

Feynman Rules - Quantum Chromodynamics

The Lagrangian density for Quantum Chromodynamics (QCD) is given by

$$\mathcal{L} = \frac{1}{2} \sum_f i \bar{\psi}_f \not{D} \psi_f + \text{h.c.} - \sum_f m_f \bar{\psi}_f \psi_f - \frac{1}{2} \text{tr}(\mathbf{G}_{\mu\nu} \mathbf{G}^{\mu\nu}) - \frac{1}{2\xi} (\partial^\mu \mathbf{A}_\mu)^2 - \partial_\mu \bar{\mathbf{c}} \mathbf{D}^\mu \mathbf{c} \quad (1)$$

where the gauge covariant derivative is $\mathbf{D}_\mu = \mathbf{1} \partial_\mu + i g_s \mathbf{A}_\mu$ and the field strength tensor is $\mathbf{G}_{\mu\nu} = \partial_\mu \mathbf{A}_\nu - \partial_\nu \mathbf{A}_\mu$. Here m is the mass of the charged fermion. We use indices α, β for spinor space, while μ, ν for Lorentz space. Also, j, k indicate the color indices in the $\mathbf{3}$ representation of $\text{SU}(3)_c$, and a, b are the indices in the $\mathbf{8}$ of $\text{SU}(3)_c$. Therefore, the components of the fermion field are $(\psi_f)_f^\alpha$, for the gluon field $(A_\mu)_{jk} = A_\mu^a (T_a)_{jk}$ where $T_a = \lambda_a/2$, and for the ghost field c_a .

Feynman Rules

Here we give the Feynman rules for the scattering amplitude \mathcal{M} ,

$$i\mathcal{M} = \text{sum of all connected, amputated diagrams},$$

where the diagrams are evaluated according to the following rules:

- Draw all topologically distinct diagrams at a given order;
- For each internal gluon line, attach a propagator

$$\mu, a \xrightarrow[p]{\text{wavy}} \nu, b = \frac{-i\delta_{ab}}{p^2 + i\epsilon} \left(g_{\mu\nu} - (1 - \xi) \frac{p_\mu p_\nu}{p^2} \right);$$

- For each internal quark line, attach a propagator

$$\alpha, j \xrightarrow[p]{\text{solid}} \beta, k = \delta_{jk} \frac{i(\not{p} + m_f)_{\beta\alpha}}{p^2 - m_f^2 + i\epsilon};$$

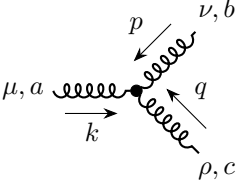
- For each internal ghost line, attach a propagator

$$j \xrightarrow[p]{\text{dotted}} k = \delta_{jk} \frac{-i}{p^2 + i\epsilon};$$

- For each quark-gluon vertex, assign

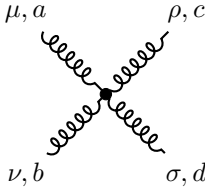
$$\begin{array}{c} \beta, k \\ \nearrow \\ \mu, a \text{ wavy} \bullet \\ \searrow \\ \alpha, j \end{array} = -i g_s (\gamma^\mu)_{\beta\alpha} (T_a)_{jk};$$

- For each three-gluon vertex, assign



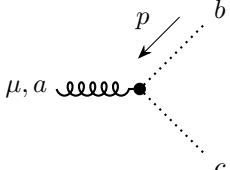
$$= -ig_s(-if_{abc}) \left[(k-p)^\rho g^{\mu\nu} + (p-q)^\mu g^{\nu\rho} + (q-k)^\nu g^{\mu\rho} \right];$$

- For each four-gluon vertex, assign



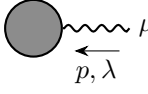
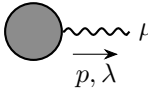
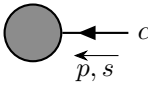
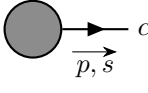
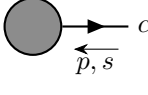
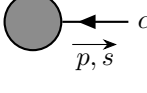
$$= -ig_s^2 \left[\begin{aligned} &f_{abe}f_{cde}(g_{\mu\rho}g_{\nu\sigma} - g_{\mu\sigma}g_{\nu\rho}) \\ &+ f_{ace}f_{bde}(g_{\mu\nu}g_{\rho\sigma} - g_{\mu\sigma}g_{\nu\rho}) \\ &+ f_{ade}f_{bce}(g_{\mu\nu}g_{\rho\sigma} - g_{\mu\rho}g_{\nu\sigma}) \end{aligned} \right];$$

- For each ghost-gluon vertex, assign



$$= -ig_s(-if_{abc})p^\mu;$$

- For each external line, place the particle on the “mass-shell”, $p^2 = m_q^2$ for the quark and $p^2 = 0$ for the gluon, and attach a wavefunction factor

“incoming photon”		$= \epsilon_\mu(p, \lambda);$
“outgoing photon”		$= \epsilon_\mu^*(p, \lambda);$
“incoming fermion”		$= u_\alpha(p, s);$
“outgoing fermion”		$= \bar{u}_\alpha(p, s);$
“incoming anti-fermion”		$= \bar{v}_\alpha(p, s);$
“outgoing anti-fermion”		$= v_\alpha(p, s);$

- Impose momentum conservation at each vertex;
- For each internal loop momentum k not fixed by momentum conservation, integrate $\int \frac{d^4 k}{(2\pi)^4}$;
- For each fermion loop, multiply the diagram by (-1) ;
- For each set of diagram which are only distinguished by interchanging two external fermion lines, multiply one of the diagrams by (-1) ;