orbit interactions

$$\ell = 0, s = 0 \text{ or } 1 \implies J = s$$

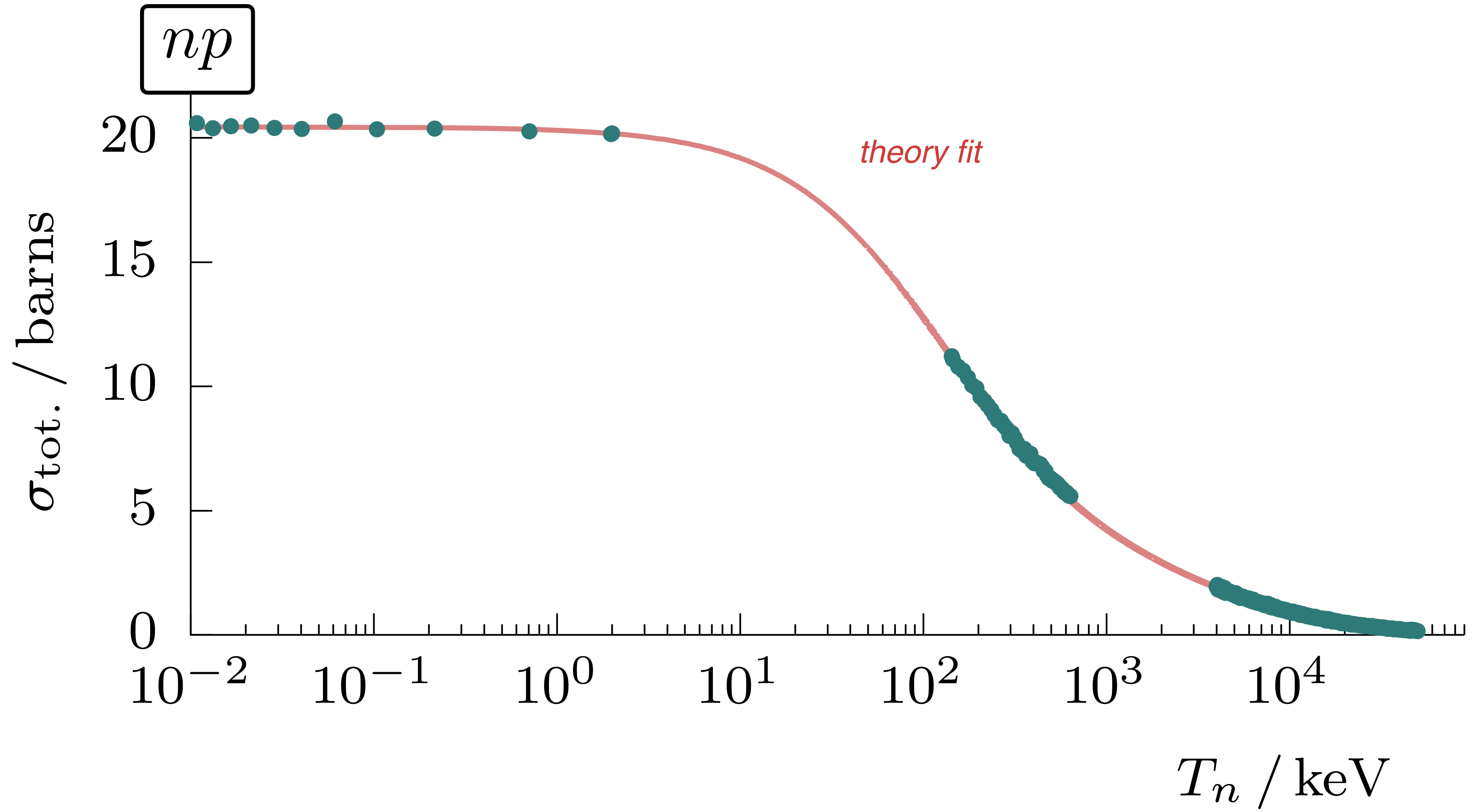
Neutron-Proton Scattering — Cross-section

$$\sigma_x = \frac{|\mathcal{M}_x|^2}{16\pi E^{\star\,2}}$$

$$\mathcal{M}_x = \frac{8\pi E^\star}{q^\star \cot\delta_x(q^\star) - iq^\star}$$

Have enough tools to analyze data

$$\sigma = \frac{1}{4}\,\sigma_s + \frac{3}{4}\,\sigma_t$$



Neutron-Proton Scattering — Cross-section

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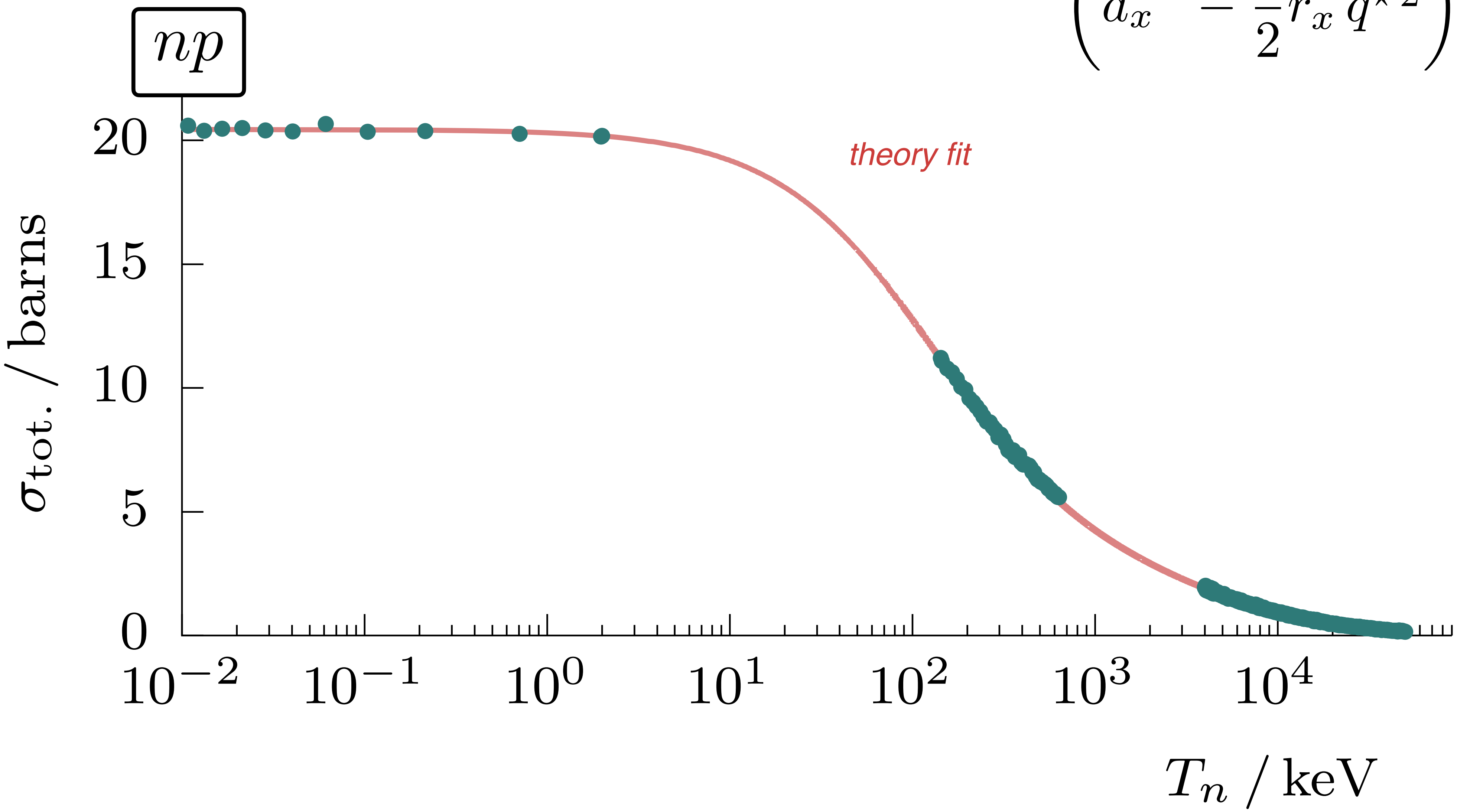
Have enough tools to analyze data

$$\sigma = \frac{1}{4}\sigma_s + \frac{3}{4}\sigma_t$$

Low-energy shape-independent approximation

$$\sigma_x = \frac{4\pi}{\left(a_x^{-1} - \frac{1}{2}r_x\, q^{\star\,2}\right)^2 + q^{\star\,2}}$$

$$q^{\star\,2} = \frac{1}{2}mT_n$$



Neutron-Proton Scattering — Cross-section

$m \approx 940\ \mathrm{MeV}$

$1\ \mathrm{fm} \approx \dfrac{1}{200\ \mathrm{MeV}}$

$1\,\mathrm{b} = 100\,\mathrm{fm}^2\ (= 10^{-28}\,\mathrm{m}^2)$

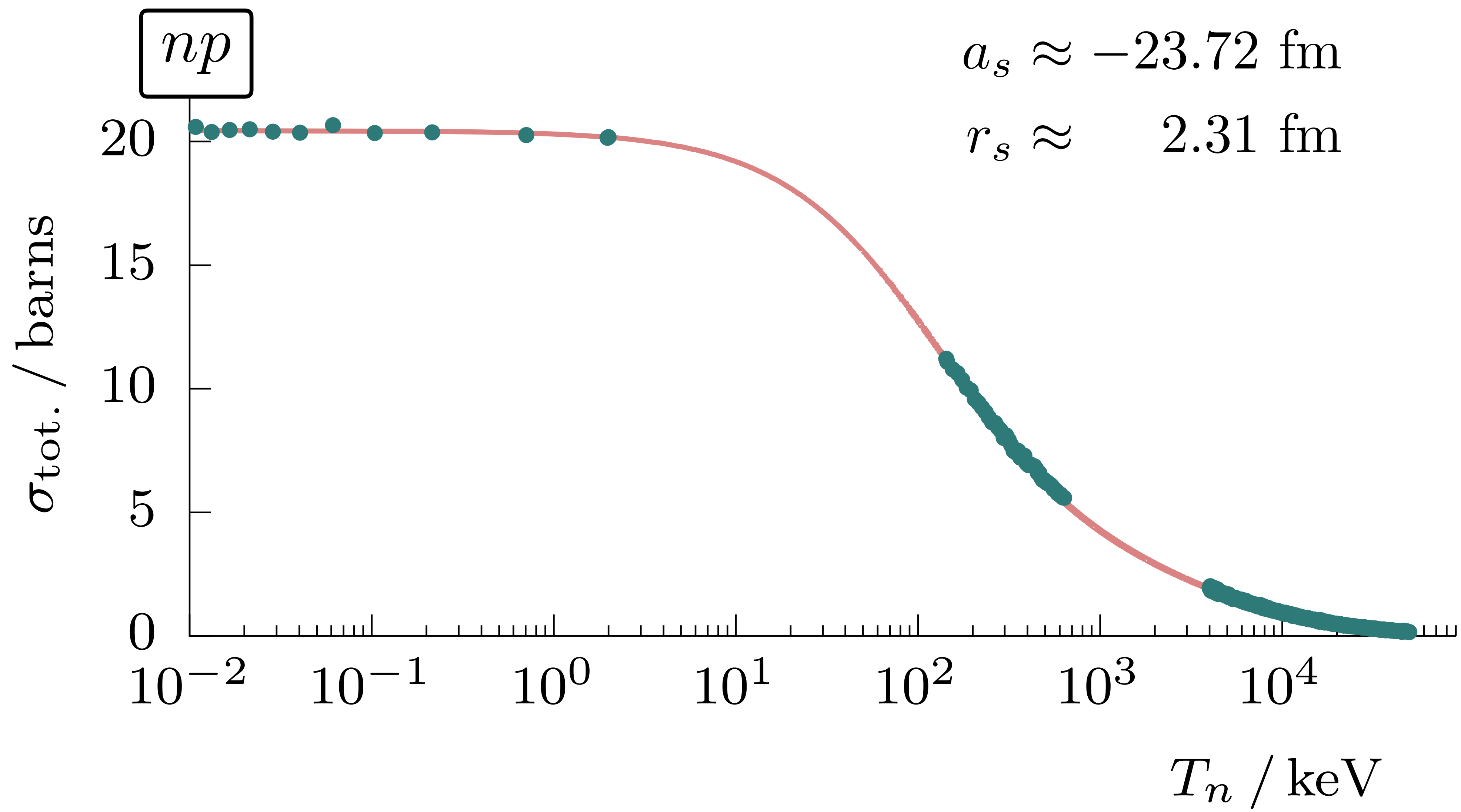
Have enough tools to analyze data

$$\chi^2 = \sum \left(\frac{\sigma_{\mathrm{exp.}} - \sigma_{\mathrm{th}}}{\Delta\sigma_{\mathrm{exp.}}} \right)^2$$

My simple fit

$$\chi^2/\nu = 472.9/(486-4) \approx 0.98$$

$$\begin{aligned} a_t &\approx 5.41\ \mathrm{fm} \\ r_t &\approx 1.62\ \mathrm{fm} \\ a_s &\approx -23.72\ \mathrm{fm} \\ r_s &\approx 2.31\ \mathrm{fm} \end{aligned}$$



Neutron-Proton Scattering — Cross-section

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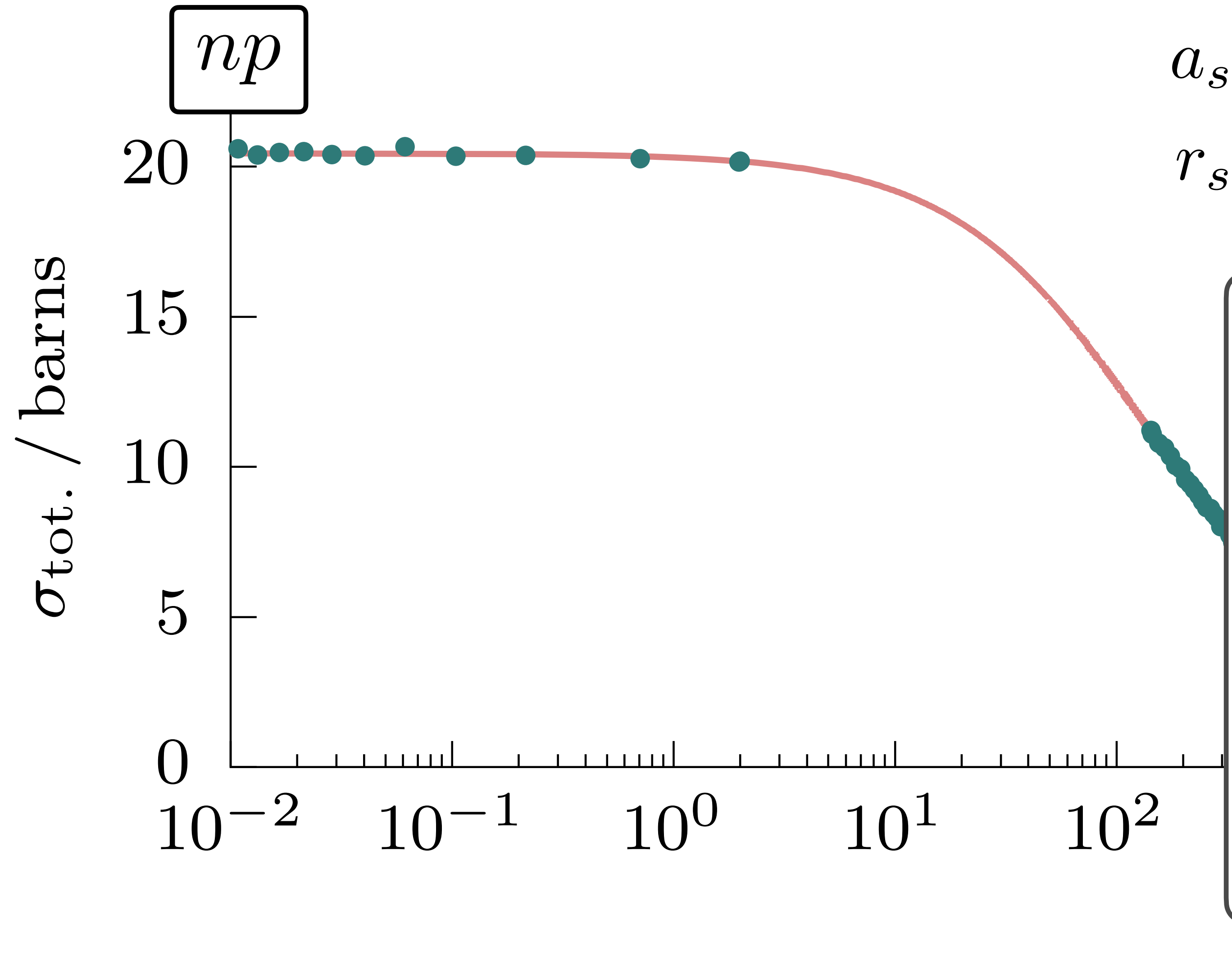

Compare to Hackenberg (2006)

$$\begin{aligned} a_t &= 5.4114 \pm 0.0015\,\text{fm} \\ r_t &= 1.7468 \pm 0.0019\,\text{fm} \\ a_s &= -23.7153 \pm 0.0043\,\text{fm} \\ r_s &= 2.754 \pm 0.059\ \ \text{fm} \end{aligned}$$

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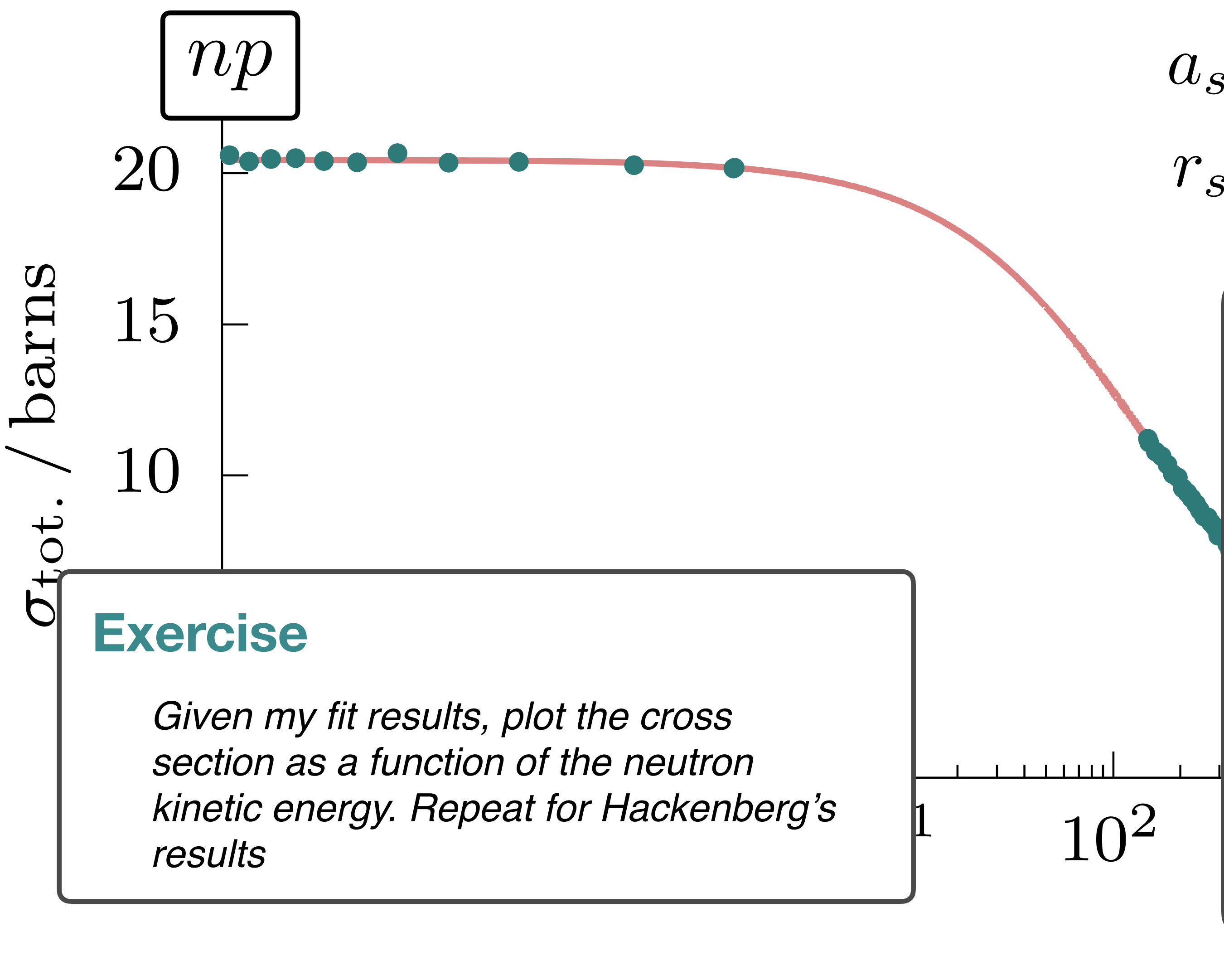

Exercise

Given my fit results, plot the cross section as a function of the neutron kinetic energy. Repeat for Hackenberg's results

Compare to Hackenberg (2006)

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Neutron-Proton Scattering — the Deuteron

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Examine bound-state spectrum

- Focus on spin-triplet results

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$$\mathcal{M}^{-1} = 0 \qquad \Longrightarrow \qquad -\frac{1}{a} + \frac{1}{2} r q^{\star\,2} - i q^{\star} = 0$$

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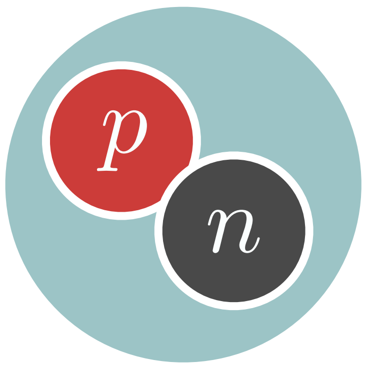
Find a shallow bound state (with my numbers) — the deuteron

$$M_d \approx 1878 \text{ MeV}$$

$$\Gamma_d = 0$$

$$\text{B.E.} = 2.1 \text{ MeV}$$



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Exercise

Given my fit results, compute the deuteron mass. Repeat the exercise for Hackenberg's results

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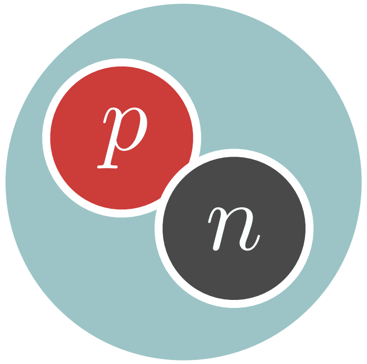
Bound state spectrum determined from scattering analysis

The deuteron is a shallow bound state of triplet S-wave np scattering



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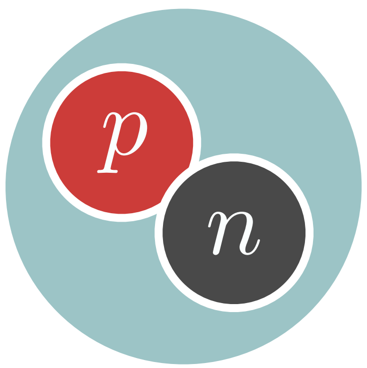

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Spectral properties checklist

- [x] Mass
- [x] Lifetime
- [x] Spin
- [x] Parity
- [x] Charge

...