1. Including anti-particles, how many fundamental particles are described within the Standard Model?

Solution: Let us first consider the number of gauge bosons, N_{gauge}. There is 1 photon (γ), 3 intermediate vector bosons ($W^{\pm}, Z^0$), and 8 gluons (g). So, $N_{\text{gauge}} = 1 + 3 + 8 = 12$.

For the number of quarks, N_{quark}, we include antimatter partners ($\times 2$) and color degrees of freedom ($\times 3$). There are three generations for both the up-type and down-type quarks (6 total types). So, $N_{\text{quark}} = 2 \times 3 \times 6 = 36$.

The number of leptons is $N_{\text{lepton}} = 2 \times 6 = 12$, which includes the antiparticle partner for each of the 6 types of leptons (three generations of charged and neutral leptons).

Finally, there is 1 Higgs boson, $N_{\text{Higgs}} = 1$. Therefore, the total number of particles in the Standard Model is

$$\begin{aligned} N_{\text{SM}} &= N_{\text{gauge}} + N_{\text{quark}} + N_{\text{lepton}} + N_{\text{Higgs}}\,, \\ &= 12 + 36 + 12 + 1 = 61\,. \end{aligned}$$

Note: Our counting is not unique, one could include spin-degrees of freedom, or chirality of the matter particles.

2. In units with $\hbar = c = 1$, show that $1\,\text{kg} \approx 5.61 \times 10^{26}$ GeV, $1\,\text{m} \approx 5.07 \times 10^{15}\ \text{GeV}^{-1}$, and $1\,\text{s} \approx 1.52 \times 10^{24}\ \text{GeV}^{-1}$.

Solution: Let $\mathcal{Q}$ be some physical quantity. The dimensions of the physical quantity can be written in the usual SI dimensions

$$[\mathcal{Q}] = \mathsf{M}^{\alpha}\mathsf{L}^{\beta}\mathsf{T}^{\gamma}\,.$$

In natural units, we trade mass (M), length (L), and time (T) dimensions for energy (E) as well as factors of $\hbar$ and c,

$$[\mathcal{Q}] = \mathsf{E}^{A}[\hbar]^{B}[c]^{C}\,.$$

In natural units, we set $\hbar = c = 1$, therefore all mechanical units can be expressed in terms of an energy scale E. Here we choose $\mathsf{E} =$ GeV. We can therefore convert any physical quantity expressed in SI units to natural units be multiplying by an appropriate conversion factor. To find that factor, we use the numerical value of physical constants given in, e.g., the Review of Particle Physics (https://pdg.lbl.gov/2023/reviews/rpp2023-rev-phys-constants.pdf), which gives $1\,\text{GeV} = 1.6022 \times 10^{-10}\,\text{J}$, $\hbar = 1.0546 \times 10^{-34}\,\text{J}\cdot\text{s}$, $c = 2.9979 \times 10^{8}\ \text{m}/\text{s}$, where $1\,\text{J} = 1\,\text{kg}\cdot\text{m}^2/\text{s}^2$, and solve for A, B, and C in terms of α, β, and γ,

$$\mathsf{M}^{\alpha}\mathsf{L}^{\beta}\mathsf{T}^{\gamma} = \mathsf{E}^{A}[\hbar]^{B}[c]^{C}\,.$$

Using $\mathsf{E} = \mathsf{M}\mathsf{L}^2\mathsf{T}^{-2}$, $[\hbar] = \mathsf{E}\mathsf{T}$, and $[c] = \mathsf{L}\mathsf{T}^{-1}$, we find $A = \alpha - \beta - \gamma$, $B = \beta + \gamma$, $C = \beta - 2\alpha$, so

$$\begin{aligned} [\mathcal{Q}] &= \mathsf{E}^{\alpha-\beta-\gamma}[\hbar]^{\beta+\gamma}[c]^{\beta-2\alpha}\,, \\ &= \mathsf{E}^{\alpha-\beta-\gamma}\,. \end{aligned}$$

We define the *true dimension* of the quantity as

$$\text{True Dimension of } \mathcal{Q} \equiv \hbar^{\beta+\gamma} c^{\beta-2\alpha}\ \text{GeV}^{\alpha-\beta-\gamma}$$

The conversion between SI units and natural units is then

$$\begin{aligned}
\text{kg}^{\alpha}\,\text{m}^{\beta}\,\text{s}^{\gamma} =&\ \text{GeV}^{\alpha-\beta-\gamma}\cdot\frac{\text{kg}^{\alpha}\,\text{m}^{\beta}\,\text{s}^{\gamma}}{\text{GeV}^{\alpha-\beta-\gamma}\hbar^{\beta+\gamma}c^{\beta-2\alpha}}\,,\\
\equiv&\ \text{GeV}^{\alpha-\beta-\gamma}\cdot\mathcal{F}^{(\alpha,\beta,\gamma)}\,,
\end{aligned}$$

where $\mathcal{F}^{(\alpha,\beta,\gamma)}$ is defined as the conversion factor.

Therefore, a physical quantity in SI, $\mathcal{Q}_{\text{SI}}$, is given in terms of the natural units $\mathcal{Q}_{\text{Nat.}}$ by

$$\mathcal{Q}_{\text{SI}}^{(\alpha,\beta,\gamma)} = \mathcal{Q}_{\text{Nat.}}^{(\alpha,\beta,\gamma)}\,\mathcal{F}^{(\alpha,\beta,\gamma)}\,.$$

First consider $1\,\text{kg}$, so $\alpha=1$, $\beta=\gamma=0$. So,

$$\begin{aligned}
1\,\text{kg} =&\ \text{GeV}^{1-0-0}\cdot\mathcal{F}^{(1,0,0)}\,,\\
=&\ \text{GeV}\left(\frac{\text{kg}^1}{\text{GeV}^1\hbar^0 c^{-2}}\right)\,,\\
=&\ \text{GeV}\left(\frac{(2.9979\times10^{8})^2\,\text{kg}\cdot\,\text{m}^2/\,\text{s}^2}{1.6022\times10^{-10}\text{J}}\right)\,,\\
=&\ 5.61\times10^{26}\,\text{GeV}\,,
\end{aligned}$$

where in going from the second line to the third we used $1\,\text{GeV} = 1.6022\times10^{-10}\,\text{J}$, $\hbar = 1.0546\times10^{-34}\,\text{J}\cdot\text{s}$, $c = 2.9979\times10^{8}\ \text{m}/\,\text{s}$ and $\text{J} = \text{kg}\cdot\,\text{m}^2/\,\text{s}^2$.
Repeating for $1\,\text{m}$, where $\beta=1$ and $\alpha=\gamma=0$, we find

$$\begin{aligned}
1\,\text{m} =&\ \text{GeV}^{0-1-0}\cdot\mathcal{F}^{(0,1,0)}\,,\\
=&\ \text{GeV}^{-1}\left(\frac{\text{m}^1}{\text{GeV}^{-1}\hbar^1 c^1}\right)\,,\\
=&\ \text{GeV}^{-1}\left(\frac{1.6022\times10^{-10}\text{J}\cdot\,\text{m}}{(1.0546\times10^{-34}\,\text{J}\cdot\,\text{s})(2.9979\times10^{8}\,\text{m}/\,\text{s})}\right)\,,\\
=&\ 5.07\times10^{15}\,\text{GeV}^{-1}\,.
\end{aligned}$$

Finally, for $1\,\text{s}$ we have $\gamma=1$ and $\alpha=\beta=0$, finding

$$\begin{aligned}
1\,\text{s} =&\ \text{GeV}^{0-0-1}\cdot\mathcal{F}^{(0,0,1)}\,,\\
=&\ \text{GeV}^{-1}\left(\frac{\text{s}^1}{\text{GeV}^{-1}\hbar^1}\right)\,,\\
=&\ \text{GeV}^{-1}\left(\frac{1.6022\times10^{-10}\text{J}\cdot\,\text{s}}{1.0546\times10^{-34}\,\text{J}\cdot\,\text{s}}\right)\,,\\
=&\ 1.52\times10^{24}\,\text{GeV}^{-1}\,.
\end{aligned}$$

3. Starting from SI units and setting $\hbar = c = \epsilon_0 = k = 1$, complete Table 1 using only powers of GeV as your units and keeping only three significant figures (k is Boltzmann's constant).

Solution: We follow the same procedure as problem 1, but must extend to include two additional dimensions involving temperature and electric charge. A physical quantity $\mathcal{Q}$ thus has dimensions

$$[\mathcal{Q}] = \mathsf{M}^\alpha \mathsf{L}^\beta \mathsf{T}^\gamma \Theta^\delta \mathsf{Q}^\epsilon \,,$$

where the dimensions are mass (M), length (L), time (T), temperature (Θ), and charge (Q). Converting to natural units, the dimension of interest is energy (E), as well as factors of $\hbar$, c, k, and ϵ_0,

$$[\mathcal{Q}] = \mathsf{E}^A [\hbar]^B [c]^C [k]^D [\epsilon_0]^E \,,$$

where $k = 1.3806 \times 10^{-23}\,\mathrm{J}\cdot\mathrm{K}^{-1}$, $\epsilon_0 = 8.8542 \times 10^{-12}\,\mathrm{C}^2\cdot\mathrm{s}^2\cdot\mathrm{kg}^{-1}\cdot\mathrm{m}^{-3}$. Again, we equate $\mathsf{M}^\alpha \mathsf{L}^\beta \mathsf{T}^\gamma \Theta^\delta \mathsf{Q}^\epsilon = \mathsf{E}^A [\hbar]^B [c]^C [k]^D [\epsilon_0]^E$, and use $\mathsf{E} = \mathsf{M}\mathsf{L}^2\mathsf{T}^2$, $[\hbar] = \mathsf{E}\mathsf{T}$, $[c] = \mathsf{L}\mathsf{T}^{-1}$, $[k] = \mathsf{E}\Theta^{-1}$, and $[\epsilon_0] = \mathsf{Q}^2\mathsf{T}^2\mathsf{M}^{-1}\mathsf{L}^{-3}$, we find

$$[\mathcal{Q}] = \mathsf{E}^{\alpha-\beta-\gamma+\delta}[\hbar]^{\beta+\gamma+\epsilon/2}[c]^{\beta-2\alpha+\epsilon/2}[k]^{-\delta}[\epsilon_0]^{\epsilon/2} \,.$$

The conversion from natural units to SI units is given by

$$\mathcal{Q}_{\mathrm{SI}}^{(\alpha,\beta,\gamma,\delta,\epsilon)} = \mathcal{Q}_{\mathrm{Nat.}}^{(\alpha,\beta,\gamma,\delta,\epsilon)} \mathcal{F}^{(\alpha,\beta,\gamma,\delta,\epsilon)} \,,$$

where the conversion factor is

$$\mathcal{F}^{(\alpha,\beta,\gamma,\delta,\epsilon)} = \frac{\mathrm{kg}^\alpha\,\mathrm{m}^\beta\,\mathrm{s}^\gamma\,\mathrm{K}^\delta\,\mathrm{C}^\epsilon}{\mathrm{GeV}^{\alpha-\beta-\gamma+\delta}\hbar^{\beta+\gamma+\epsilon/2}c^{\beta-2\alpha+\epsilon/2}k^{-\delta}\epsilon_0^{\epsilon/2}} \,.$$

We give two simple examples and leave the rest for an exercise and show the results in the table. Consider $1\,\mathrm{K}$, so $\delta = 1$ and $\alpha = \beta = \gamma = \epsilon = 0$. So,

$$\begin{aligned}
1\,\mathrm{K} =&\ \mathrm{GeV}^{0-0-0+1}\cdot\mathcal{F}^{(0,0,0,1,0)} \,, \\
=&\ \mathrm{GeV}\left(\frac{\mathrm{K}^1}{\mathrm{GeV}^1\hbar^0 c^0 k^{-1}\epsilon_0^0}\right) \,, \\
=&\ \mathrm{GeV}\left(\frac{1.3806\times 10^{-23}\,\mathrm{J}\cdot\mathrm{K}^{-1}\cdot\mathrm{K}}{1.6022\times 10^{-10}\,\mathrm{J}}\right) \,, \\
=&\ 8.62\times 10^{-14}\,\mathrm{GeV} \,.
\end{aligned}$$

Next, consider 1C, with $\epsilon = 1$ and $\alpha = \beta = \gamma = \delta = 0$. Here, we have

$$\begin{aligned}
1\,\mathrm{C} =&\ \mathrm{GeV}^{0-0-0+0}\cdot\mathcal{F}^{(0,0,0,0,1)} \,, \\
=&\ \mathrm{GeV}^0\left(\frac{\mathrm{C}^1}{\mathrm{GeV}^0\hbar^{1/2}c^{1/2}k^0\epsilon_0^{1/2}}\right) \,, \\
=&\ \frac{1\,\mathrm{C}}{(1.0546\times 10^{-34}\,\mathrm{J}\cdot\mathrm{s})^{1/2}(2.9979\times 10^8\,\mathrm{m}/\mathrm{s})^{1/2}(8.8542\times 10^{-12}\,\mathrm{C}^2\cdot\mathrm{s}^2\cdot\mathrm{kg}^{-1}\cdot\mathrm{m}^{-3})^{1/2}} \,, \\
=&\ 1.89\times 10^{18} \,.
\end{aligned}$$

4. Starting from SI units and setting $\hbar = c = \epsilon_0 = k = 1$, complete Table 2 using only powers of GeV as your units and keeping only three significant figures.

Solution: In the same way as a physical quantity, a *physical constant* $\mathcal{C}$ expressed in SI units can be converted to natural units. We have worked out the conversion between the units in the previous problems, therefore we can multiply by "1" in the form

$$1 = \frac{\text{GeV}^{\alpha-\beta-\gamma+\delta}}{\text{GeV}^{\alpha-\beta-\gamma+\delta}\hbar^{\beta+\gamma+\epsilon/2}c^{\beta-2\alpha+\epsilon/2}k^{-\delta}\epsilon_0^{\epsilon/2}}\,,$$

where the constants $\hbar = c = k = \epsilon_0 = 1$ in the numerator. So, we can write the constant in natural units as

$$\mathcal{C}_{\text{Nat.}}^{(\alpha,\beta,\gamma,\delta,\epsilon)} = \mathcal{C}_{\text{SI}}^{(\alpha,\beta,\gamma,\delta,\epsilon)}\left(\frac{\text{GeV}^{\alpha-\beta-\gamma+\delta}}{\text{GeV}^{\alpha-\beta-\gamma+\delta}\hbar^{\beta+\gamma+\epsilon/2}c^{\beta-2\alpha+\epsilon/2}k^{-\delta}\epsilon_0^{\epsilon/2}}\right).$$

Let's look at two non-trivial examples. First is Newton's constant for universal gravitation, $G_N|_{\text{SI}} = 6.67\times10^{-11}\,\text{m}^3\cdot\text{kg}^{-1}\cdot\text{s}^{-2}$. We have

$$\begin{aligned}
G_N\Big|_{\text{Nat.}} &= G_N\Big|_{\text{SI}}\left(\frac{\text{GeV}^{-1-3+2+0}}{\text{GeV}^{-1-3+2+0}\hbar^{3-2+0/2}c^{3+2+0/2}k^0\epsilon_0^0}\right),\\
&= \left(6.67\times10^{-11}\,\frac{\text{m}^3}{\text{kg}\cdot\text{s}^2}\right)\left(\frac{\text{GeV}^{-2}}{\text{GeV}^{-2}\hbar^1c^5}\right),\\
&= \frac{6.67\times10^{-11}\,\text{m}^3\cdot\text{kg}^{-1}\cdot\text{s}^{-2}\cdot\text{GeV}^{-2}}{(1.6022\times10^{-10}\,\text{J})^{-2}(1.0546\times10^{-34}\,\text{J}\cdot\text{s})(2.9979\times10^8\,\text{m/s})^5}\,,\\
&= 6.70\times10^{-39}\,\text{GeV}^{-2}\,.
\end{aligned}$$

So, in natural units, $G_N|_{\text{Nat.}} = 6.70\times10^{-39}\,\text{GeV}^{-2}$ and its true dimension is $\hbar c^5\cdot\text{GeV}^{-2}$.

A second example is the fundamental charge, $e|_{\text{SI}} \approx 1.602\times10^{-19}\,\text{C}$. In natural units, we find

$$\begin{aligned}
e\Big|_{\text{Nat.}} &= e\Big|_{\text{SI}}\left(\frac{\text{GeV}^{0-0-0+0}}{\text{GeV}^{0-0-0+0}\hbar^{0+0+1/2}c^{0-0+1/2}k^0\epsilon_0^{1/2}}\right),\\
&= (1.602\times10^{-19}\,\text{C})\left(\frac{1}{(\hbar c\epsilon_0)^{1/2}}\right),\\
&= \frac{1.602\times10^{-19}\,\text{C}}{(1.0546\times10^{-34}\,\text{J}\cdot\text{s})^{1/2}(2.9979\times10^8\,\text{m/s})^{1/2}(8.8542\times10^{-12}\,\text{C}^2\cdot\text{s}^2\cdot\text{kg}^{-1}\cdot\text{m}^{-3})^{1/2}}\,,\\
&= 3.02\times10^{-1}\,.
\end{aligned}$$

Therefore, the elementary charge in natural units is dimensionless. All the remaining constants in Table 2 can be found in the same way.

5. Using the current *Review of Particle Physics* particle listings or the summary tables (Particle Data

Group, https://pdg.lbl.gov), answer the following questions. Quote uncertainties for all data where it is given.

(a) What are the valence quarks of the D^+ meson? What is its spin-parity (J^P) quantum numbers? What is its mass in MeV? What is its lifetime τ in seconds? Convert the lifetime to a decay width Γ and quote your answer in MeV ($\Gamma = \hbar/\tau$). What is the significance of the quantity $c\tau = 309.8\,\mu\text{m}$ quoted in the PDG listings for experiments?

Solution: Consulting the Charm Meson Summary Table (https://pdg.lbl.gov/2023/tables/rpp2023-tab-mesons-charm.pdf), we find for the D^+ the following: The D^+ has $c\bar{d}$ valence quark content, and is a pseudoscalar meson, $J^P = 0^-$. It's mass is $m_{D^+} = 1869.66 \pm 0.05\,\text{MeV}$, while it's lifetime is $\tau = (1033 \pm 5) \times 10^{-15}\,\text{s}$. It's decay width is given by

$$\Gamma = \frac{\hbar}{\tau} = \frac{6.58 \times 10^{-22}\,\text{MeV} \cdot s}{1033 \times 10^{-15}\,\text{s}} = 6.37 \times 10^{-10}\,\text{MeV}\,,$$

with an uncertainty

$$\delta\Gamma = \Gamma \cdot \frac{\delta\tau}{\tau} = (6.37 \times 10^{-10}\,\text{MeV}) \cdot \frac{5}{1033} = 3 \times 10^{-12}\,\text{MeV}\,.$$

This gives $\Gamma = (6.37 \pm 0.03) \times 10^{-10}\,\text{MeV}$. A D^+ produced in a modern collider will be relativistic. Thus its speed will be $v \approx c$, and $c\tau$ represents the distance it will cover in its lifetime. For the D^+, $c\tau = 309.8\,\mu\text{m}$, which is long enough for a modern vertex detector to be sensitive to it, although it is near the limit of capabilities.

(b) What are the branching ratios $\text{BR}(\tau^- \to \mu^-\bar{\nu}_\mu\nu_\tau)$ and $\text{BR}(\tau^- \to e^-\bar{\nu}_e\nu_\tau)$? Note that the branching ratio is defined as the ratio $\text{BR}(X \to R_1R_2\cdots) \equiv \Gamma(X \to R_1R_2\cdots)/\Gamma$, where $\Gamma(X \to R_1R_2\cdots)$ is the *partial decay width* and Γ is the total decay width. What are the partial decay widths $\Gamma(\mu^- \to e^-\bar{\nu}_e\nu_\mu)$ and $\Gamma(\mu^- \to e^-\bar{\nu}_e\nu_\mu\gamma)$ in MeV?

Solution: From the Leptons Summary Table (https://pdg.lbl.gov/2023/tables/rpp2023-sum-leptons.pdf), the branching ratios are

$$\text{BR}(\tau^- \to \mu^-\bar{\nu}_\mu\nu_\tau) = (17.39 \pm 0.04)\%\,,$$

$$\text{BR}(\tau^- \to e^-\bar{\nu}_e\nu_\tau) = (17.82 \pm 0.04)\%\,.$$

To find the partial decay widths, we need the total decay width. This is given by $\Gamma = \hbar/\tau$, where τ is the lifetime. The summary table gives the mean lifetime as $(290.3\pm0.5)\times10^{-15}\,\text{s}$. The decay width is then

$$\Gamma = \frac{\hbar}{\tau} = \frac{6.58 \times 10^{-22}\,\text{MeV} \cdot s}{290.3 \pm 0.5) \times 10^{-15}\,\text{s}} = 2.267 \times 10^{-9}\,\text{MeV}\,,$$

and

$$\delta\Gamma = \Gamma \cdot \frac{\delta\tau}{\tau} = (2.27 \times 10^{-9}\,\text{MeV}) \cdot \frac{0.5}{290.3} = 4 \times 10^{-12}\,\text{MeV}\,,$$

or $\Gamma = (2.267 \pm 0.004) \times 10^{-9}\,\text{MeV}$. The the partial decay widths are

$$\Gamma(\tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau) = \text{BR}(\tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau) \cdot \Gamma = (3.94 \pm 0.01) \times 10^{-8}\,\text{MeV}\,,$$

$$\Gamma(\tau^- \to e^- \bar{\nu}_e \nu_\tau) = \text{BR}(\tau^- \to e^- \bar{\nu}_e \nu_\tau) \cdot \Gamma = (4.04 \pm 0.01) \times 10^{-8}\,\text{MeV}\,,$$

where the error was computed as

$$\delta\Gamma(\tau^- \to \ell^- \nu_\ell \nu_\tau) = \Gamma(\tau^- \to \ell^- \nu_\ell \nu_\tau) \cdot \sqrt{\left(\frac{\delta \text{BR}(\tau^- \to \ell^- \nu_\ell \nu_\tau)}{\text{BR}(\tau^- \to \ell^- \nu_\ell \nu_\tau)}\right)^2 + \left(\frac{\delta\Gamma}{\Gamma}\right)^2}\,.$$

For the partial widths of $\mu^- \to e^- \bar{\nu}_e \nu_\mu$ and $\mu^- \to e^- \bar{\nu}_e \nu_\mu \gamma$, we repeat the same procedure as above. Note that the lifetime of the μ^- is $\tau = (2.1969811 \pm 0.0000022) \times 10^{-6}$ s, so the total decay width is

$$\Gamma = \frac{\hbar}{\tau} = \frac{6.58 \times 10^{-22}\,\text{MeV} \cdot s}{2.1969811 \times 10^{-6}\,\text{s}} = 2.995940 \times 10^{-16}\,\text{MeV}\,,$$

while its uncertainty is

$$\delta\Gamma = \Gamma \cdot \frac{\delta\tau}{\tau} = (2.995940 \times 10^{-16}\,\text{MeV}) \cdot \frac{0.0000022}{2.1969811} = 3.0 \times 10^{-22}\,\text{MeV}\,.$$

The branching ratios for the two decay modes are reported as

$$\text{BR}(\mu^- \to e^- \bar{\nu}_e \nu_\mu) \approx 100\%\,,$$

$$\text{BR}(\mu^- \to e^- \bar{\nu}_e \nu_\mu \gamma) = (6.0 \pm 0.5) \times 10^{-8}\,.$$

So, the partial widths are

$$\Gamma(\mu^- \to e^- \bar{\nu}_e \nu_\mu) = \text{BR}(\mu^- \to e^- \bar{\nu}_e \nu_\mu) \cdot \Gamma \approx (2.995940 \pm 0.000003) \times 10^{-16}\,\text{MeV}\,,$$

$$\Gamma(\mu^- \to e^- \bar{\nu}_e \nu_\mu \gamma) = \text{BR}(\mu^- \to e^- \bar{\nu}_e \nu_\mu \gamma) \cdot \Gamma = (1.80 \pm 0.06) \times 10^{-23}\,\text{MeV}\,.$$

(c) What are the mass and width of the top quark? Does it decay via the strong, weak, or electromagnetic interactions? Why are there no listings for mesons and baryons containing top quarks?

Solution: From the Quark Summary Table (https://pdg.lbl.gov/2023/tables/rpp2023-sum-quarks.pdf), the top quark mass is $m_t = 172.69 \pm 0.30\,\text{GeV}$ (quoting the *direct measurement* entry), while the width is $\Gamma_t = 1.42^{+0.19}_{-0.15}\,\text{GeV}$. The only observed decay mode is $t \to Wb$, which is a weak interaction. The lifetime of the top quark is $\tau = \hbar/\Gamma = 6.58 \times 10^{-25}\,\text{GeV} \cdot s/1.42\,\text{GeV} = 4.63 \times 10^{-25}\,\text{s}$. The typical timescale for strong interaction is $t_{\text{strong}} \sim 10^{-23}\,\text{s}$, which is two orders of magnitude larger than the $t \to Wb$ decay. This means that the top quark decays before it can *hadronize* into a QCD bound state.

(d) What are the valence quarks of the Λ^0 and Λ_c^+ baryons? What are their spins and parities? What are the branching ratios for their semileptonic decay modes into muons and electrons? Can you explain why these branching fractions are very similar for Λ_c^+ and very different for Λ^0?

Solution: From the Baryons Summary Table (https://pdg.lbl.gov/2023/tables/rpp2023-sum-baryons.pdf), the valence quarks of Λ^0 are uds while those for the Λ_c^+ are udc. Both baryons are spin-1/2 fermions, $J^P = 1/2^+$. Each of these baryons have two semileptonic decay modes. For Λ^0 we find $\Lambda^0 \to p\ell^-\bar{\nu}_\ell$ with $\ell = \{e, \mu\}$, while for Λ_c^+ we have $\Lambda^+ \to \Lambda^0\ell^+\nu_\ell$ with $\ell = \{e, \mu\}$. The branching ratios are observed to be

$$\mathrm{BR}(\Lambda^0 \to pe^-\bar{\nu}_e) = (8.34 \pm 0.14) \times 10^{-4}\,,$$

$$\mathrm{BR}(\Lambda^0 \to p\mu^-\bar{\nu}_\mu) = (1.51 \pm 0.19) \times 10^{-4}\,,$$

$$\mathrm{BR}(\Lambda^+ \to \Lambda^0 e^+\nu_e) = (3.56 \pm 0.13)\% = (3.56 \pm 0.13) \times 10^{-2}\,,$$

$$\mathrm{BR}(\Lambda^+ \to \Lambda^0 \mu^+\nu_\mu) = (3.5 \pm 0.5)\% = (3.5 \pm 0.5) \times 10^{-2}\,.$$

All of these decays are weak decays, which have a universal coupling, and therefore cannot be the origin of the two orders of magnitude in the branching ratios. The reason for the difference is the phase space of the decay, due to the mass scales. Consider first $m_{\Lambda^0} - m_p \approx 180\,\mathrm{MeV} \sim m_\mu$, while $m_{\Lambda_c^+} - m_{\Lambda^0} \approx 1.170\,\mathrm{GeV} \gg m_\mu$. Therefore, the Λ^0 semileptonic decays are kinematically suppressed, whereas the Λ_c^+ modes have a large phase space.

(e) What is the branching ratio for the Higgs boson decay into two photons? Estimate the partial decay width $\Gamma(H^0 \to \gamma\gamma)$. What is the significance of the quantity $P = 62\,625$ MeV/c quoted in the PDG listing? The Higgs was first discovered in this channel at the Large Hadron Collider, even though its branching ratio is very small. Can you explain why this was so?

Solution: From the Gauge and Higgs boson Summary Table (https://pdg.lbl.gov/2023/tables/rpp2023-sum-gauge-higgs-bosons.pdf), the branching ratio for a Higgs decay to two photons is

$$\mathrm{BR}(H^0 \to \gamma\gamma) = (2.50 \pm 0.20) \times 10^{-3}.$$

Since the full width is measured to be $\Gamma = 3.2^{+2.4}_{-1.7}\,\mathrm{MeV}$, we can estimate the partial decay width of $H^0 \to \gamma\gamma$ to be

$$\Gamma(H^0 \to \gamma\gamma) = \mathrm{BR}(H^0 \to \gamma\gamma) \cdot \Gamma = (8.0) \times 10^{-3}\,\mathrm{MeV}$$

The quantity $P = 62\,625\,\mathrm{MeV}/c$ is the momentum of a daughter photon in the rest frame of the Higgs. That is, by conservation of energy we know $m_{H^0} = 2E_\gamma$ and $E_\gamma = cP$ for a photon. Since $m_{H^0} \approx 125.25\,\mathrm{GeV}$, we know $P = m_{H^0}/(2c) \approx 62.625\,\mathrm{GeV}/c = 62\,625\,\mathrm{MeV}/c$.

At the Large Hadron Collider, two proton beams collide in their CM frame at $s \approx 14$TeV. At such energetic collisions, many particles are produced which subsequently decay by various hadronic and electroweak modes. For $H^0 \to \gamma\gamma$, there is low background noise from other decay products, making this channel relatively clean to discover the Higgs boson despite its low branching ratio.

Table 1: Physical quantities in SI and in natural units.

Physical Quantity	**SI Unit**	**Natural Unit**	**True Dimension**
mass	kg	$5.61 \times 10^{26}\,\mathrm{GeV}$	$c^{-2} \cdot \mathrm{GeV}$
length	m	$5.07 \times 10^{15}\,\mathrm{GeV}^{-1}$	$\hbar c \cdot \mathrm{GeV}^{-1}$
time	s	$1.52 \times 10^{24}\,\mathrm{GeV}^{-1}$	$\hbar \cdot \mathrm{GeV}^{-1}$
charge	C	1.89×10^{18}	$(\epsilon_0 \hbar c)^{1/2}$
temperature	K	$8.62 \times 10^{-14}\,\mathrm{GeV}$	$k^{-1} \cdot \mathrm{GeV}$
frequency	Hz	$6.58 \times 10^{-25}\,\mathrm{GeV}$	$\hbar^{-1} \cdot \mathrm{GeV}$
force	N	$1.23 \times 10^{-6}\,\mathrm{GeV}^2$	$(\hbar c)^{-1} \cdot \mathrm{GeV}^2$
pressure	Pa	$4.80 \times 10^{-38}\,\mathrm{GeV}^4$	$(\hbar c)^3 \cdot \mathrm{GeV}^4$
angular momentum	$\mathrm{kg} \cdot \mathrm{m}^2/\mathrm{s}$	9.48×10^{33}	$\hbar$
energy	J	$6.24 \times 10^{9}\,\mathrm{GeV}$	GeV
power	W	$4.11 \times 10^{-15}\,\mathrm{GeV}^2$	$\hbar^{-1} \cdot \mathrm{GeV}^2$
scalar potential	V	$3.30 \times 10^{-9}\,\mathrm{GeV}$	$(\epsilon_0 \hbar c)^{-1/2} \cdot \mathrm{GeV}$
electric field	V/m	$6.52 \times 10^{-25}\,\mathrm{GeV}^2$	$[\epsilon_0 (\hbar c)^3]^{-1/2} \cdot \mathrm{GeV}^2$
current	A	$1.24 \times 10^{-6}\,\mathrm{GeV}$	$[\epsilon_0 \hbar^{-1} c]^{1/2} \cdot \mathrm{GeV}$
vector potential	$\mathrm{T} \cdot \mathrm{m}$	$9.90 \times 10^{-1}\,\mathrm{GeV}$	$(\epsilon_0 \hbar c^3)^{-1/2} \cdot \mathrm{GeV}$
magnetic induction	T	$1.95 \times 10^{-16}\,\mathrm{GeV}^2$	$(\epsilon_0 \hbar^3 c^5)^{-1/2} \cdot \mathrm{GeV}^2$
magnetic flux	Wb	5.02×10^{15}	$[(\epsilon_0 c)^{-1} \hbar]^{1/2}$
resistance	Ω	2.65×10^{-3}	$(\epsilon_0 c)^{-1}$
capacitance	F	$5.72 \times 10^{26}\,\mathrm{GeV}^{-1}$	$\epsilon_0 \hbar c \cdot \mathrm{GeV}^{-1}$
inductance	H	$4.03 \times 10^{21}\,\mathrm{GeV}^{-1}$	$\hbar (\epsilon_0 c)^{-1} \cdot \mathrm{GeV}^{-1}$

Table 2: Physical constants in SI and in natural units.

Physical Quantity	**Value (SI Units)**	**Value (Natural Units)**	**True Dimension**
Planck constant, $\hbar$	$1.05 \times 10^{34}\,\mathrm{J} \cdot \mathrm{s}$	1	$\hbar$
speed of light, c	$3.00 \times 10^{8}\,\mathrm{m/s}$	1	c
permittivity, ϵ_0	$8.85 \times 10^{-12}\,\mathrm{F/m}$	1	ϵ_0
Boltzmann constant, k	$1.38 \times 10^{-23}\,\mathrm{J/K}$	1	k
Planck constant, h	$2\pi\,\hbar = 6.63 \times 10^{34}\,\mathrm{J} \cdot \mathrm{s}$	2π	$\hbar$
Newton constant, G_N	$6.67 \times 10^{-11}\,\mathrm{m}^3 \cdot \mathrm{kg}^{-1} \cdot \mathrm{s}^{-2}$	$6.70 \times 10^{-39}\,\mathrm{GeV}^{-2}$	$\hbar c^5 \cdot \mathrm{GeV}^{-2}$
$m_{\mathrm{Planck}} = \sqrt{\hbar c/G_N}$	$2.18 \times 10^{-8}\,\mathrm{kg}$	$1.22 \times 10^{19}\,\mathrm{GeV}$	$c^{-2} \cdot \mathrm{GeV}$
$l_{\mathrm{Planck}} = \sqrt{\hbar G_N/c^3}$	$1.62 \times 10^{-35}\,\mathrm{m}$	$8.19 \times 10^{-20}\,\mathrm{GeV}^{-1}$	$\hbar c \cdot \mathrm{GeV}^{-1}$
$t_{\mathrm{Planck}} = \sqrt{\hbar G_N/c^5}$	$5.39 \times 10^{-44}\,\mathrm{s}$	$8.19 \times 10^{-20}\,\mathrm{GeV}^{-1}$	$\hbar \cdot \mathrm{GeV}^{-1}$
$\rho_{\mathrm{Planck}} = c^5/\hbar G_N^2$	$5.16 \times 10^{96}\,\mathrm{kg} \cdot \mathrm{m}^{-3}$	$2.22 \times 10^{76}\,\mathrm{GeV}^4$	$\hbar^{-3} c^{-5} \cdot \mathrm{GeV}^4$
permeability, μ_0	$1.26 \times 10^{-6}\,\mathrm{N} \cdot \mathrm{A}^{-2}$	1	$\epsilon_0^{-1} c^{-2}$
electron charge, e	$1.60 \times 10^{-19}\,\mathrm{C}$	3.02×10^{-1}	$(\epsilon_0 \hbar c)^{1/2}$
fine structure constant, α	7.30×10^{-3}	7.30×10^{-3}	1
electron mass, m_e	$9.11 \times 10^{-31}\,\mathrm{kg}$	$5.11 \times 10^{-4}\,\mathrm{GeV}$	$c^{-2} \cdot \mathrm{GeV}$
proton mass, m_p	$1.67 \times 10^{-27}\,\mathrm{kg}$	$9.37 \times 10^{-1}\,\mathrm{GeV}$	$c^{-2} \cdot \mathrm{GeV}$
Bohr radius, a_0	$5.29 \times 10^{-11}\,\mathrm{m}$	$2.68 \times 10^{5}\,\mathrm{GeV}^{-1}$	$\hbar c \cdot \mathrm{GeV}^{-1}$
Rydberg, Ry	$2.18 \times 10^{-18}\,\mathrm{J}$	$1.36 \times 10^{-8}\,\mathrm{GeV}$	GeV
Bohr magneton, μ_e	$9.27 \times 10^{-24}\,\mathrm{J/T}$	$2.96 \times 10^{2}\,\mathrm{GeV}^{-1}$	$[\epsilon_0 \hbar^3 c^5]^{1/2} \cdot \mathrm{GeV}^{-1}$
nuclear magneton, μ_p	$5.05 \times 10^{-27}\,\mathrm{J/T}$	$1.61 \times 10^{-1}\,\mathrm{GeV}^{-1}$	$[\epsilon_0 \hbar^3 c^5]^{1/2} \cdot \mathrm{GeV}^{-1}$
Stefan constant, σ	$5.67 \times 10^{-8}\,\mathrm{W} \cdot \mathrm{m}^{-2} \cdot \mathrm{K}^{-4}$	1.64×10^{-1}	$k^4 \hbar^{-3} c^{-2}$
Fermi constant, $G_F/\hbar^3 c^3$	$3.68 \times 10^{48}\,\mathrm{kg}^{-2}$	$1.17 \times 10^{-5}\,\mathrm{GeV}^{-2}$	GeV^{-2}
Higgs condensate, $v = \sqrt{(\hbar c)^3/\sqrt{2} G_F}$	$3.94 \times 10^{-8}\,\mathrm{J}$	246 GeV	GeV