Feynman Rules - Quantum Electrodynamics

The Lagrangian density for a Quantum Electrodynamics (QED) is given by

$$\mathcal{L} = \frac{1}{2} i\bar{\psi}\not{D}\psi + \text{h.c.} - m\bar{\psi}\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2\xi}(\partial^\mu A_\mu)^2 \tag{1}$$

where m is the mass of the fermion, D_μ is the gauge covariant derivative is $D_\mu = \partial_\mu + iqA_\mu$, and the field strength tensor is $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. The charge of the fermion is q, e.g., for the electron $q = -e$ while the up-quark has $q = +2e/3$, where $e = \sqrt{4\pi\alpha}$ is the magnitude of the electron charge in natural units, and α is the fine-structure constant. The CODATA value for the fine-structure constant is $\alpha = 7.297\,352\,5693(11) \times 10^{-3}$. The gauge fixing parameter ξ is arbitrary, and physical observables must be independent of ξ. The Fermi-Feynman gauge, $\xi = 1$, is a common choice especially for tree-level calculations. Note below we use indices α, β for spinor space, while μ, ν for Lorentz space.

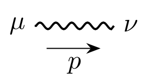
Feynman Rules

Here we give the Feynman rules for the scattering amplitude $\mathcal{M}$,

$$i\mathcal{M} = \text{sum of all connected, amputated diagrams}\,,$$

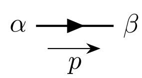
where the diagrams are evaluated according to the following rules:

- Draw all topologically distinct diagrams at a given order;
- For each internal photon line, attach a propagator



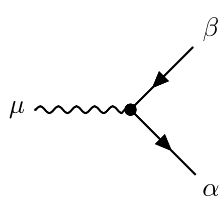
$$= \frac{-i}{p^2 + i\epsilon}\left(g_{\mu\nu} - (1-\xi)\frac{p_\mu p_\nu}{p^2}\right)\,;$$

- For each internal spinor line, attach a propagator



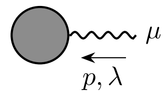
$$= \frac{i(\not{p} + M)_{\alpha\beta}}{p^2 - M^2 + i\epsilon}\,;$$

- For each vertex, assign

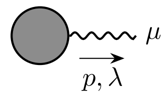


$$= -iq(\gamma^\mu)_{\beta\alpha}\,;$$

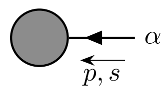
- For each external line, place the particle on the mass-shell, $p^2 = m^2$ for the fermion and $p^2 = 0$ for the photon, and attach a wavefunction factor

"incoming photon"


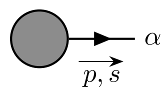
$= \epsilon_\mu(p,\lambda)\,;$

"outgoing photon"


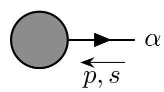
$= \epsilon^*_\mu(p,\lambda)\,;$

"incoming fermion"


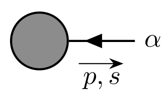
$= u_\alpha(p,s)\,;$

"outgoing fermion"


$= \bar{u}_\alpha(p,s)\,;$

"incoming anti-fermion"


$= v_\alpha(p,s)\,;$

"outgoing anti-fermion"


$= \bar{v}_\alpha(p,s)\,;$

- Impose momentum conservation at each vertex;
- For each internal loop momentum k not fixed by momentum conservation, integrate $\displaystyle\int \frac{\mathrm{d}^4 k}{(2\pi)^4}$;
- For each fermion loop, multiply the diagram by (-1);
- For each set of diagram which are only distinguished by interchanging two external fermion lines, multiply one of the diagrams by (-1);