

Feynman Rules - Self-Interacting Complex Scalar theory

The Lagrangian density for a self-interacting complex-scalar field theory is given by

$$\mathcal{L} = \partial_\mu \varphi^\dagger \partial^\mu \varphi - m^2 \varphi^\dagger \varphi - \lambda (\varphi^\dagger \varphi)^2$$

where m is the mass parameter and λ is the coupling.

Feynman Rules

Here we give the Feynman rules for the scattering amplitude \mathcal{M} ,

$$i\mathcal{M} = \text{sum of all connected, amputated diagrams,}$$

where the diagrams are evaluated according to the following rules:

- Draw all topologically distinct diagrams at a given order;
- For each internal scalar line, attach a propagator

$$\begin{array}{c} \text{---}\overrightarrow{\hspace{1cm}}\text{---} \\ p \end{array} = \frac{i}{p^2 - m^2 + i\epsilon};$$

- For each vertex, assign

$$\begin{array}{c} \diagup \quad \diagdown \\ \diagdown \quad \diagup \end{array} = -i\lambda;$$

- For each external line, place the particle on the mass-shell $p^2 = m^2$ and attach a wavefunction factor

$$\text{“incoming scalar”} \quad \begin{array}{c} \text{---}\overleftarrow{\hspace{1cm}}\text{---} \\ p \end{array} = 1;$$

$$\text{“outgoing scalar”} \quad \begin{array}{c} \text{---}\overrightarrow{\hspace{1cm}}\text{---} \\ p \end{array} = 1;$$

- Impose momentum conservation at each vertex;
- For each internal loop momentum k not fixed by momentum conservation, integrate $\int \frac{d^4 k}{(2\pi)^4}$;
- Multiply the contribution for each diagram by an appropriate symmetry factor \mathcal{S}^{-1} .