Feynman Rules - Self-Interacting Complex Scalar theory

The Lagrangian density for a self-interacting complex-scalar field theory is given by

$$\mathcal{L} = (D_\mu\varphi)^\dagger (D^\mu\varphi) - m^2\varphi^\dagger\varphi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2\xi}(\partial_\mu A^\mu)^2$$

where m is the mass of the boson, D_μ is the gauge covariant derivative is $D_\mu = \partial_\mu + iqA_\mu$, and the field strength tensor is $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. The charge of the boson is q, e.g., for the π^+ $q = +e$ while the K^- has $q = -e$, where $e = \sqrt{4\pi\alpha}$ is the magnitude of the electron charge in natural units, and α is the fine-structure constant. The CODATA value for the fine-structure constant is $\alpha = 7.297\,352\,5693(11) \times 10^{-3}$. The gauge fixing parameter ξ is arbitrary, and physical observables must be independent of ξ. The Fermi-Feynman gauge, $\xi = 1$, is a common choice especially for tree-level calculations.

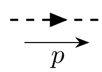
Feynman Rules

Here we give the Feynman rules for the scattering amplitude $\mathcal{M}$,

$$i\mathcal{M} = \text{sum of all connected, amputated diagrams}\,,$$

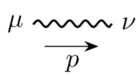
where the diagrams are evaluated according to the following rules:

- Draw all topologically distinct diagrams at a given order;
- For each internal scalar line, attach a propagator



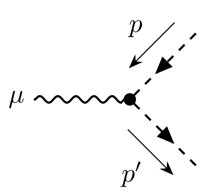
$$= \frac{i}{p^2 - m^2 + i\epsilon}\,;$$

- For each internal photon line, attach a propagator



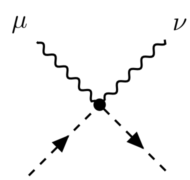
$$= \frac{-i}{p^2 + i\epsilon}\left(g_{\mu\nu} - (1-\xi)\frac{p_\mu p_\nu}{p^2}\right)\,;$$

- For each 3-point vertex, assign



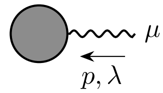
$$= -iq(p + p')^\mu\,;$$

- For each 4-point vertex, assign

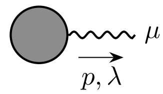


$$= 2iq^2\, g^{\mu\nu}; \tag{1}$$

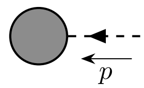
- For each external line, place the particle on the mass-shell, $p^2 = m^2$ for the boson and $p^2 = 0$ for the photon, and attach a wavefunction factor

"incoming photon" 

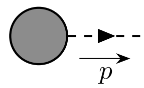
$= \epsilon_\mu(p, \lambda)\,;$

"outgoing photon" 

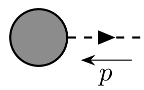
$= \epsilon^*_\mu(p, \lambda)\,;$

"incoming boson" 

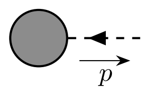
$= 1\,;$

"outgoing boson" 

$= 1\,;$

"incoming anti-boson" 

$= 1\,;$

"outgoing anti-boson" 

$= 1\,;$

- Impose momentum conservation at each vertex;
- For each internal loop momentum k not fixed by momentum conservation, integrate $\int \frac{\mathrm{d}^4 k}{(2\pi)^4}$;
- Multiply the contribution for each diagram by an appropriate symmetry factor $\mathcal{S}^{-1}$.