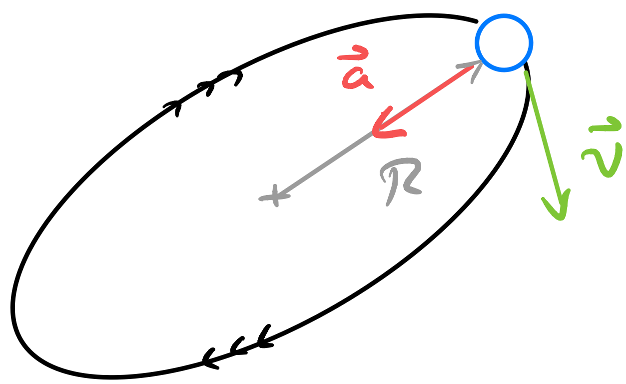
Physics 101 P

General Physics I

Problem Sessions - Week 4

A.W. Jackura —— William & Mary

Circular Motion



Uniform circular motion

- constant speed

$$v = \frac{2\pi R}{T}$$

- $\vec{v}$ tangent to path
- $\vec{a}$ points toward center
- $$a_c = \frac{v^2}{R}$$

Centripetal force

$$F_c = m a_c = \frac{m v^2}{R}$$

Work & Energy

$$W = \int_{\text{path } A \to B} \vec{F} \cdot d\vec{r} = K_B - K_A = -(U_B - U_A)$$

Conservation of Energy : $K_A + U_A = K_B + U_B$

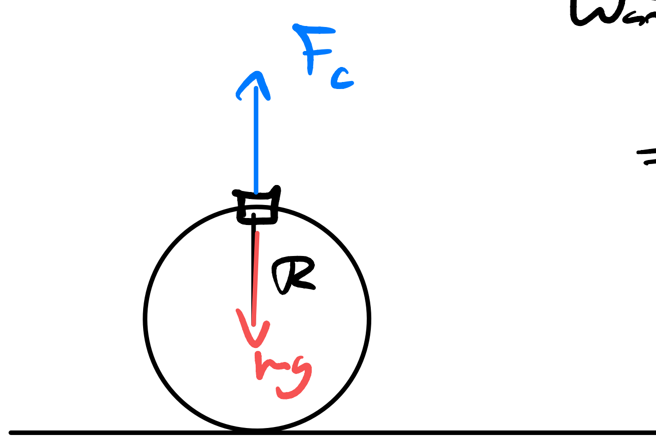
Example

Roller coasters have vertical loops.
the radius of curvature is smaller at the top than the sides — why?

Solution

Since $a_c = \frac{v^2}{R}$, & $F = ma_c$

the small R is such that to ensure the centripetal force at the top is greater than gravity



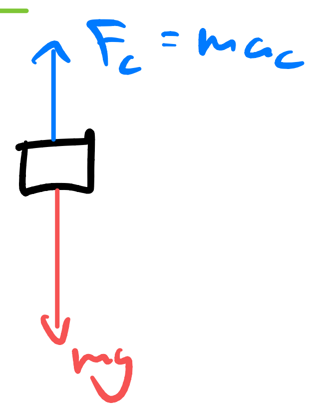
Want $F_c > mg$

$\Rightarrow a_c > g$!

Example

What is the speed of a roller coaster at the top of a loop if the radius of curvature there is 15.0 m & the downward acceleration of the car is 1.50g?

Solution



$a_c = 1.5g$

$a_c > g \Rightarrow$ car is on track

Now, $a_c = \dfrac{v^2}{R} \Rightarrow v = \sqrt{R a_c}$

$= \sqrt{1.5 R g}$

$= \sqrt{1.5 \cdot 15 \cdot 9.8}$

$\simeq 14.8$ m/s ∎

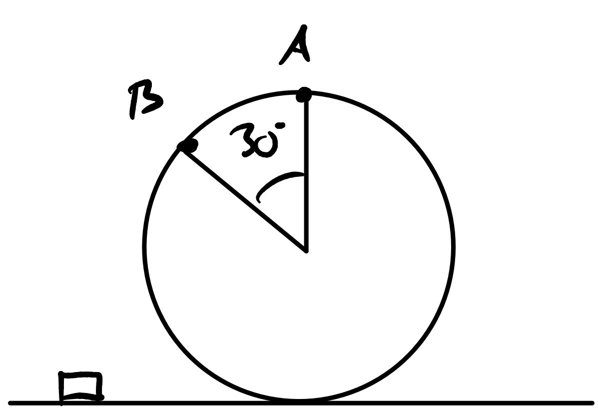
Example

A child of mass 40 kg is in a roller coaster car that travels in a loop of radius 7.00 m. At point A the speed of the car is 10.0 m/s, & at point B the speed is 10.5 m/s. Assume the child is not holding on and does not wear a seat belt.

(a) What is the force of the car seat on the child at point A?

(b) What is the force of the car seat on the child at point B?

(c) What minimum speed is required to keep the child in his seat at point A?

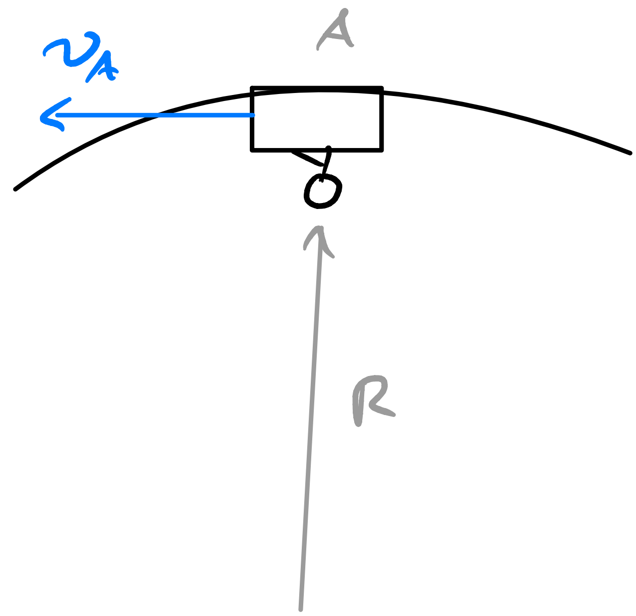


Solution

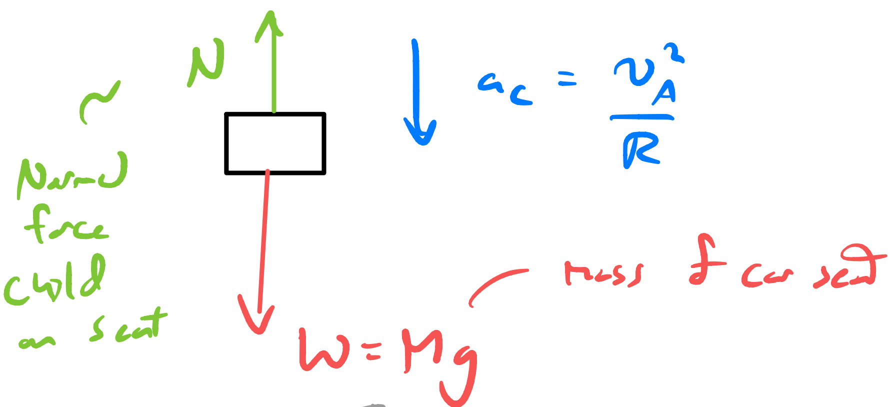
(a) $v_A = 10.0\ \text{m/s}$

$R = 7.00\ \text{m}$

$m = 40\ \text{kg}$



FBD Car seat ①



Didn't know M

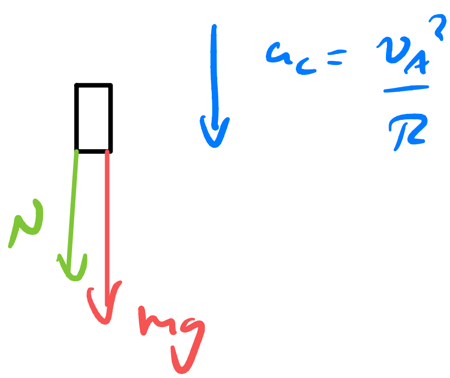
$$\sum \vec{F} = m\vec{a}$$

$\uparrow^{+}$

① $N - Mg = -M\dfrac{v_A^2}{R}$

② $-N - mg = -m\dfrac{v_A^2}{R}$

FBD Child ②



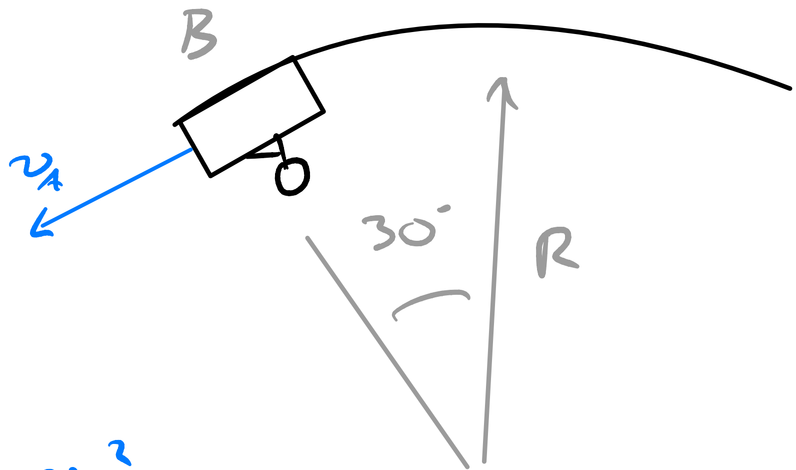
From ②

$$N = m\frac{v_A^2}{R} - mg$$

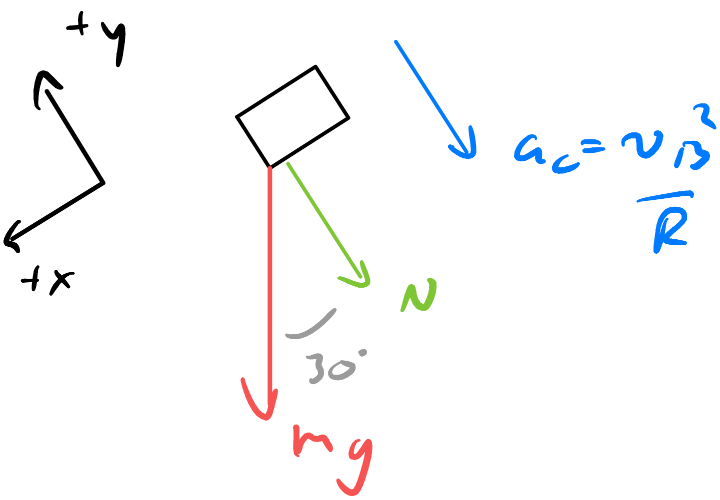
$$= m\left(\frac{v_A^2}{R} - g\right)$$

$$\simeq 179.4\ \text{N}$$

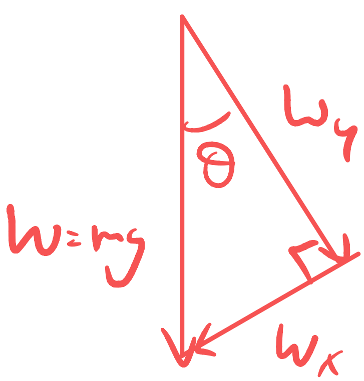
(b) $v_B = 10.5$ m/s



FBD of child



$\theta = 30°$



$W_y = W\cos\theta$

$\sum \vec{F} = m\vec{a}$

$$y: -N - mg\cos\theta = -ma_c$$

$$\Rightarrow N = ma_c - mg\cos\theta$$

$$= m\left(\frac{v_B^2}{R} - g\cos\theta\right)$$

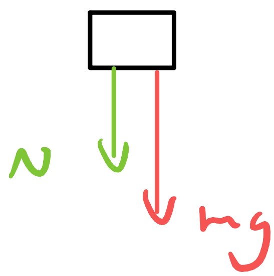
$$\simeq 240.2 \text{ N} \blacksquare$$

(c) Minimum speed at point A ?

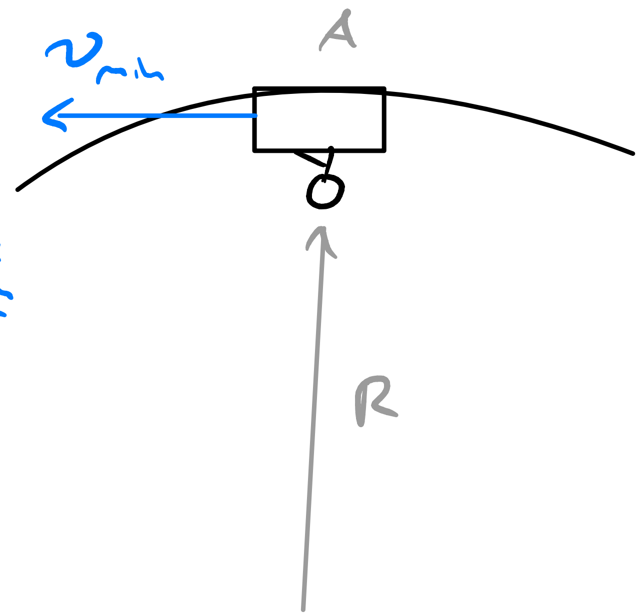
v_{min} is such that child <u>just</u>

touches the seat ⇒ No normal force

<u>FBD child</u>



$\downarrow a_c = \frac{v_{min}^2}{R}$



$$\uparrow^{+} \quad \sum \vec{F} = m\vec{a}$$

$$-N - mg = -m a_c$$

But, $N = 0 \Rightarrow g = a_c = \frac{v_{min}^2}{R}$

$$\Rightarrow v_{min} = \sqrt{gR}$$

$$\simeq 8.3 \text{ m/s} \quad \blacksquare$$

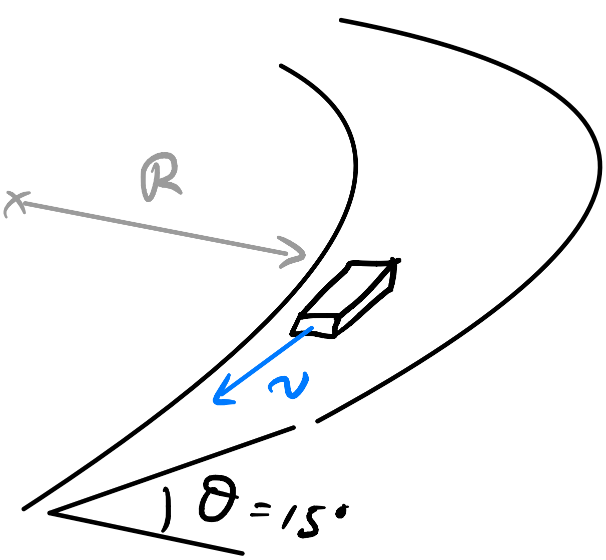
Example

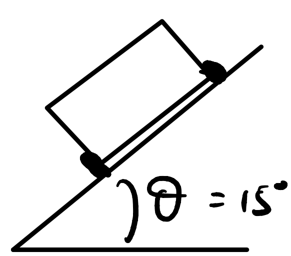
If a car takes a banked curve at less than ideal speed, friction is needed to keep it from sliding toward the inside of the curve.

(a) Calculate the ideal speed to take a 100.0 m radius curve banked at 15°.

(b) What is the minimum coefficient of friction needed for a driver taking the same curve at 20.0 km/h?

Solution

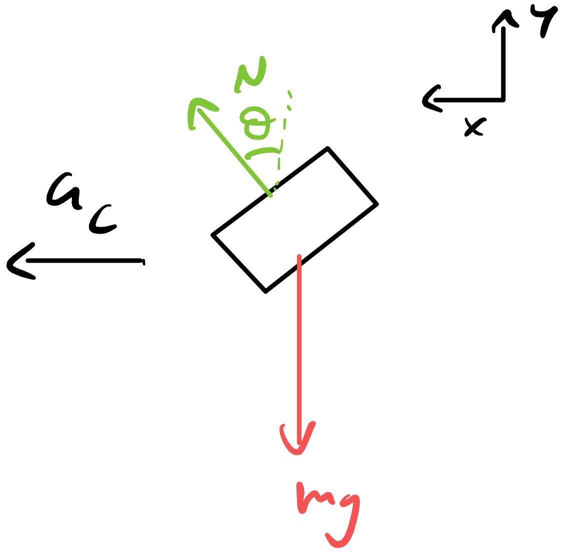




Front view

(a)

FBD car



$\sum \vec{F} = m\vec{a}$

$x: \quad N\sin\theta = m a_c$

$y: \quad N\cos\theta - mg = 0$

$$\Rightarrow \quad N = \frac{mg}{\cos\theta}$$

So, $\quad a_c = g\tan\theta$

But, $\quad a_c = \frac{v^2}{R}$

$$\Rightarrow \quad v = \sqrt{gR\tan\theta}$$

$$= \sqrt{9.8\,\text{m/s}^2 \cdot 100\,\text{m} \cdot \tan 15^\circ}$$

$$\approx 16.2\ \text{m/s} \quad \blacksquare$$

$$= 16.2\,\frac{\text{m}}{\text{s}} \cdot \left(\frac{3600\,\text{s}}{\text{h}}\right) \cdot \left(\frac{1\,\text{km}}{1000\,\text{m}}\right)$$

$$\approx 58.3\ \frac{\text{km}}{\text{h}} \quad \blacksquare$$

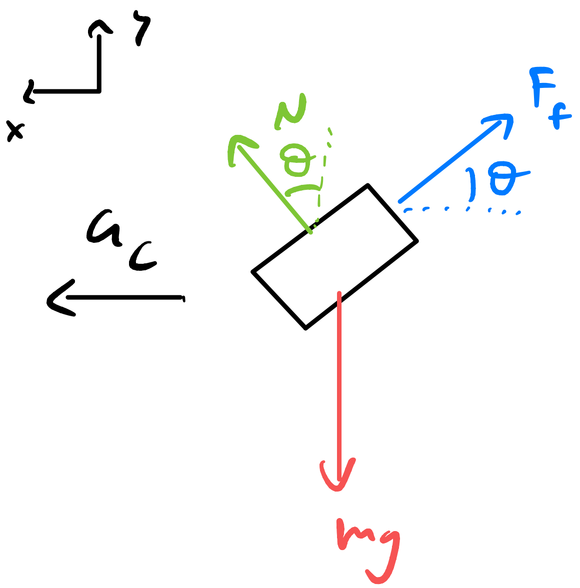
(b) Now, for very low speed car

$$v = 20 \frac{km}{h}$$

$$= 20 \frac{km}{h} \cdot \left(\frac{1 h}{3600 s}\right) \cdot \left(\frac{1000 m}{1 km}\right)$$

$$= 5.6 \ m/s$$

FBD



$$\sum \vec{F} = m\vec{a}$$

x: $N \sin\theta - F_f \cos\theta = m a_c$

y: $N \cos\theta + F_f \sin\theta - mg = 0$

also, $F_f = \mu N$

and $a_c = \frac{v^2}{R}$

$$N \sin\theta - \mu N \cos\theta = \frac{m v^2}{R} \qquad (1)$$

$$N \cos\theta + \mu N \sin\theta = mg \qquad (2)$$

$$N\sin\theta - \mu N\cos\theta = \frac{mv^2}{R} \quad (1)$$

$$N\cos\theta + \mu N\sin\theta = mg \quad (2)$$

Solve (1) for N,

$$N = \frac{mv^2}{R} \frac{1}{\sin\theta - \mu\cos\theta}$$

Solve (2) for μ

$$\mu = \frac{mg - N\cos\theta}{N\sin\theta}$$

$$= \frac{1}{N}\frac{mg}{\sin\theta} - \frac{1}{\tan\theta}$$

$$\mu = \frac{R}{\cancel{m}v^2} \cdot \frac{\cancel{m}g}{\cancel{\sin\theta}} \cdot (\overset{1}{\cancel{\sin\theta}} - \mu \overset{\frac{1}{\tan\theta}}{\cancel{\cos\theta}}) - \frac{1}{\tan\theta}$$

$$= \frac{Rg}{v^2}\left(1 - \frac{\mu}{\tan\theta}\right) - \frac{1}{\tan\theta}$$

$$\Rightarrow \mu\left[1 + \frac{Rg}{v^2\tan\theta}\right] = \frac{Rg}{v^2} - \frac{1}{\tan\theta}$$

$$\mu\left[1+\frac{Rg}{v^2\tan\theta}\right]=\frac{Rg}{v^2}-\frac{1}{\tan\theta}$$

$$\mu=\frac{\dfrac{Rg}{v^2}-\dfrac{1}{\tan\theta}}{1+\dfrac{Rg}{v^2\tan\theta}}$$

$$=\frac{Rg\tan\theta-v^2}{Rg+v^2\tan\theta}$$

$$\Rightarrow\qquad \mu=\frac{Rg\tan\theta-v^2}{Rg+v^2\tan\theta}$$

$$\simeq 0.234 \qquad \blacksquare$$

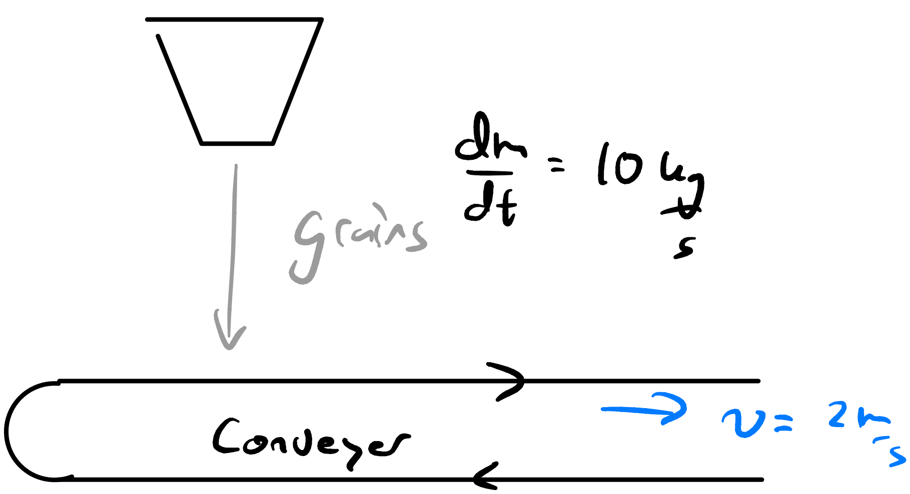
Example

Grains from a hopper falls at a rate of 10 kg/s vertically onto a conveyer belt that is moving horizontally at a constant speed of 2 m/s.

(a) What is the force needed to keep the conveyer belt moving at the constant velocity?

(b) What is the minimum power of the motor driving the conveyer belt?

Solution



$$\frac{dm}{dt} = 10\,\frac{kg}{s}$$

(a)

N.B.

Really, should introduce concept of momentum to fully understand / appreciate this problem.

$$\vec{p} = m\vec{v}$$

& NII says $\vec{F} = \frac{d\vec{p}}{dt}$

But, here $\vec{v}$ = constant, $m \neq$ constant

$$\Rightarrow \frac{d}{dt}(m\vec{v}) = \frac{dm}{dt}\vec{v} + \cancel{m\frac{d\vec{v}}{dt}}^{\,0}$$

$$\Rightarrow \vec{F} = \frac{dm}{dt}\vec{v}$$

$$F = \frac{dm}{dt} \cdot v = 10\,\frac{kg}{s} \cdot (2\,m/s)$$

$$\Rightarrow F = 20\,N$$

(b)

Power $P = F \cdot v$

$$= \frac{dm}{dt} v^2$$

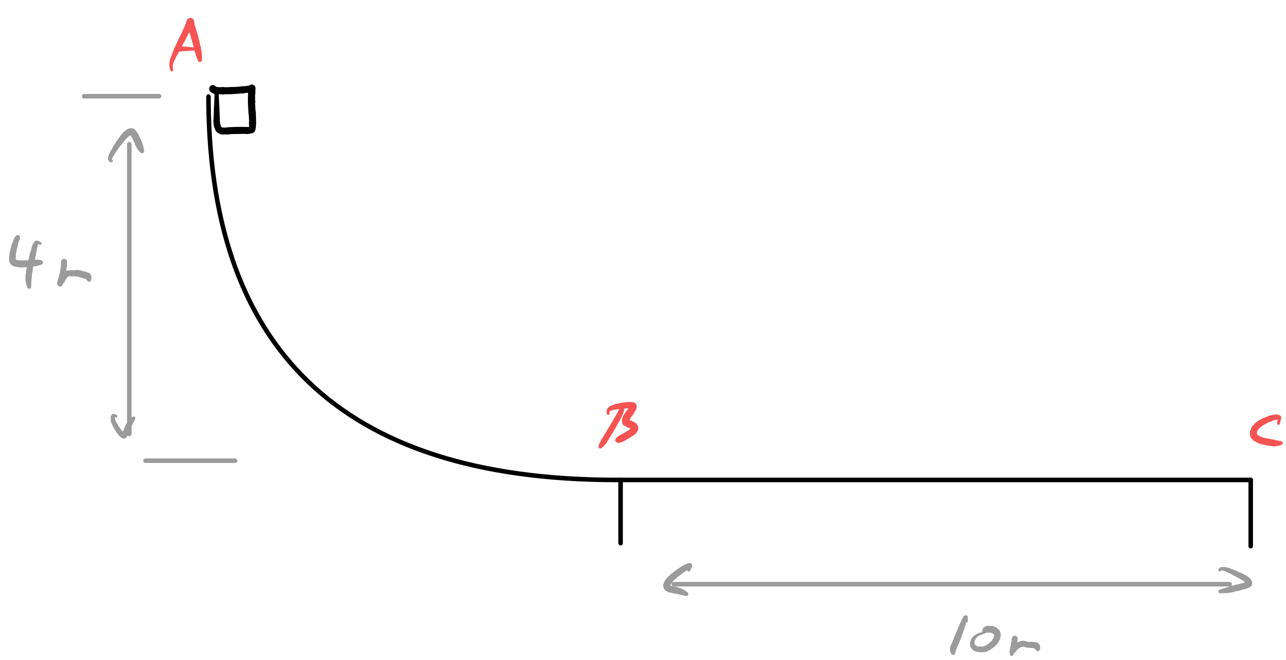
$\Rightarrow$ $P = 40 \text{ W}$

Example

A small block of mass 200 g starts at rest at A, slides to B where its speed is $v_B = 8.0$ m/s, then slides along the horizontal surface a distance 10 m before coming to rest at C.

(a) What is the work of friction along the curved surface?

(b) What is the coefficient of kinetic friction along the horizontal surface?



Solution

(a) Work of friction on curved surface

$$W = \int \vec{F} \cdot d\vec{r}$$

$$= \int \vec{F}_g \cdot d\vec{r} + \int F_f \cdot d\vec{r}$$

$$= -(U_B - U_A) + W_{fr}$$

But, also $W = K_B - K_A$

$$\Rightarrow \quad K_B - K_A = -U_B + U_A + W_{fr}$$

$$\Rightarrow \quad W_{fr}\Big|_{A\to B} = K_B + U_B - (K_A + U_A)$$

Now,

A	B
$K_A = 0$	$K_B = \frac{1}{2} m v_B^2$
$U_A = mgh$	$U_B = 0$

So,

$$W_{fr} = \frac{1}{2} m v_B^2 - mgh$$

$$= \frac{1}{2}(0.2\,kg)(8\,\tfrac{m}{s})^2 - (0.2\,kg)\cdot(9.8\,\tfrac{m}{s^2})\cdot(4\,m)$$

$$= -1.44\ J \quad \blacksquare$$

(b) $$W_{fr}\Big|_{B\to C} = \cancelto{0}{K_C} - K_B$$

$$= -\frac{1}{2} m v_B^2$$

$$= -6.4\ J$$

But,

$$W_{fr} = -F_f\,\Delta x$$

$$\left\{ \begin{array}{l} F_f = \mu N \\ N = mg \end{array} \right.$$

$$= -\mu mg\,\Delta x$$

$$\Rightarrow \mu = \frac{-W_{fr}}{mg\,\Delta x} = \frac{+6.4\ J}{(0.2\,kg)(9.8\,\tfrac{m}{s^2})\cdot(10\,m)}$$

$$\Rightarrow \mu \simeq 0.33 \quad \blacksquare$$