Feynman Rules - Quartic Scalar theory

The Lagrangian density for a self-interacting real-scalar field theory is given by

$$\mathcal{L} = \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - \frac{1}{2}m^2\varphi^2 - \frac{1}{4!}\lambda\varphi^4 \tag{1}$$

where m is the mass parameter and λ is the quartic coupling.

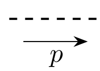
Feynman Rules - Momentum space

- Draw all topologically distinct connected diagrams at a given order.
- In each diagram, attach a propagator $i\Delta(p)$ to each internal scalar line.
- To each vertex, assign a vertex function $i\Gamma$. Each vertex conserves four-momentum.
- Each internal momentum k not fixed by momentum conservation at the vertices, integrate $\displaystyle\int \frac{\mathrm{d}^4k}{(2\pi)^4}$.
- Multiply the contribution for each diagram by an appropriate symmetry factor S^{-1} given by

  $$S^{-1} = g \prod_{n=2,3,\ldots} 2^\beta (n!)^{\alpha_n}\,,$$

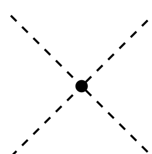
  where α_n is the number of *pairs* of vertices connected by n identical self-conjugate lines, β is the number of lines connecting a vertex with itself, and g is the number of permutations of vertices which leave the diagram unchanged with fixed external lines.
- For scattering amplitudes, place all external lines on their mass-shell $p^2 = m^2$ and multiply by the scalar wavefunction "1".

Propagators



$$i\Delta(p) = \frac{i}{p^2 - m^2 + i\epsilon} \tag{2}$$

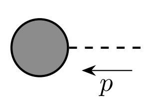
Vertex



$$i\Gamma = -i\lambda \tag{3}$$

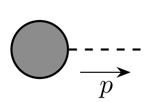
External Lines

"incoming scalar"



$$1 \tag{4}$$

"outgoing scalar"



$$1 \tag{5}$$