Physics 101 P

General Physics I

Problem Sessions - Week 12

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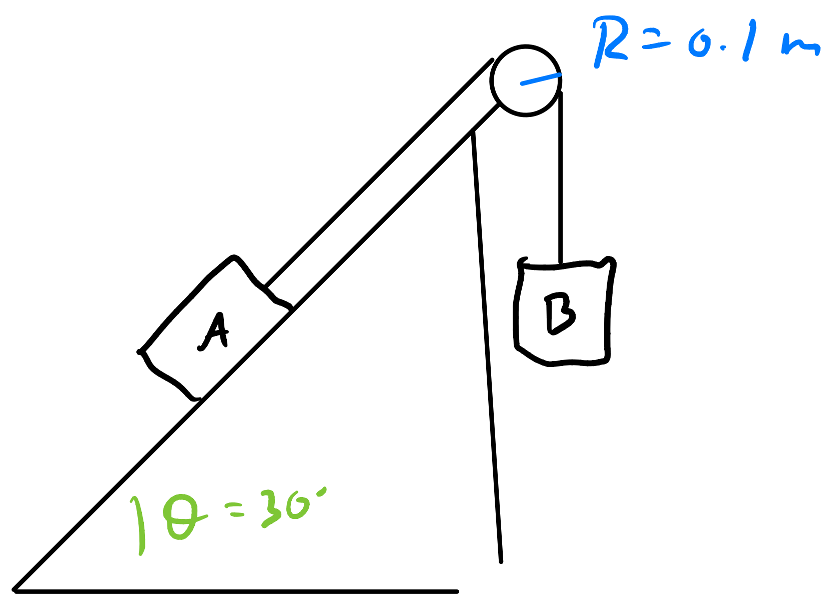
Example

Box A, mass $m_A = 10$ kg, rests on a surface inclined at $\theta = 30°$ to the horizontal. It is connected by a lightweight cord, which passes over a pulley of mass 5 kg and radius 0.1 m, to box B which hangs freely.

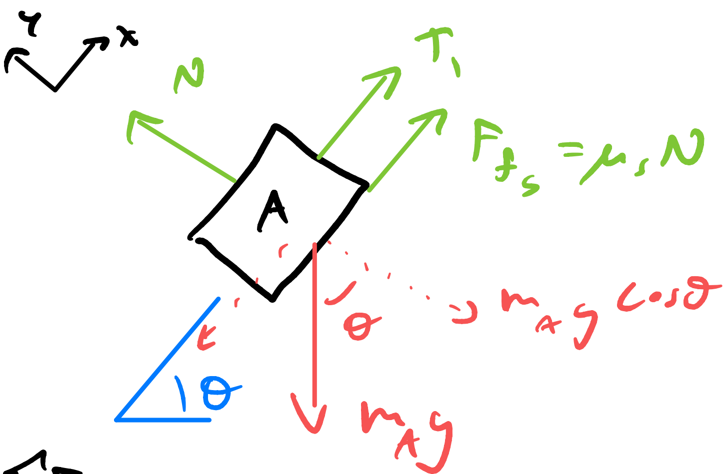
(a) if the coefficient of static friction is $\mu_s = 0.40$, determine the minimum mass of B before it slides down the incline. $I_{disk,cm} = \frac{1}{2}MR^2$



Solution

$$\sum \vec{F} = \vec{0}, \quad \sum \tau = 0$$

FBD A

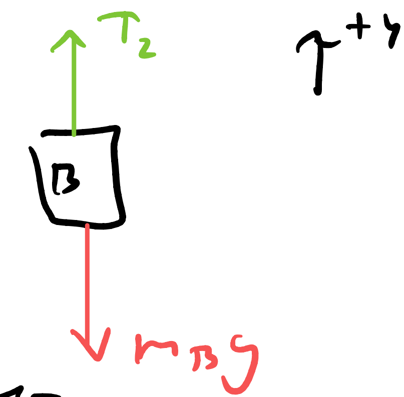


$\sum F = 0$

x: $T_1 + F_{f_s} - m_A g \sin\theta = 0$

y: $N - m_A g \cos\theta = 0$

FBD B

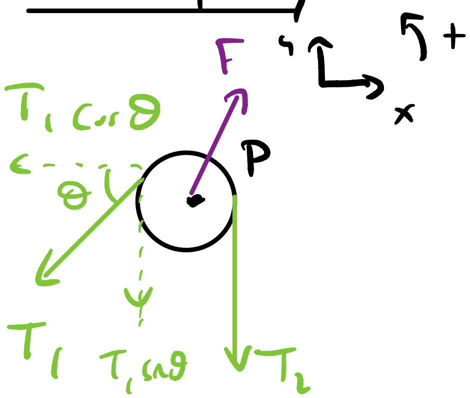


$\sum F = 0$

$T_2 - m_B g = 0$

$\Rightarrow m_B = \frac{T_2}{g}$

FBD pulley



$\sum \tau_p = 0$

$-T_2 R + T_1 R = 0$

$\Rightarrow T_1 = T_2$

So,

$$m_B = \frac{T_2}{g}$$

$$T_1 = T_2$$

$$T_1 + \mu_s N = m_A g \sin\theta$$

$$N = m_A g \cos\theta$$

$$\Rightarrow \quad T_1 = m_A g (\sin\theta - \mu_s \cos\theta)$$

So,

$$m_B = m_A(\sin\theta - \mu_s \cos\theta)$$

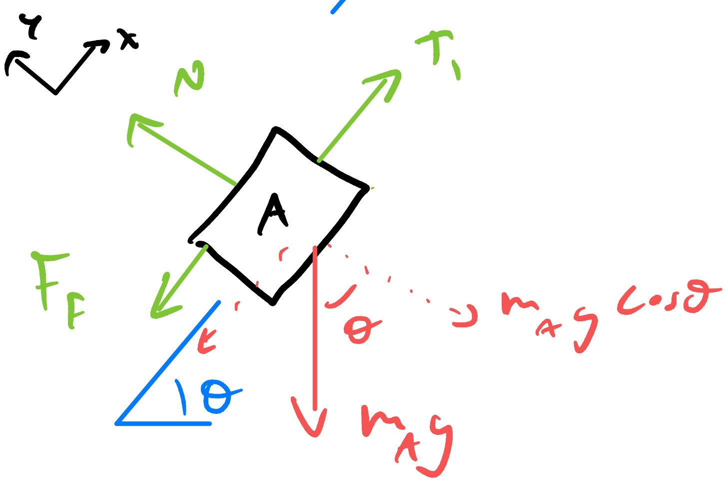
$$\simeq 1.54 \text{ kg} \quad \blacksquare$$

(b) If $m_B = 10$ kg and kinetic friction coefficient is $\mu_k = 0.2$, determine the accelerations of boxes A and B.

Solution

$$a_A = a_B \equiv a$$

FBD_A



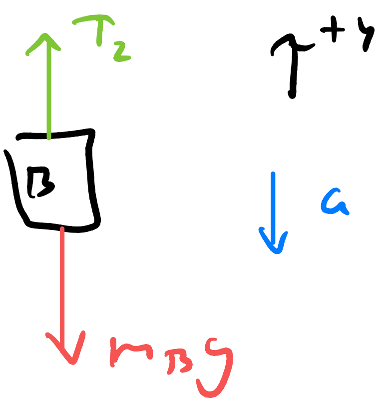
$$\sum F_x = m_A a$$

x: $T_1 - F_f - m_A g \sin\theta = m_A a$

y: $N - m_A g \cos\theta = 0$

and $F_f = \mu_a N$

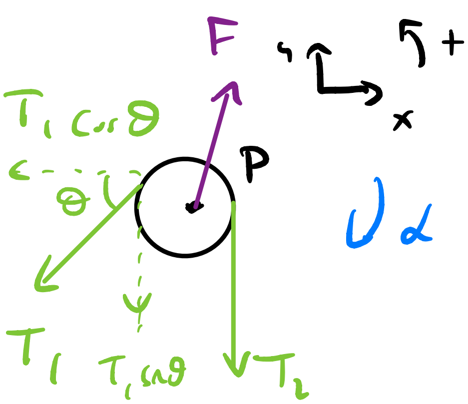
FBD_B



$$\sum F_B = m_B a$$

$$T_2 - m_B g = -m_B a$$

FBD_{pulley}



$$\sum \tau_p = I\alpha$$

$$+T_1 R - T_2 R = -I\alpha$$

$$\begin{cases} T_1 - F_f - m_A g \sin\theta = m_A a \\ N - m_A g \cos\theta = 0 \\ T_2 - m_B g = -m_B a \\ +T_1 R - T_2 R = -I\alpha \end{cases}$$

$\Rightarrow T_1 = m_A a + m_A g \sin\theta$ $+ \mu_k N$

$\Rightarrow N = m_A g \cos\theta$

$\Rightarrow T_2 = m_B g - m_B a$

Now, $F_f = \mu_k N$

$a = R\alpha$

$I = \frac{1}{2} M R^2$

So,

$$\alpha = \frac{(T_2 - T_1)R}{I}$$

$$\Rightarrow \frac{a}{R} = \frac{(T_2 - T_1)}{\frac{1}{2} M R} \Rightarrow \frac{1}{2} M a = T_2 - T_1$$

So, $T_1 = m_A a + m_A g \sin\theta + \mu_k m_A g \cos\theta$

$T_2 = m_B g - m_B a$

So, $\frac{1}{2}Ma = m_B g - m_B a$

$$- (m_A a + m_A g \sin\theta + \mu_k m_A g \cos\theta)$$

$$\Rightarrow -\frac{1}{2}Ma = (m_A + m_B) a + m_A g (\sin\theta + \mu_k \cos\theta) - m_B g$$

$$\Rightarrow \left(m_A + m_B + \frac{1}{2}M\right) a = g\left[m_B - m_A (\sin\theta + \mu_k \cos\theta)\right]$$

$$\Rightarrow a = \frac{g\left[m_B - m_A (\sin\theta + \mu_k \cos\theta)\right]}{m_A + m_B + \frac{1}{2}M}$$

$$\simeq 1.42 \ m/s^2 \quad \blacksquare$$

(c) What is α?

Solution

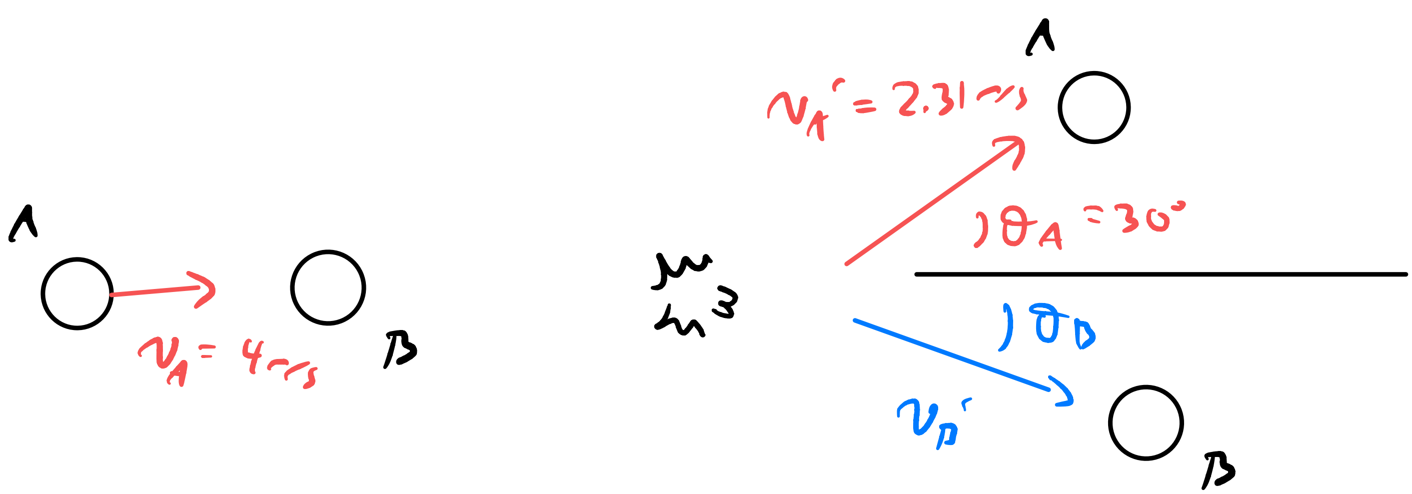
$$\alpha = \frac{a}{R} \simeq 14.2 \ \frac{rad}{s^2} \quad \blacksquare$$

Example

A cue ball with an initial velocity 4.0 m/s is headed toward a stationary billiard ball. After they collide, the cue ball has a velocity of 2.31 m/s in a direction 30° w.r.t. the original direction of the cue ball. The mass of each ball is $m = 0.17$ kg.

(a) Assuming the collision is elastic, what is the final speed of the billiard ball?

(b) What angle does it make w.r.t. the original direction of the cue ball?



Solution

(a) Conservation of momentum

$$x: \quad m v_A = m v_A' \cos\theta_A' + m v_B' \cos\theta_B$$

$$y: \quad 0 = m v_A' \sin\theta_A - m v_B' \sin\theta_B$$

$$KE: \quad v_B'^2 = v_A^2 - v_A'^2$$

$$\Rightarrow \quad v_B' = 3.27 \text{ m/s}$$

(b) $$v_A' \sin\theta_A = v_B' \sin\theta_B$$

$$\Rightarrow \quad \sin\theta_B = \frac{v_A'}{v_B'} \sin\theta_A$$

$$= \frac{2.31}{3.27} \sin 30^\circ$$

$$\simeq 0.35$$

$$\Rightarrow \quad \theta_B = \sin^{-1}(0.35)$$

$$\simeq 20.5^\circ$$

Example

A mass $m = 0.6$ kg hangs at the end of a vertical spring whose top end is fixed to the ceiling. The spring has spring constant $k = 55$ N/m and negligible mass. The mass undergoes simple harmonic motion when placed in vertical motion, with its position given as a function of time by

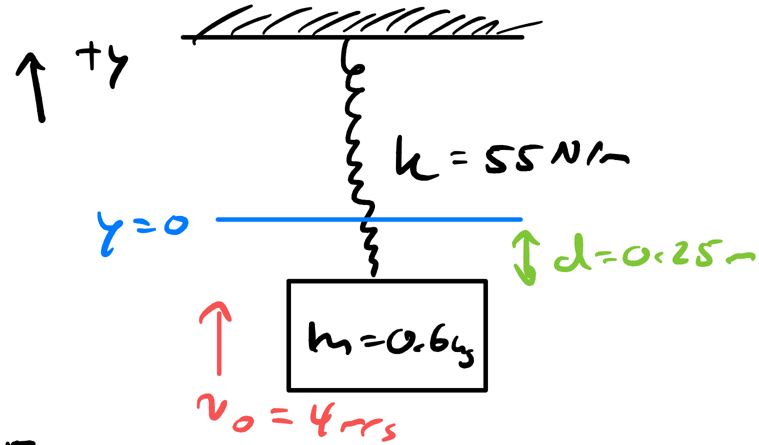
$y(t) = A\cos(\omega t - \varphi)$, with positive y-axis pointing upward. At time $t = 0$, the mass is observed to be at a distance $d = 0.25$ m below its equilibrium height with an upward speed of $v_0 = 4$ m/s.

(a) What is ω?

(b) What is φ and A?

(c) What is its acceleration at $t = 2$ s?

Solution



$$y(t) = A\cos(\omega t - \varphi)$$

(a) Now, $\omega = \sqrt{\frac{k}{m}}$

$$\simeq 9.57 \text{ rad/s}$$

(b) We want A and φ given d and v_0

here, $y(t=0) = -d$

$$v(t=0) = \left.\frac{dy}{dt}\right|_{t=0} = +v_0$$

What is $v(t)$?

$$v(t) = \frac{dy}{dt} = \frac{d}{dt}\left(A\cos(\omega t - \varphi)\right)$$

$$= -A\omega \sin(\omega t - \varphi)$$

So, set up constraints

$$y(t=0) = -d = A\cos(-\varphi)$$

$$v(t) = +v_0 = -A\omega \sin(-\varphi)$$

$$\begin{cases} -d = A\cos(-\varphi) \\ v_0 = -A\omega\sin(-\varphi) \end{cases}$$

Note: $\cos(-x) = \cos x$

$\sin(-x) = -\sin x$

$$\Rightarrow \begin{cases} -d = A\cos\varphi & (1) \\ v_0 = A\omega\sin\varphi & (2) \end{cases}$$

two eqns, two unknowns

Divide (2) by (1)

$$\frac{v_0}{-d} = \frac{A\omega\sin\varphi}{A\cos\varphi} = \omega\tan\varphi$$

$$\Rightarrow \quad \varphi = \tan^{-1}\left(\frac{v_0}{-d\omega}\right)$$

$$\simeq -59.1^\circ$$

$$\simeq -1.03 \text{ rad} \quad \blacksquare$$

Now,

$$A\cos\varphi = -d$$

$$\Rightarrow A = \frac{-d}{\cos\varphi}$$

$$\simeq -0.487\ \text{m} \ \blacksquare$$

(c) $$a(t) = \frac{dv}{dt} = \frac{d}{dt}\left(-A\omega\sin(\omega t - \varphi)\right)$$

$$= -A\omega^2\cos(\omega t - \varphi)$$

So, $$a(t = 2\,\text{s}) = (0.487\ \text{m})\left(9.57\,\tfrac{\text{rad}}{\text{s}}\right)^2$$

$$\times \cos\Big(\underbrace{\left(9.57\,\tfrac{\text{rad}}{\text{s}}\right)2\,\text{s} + 1.03\,\text{rad}}_{\text{Careful w/ calculators!}}\Big)$$

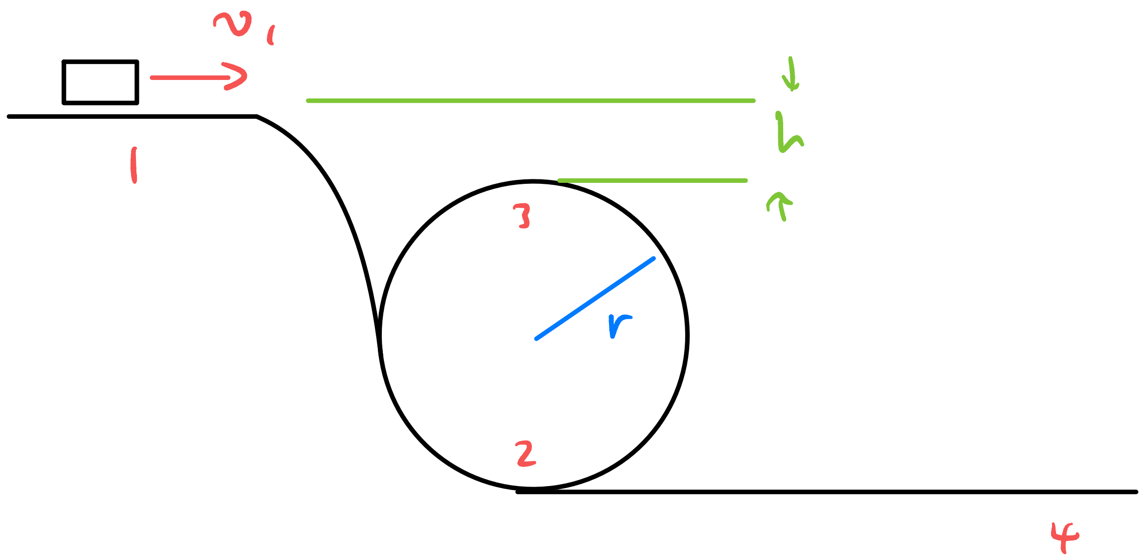
$$\simeq 11.05\ \tfrac{\text{m}}{\text{s}^2} \ \blacksquare$$

Example

A toy car starts at the top of a loop-the-loop at a speed of 2 cm/s. It then rolls without friction along the track (specified by the points 1 2 3 2 4). The car has a mass $m = 200\ g$. The loop has a radius $r = 15\ cm$, and the distance between the initial position and the top of the loop is h.

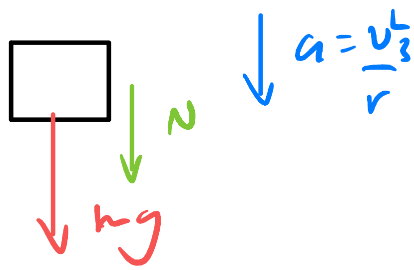
(a) Find the minimum height at which the car can ~~start~~ without falling off the track.

(b) What speed does the car have at position 4, if the height $h = r$.



Solution

At point 3



For minimum length,

$N = 0$

$\Rightarrow \quad \frac{m v_3^2}{r} = mg$

$\Rightarrow \quad v_3^2 = gr$

Now, Conservation of energy

$$E_1 = E_3$$

$$\Rightarrow \quad K_1 + U_1 = K_3 + U_3$$

$$\Rightarrow \quad \frac{1}{2} m v_1^2 + mgh = \frac{1}{2} m v_3^2 + 0$$

$$\Rightarrow \quad h = \frac{1}{2g}\left(v_3^2 - v_1^2\right) = \frac{1}{2g}\left(gr - v_1^2\right)$$

$$h = \frac{1}{2g}\left(gr - v_1^2\right)$$

$$= \frac{r}{2} - \frac{v_1^2}{2g}$$

$$= \frac{15\,cm}{2} - \frac{(2\,cm/s)^2}{2(980\,cm/s^2)}$$

$$\simeq 7.49\,cm \ \blacksquare$$

$$g = 9.8\,m/s^2$$

$$= 9.8\,\frac{m}{s^2}\cdot\left(\frac{100\,cm}{1\,m}\right)$$

$$= 980\,\frac{cm}{s^2}$$

(b)

$$K_1 + U_1 = K_4 + U_4$$

$$\Rightarrow \quad \frac{1}{2}mv_1^2 + mg\Delta y = \frac{1}{2}mv_4^2 + 0$$

$$\Delta y = h + 2r = 3r$$

$$\Rightarrow \quad \frac{1}{2}mv_1^2 + 3mgr = \frac{1}{2}mv_4^2$$

$$\Rightarrow \quad v_4 = \sqrt{v_1^2 + 6gr}$$

$$\simeq \sqrt{(2\,cm/s)^2 + 6(980\,cm/s^2)(15\,cm)}$$

$$\simeq 296.9\,cm/s$$

$$\simeq 2.97\,m/s \ \blacksquare$$