Beyond the Standard Model

The Minimal Standard Model is extremely successful, & theoretical consistent. However, we know it is incomplete from various phenomenological and theoretical issues.

Phenomenological Issues

- Neutrino flavor oscillations have been observed, indicating that neutrinos must have mass.
- We have observed it accounts for only ~4.6 % of matter in the universe
- Struggles to explain the observed matter-antimatter asymmetry.
- ...

Theoretical Issues

- No accepted unification with gravity
- hierarchy problems, fine-tuning, Λ cosmological constant
- CP problem
- ...

It is generally accepted that the SM is a low-energy approximation to a more fundamental theory,

$$\mathcal{L} = \mathcal{L}_{SM} + \mathcal{L}_{EH} + \delta\mathcal{L}_{BSM}$$

Einstein-Hilbert for classical gravity

Beyond the Standard Model

There are many theoretical approaches in looking for BSM physics

- More symmetry (GUTS, SUSY, ...)
- More d.o.f.
- Less symmetry (e.g., Lorentz violation)
- More dimensions
- Extended fundamental d.o.f.
- ...

Here we will focus on one of the more well-known extensions BSM — Neutrino masses.

Neutrino Physics

Let's go through a brief history of the neutrino

- pre-ν, β-decay spectrum was puzzling.

  Observed ${}^{A}_{Z}N \to {}^{A}_{Z+1}N' + e^-$,

  TBD, spectrum was continuous

Pauli proposed a light fermion which interacts weakly called the neutrino ν

$${}^{A}_{Z}N \to {}^{A}_{Z+1}N' + e^- + \bar{\nu}_e$$

- 1956 ν_e was discovered
- 1962 ν_μ was discovered
- 1974–1977 ν_τ evidence

  (2001) ν_τ detected (DONUT @ Fermilab)

Neutrinos exhibit parity violation (Wu 1956), are left-handed (Goldhaber 1958), and Z-decays indicate that there are 3 light-families. All consistent with SM so far.

Between 1970 & 1994, many observations of solar neutrinos indicated a neutrino deficit

$$4p \to {}^4He + 2e^+ + \underbrace{2\nu_e}_{\text{flux predicted}} \quad \text{reaction in sun}$$

Experimentally observe ν_e-flux $\sim \frac{1}{3}$ (theory prediction)

Experiments of SNO (2001) and Super K (1998) found flux $(\nu_e + \nu_\mu) \approx$ theoretical flux. This led to the hypothesis that neutrinos exhibit flavor transmutations, i.e., weak eigenstates $\neq$ propagating states.

(cf. CKM matrix & neutral meson oscillations)

This mixing suggests neutrinos have mass!

Let's look at a simple Quantum mechanical model, with no spinor structure & only 2 flavors

$|\alpha\rangle$, $\alpha = e, \mu$ produced by weak interactions

$|j\rangle$, $j = 1, 2$ propagating states

i.e.,

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \underbrace{\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}}_{\text{Mixing matrix}} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

Consider a fixed p-state $|\nu_e\rangle \equiv |\nu_e(\vec{p})\rangle$

At $t > 0$,

$$\begin{aligned} |\nu_e(t)\rangle &= e^{-iHt} |\nu_e\rangle \\ &= e^{-iHt} \left(\cos\theta |1\rangle - \sin\theta |2\rangle \right) \\ &= e^{-iE_1 t} \cos\theta |1\rangle - e^{-iE_2 t} \sin\theta |2\rangle \end{aligned}$$

Probability to detect $|\nu_e\rangle$ at t is

$$\begin{aligned} P_{e\to e}(t) &= |\langle \nu_e | \nu_e(t) \rangle|^2 \\ &= \left| e^{-iE_1 t} \cos^2\theta + e^{-iE_2 t} \sin^2\theta \right|^2 \\ &= \cdots \\ &= 1 - \sin^2 2\theta \, \sin^2\left((E_2 - E_1) t/2 \right) \end{aligned}$$

In the ultrarelativistic limit, $|\vec{p}| \gg m_j \Rightarrow |\vec{p}| \sim E$
$t \sim L$

$$E_{\nu} \simeq |\vec{p}| + \frac{1}{2}\frac{m_j^2}{|\vec{p}|}$$

some distance at time t,

$$\Rightarrow \boxed{P_{e\to e}(t) = 1 - \sin^2 2\theta \, \sin^2\left(\frac{\Delta m_{21}^2}{4E} L\right)}$$

where $\Delta m_{21}^2 \equiv m_2^2 - m_1^2$

Similar analysis gives

$$\boxed{P_{e\to\mu}(t) = \sin^2 2\theta \, \sin^2\left(\frac{\Delta m_{21}^2}{4E} L\right)}$$

Notice that we are NOT sensitive to m_j, just Δm_{21}^2.
But, experimentally

$$\sqrt{\Delta m_{21}^2} = 9 \times 10^{-3} \, eV$$
$$\sqrt{\Delta m_{32}^2} = 5 \times 10^{-2} \, eV$$

Cosmological observation & direct measurements give

$$m_{\nu} \lesssim 2 \, eV$$

Also notice that we need both $\theta \neq 0$ & $\Delta m \neq 0$

$\Rightarrow$ Need flavour mixing & neutrino masses!

The total probability is 1, $P_{e\to e} + P_{e\to\mu} = 1 \quad \forall\, t$.

This model holds in vacuum, and assumes coherent and monochromatic neutrinos (although could extend argument to wave packets, & also QFT).

So, in order to study neutrinos within the SM, we need to extend it with neutrino masses. Our goal is to maintain gauge-structure of MSM, with renormalizable interactions & aim for minimal changes.

Dirac Mass

Recall that $\bar{\psi}_R \psi_R = 0 = \bar{\psi}_L \psi_L$

To add a mass term, we need a new field,

$$\mathcal{L}^{D}_{mass} \sim m, \langle \phi \rangle \bar{\psi}_R \psi_L + h.c.$$

$\Rightarrow$ Need new d.o.f. to MSM, ν_R.

$$(\underset{\sim}{1}, \underset{\sim}{1})_0$$

$\longrightarrow$ Can argue this by anomaly cancellation

No strong int. $\longleftarrow$

$\hookrightarrow$ 1 new field (Not doublet)

Try Yukawa-Higgs term

$$G'(\bar{L}\phi)\nu_R + h.c. \qquad SU(3),\ SU(2) \text{ singlet } \checkmark$$

$$Y: \quad +1 \quad +1 \quad \boxed{-2} \quad \times \qquad \nu_R \text{ should be } Y=0$$

↳ Does not transform correctly under Hypercharge!

Try conjugate-Higgs

$$G(\bar{L}\phi^c)\nu_R \ + h.c. \qquad SU(3),\ SU(2) \text{ singlets } \checkmark$$

$$Y: \quad +1 \quad -1 \quad 0 \quad \checkmark$$

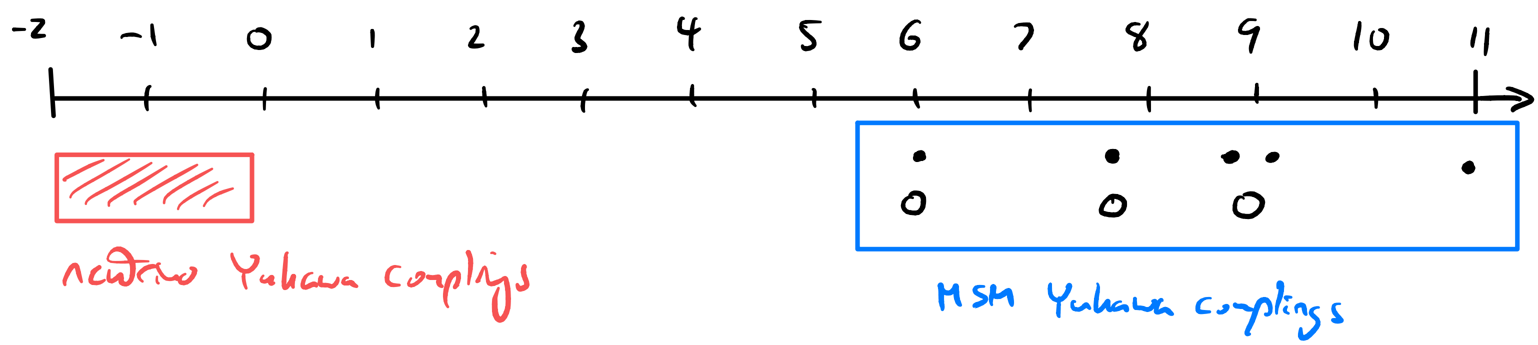
$$Q = T_3 + \tfrac{1}{2}Y = 0 + \tfrac{1}{2}(0) = 0 \quad \checkmark$$

$$\Rightarrow \quad \boxed{G(\bar{L}\phi^c)\nu_R \ + h.c.} \ \text{gives mass to } \nu!$$

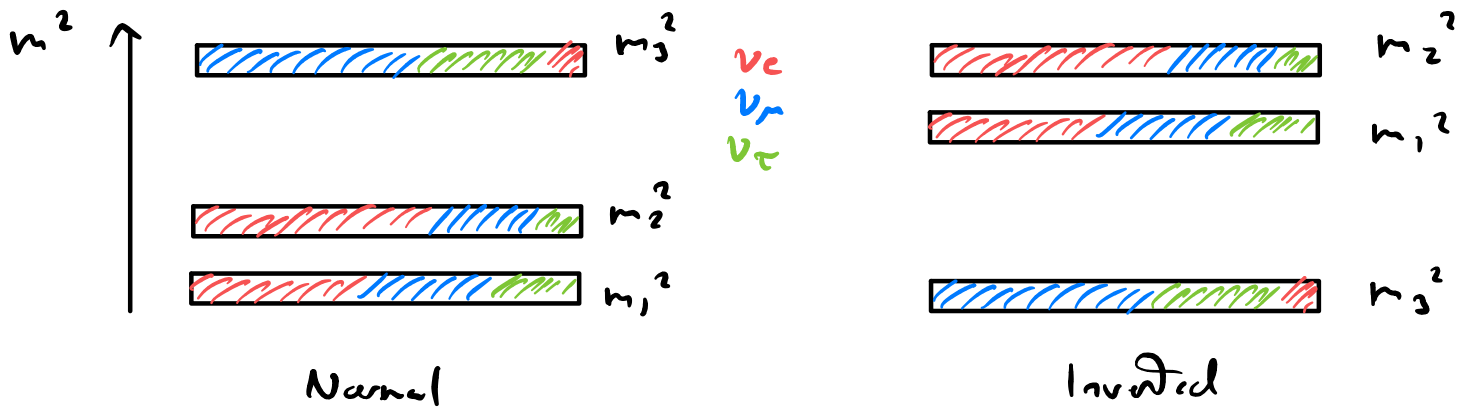
So, add new field to SM, $N = \nu_R$, $(\underset{\sim}{1}, \underset{\sim}{1})_0$

Yukawa couplings are tiny ⇒ New heirarchy problem?

mass $\log_{10}$ eV



Since only have Δm^2, two possibilities for mass spectrum



Have neutrino mass, what about mixing?

Let's revisit Yukawa mixings for leptons. Now that there is a right-handed neutrino, cannot perform diagonalization trick.

$$\ell_L = (U_L^\ell)\,\hat{\ell}_L \qquad \nu_L = (U_L^\nu)\,\hat{\nu}_L$$

$$\ell_R = (U_R^\ell)\,\hat{\ell}_R \qquad \nu_R = (U_R^\nu)\,\hat{\nu}_R$$

So,

$$\mathcal{L}^{\text{leptons}}_{\text{mass}} = -\frac{1}{\sqrt{2}} G^L \bar{\ell}_R (0,a)\begin{pmatrix}\nu_L \\ \ell_L\end{pmatrix} + h.c.$$

$$-\frac{1}{\sqrt{2}} G \bar{\nu}_R (a,0)\begin{pmatrix}\nu_L \\ \ell_L\end{pmatrix} + h.c.$$

$$= -\frac{1}{\sqrt{2}} \bar{\hat{\ell}}_R \underbrace{(U_R^\ell)^\dagger G^L (U_L^\ell)} (0,a)\begin{pmatrix}(U_L^\ell)^\dagger (U_L^\nu)\hat{\nu}_L \\ \hat{\ell}_L\end{pmatrix} + h.c.$$

$$-\frac{1}{\sqrt{2}} \bar{\hat{\nu}}_R \underbrace{(U_R^\nu)^\dagger G (U_R^\nu)}_{\text{choose s.t. diagonal}} (a,0)\begin{pmatrix}\hat{\nu}_L \\ (U_L^\nu)^\dagger (U_L^\ell)\hat{\ell}_L\end{pmatrix} + h.c.$$

$$\supset -m_e \bar{\hat{\ell}}_R \hat{\ell}_L + h.c. \quad - m_\nu \bar{\hat{\nu}}_R \hat{\nu}_L + h.c.$$

As with the CKM matrix, this lepton mixing affects interactions with $W^{\pm}$,

$$\bar{L} \not{D} L = (\bar{\nu}_L, \bar{\ell}_L)\gamma^\mu \left(\begin{array}{c|c} \# & -\frac{1}{\sqrt{2}} i g W_\mu^+ \\ \hline -\frac{1}{\sqrt{2}} i g W_\mu^- & \# \end{array}\right)\begin{pmatrix} \nu_L \\ \ell_L \end{pmatrix}$$

$$\Rightarrow \bar{L}\not{D} L \supset \bar{\hat{\nu}}_L (U_L^\nu)^\dagger \left(-\frac{1}{\sqrt{2}} i g W_\mu^+\right)(U_L^\ell)\hat{\ell}$$
$$+ \bar{\hat{\ell}}_L (U_L^\ell)^\dagger \left(-\frac{1}{\sqrt{2}} i g W_\mu^-\right)(U_L^\nu)\hat{\nu}_L$$

Define $\boxed{U_{PMNS} \equiv (U_L^\ell)^\dagger (U_L^\nu)}$

Pontecorvo-Maki-Nakagawa-Sakata (PMNS) mixing matrix

Mixing angle counting is same as CKM

$$\theta'_{12} = 33.41^\circ \,{}^{+0.75^\circ}_{-0.72^\circ}$$ Assumes normal ordering

$$\theta'_{23} = 49.1^\circ \,{}^{+1.0^\circ}_{-1.3^\circ}$$

$$\theta'_{13} = 8.54^\circ \,{}^{+0.11^\circ}_{-0.12^\circ}$$

$$\delta' = 197^\circ \,{}^{+42^\circ}_{-25^\circ}$$ $\longleftarrow$ very difficult to measure

Compare with CKM: $\theta_{12} \sim 13^\circ$, $\theta_{23} \sim 2^\circ$, $\theta_{13} \sim 0.2^\circ$, $\delta \sim 70^\circ$

So, adding neutrino masses "seems" simple, take MSM with 19 parameters, add right-handed neutrinos with 3 mass parameters + 4 PMNS parameters. Why is this not accepted as "SM", and considered as "BSM"?

It turns out since neutrinos are neutral, there is an alternative way to add neutrino masses with SM fields.

Majorana Mass

Recall for scalar fields,

$$\varphi^* \varphi \qquad \text{vs.} \qquad \varphi \varphi$$

↳ has conserved current

↳ no conserved current

Both are mass terms! For spin-$\frac{1}{2}$, have similar structures

$$\bar{\psi} \psi \qquad \text{vs.} \qquad \bar{\psi} \psi^c$$

← complex conjugate

↳ has conserved current

↳ no conserved current (not complex conj) ⇒ still Lorentz invariant

Recall: $\psi^c = C \bar{\psi}^T$

$\bar{\psi} \psi^c = \bar{\psi} C \bar{\psi}^T$ are called Majorana mass terms.

Propose new mass terms

$$\mathcal{L}^{M}_{mass} \sim (\quad)\, \bar{\nu}_L \nu_L^c = (\quad)\, \bar{\nu}_L C \bar{\nu}_L^T$$

Some ϕ field mechanism

Let's try various contractions

- $d=3$ terms

$$\bar{\nu}_L C \bar{\nu}_L^T \quad , \quad \bar{L} C \bar{L}^T$$

hypercharge $Y = \quad +1 \quad +1 \qquad\qquad +1 \quad +1 \quad \Rightarrow Y \neq 0$ ✗

- $d=4$ terms, need one Higgs doublet

$\Rightarrow$ still 1 unaccounted $SU(2)$ index!

$$(\bar{L}\phi) C \bar{L}^T \quad , \quad (\bar{L}\phi^c) C L^T \qquad ✗$$

No $SU(2)$ index

$SU(2)$ index

- $d=5$, two Higgs fields

We know that $\bar{L}\phi^c$ is singlet under all gauge groups of MSM

$$Y = +1 -1 = 0 \ ✓$$

acts on spinor indices

try

$$\frac{1}{M} (\bar{L}\phi^c) C \underbrace{(\bar{L}\phi^c)^T}_{= \phi^{cT} \bar{L}^T} + h.c.$$

$[M] = \text{GeV}$

After SSB, $\Phi^c = \frac{a}{\sqrt{2}}\begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$$\Rightarrow \frac{1}{M} \underbrace{(\bar{L}\Phi^c)}_{\bar{\nu}_L \frac{a}{\sqrt{2}}} C \underbrace{(\Phi^{cT}\bar{L}^T)}_{\bar{\nu}_L^T \frac{a}{\sqrt{2}}} + h.c$$

$$= \frac{a^2}{2M} \underbrace{\bar{\nu}_L C \bar{\nu}_L^T}_{\hookrightarrow \text{ Majorana mass term!}} + h.c.$$

Notice no mass term generated for l-leptons.

Suppose $M \sim 10^{16}$ GeV (GUT scale)

$$\Rightarrow m_\nu \sim \frac{(246 \text{ GeV})^2}{10^{16} \text{ GeV}} \simeq 3\times 10^{-3} \text{ eV } !$$

So, $\boxed{\mathcal{L}^M_{mass} = -\frac{1}{M}(\bar{L}\Phi^c) C (\bar{L}\Phi^c)^T + h.c.}$

No new particle & no new hierarchy problem!

Can extend to 3 families. The $d=5$ operator is called "Weinberg operator" (1979), and is usually interpreted as an effective interaction.

Note also that lepton # is not conserved!

$$\nu_L \to e^{iQ_L\alpha}\nu_L$$
$$\bar{\nu}_L \to e^{-iQ_L\alpha}\bar{\nu}_L$$

$$\Rightarrow \quad \bar{\nu}_L C \bar{\nu}_L^T \to e^{-2iQ_L\alpha}\bar{\nu}_L C \bar{\nu}_L^T$$

violates lepton number by 2 units

$$\boxed{\begin{array}{l} \text{MSM} \to \text{each lepton \# is conserved} \\ \text{Dirac } \nu + \text{MSM} \to \text{Total lepton \# is conserved} \\ \text{Majorana } \nu + \text{MSM} \to \text{lepton \# \underline{not} conserved} \end{array}}$$

To include mixing, $\frac{1}{M} \to \left(\frac{1}{M}\right)_{AB} \equiv H_{AB}$ $\quad [H] = \text{GeV}^{-1}$

$\hookrightarrow$ family index

flavor space

Consider $\bar{\nu}_L^A C \bar{\nu}_L^{BT} = (\bar{\nu}_L^A C \bar{\nu}_L^{BT})^T$

$$= -\bar{\nu}_L^B C^T \bar{\nu}_L^{AT} \quad , \quad C^T = -C$$

$$= \bar{\nu}_L^B C \bar{\nu}_L^{AT} \Rightarrow \text{Symmetric!}$$

So, $\mathcal{L}^M_{mass} \sim H_{AB}\,\bar{\nu}_L^A C \bar{\nu}_L^{BT}$

$\Rightarrow$ H_{AB} is symmetric! $H^T = H$ (unlike Yukawa in MSM)

Note! H not Hermitian!

Diagonalize, $U H U^T = $ diagonal

Let $J = H H^\dagger \Rightarrow J = J^\dagger$ Hermitian

So, $V^\dagger J V = $ diagonal, real eigenvalues

Let $A = V^T H V$, $A^T = V^T H^T V = A$ since $H^T = H$

So,

$$A^\dagger A = V^\dagger H^\dagger \underbrace{V^* V^T}_{(V V^\dagger)^* = \mathbb{1}} H V$$

$$= V^\dagger H^\dagger H V \quad \text{diagonal, } \lambda \text{ real}$$

Let $A = X + iY$, X, Y real matrices ($X^T = X$, $Y^T = Y$)

$$A^\dagger A = X^2 - Y^2 + X^T i Y - i Y^T X$$
$$= X^2 - Y^2 + i[X, Y]$$

Since $A^\dagger A$ is real $\Rightarrow [X, Y] = 0$

$\Rightarrow X, Y$ simultaneously diagonalizable

$W X W^T$ diagonal

$W Y W^T$ diagonal

$$\Rightarrow \quad U H U^T = W \underbrace{V^T H V}_{A} W^T$$

w/ $U = W V^T$

$$= W A W^T$$

$$= W(X + iY) W^T$$

$$= \text{diagonal} \; !$$

So, can diagonalize to get mass terms.

Define $\bar{\nu}_L = \hat{\bar{\nu}}_L (U_L^{\nu})$, thus as before

Notice, defined with $\bar{\nu}$!

$$\mathcal{L}_{\text{CC}} = (\bar{\nu}_L, \bar{\ell}_L) \gamma^\mu \left(\begin{array}{c|c} \# & -\frac{1}{\sqrt{2}} i g W_\mu^+ \\ \hline -\frac{1}{\sqrt{2}} i g W_\mu^- & \# \end{array} \right) \begin{pmatrix} \nu_L \\ \ell_L \end{pmatrix}$$

$$= \text{diagonal terms}$$

$$-\frac{1}{\sqrt{2}} i g \, \hat{\bar{\nu}}_L (U_L^{\nu}) \gamma^\mu W_\mu^+ (U_L^{\ell}) \hat{\ell}_L$$

$$-\frac{1}{\sqrt{2}} i g \, \hat{\bar{\ell}}_L (U_L^{\ell})^\dagger \gamma^\mu W_\mu^- (U_L^{\nu}) \hat{\nu}_L$$

Define $U_\nu \equiv (U_L^{\nu})(U_L^{\ell})$

$$U_\nu^\dagger = (U_L^{\ell})^\dagger (U_L^{\nu})^\dagger \equiv U'_{\text{PMNS}}$$