

## Leptonic Electroweak Model

Let's consider the gauge theory of weak interactions.

This is a "simple" model of leptons, Higgs, and EW bosons. The model is based on the gauge field theory with product group  $SU(2)_L \times U(1)_Y$

generators of  $SU(2)_L$ :  $T^a$ ,  $a=1,2,3$

$$c_{ij}, \text{ for } \tilde{\chi}, T^a = \frac{1}{2} \sigma^a \quad \text{weak isospin}$$

generators of  $U(1)_Y$ :  $Y$       weak hypercharge

the algebra or  $SU(2)_L \oplus U(1)_Y$

$$[T^a, T^b] = i \epsilon^{abc} T^c$$

$$[Y, Y] = 0$$

$$[T^a, Y] = 0$$

We can define the electric charge generator  $Q$  that generates the  $U(1)_Q$  of QED. This is given by "Gell-Mann - Nishijima relation"

$$Q = T^3 + \frac{1}{2} Y$$

Not related to  
strong interaction  
version

The algebra is then

$$[Q, Q] = 0$$

$$[Y, Q] = 0$$

$$[T^a, Q] = [T^a, T^3] = i \epsilon^{abc} T^c$$

$$\Rightarrow [T^3, Q] = 0, [T^1, Q] = -iT^2, [T^2, Q] = iT^1$$

Basic particles in leptonic EW model

① Spin- $\frac{1}{2}$  leptons :  $e, \nu_e, \mu, \nu_\mu, \tau, \nu_\tau$

② Spin-1 gauge bosons :  $A_\mu, W_\mu^\pm, Z_\mu^-$

③ Spin-0 Higgs boson :  $h$

No quarks or strong interactions (will add later)

In 1957, Wu et al. found electrons emitted in the nuclear  $\beta$ -decay of  $^{60}\text{Co} \rightarrow ^{60}\text{Ni}^+ + e^- + \bar{\nu}_e$ , in a preferred direction opposite to the nuclear spin.

Can conclude that parity is violated in weak processes.

Observe that parity is maximally violated, i.e., left-handed particles experience weak interaction, but not right-handed.

$\Rightarrow$  Matter fields must be contragredient to respect this.

① Spinor fields for leptons labeled by "genus"

Charged leptons :  $l_A = (e, \mu, \tau)$ ,  $A=1, 2, 3 \sim e, \mu, \tau$

neutrino fields :  $\nu_\lambda = (\nu_e, \nu_\mu, \nu_\tau)$

Introduce left- and right-handed fields,

$$\Psi_L = \frac{1}{2} (1 + r_5) \Psi .$$

For each family  $\ell_A, v_A$  in question.

Dfine

$$L_A = \begin{pmatrix} v_{LA} \\ l_{LA} \end{pmatrix}, \quad R_A = l_{R_A}$$

↑  
will be a  $SU(2)_L$  singlet

will be a  $SU(2)_L$  doublet

For basic EW model, no right-handed neutrinos

$\Rightarrow$  No  $v_{R_A}$  fields  $\Rightarrow$  massless neutrinos

② Vector fields are all gauge fields

$$SU(2)_L \otimes U(1)_Y$$

↑  
3 gers.

↑  
1 gen.

⇒ 4 gauge fields

w<sup>a</sup>

B<sub>p</sub>

$\omega_\mu^c$  is a  $SU(2)_L$  triplet.  $B_\mu$  is a  $U(1)_Y$  singlet.

Useful combinations:  $\omega_\mu^\pm = \frac{1}{\sqrt{2}} (\omega_\mu^1 \pm i \omega_\mu^2)$

$$\omega_\mu^- = (\omega_\mu^+)^*$$

We will see later that  $A_\mu$  &  $Z_\mu^*$  are linear combos of  $\omega_\mu^3$  &  $B_\mu$ .

③ Scalar fields are Higgs fields.

EW model has 4 real Higgs fields to give mass to  $\omega_\mu^\pm$ ,  $Z_\mu^*$ . Write these as 2 complex scalars

$$\phi^+, \phi^0$$

and define complex conjugates

$$\phi^- = (\phi^+)^*, \quad \phi^{0*}$$

Useful to define  $\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$ , which is an  $SU(2)_L$  doublet.

$$\Rightarrow \Phi^T = (\phi^-, \phi^{0*}) = \Phi^{*T}$$

$$2 \quad \Phi^c = i\sigma^2 \Phi^* = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} \phi^- \\ \phi^{0*} \end{pmatrix} = \begin{pmatrix} \phi^{0*} \\ -\phi^- \end{pmatrix} \in \mathbb{Z} \otimes SU(2)_L$$

Warning:  $\bar{\psi}_L$  is understood as  $\overline{(\psi_L)}$ ,  
which is right-handed! (see below)

## Representations of Fields

Field  $SU(2)_L U(1)_Y \quad SU(2)_L : T \quad SU(2)_C : T_3 \quad U(1)_Y : Y \quad U(1)_Q : Q$

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$$L = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \quad \begin{matrix} 2_{-1} & \frac{1}{2} & +\frac{1}{2} & -1 & 0 \\ & \frac{1}{2} & -\frac{1}{2} & -1 & -1 \end{matrix}$$

$$R_A = \begin{pmatrix} R_A \\ 1 \end{pmatrix} \quad \begin{matrix} 1_{-2} & 0 & 0 & -2 & -1 \end{matrix}$$

$$W_\mu = \begin{pmatrix} W_\mu^+ \\ W_\mu^0 \\ W_\mu^- \end{pmatrix} \quad \begin{matrix} 3_0 & 1 & +1 & 0 & +1 \\ & 1 & 0 & 0 & 0 \\ & & -1 & 0 & -1 \end{matrix}$$

$$B_\mu \quad \begin{matrix} 1_0 & 0 & 0 & 0 & 0 \end{matrix}$$

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \quad \begin{matrix} 2_{+1} & \nu_2 & +\nu_2 & +1 & +1 \\ & \nu_2 & -\nu_2 & +1 & 0 \end{matrix}$$


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The covariant derivative is

$$(D_\mu)_{jk} = \delta_{jk} \partial_\mu + ig \omega_\mu^a (T_a)_{jk} + i(\frac{1}{2}g') B_\mu Y \delta_{jk}$$

↑  
 SU(2)<sub>L</sub>  
 Coupling

↑  
 U(1)<sub>Y</sub>  
 Coupling

↓ Standard coupling

Generators are for following multiplets

Multiplet	$(T^a)_{jk}$	$Y$
$L_1$	$\frac{1}{2}(\sigma^a)_{jk}$	-1
$R_1$	0	-2
$\Phi$	$\frac{1}{2}(\sigma^a)_{jk}$	-1

The gauge field strengths are

$$\omega_{\mu\nu}^a = \partial_\mu \omega_\nu^a - \partial_\nu \omega_\mu^a - g \epsilon^{abc} \omega_\mu^b \omega_\nu^c$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

The leptonic EW model is then given by

$$\begin{aligned}
 \mathcal{L} = & -\frac{1}{4} \omega_{\mu\nu}^a \omega^{a\nu} - \frac{1}{4} B_{\mu\nu} B^{a\nu} \\
 & + \frac{1}{2} i \bar{L}_A D_\mu L_A + \frac{1}{2} i \bar{R}_A D_\mu R_A + \text{h.c.} \\
 & + (D_\mu \phi)^+ (D^\mu \phi) + \mu^2 \phi^+ \phi - \frac{\lambda}{3!} (\phi^+ \phi)^2 \\
 & - G_A^L [\bar{R}_A (\phi^+ L_A) + (\bar{L}_A \phi) R_A]
 \end{aligned}$$

Gauge KE  
 Fermion KE  
 Higgs KE  
 + (unstable pJW)

Yukawa coupling  
 ( $\text{sum over } A$ )

All terms are individually  $SU(2)_L \otimes U(1)_Y$  invariant.

Note that  $D_\mu$  means different things acting on different fields. Note that  $\phi^+ L_A$  is understood as product

$$(\phi^+, \phi^{+*}) \begin{pmatrix} v_{L_A} \\ l_{L_A} \end{pmatrix} \text{ in } SU(2)_L \text{ space.}$$

This is (almost) the most general  $\mathcal{L}$ . Notice that there are no mass terms for fermions (would break chiral  $SU(2)_L$ ). Also notice that  $G_A^L$  could be a matrix in general. For this model, we can diagonalize. Later in SM, we will see mass mixing.

The number of free parameters is 7!

$$g, g', \mu^2, \lambda, G_c^L, G_\mu^L, G_\tau^L$$

These need to be measured in order to make predictions. The potential for the Higgs is most general for this model, since  $\Phi^+ \Phi^c = \Phi^+ \Phi$ . Also, no Majorana mass terms are compatible for  $V$  are compatible with  $SU(2)_L \otimes U(1)_Y$ . Turns out to be tricky to add  $V$  masses (see later).

The Higgs potential induces SSB.

To see what happens write  $\Phi$  as

$$\Phi(x) = \begin{pmatrix} \Phi^+(x) \\ \Phi^0(x) \end{pmatrix} = \frac{1}{\sqrt{2}} \exp\left(\frac{1}{2} i \Theta^a(x) \sigma^a\right) \begin{pmatrix} 0 \\ v(x) \end{pmatrix}$$

$\downarrow$  4 real fields                             $\downarrow$  3 real fields                             $\downarrow$  1 real field

Next, choose unitary gauge to interpret, where  $SU(2)_L$  gauge parameters are  $\alpha^a(x) = -\Theta^a(x)$

$\Rightarrow$  3 of 4 components of Higgs "disappear"

$$\text{so, unitary gauge } \Rightarrow \phi \rightarrow \exp(-\frac{1}{2}i\theta^a(\omega) \sigma^a) \phi \\ = \frac{1}{\sqrt{2}} \left( \begin{array}{c} 0 \\ r \end{array} \right)$$

So,

$$\mathcal{L}_{Higgs} = \frac{1}{2} \left[ D_r \left( \begin{array}{c} 0 \\ r \end{array} \right) \right]^+ \left[ D^r \left( \begin{array}{c} 0 \\ r \end{array} \right) \right] - U(r)$$

$$\text{where } U(r) = -\frac{\lambda}{4!} (r^2 - a^2)^2, \quad a = \sqrt{\frac{6r^2}{\lambda}}$$

The minimum of the potential is at  $r=a$ . So, define

shifted field

$$v(x) = a + h(x), \quad \downarrow \text{quanta gives Higgs } H^0$$

with all others not shifted

Notice that it is a real  $U(1)_Q$ -neutral field

$v(x)$  that is shifting  $\Rightarrow$  All generators of  $SU(2)_L \otimes U(1)_Y$  break except  $U(1)_Q$  const.

The covariant derivative is

$$D_r \left( \begin{matrix} 0 \\ r \end{matrix} \right) = \left( \partial_r + \frac{1}{2} i g w_r^\alpha \sigma^\alpha + \frac{1}{2} i g' B_r \right) \left( \begin{matrix} 0 \\ r \end{matrix} \right)$$

$$= \left( \begin{array}{c|c} \partial_r + \frac{1}{2} i g w_r^3 + \frac{1}{2} i g' B_r & \frac{1}{2} i g \sqrt{2} w_r^+ \\ \hline \frac{1}{2} i g \sqrt{2} w_r^- & \partial_r - \frac{1}{2} i g w_r^3 + \frac{1}{2} i g' B_r \end{array} \right) \left( \begin{matrix} 0 \\ r \end{matrix} \right)$$

$$= \left( \begin{array}{c} \frac{1}{\sqrt{2}} i g w_r^+ r \\ \partial_r r - \frac{1}{2} i g w_r^3 r + \frac{1}{2} i g' B_r r \end{array} \right)$$

likewise,  $[D_r \left( \begin{matrix} 0 \\ r \end{matrix} \right)]^+$  can be found.

Find for the Higgs KE term

$$\begin{aligned} \frac{1}{2} [D_r \left( \begin{matrix} 0 \\ r \end{matrix} \right)]^+ [D^r \left( \begin{matrix} 0 \\ r \end{matrix} \right)] = & \frac{1}{4} g^2 w_r^+ w_r^- r^2 + \frac{1}{2} \partial_r r \partial^r r \\ & + \frac{1}{8} r^2 (g w_r^3 - g' B_r)^2 \end{aligned}$$

If  $r(x) = a + bx$ , then about true values

$$\begin{aligned} \mathcal{L}_{Higgs} &= \frac{1}{2} \partial_\mu h \partial^\mu h \\ &+ \frac{1}{2} (a+b)^2 \left[ \frac{1}{2} g^2 W_\mu^+ W^\mu_- + \frac{1}{4} (g W_\mu^3 - g' B_\mu)^2 \right] \\ &- \frac{a^2 \lambda}{6} h^2 - \frac{a \lambda}{6} h^3 - \frac{\lambda}{4!} h^4 \end{aligned}$$

We see that  $m_1 = \sqrt{2} \mu$ , as usual.

Since  $W_\mu^- = (W_\mu^+)^*$ , see the mass for  $W_\mu^\pm$  is

$$m_{W^\pm} = \frac{1}{2} a g$$

For  $W_\mu^3, B_\mu$ , the given term has mass mixing!

$\Rightarrow$  Must diagonalize. This is already done,  $\mathcal{L}_{Higgs}$  is the massive boson

Define:

$$Z_\mu^0 = \frac{g W_\mu^3 - g' B_\mu}{\sqrt{g^2 + g'^2}}$$

$$A_\mu = \frac{g' W_\mu^3 + g B_\mu}{\sqrt{g^2 + g'^2}}$$

$$\text{So, } Z_{H^0} \geq \frac{1}{2} a^2 \left( \frac{1}{4} (g \omega_r^2 - g' B_r)^2 \right)$$

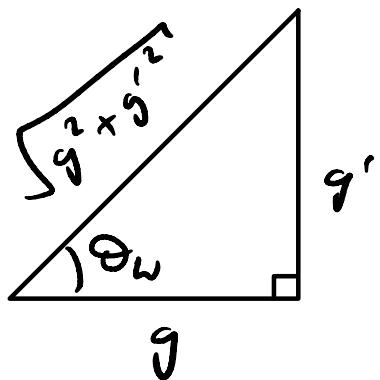
$$= \frac{1}{2} \frac{1}{4} a^2 (g^2 + g'^2) Z_{\mu^0}^2 + 0 \cdot A_r A'$$

So,

$$m_A = 0$$

$$m_{Z^0} = \frac{1}{2} a \sqrt{g^2 + g'^2}$$

Define the weak mixing angle  $\theta_W$



so that

$$\begin{pmatrix} A_r \\ Z_{\mu^0} \end{pmatrix} = \begin{pmatrix} \cos \theta_W & \sin \theta_W \\ -\sin \theta_W & \cos \theta_W \end{pmatrix} \begin{pmatrix} B_r \\ \omega_r^2 \end{pmatrix}$$

$$\Rightarrow m_{W^\pm} = m_{Z^0} \cos \theta_W, \quad g' = g \tan \theta_W$$

$S_0$ , complete Higgs Lagrangian

$$\mathcal{L}_{HSS} = \frac{1}{2} \partial_\mu h \partial^\mu h$$

$$+ \frac{1}{2} (a+h)^2 \left[ \frac{1}{2} g^2 W_\mu^+ W^\mu_- + \frac{1}{4} (g^2 + g'^2) Z_\mu^+ Z^\mu_- \right]$$

$$- \frac{a^2 \lambda}{6} h^2 - \frac{a \lambda}{6} h^3 - \frac{\lambda}{4!} h^4$$

$$= \frac{1}{2} \partial_\mu h \partial^\mu h - \frac{1}{2} m_h^2 h^2$$

$$+ m_{W^\pm}^2 W_\mu^+ W^\mu_- + \frac{1}{2} m_{Z'}^2 Z_\mu^+ Z^\mu_-$$

$$+ \frac{1}{4} g^2 h^2 W_\mu^+ W^\mu_- + \frac{1}{8} (g^2 + g'^2) h^2 Z_\mu^+ Z^\mu_-$$

$$+ \frac{1}{2} a g^2 h W_\mu^+ W^\mu_- + \frac{1}{4} a (g^2 + g'^2) h Z_\mu^+ Z^\mu_-$$

$$- \frac{a \lambda}{6} h^3 - \frac{\lambda}{4!} h^4$$

With  $m_{W^\pm} = \frac{1}{2} a g$ ,  $m_{Z'} = \frac{1}{2} a \sqrt{g^2 + g'^2}$

$$m_h = \sqrt{2} \mu, \quad \mu = \frac{a \lambda}{56}$$

$$g' = g \tan \theta_W, \quad m_{W^\pm} = m_{Z'} \cos \theta_W$$

What about other terms in the EW Lagrange density?

Consider first gauge KE terms,

$$L_{\text{gauge}} = -\frac{1}{4} \omega_{\mu\nu}^{\alpha\beta} \omega^{\mu\nu} - \frac{1}{4} B_{\mu\nu}^{\alpha\beta} B^{\mu\nu}$$

transforming the fields,

$$\begin{pmatrix} B_\mu \\ \omega_\mu^\beta \end{pmatrix} = \begin{pmatrix} \cos\theta_W & -\sin\theta_W \\ \sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} A_\mu \\ Z_\mu^\beta \end{pmatrix}$$

After considerable algebra, find

$$\begin{aligned} L_{\text{gauge}} = & -\frac{1}{4} \left( \partial_\mu Z_\nu - \partial_\nu Z_\mu - ie \cos\theta_W (\omega_\mu^+ \omega_\nu^- - \omega_\mu^- \omega_\nu^+) \right)^2 \\ & - \frac{1}{4} \left( \partial_\mu A_\nu - \partial_\nu A_\mu - ie (\omega_\mu^+ \omega_\nu^- - \omega_\mu^- \omega_\nu^+) \right)^2 \\ & - \frac{1}{2} \left| \partial_\nu \omega_\mu^+ - \partial_\mu \omega_\nu^+ + ie \cos\theta_W (\omega_\mu^+ Z_\nu^\beta - \omega_\nu^+ Z_\mu^\beta) \right. \\ & \quad \left. + ie (\omega_\mu^+ A_\nu - \omega_\nu^+ A_\mu) \right|^2 \end{aligned}$$

where  $e = g \sin\theta_W = g' \cos\theta_W$

↳ electric charge!

