

# Physics 101 P

## General Physics I

Problem Sessions - Week 10

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## Example

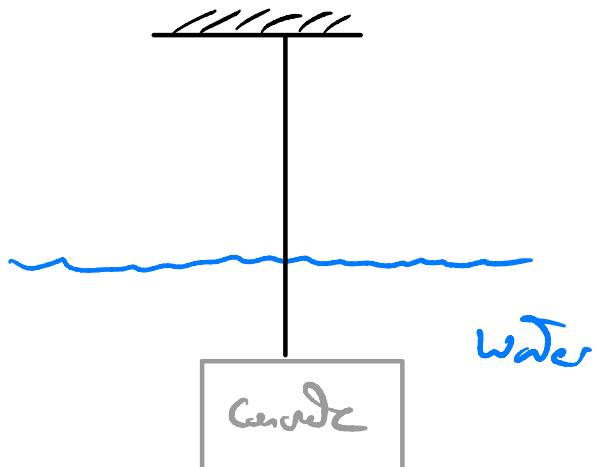
A 500 kg solid block of concrete is submerged under water and held by an ideal cable, as shown. The density of concrete is  $2300 \text{ kg/m}^3$ , & the density of water is  $1000 \text{ kg/m}^3$ . What is the buoyant force on the block? What is the tension in the cable? Note that the tension measures the apparent weight.

## Solution

The buoyant force is given by

$$B = \rho_{H_2O} V g$$

where  $V$  is volume of displaced water



Nay, the volume of concrete is the same volume of water displaced as the concrete is fully submerged

$\Rightarrow$  Archimedes principle

$$\Rightarrow m = \rho_{\text{concrete}} V \Rightarrow V = \frac{m}{\rho_{\text{concrete}}} = 0.217 \text{ m}^3$$

$$S_r' \quad T_B = \rho_{H_2O} V g$$

$$= \frac{\rho_{H_2O} mg}{\rho_{\text{concrete}}}$$

$$\approx \frac{mg}{2.3}$$

$$\approx 0.43 mg = 2,170 \text{ N } \blacksquare$$



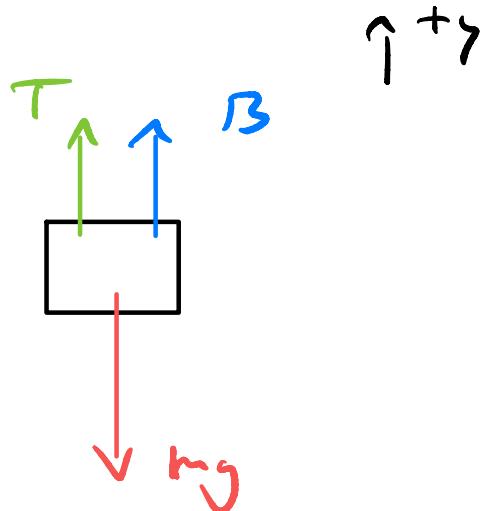
43% of weight of concrete

The tension on cable can be found

by  $\sum \vec{F} = 0$

$\sum \vec{F} = 0$

$\gamma: T + B - mg = 0$



$\Rightarrow T = mg - B$

$$= mg \left( 1 - \frac{\rho_{H_2O}}{\rho_{concrete}} \right)$$

$$= 0.56 mg \quad \leftarrow 56\% \text{ f weight}$$

$$\approx 2800 \text{ N} \quad \bullet \quad \delta \text{ concrete}$$

What if not  $H_2O$ ,  $\hookrightarrow$  air

$$\rho_{air} = 1.3 \frac{kg}{m^3}$$

$$\Rightarrow T = \left( 1 - \frac{\rho_{air}}{\rho_{concrete}} \right) mg$$

$$\approx 0.9994 mg$$

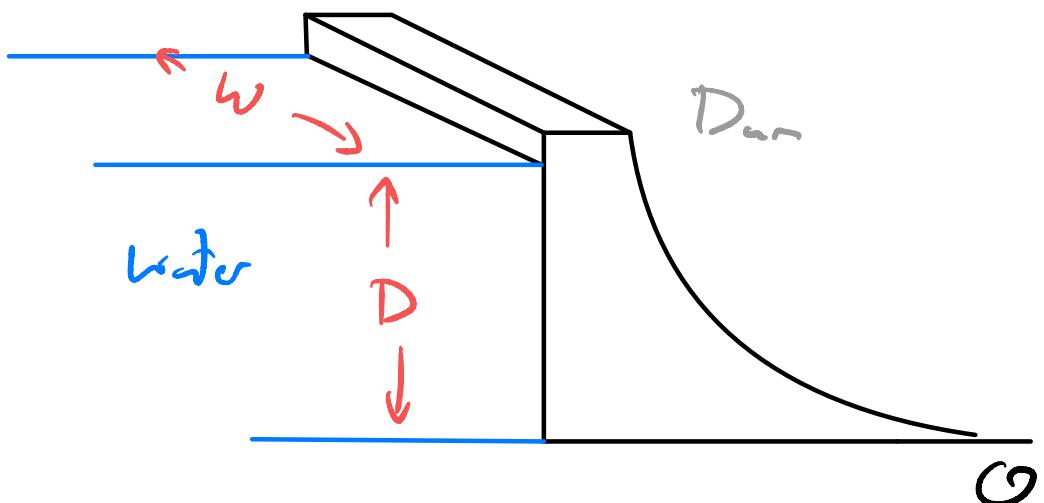
$\hookrightarrow$  negligible effect

## Example

Water stands at a depth  $D$  behind the vertical upstream face of a dam.

Let  $W$  be the width of the dam.

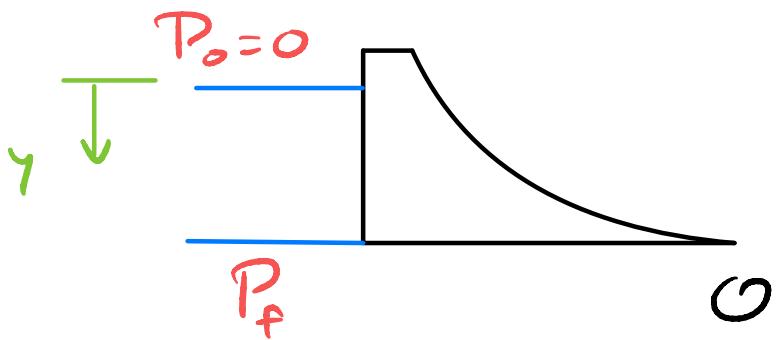
- Find the resultant (horizontal) force exerted on the dam by the gauge pressure of the water.
- Find the net torque due to the gauge pressure of the water exerted about a line through  $O$  parallel to the width of the dam.



## Solutions

Gauge pressure = pressure relative to  
atmospheric

(or)



$$\text{Now, } P - P_o = \rho g (y - y_o)$$

$$\text{at } y_o = 0, P_o = 0$$

$$\Rightarrow P = \rho g y$$

Look at infinitesimal area on dam



$$dF = P dA$$

So,

$$F = \int P dx dy$$
$$= \rho g \int_0^w dx \int_0^D \gamma dy$$

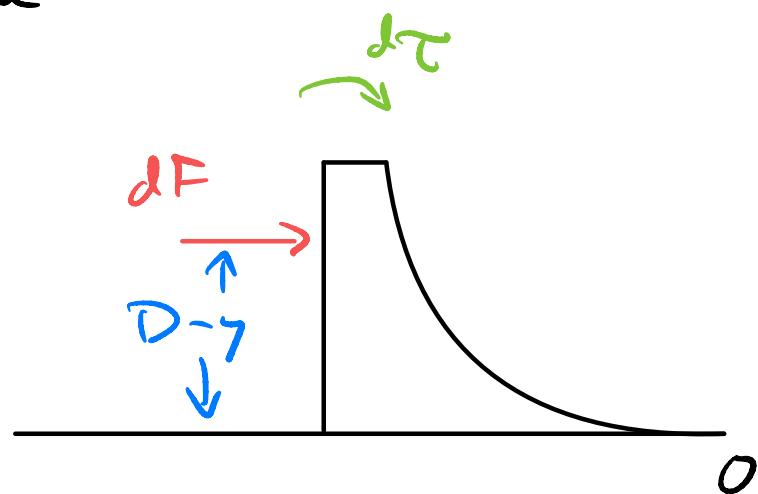
$$= \rho g w \frac{\gamma^2}{2} \Big|_0^D$$
$$= \frac{1}{2} \rho g w D^2$$

As we go deeper, larger force

$\Rightarrow$  Dam needs to be thicker

(6)

target

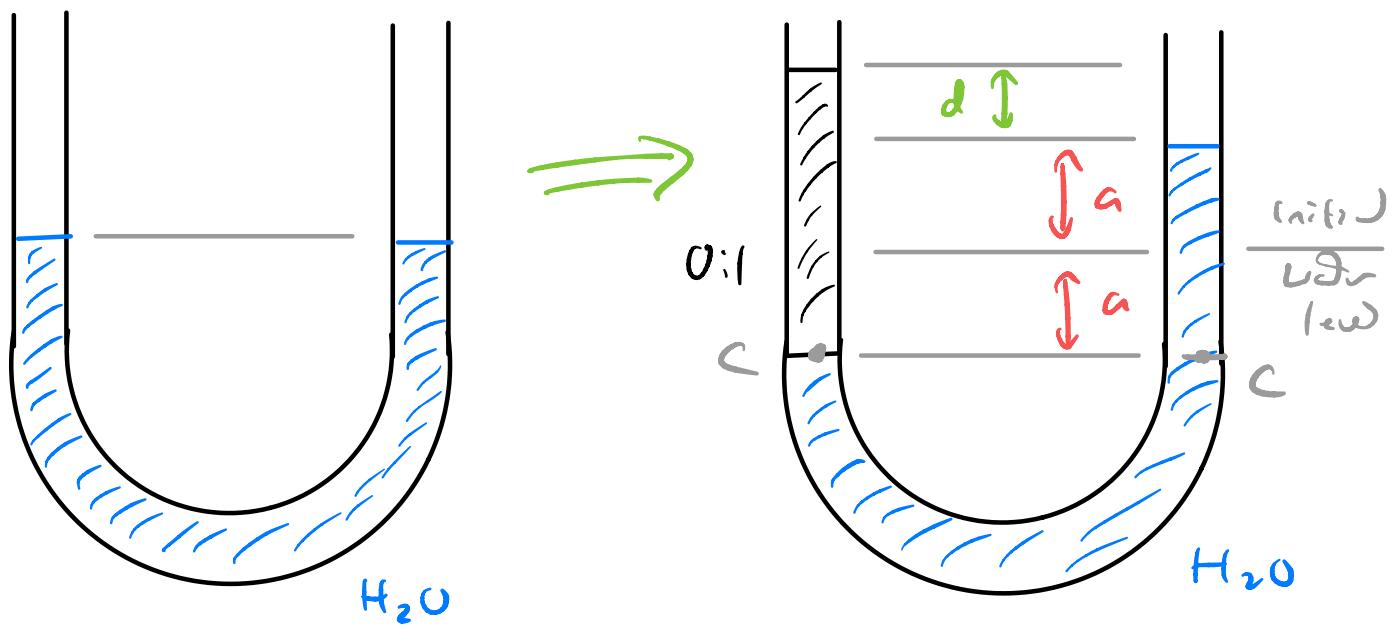


$$d\tau = (D-\gamma) dF$$

$$\begin{aligned} \Rightarrow \tau &= \int (D-\gamma) P dx dy \\ &= \rho g \int_0^w dx \int_0^D (D-\gamma) \gamma dy \\ &= \rho g w \left[ \frac{D\gamma^2}{2} - \frac{\gamma^3}{3} \right]_0^D \\ &= \rho g w \left( \frac{D^3}{2} - \frac{D^3}{3} \right) \\ &= \frac{1}{6} \rho g w D^3 \end{aligned}$$

## Example

A U-tube, in which both ends are open to the atmosphere, is partly filled with water. Oil, which does not mix with water, is poured into one side until it stands a distance  $d = 12.3 \text{ mm}$  above the water level on the other side, which has meanwhile risen a distance  $a = 67.5 \text{ mm}$  from its original level. Find the density of the oil.



## Solution

Points C are at the same pressure.

the pressure change to C from the water side

$$\Delta P = \rho_w g (z_a)$$

the pressure drop from Oil side to C

$$\Delta P = \rho_{oil} g (z_a + d)$$

Now, pressure drop to C must be equal

$$\Rightarrow \rho_w g (z_a) = \rho_{oil} g (z_a + d)$$

Solve for density of oil

$$\rho_{oil} = \rho_w \frac{z_a}{z_a + d}$$

$$= \rho_w \frac{1}{1 + \frac{d}{z_a}} \approx 916 \text{ kg/m}^3$$

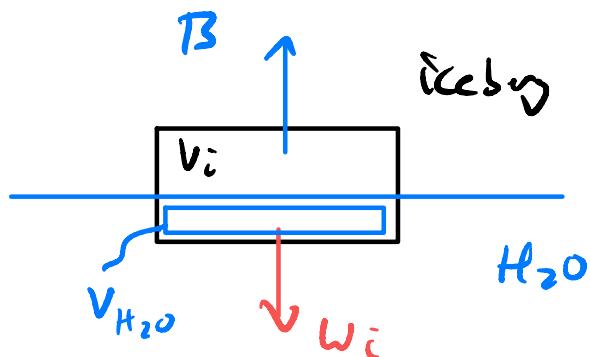
## Example

What fraction of the total volume of an iceberg is exposed?

## Solution

Weight of iceberg

$$W_i = \rho_i V_i g$$



Buoyant force

$$B = \rho_{H_2O} V_{H_2O} g$$

Density of ice =  $917 \text{ kg/m}^3$

Density of seawater =  $1024 \text{ kg/m}^3$

Volume of water displaced

Static equilibrium

$$\Rightarrow \sum \vec{F} = 0 \Rightarrow B = W_i$$

$$\therefore \rho_{H_2O} V_{H_2O} g = \rho_i V_i g$$

$$\Rightarrow \frac{V_{H_2O}}{V_i} = \frac{\rho_i}{\rho_{H_2O}} = 0.896 \Rightarrow 89.6\% \quad \blacksquare$$

$\Rightarrow 10.4\%$  Exposed  $\blacksquare$

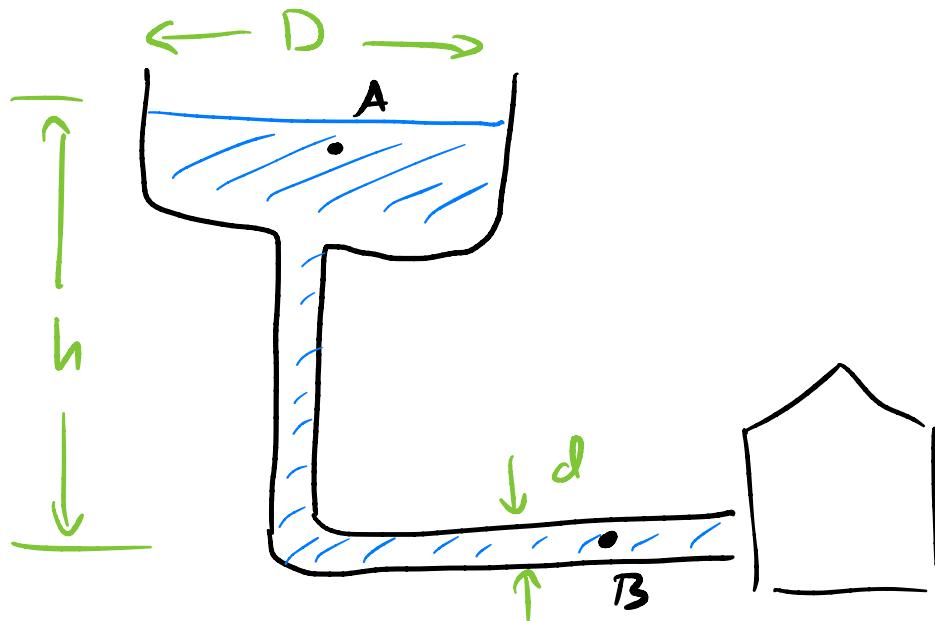
## Example

A storage tower of height  $h = 32 \text{ m}$  & diameter  $D = 3 \text{ m}$  supplies water to a house.

A horizontal pipe at the base of the tower has a diameter  $d = 2.54 \text{ cm}$ .

To satisfy the needs of the house, the supply pipe must deliver water at a rate  $R = 0.0025 \text{ m}^3/\text{s}$ .

If water were flowing at maximum rate, what is the pressure in the horizontal pipe?



## Solution

use Bernoulli's equation between A & B

$$P_A + \frac{1}{2} \rho v_A^2 + \rho g \gamma_A = P_B + \frac{1}{2} \rho v_B^2 + \rho g \gamma_B$$

At point A,

$$P_A = P_0 = P_{\text{atmosphere}}$$

$$\gamma_A = h$$

So, with  $\gamma_B = 0$

$$\Rightarrow P_B = P_0 + \rho gh + \frac{1}{2} \rho (v_A^2 - v_B^2)$$

To get  $v_A, v_B$ , use conservation of mass

$$\Rightarrow \frac{dm}{dt} = \rho R = \text{constant}$$

$$= \rho v_A A_A = \rho v_B A_B$$

$$So, \quad v_A = \frac{R}{A_A} = \frac{R}{\pi r_A^2} = 3.5 \times 10^{-4} \text{ m/s}$$

$$v_B = \frac{R}{A_B} = \frac{R}{\pi r_B^2} = 4.9 \text{ m/s}$$

Since,  $v_A \ll v_B$

$$\Rightarrow \frac{1}{2} \rho (v_A^2 - v_B^2) \approx -\frac{1}{2} \rho v_B^2$$

$$\Rightarrow P = P_0 + \rho gh - \frac{1}{2} \rho v_B^2$$

$$P_0 = 1.01 \times 10^5 \text{ Pa}$$

$$\rho = 1000 \text{ kg/m}^3$$

$$\Rightarrow P \approx 4.03 \times 10^5 \text{ Pa} = 4 \text{ atm}$$

## Example

Water emerges from a faucet & "nudges down" as it falls. The cross-section area

$A_1$  is  $1.2 \text{ cm}^2$ , &  $A_2$  is  $0.35 \text{ cm}^2$ .

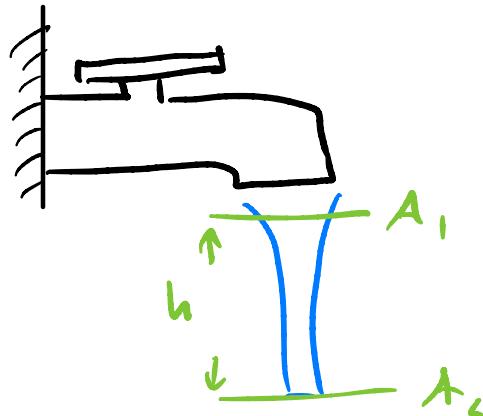
The two levels are separated by a vertical distance  $h = 45 \text{ mm}$ .

How long does it take to fill a  $100 \text{ mL}$  beaker?

## Solution

Conservation of mass

$$\frac{dm}{dt} = \rho A v = \text{constant}$$



so,

$$A_1 v_1 = A_2 v_2$$



Apply conservation of energy on fluid element

$$K_1 + U_1 = K_2 + U_2$$

$$\Rightarrow \frac{1}{2} m v_2^2 = \frac{1}{2} m v_1^2 + mgh$$

∴

$$v_2^2 = v_1^2 + 2gh$$

Now, solve for  $v_1$ ,

$$v_2 = \frac{A_1}{A_2} v_1$$

$$\Rightarrow v_1 = \sqrt{\frac{2gh A_2^2}{A_1^2 - A_2^2}}$$

$$\approx 0.286 \text{ m/s} \approx 28.6 \text{ cm/s}$$

∴

$$R = A_1 v_1 = 34 \text{ cm}^3/\text{s}$$

Now, volume of beaker

$$V = RT$$

$$\Rightarrow T = \frac{V}{R} = \frac{100 - L}{34 \text{ cm}^3/\text{s}} = \frac{100 \text{ cm}^3}{34 \text{ cm}^3/\text{s}}$$

$$\approx 2.9 \text{ s} \quad \blacksquare$$