Hadrons

Beginning in the 1950's, there were 100's of particles being discovered. It was observed that these new particles did not interact like electrons or photons, and had features similar to the proton and neutron. These particles were called hadrons. Attempts to understand the "Hadron zoo" led to the applications of symmetry to understand their fundamental interactions.

Hadrons come in two types: Mesons and Baryons

To distinguish these two, we introduce the quantum number Baryon number B_n

We assign $$B_n = \begin{cases} +1 & \text{for Baryons} \\ -1 & \text{for Anti Baryons} \end{cases}$$

Mesons have $B_n = 0$.

Baryon number is always conserved!

$$\boxed{\Delta B_n = 0}$$

In the SM, Baryon number is represented by a global $U(1)$ symmetry of QCD.

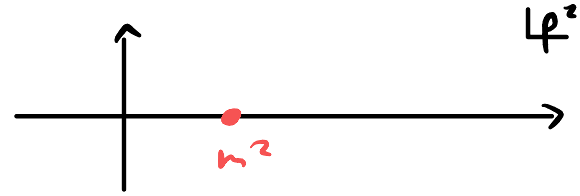
The fact that $\Delta B_n = 0$ in reactions means that reactions like $p \to e^+ + \pi^0$ are not observed!

In the universe we observe more matter than antimatter. Baryogenesis is the generation of this asymmetry. One of the conditions for such a process is $\Delta B_n \neq 0$ (along with C & CP violation).

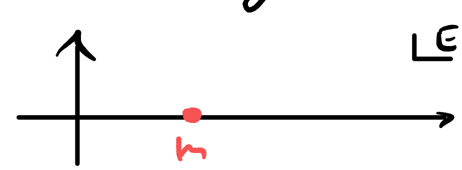
Resonances

How do we detect these short-lived hadrons? Strongly interacting hadrons are resonances of scattering processes. Consider a propagator for a stable particle,

$$i\Delta = \frac{i}{p^2 - m^2}$$



The associated wave function is $\langle 0 | \hat{\varphi}(x) | \vec{p} \rangle = e^{-ip\cdot x}$. For a state at rest, $p = (E, \vec{0})$, and for energies near the pole, $i\Delta \sim \dfrac{i}{2m(E-m)}$



In NRQM, the wave function gives probability amplitude,

$$\psi^* \psi \sim e^{imt} e^{-imt} = 1$$

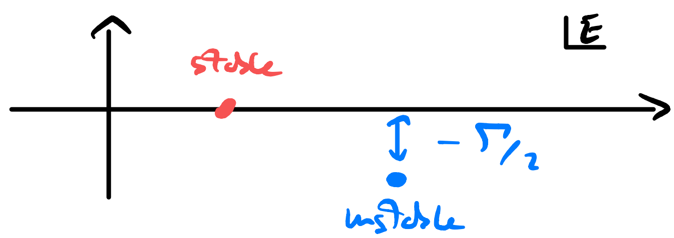
What if a particle decay?

Expect $\psi^* \psi \sim e^{-t/\tau}$ ← Lifetime of the particle

This effectively shifts the pole into the complex energy plane

$$i\Delta \sim \frac{i}{2m(E - m + i\Gamma/2)}$$

where Γ is the decay width, $\Gamma = 1/\tau$



wave function, $\psi = e^{-iE_{\text{pole}}} = e^{-i(m - i\Gamma/2)}$

$$\Rightarrow \quad \psi^* \psi = e^{i(m+i\Gamma/2)t} e^{-i(m-i\Gamma/2)t}$$

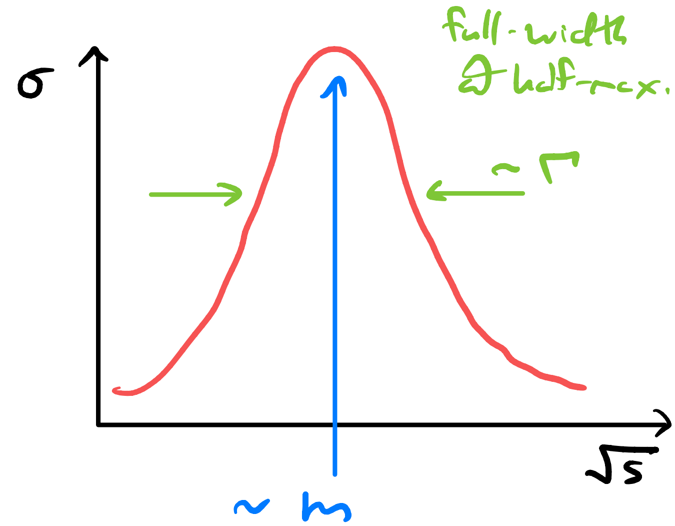
$$= e^{-\Gamma/2\, t} e^{-\Gamma/2\, t}$$

$$= e^{-\Gamma t} = e^{-t/\tau} \quad ✓$$

⇒ Unstable hadrons are pole singularities of Green's functions or scattering amplitudes in the complex energy plane

For a narrow, isolated resonance, the Breit-Wigner amplitude phenomenologically parameterizes the amplitude

$$i\mathcal{M}_{BW} \sim \frac{1}{s - m^2 + im\Gamma}$$



So that

some constant

$$\sigma_{BW} = \frac{A}{(s-m^2)^2 + m^2\Gamma^2}$$

Note: $s = m^2 - im\Gamma$

$$\Rightarrow E = m\sqrt{1 - i\Gamma/m}$$

$$\simeq m - i\Gamma/2$$ assumes $m \ll \Gamma$

Isospin

There are other useful "symmetries" to unravel hadron physics. Let's consider first the proton and the neutron.

It is observed that $m_p \simeq m_n$

$$\left.\begin{array}{l} m_p \simeq 938.3 \text{ MeV} \\ m_n \simeq 939.6 \text{ MeV} \end{array}\right\} \Rightarrow \frac{\Delta m}{m} \sim 0.14\ \%$$

This suggests that p, n as having an "approximate" symmetry under strong interaction properties

$\Rightarrow$ p, n are 2 aspects of "single" particle N, "nucleon"

$$N = \begin{pmatrix} p \\ n \end{pmatrix}$$ a doublet

Similarly: $\pi^+ \quad \pi^0 \quad \pi^- \Rightarrow \frac{\Delta m}{m} \sim 3.3\%$

$\Delta^{++} \quad \Delta^+ \quad \Delta^0 \quad \Delta^- \Rightarrow \frac{\Delta m}{m} \sim 1\%$

etc.

It is observed that the strong interaction cannot distinguish components of these "grouped" particles, e.g.,

$$\pi^+ p \sim \pi^+ n$$
$$\sim \pi^0 p \sim \pi^0 n$$
$$\sim \pi^- p \sim \pi^- n$$

To distinguish the states, introduce the idea of isospin $\vec{I} = (I_1, I_2, I_3)$, generated by su(2) algebra $\boxed{[I_j, I_k] = i\epsilon_{jkl} I_l.}$

States labeled by eigenvalues of $\vec{I}^2$ $(i(i+1))$ and I_3 (i_3)

Explicitly, $N \quad i = \frac{1}{2} \quad i_3 = \begin{cases} +\frac{1}{2} & p \\ -\frac{1}{2} & n \end{cases}$

$\pi \quad i = 1 \quad i_3 = \begin{cases} +1 & \pi^+ \\ 0 & \pi^0 \\ -1 & \pi^- \end{cases}$

Strong isospin only applies to hadrons. For non-strange hadrons,

$$Q = I_3 + \frac{1}{2} B_n$$

electric charge ↲ ↳ Baryon number

Can learn a lot about reactions just by isospin considerations. If SM Hamiltonian is decomposed as $H_{SM} = H_s + H_{EM} + H_w$

then $[H_s, \vec{I}] = 0$ since we defined isospin to be conserved in strong interactions. Now, consider the Nucleon N, $N = (p, n)$. Isospin is broken explicitly by EM interactions $\Rightarrow [H_{EM}, \vec{I}] \neq 0$ since one state is charged. But, $Q = I_3 + \frac{1}{2} B$ and Q & B are good quantum numbers $\Rightarrow I_3$ is a good quantum number $\Rightarrow \underline{[H_{EM}, I_3] = 0}$!

But, it is observed that $[H_w, \vec{I}] \neq 0$ & $[H_w, I_3] \neq 0$
$\Rightarrow$ isospin is completely broken by weak interactions

Example

Consider $\Lambda^0 \to p\,\pi^-$ decay (BR ~ 64%)

Estimate lifetime of Λ^0

Consider isospin numbers

$$\begin{array}{lcccc} & \Lambda^0 & \to & p & \pi^- \\ i: & 0 & \to & \tfrac{1}{2} & 1 \\ i_3: & 0 & \to & \tfrac{1}{2} & -1 \end{array}$$

$\leftarrow \Delta I \neq 0$ since $\underset{\sim}{2} \times \underset{\sim}{3} \not\supset \underset{\sim}{1}$

$\leftarrow \Delta I_3 \neq 0$

Therefore, since $\vec{I}$ & I_3 not conserved, this must be a weak decay!

$\Rightarrow \tau_{\Lambda^0} \sim 10^{-9}$ sec $\quad (\tau_{\Lambda^0} = 2.6 \times 10^{-10}\,s)$

Example

Consider deuteron production, $NN \to \pi d$

where deuteron is a bound state of p and n.

Given that deuteron has $i_d = 0$, estimate

the ratio of cross-sections σ_a/σ_b where

$a: \; pp \to \pi^+ d$

$b: \; np \to \pi^0 d$

Recall that

$$\sigma \sim |\langle f|T|i\rangle|^2 \times (\text{kinematic factors})$$

To good approx, only difference between σ_a & σ_b comes from the amplitude. Moreover, the only difference is in $|i\rangle$ and $|f\rangle$, which is contained in Clebsch-Gordan coefficients

Here,

$$\begin{array}{ccccccc} & N & N & \rightarrow & \pi & d \\ i: & \underbrace{\tfrac{1}{2} \quad \tfrac{1}{2}}_{} & & \longrightarrow & \underbrace{1 \quad 0}_{} & \end{array}$$

restricted to $i = 1$ only $\Leftarrow$ { This means $\underset{\sim}{3} \times \underset{\sim}{1} = \underset{\sim}{1}$ only for $su(2)_I$

So, for $i = 1$ for this process, find

a:

$$|i, i_3\rangle$$

$$\begin{array}{ll} p \; p & |\tfrac{1}{2}\;\tfrac{1}{2}\rangle \times |\tfrac{1}{2}\;\tfrac{1}{2}\rangle = |1\;1\rangle \\ \pi^+ \; d & |1\;1\rangle \times |0\;0\rangle = |1\;1\rangle \end{array}$$

b:

$$\begin{array}{ll} n \; p & |\tfrac{1}{2}\;-\tfrac{1}{2}\rangle \times |\tfrac{1}{2}\;\tfrac{1}{2}\rangle = \frac{1}{\sqrt{2}}\left(|1\,0\rangle - |0\,0\rangle\right) \\ \pi^0 \; d & |1\,0\rangle \times |0\;0\rangle = |1\,0\rangle \end{array}$$

So,
$$\frac{\sigma_a}{\sigma_b} = \frac{1}{\left(\frac{1}{\sqrt{2}}\right)^2} = 2$$
, which is experimentally observed.

Isospin is part of a larger concept called Flavour symmetry. Another quantum number, Strangeness S, was assigned to hadrons produced by strong interactions, yet decayed via weak interactions.

Examples of strange hadrons include:

$K^0, K^+ - S = +1$

$\bar{K}^0, K^- - S = -1$

$$\Lambda^0 - S = -1 \Rightarrow \Lambda^0 \rightarrow \begin{cases} p + \pi^- & \sim 64\% \\ n + \pi^0 & \sim 36\% \end{cases}$$

$\Delta S \neq 0 \Rightarrow$ weak decay
$\Rightarrow$ Long lifetime

$\tau_{\Lambda^0} \sim 2.6 \times 10^{-10}\,s$

Common to combine S & B_n into Hypercharge Y

$$\boxed{Y = B_n + S}$$

There are other flavour quantum numbers, Charmness C and Bottomness B. We will see these are associated w/ the quark content. Won't discuss more on these for time.

$$Q = I_3 + \tfrac{1}{2}Y, \quad Y = B_n + S + C + B$$

G-parity

There is another discrete quantum number useful for strong interactions, G-parity. This is an extension of C-parity valid for states w/ $Q \neq 0$ ($B = S = 0$ still) and $I \neq 0$.

Def.
$$G = C \exp(i\pi I_2)$$

charge conjugation — rotation by π in isospin space around "2"-axis.

Consider some state $|i_3\rangle$

$$G|i_3\rangle = C\exp(i\pi I_2)|i_3\rangle$$
$$\propto C|-i_3\rangle$$
$$\propto |i_3\rangle$$ An eigenstate!

Example: The $\pi = (\pi^+, \pi^0, \pi^-)$ multiplet.

Since $C|\pi^0\rangle = +|\pi^0\rangle$ ← definition

$R|i\, i_3\rangle = (-1)^i |i\, -i_3\rangle$

and
$$R|\pi^0\rangle = R|1\,0\rangle = (-1)^1 |1\,0\rangle = -|1\,0\rangle = -|\pi^0\rangle$$
$$R = \exp(i\pi I_2)$$

$$\Rightarrow G|\pi^0\rangle = -|\pi^0\rangle$$

So, assign $G|\pi^\pm\rangle = -|\pi^\pm\rangle$

Therefore, we have for the π-multiplet

$$\boxed{G|\pi\rangle = -|\pi\rangle}$$

$$\Rightarrow G|n\pi\rangle = (-1)^n |n\pi\rangle$$

n = number of pions

States w/ $B_n \neq 0$ or $S \neq 0$ cannot be eigenstates of G-parity. This doesn't really add much new as it really is a consequence of isospin, but it gives a shortcut in analyzing reactions.

e.g., what type of decay is $\eta^0 \to 3\pi$?

$$\begin{array}{lcccccl} & \eta^0 & \to & \pi^+ & \pi^0 & \pi^- & \\ i_3 & 0 & \to & +1 & 0 & -1 & \Delta I_3 = 0 \end{array}$$

strong? No!

$\Rightarrow$ Not strong decay!

$$G|\eta^0\rangle = +|\eta^0\rangle, \quad G|3\pi\rangle = (-1)^3|3\pi\rangle = -|3\pi\rangle$$

$\Rightarrow$ G-parity not conserved! It is electromagnetic

To see that it is not strong w/o G-parity requires analysis of the states angular distribution under generalized Bose statistics.

Flavor SU(3)

We discussed the approximate (or broken) Isospin and Strangeness symmetries, both good for strong interactions. Suggests enlarging $su(2)_I$ to bigger group.

Particles in $su(2)_I$ multiplets are "easy" to find (masses are similar as I-spin only slightly broken). Appropriate choice for multiplets with S harder.

Accepted solution is $SU(3)_F$ (Flavor SU(3)), known as the "Eightfold way", proposed by Gell-Mann and Ne'eman in 1961.

$\boxed{\text{Note: Do not confuse with } SU(3)_c \text{ - color}}$

Essential content of $SU(3)_F$

(1) All baryons fit into $\underset{\sim}{1}$, $\underset{\sim}{8}$, $\underset{\sim}{10}$ of SU(3)

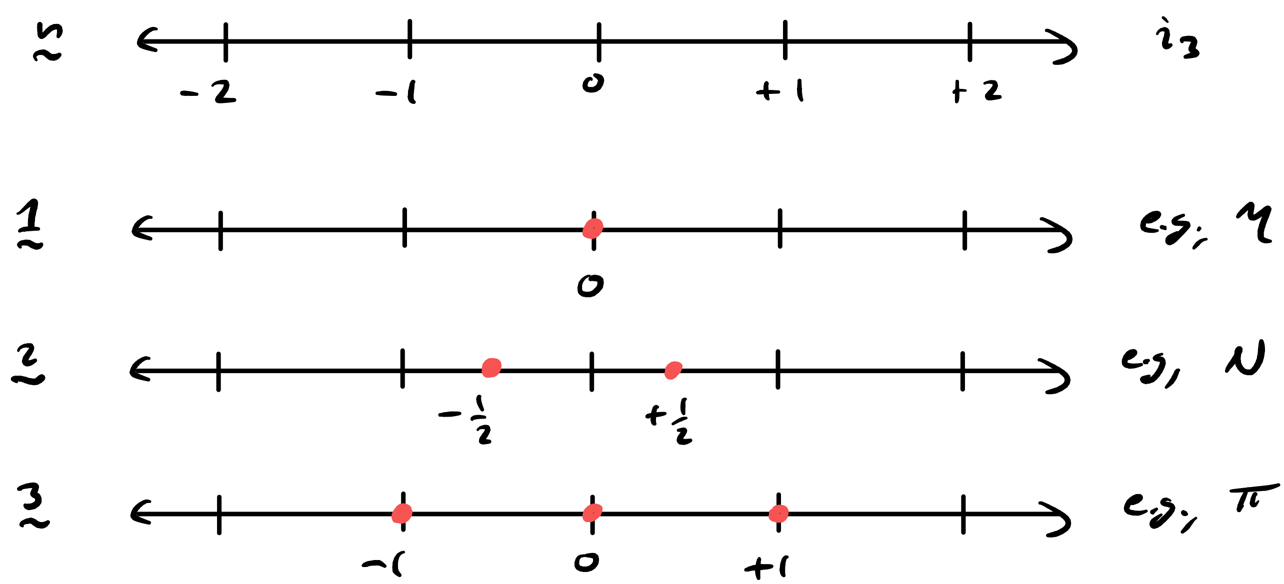
(2) All mesons fit into $\underset{\sim}{1}$, $\underset{\sim}{8}$ of SU(3)

Quantum numbers of $SU(3)_F$: I, I_3, S or Y, ...

N.B. There are two Casimirs of SU(3). Here we will focus just on these and the representation.

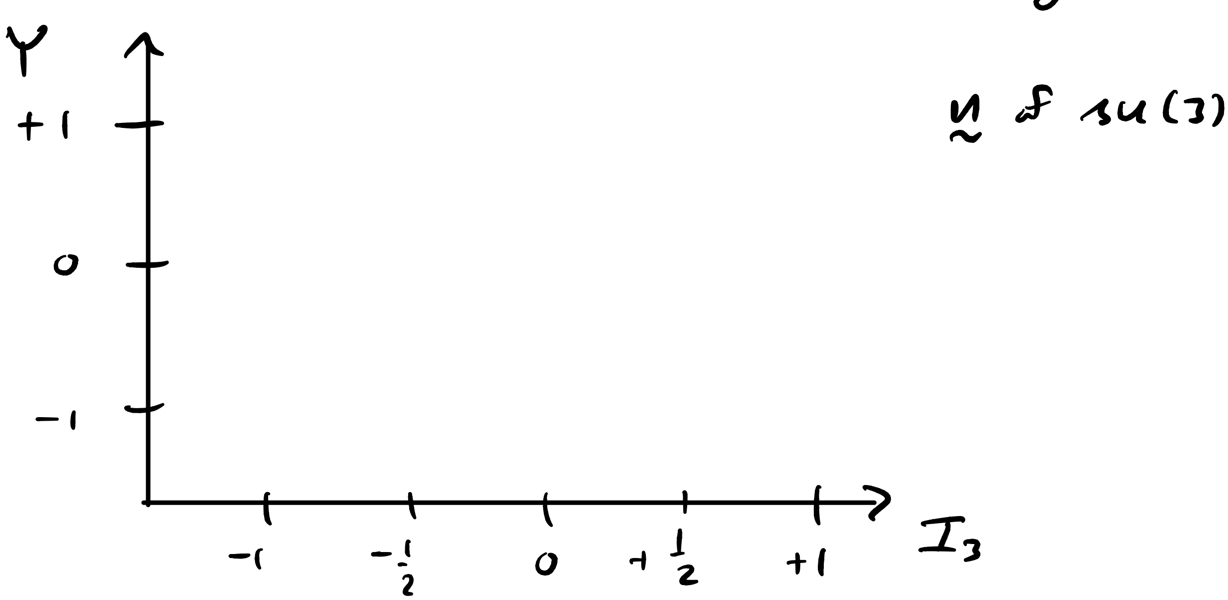
To organize states, can represent a particular rep by a "weight diagram"

For example, $SU(2) \Rightarrow$ for given i, only need one number, i_3, to classify states



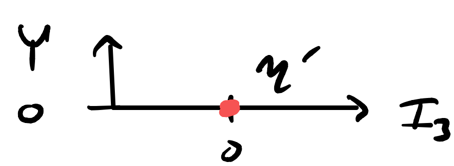
For a given rep in $SU(3)$, need two numbers.

For $SU(3)_F$, let's take I_3 and Y for a given rep.



Example: The $J^{PC} = 0^{-+}$ meson singlet

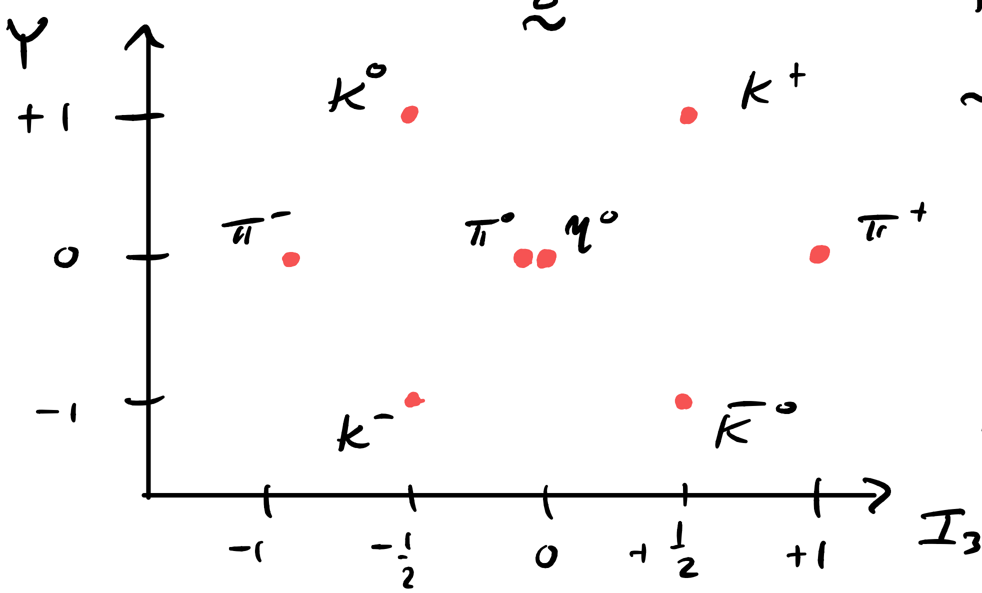
$\underset{\sim}{1}$



Mass / MeV
~ 958

Example: The $J^{PC} = 0^{-+}$ meson octet

$\underset{\sim}{8}$



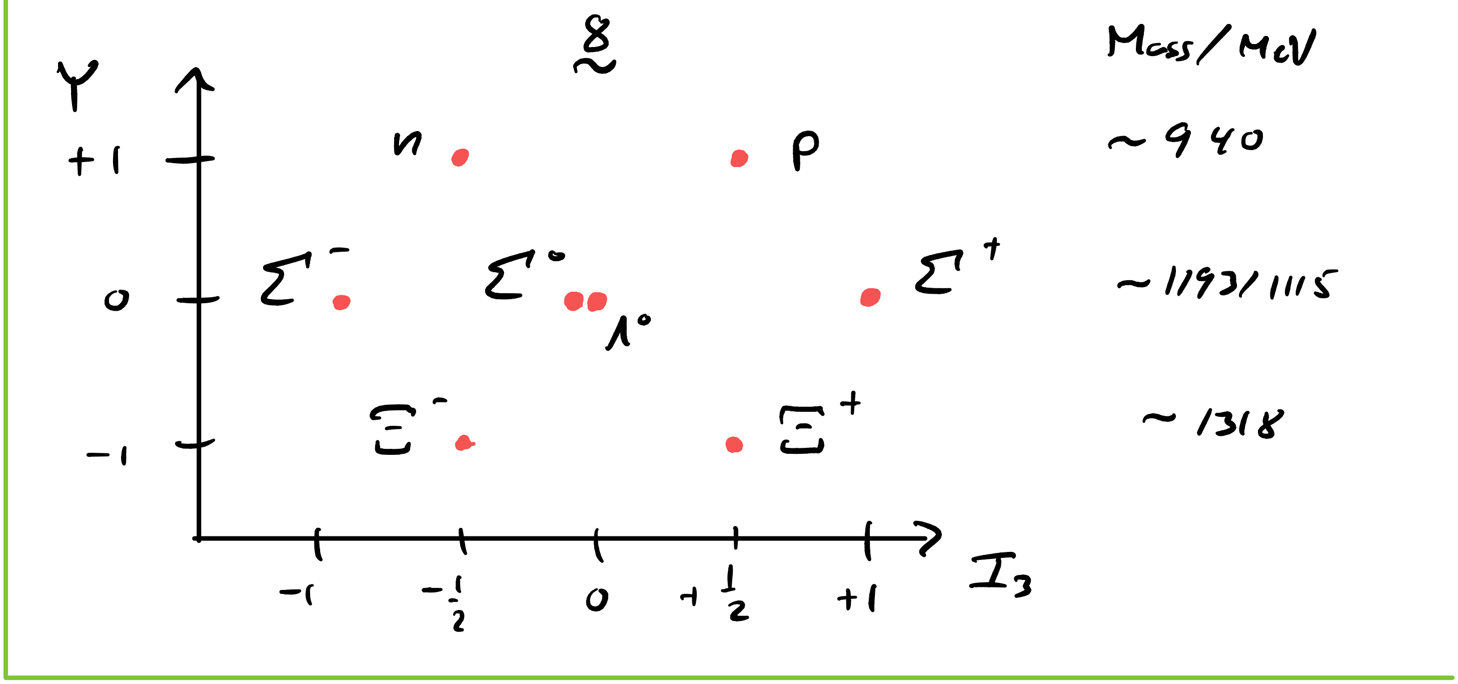
Mass / MeV
~ 493
$\sim 137 / 549$
~ 493

Notice that we have larger mass differences $\Rightarrow$ $SU(3)_F$ is "more broken" than $SU(2)_I$. Also, $SU(2)_I$ multiplets are contained in $SU(3)_F$ ones (proper subgroup).

Notice that the "Gell-Mann / Nishijima" relation holds,

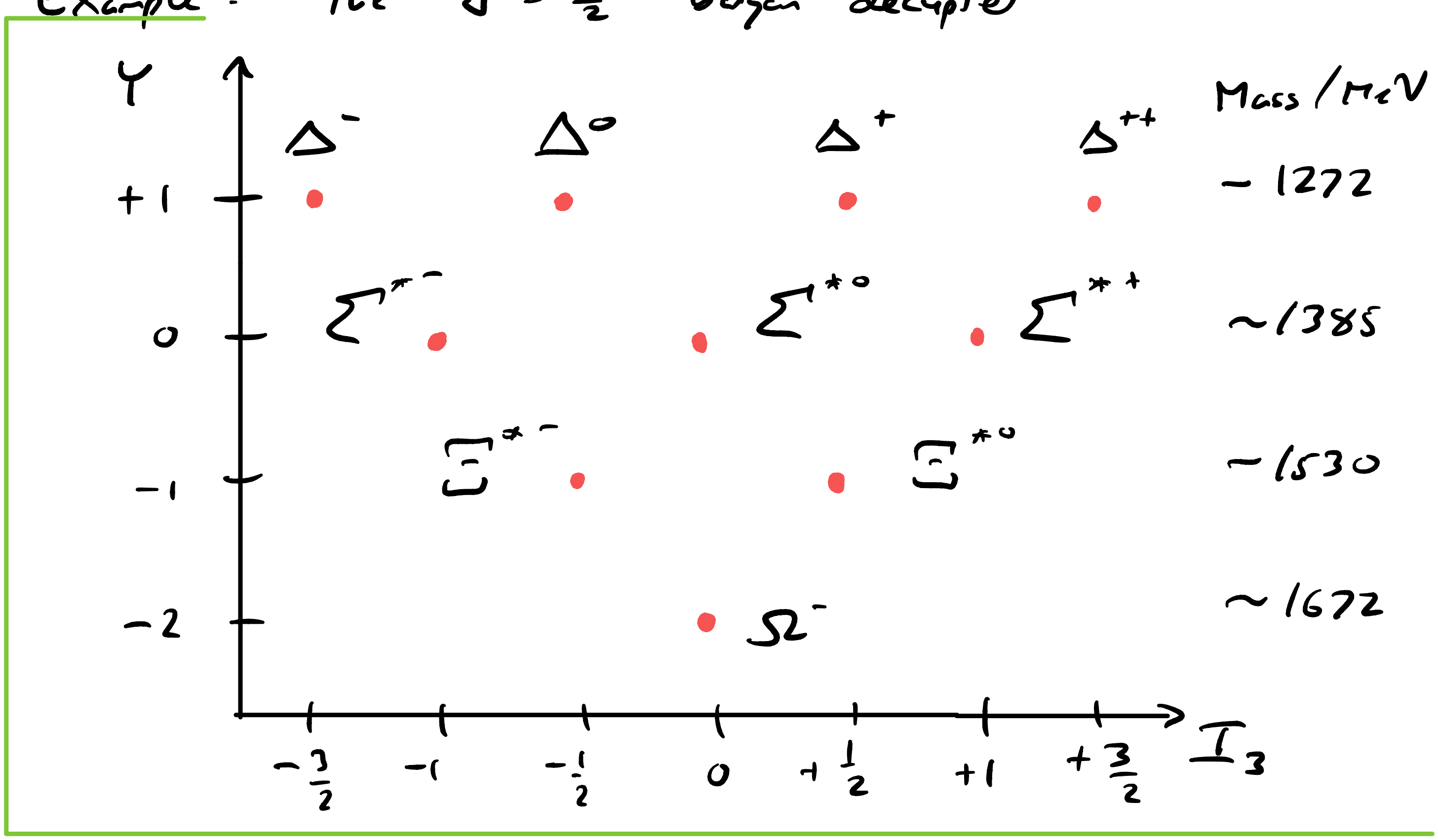
$$Q = I_3 + \tfrac{1}{2} Y \quad \text{with} \quad Y = B_n + S \qquad \text{(check)}$$

Example: The $J^P = \frac{1}{2}^+$ baryon octet



Since $B_n \neq 0$, baryons and antibaryons live in different multiplets. (Exercise – construct antibaryon octet)

Example: The $J^P = \frac{3}{2}^+$ baryon decuplet



The prediction of the Ω^- using $SU(3)_F$ in 1962 and subsequent discovery in 1964 led to the acceptance of the "Eightfold way".

$$K^- + p \to \Omega^- + K^+ + K^0$$

$$\Omega^- \to \Xi^0 + \pi^-$$

$$\Xi^0 \to \Lambda^0 + \pi^0$$

$$\pi^0 \to \gamma + \gamma, \quad \gamma \to e^- + e^+, \quad \gamma \to e^- + e^+$$

$$\Lambda^0 \to \pi^- + p$$

There are many properties of hadrons we can learn simply by examining their symmetry,

e.g., magnetic moments of hadrons,

mass relations,

selection rules in reactions,

etc.

The symmetry structures suggest that these hadrons are composite particles of some more fundamental constituents...

The Quark Model

Hypothesis: Hadrons are formed from constituent particles

Let's focus on 3-flavor hadrons.

In 1964, Gell-Mann / Zweig postulated

a triplet of fundamental particles with $B_n = 1/3$

to form all hadrons, including $p, n, \Lambda, \ldots$

This is known as the Quark Model

Basic idea,

- Baryons are bound states of qqq $(B_n = 1)$
- Antibaryons are bound states of $\bar{q}\bar{q}\bar{q}$ $(B_n = -1)$
- Mesons are bound states of $q\bar{q}$ $(B_n = 0)$