Phenomenology II - QCD

Quantum Chromodynamics (QCD) is a non-abelian gauge theory describing the color interactions of quarks and gluons. The Lagrange density is

$$\mathcal{L} = \frac{1}{2} i \sum_f \bar{q}_f \not{D} q_f + \text{h.c.} - \sum_f m_f \bar{q}_f q_f - \frac{1}{2} \mathrm{tr}\left[G_{\mu\nu} G^{\mu\nu}\right]$$

where $D_\mu = \partial_\mu + i g_s A_\mu$, $A_\mu = A_\mu^a T_a$

$\hookrightarrow = \frac{\lambda_a}{2}$

and

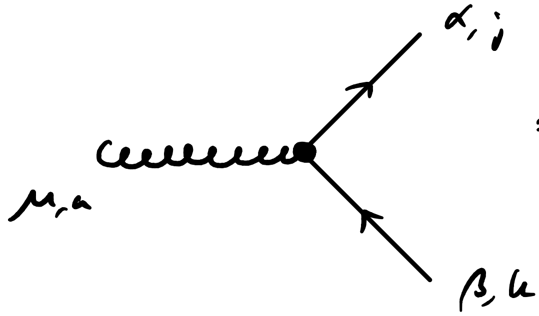
$$G_{\mu\nu} = \frac{1}{i g_s} [D_\mu, D_\nu]$$

There are 6 flavors, $f = \{u, d, s, c, b, t\}$

$\hookrightarrow$ "decays" via weak before hadronization

QCD, in principle, describes <u>all</u> hadronic and nuclear phenomena we observe in the universe. It depends on 6 quark masses, and 1 coupling, g_s. Define $\boxed{\alpha_s \equiv \frac{g_s^2}{4\pi}}$

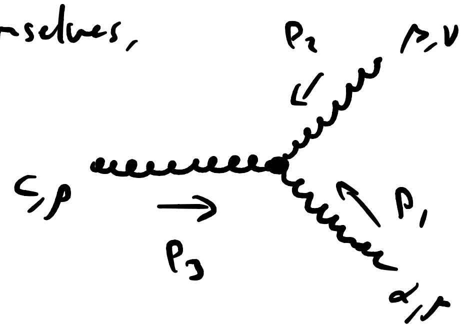
This coupling governs the interactions of massless gluons with quarks, e.g.,



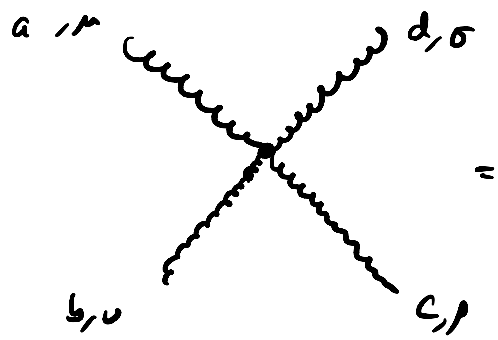
$$= -i g_s (T^a)_{jk} \gamma^\mu_{\alpha\beta}$$

$\uparrow$ carries 2 color indices

The same coupling also governs interactions of gluons with themselves,

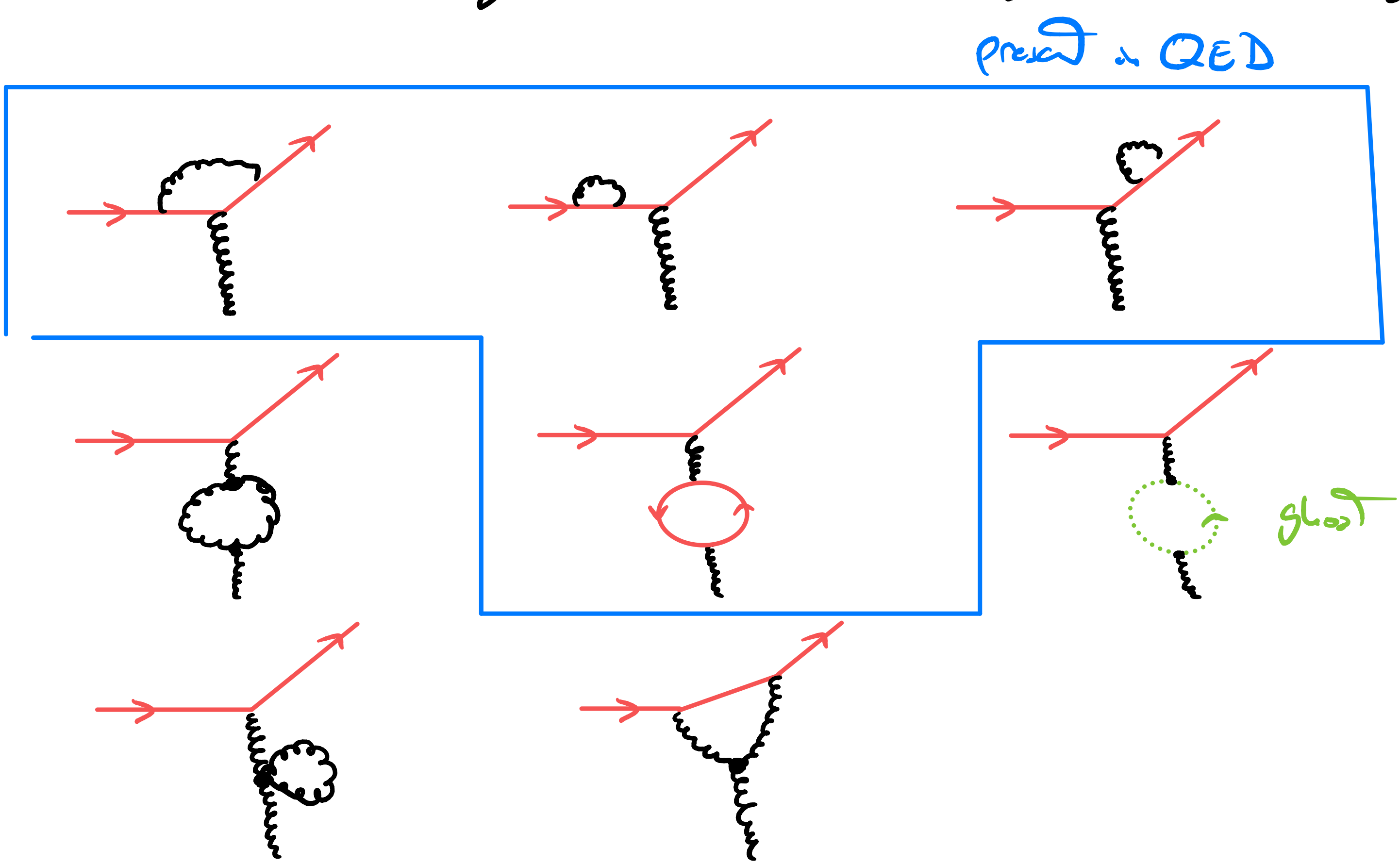


$$= -i g_s f^{abc} \left(p_3^{\mu} g^{\nu\rho} + \text{perm.} \right)$$



$$= -g_s^2 f^{abe} f^{cde} g^{\mu\rho} g^{\nu\sigma} + \text{perm.}$$

In perturbation theory, higher order corrections augment the coupling.



Can show (consult QFT II), that the coupling g_s runs with the momentum transfer Q^2, and is described by a differential equation

"beta function"

$$\frac{dg_s}{d\ln Q} = \beta(g_s)$$

# of quarks with $m_q < Q$

$$= -\frac{g_s^3}{16\pi^2}\underbrace{\left[11 - \frac{2}{3}N_f\right]}_{\text{LO } \beta_0 \text{ (1-loop)}} + \mathcal{O}(g_s^5)$$

Notice that for $N_f < 16$, $\beta(g_s) < 0$

$\Rightarrow$ g_s decreases as Q increases. This is known as $\boxed{\text{Asymptotic freedom}}$

Conversely, as Q decreases, g_s increases, and eventually perturbation theory breaks down. This indicates that low-energy physics must be computed with non-perturbative techniques. This is the $\boxed{\text{confinement}}$ region. Gluons are essential for this, cf. QED with electrons and photons

$$\frac{de}{d\ln Q} = \frac{e^3}{16\pi^2}\cdot\frac{4}{3} + \mathcal{O}(e^5) > 0$$

Can write a general for α_s,

$$\frac{d\alpha_s}{d\ln Q^2} = -\alpha_s\left[\left(\frac{\alpha_s}{4\pi}\right)\beta_0 + \left(\frac{\alpha_s}{4\pi}\right)^2\beta_1 + \cdots\right]$$

$\hookrightarrow$ $11 - 2N_f/3$

Solving the 1-loop equation, let $x = 1/\alpha_s$, $L = \ln Q^2$

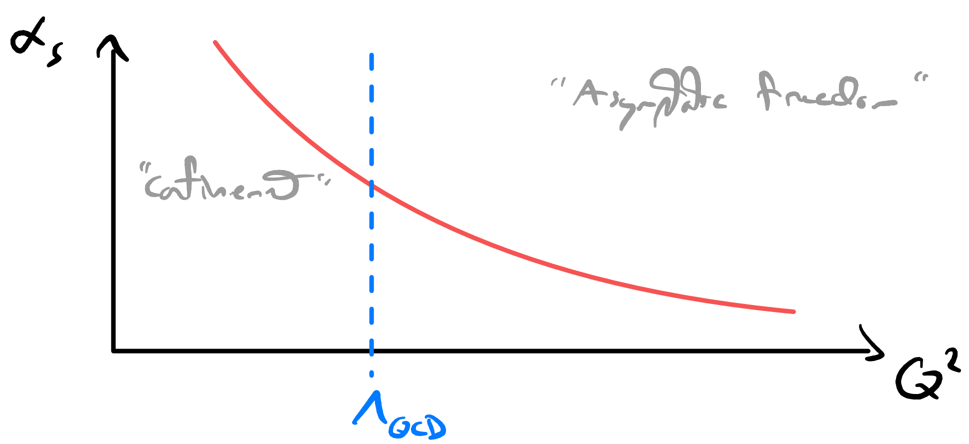
$$\Rightarrow \quad \frac{dx}{dL} = \frac{-1}{\alpha_s^2}\frac{d\alpha_s}{dL} = \frac{\beta_0}{4\pi} \quad \Rightarrow \quad x = \frac{\beta_0}{4\pi}L + \text{const.}$$

$$\Rightarrow \quad \boxed{\alpha_s(Q^2) = \frac{4\pi}{\beta_0}\frac{1}{\ln(Q^2/\Lambda^2_{QCD})}}$$

$\hookrightarrow$ integration const.

This is the running coupling.

Λ_{QCD} is value of Q at which $\alpha_s = 1$. Consider massless QCD, with only the dimensionless coupling (1 parameter). But, the running introduces a dimensionful scale. This is known as <u>dimensional transmutation</u>. In effect, Λ_{QCD} sets the scale for strong interactions involving light quarks.



To study low-energy hadrons, we need a non-perturbative technique to access observables. There is another issue we need to deal with, and that is we do not observe quarks and gluons, but observe them only as confined inside hadrons.

What are the allowed quark & gluon combinations that can give a valid hadron? We have seen from the Quark Model

$q\bar{q}$ – Mesons

qqq – Baryons, $\bar{q}\bar{q}\bar{q}$ – Antibaryons

In QCD, we need color singlet objects made from q_j, $\bar{q}_j$, and A^{μ}_{jk} (Field operators)

color indices

For hadrons to exist, there must be a color singlet operator to create it from the QCD vacuum

e.g.,

- $\bar{q}_j \gamma_5 q'_j$ for $J^{PC} = 0^{-+}$ mesons
- $\epsilon_{jkl}\, q_j q'_k q''_l$ for baryons

BUT, also can have

- $G^{\mu\nu}_{jk} G_{\nu\mu, kj}$ for scalar glueball
- $\bar{q}_j [\gamma^\mu, \gamma^\nu] G_{\mu\nu, jk} q_k$ for $q\bar{q}g$ hybrid
- $\bar{q}_i \gamma^\mu q'_j \bar{q}''_k \gamma_\mu q'''_k$ for "tetraquark"
- $\epsilon_{jkl} q_j q'_k q''_l \bar{q}'''_n q''''_n$ for "pentaquark"

... any many (∞) more.

The problem is that there is no 1-to-1 correspondence between operators and states in QFT. Operators with the same quantum numbers can create the same state

e.g.,
$$\mathcal{O}_1 = \bar{u}\gamma_5 u + \bar{d}\gamma_5 d + \bar{s}\gamma_5 s$$ looks like $\bar{q}q$
$$\mathcal{O}_2 = \epsilon_{\mu\nu\rho\sigma} G^{\mu\nu}_{jk} G^{\rho\sigma}_{jk}$$ looks like pseudoscalar glueball

Both can create η & η', $\langle 0 | \mathcal{O}_{1,2} | \eta^{(\prime)} \rangle \neq 0$

From a QFT pov, asking whether η' is a $\bar{q}q$ state or a glueball is almost meaningless.

Moreover, states created by a given operator can consist of multiple hadrons, e.g.,

$$\bar{u}\gamma_5 d\,\bar{u}\gamma_5 d\,|0\rangle \sim |\pi^+\pi^+\rangle$$

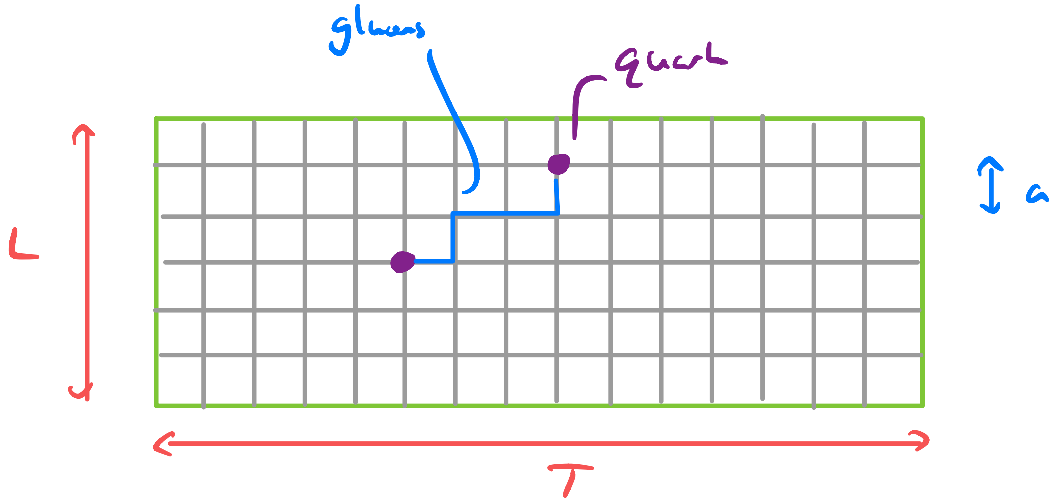
$\Rightarrow$ Existence of an operator <u>Does Not</u> imply existence of a new hadron.

How can we access such complicated non-perturbative objects from QCD? <u>Lattice QCD</u> is a numerical technique to stochastically compute the non-perturbative QCD path integral. Can now calculate light hadron masses w/ $\lesssim 1\%$ uncertainties and reaction amplitudes involving a couple of hadrons. This is the only known method for making quantitative predictions from QCD in the low-energy region.

To compute with Lattice QCD, we rotate the action S_{QCD} to Euclidean time, $t \to -i\tau_E$

$$\Rightarrow e^{-iEt} \to e^{-E\tau_E}$$

and discretize spacetime (w/ spacing a) & use a finite size box $L^3 \times T$.



The QCD path integral is formulated as

$$Z_{QCD} = \underbrace{\int \mathcal{D}q\,\mathcal{D}\bar{q}\,\mathcal{D}U}_{\text{integrate over all field "configurations"}}\; e^{-S^{(E)}_{QCD}[q,\bar{q},U]}$$

$U \to e^{i g_s a A_\mu \cdot n_\mu}$

On a lattice, this is millions of integrals. We use Monte Carlo methods to stochastically estimate correlation functions. For example, to get proton mass, want

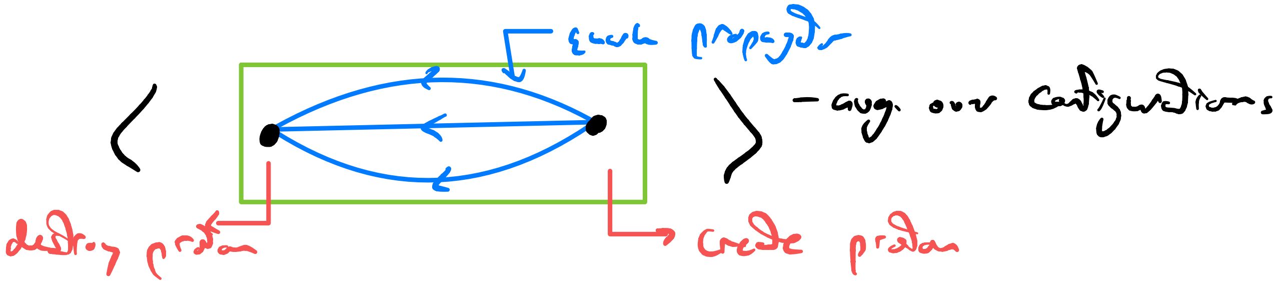
$$\langle O_p(\tau_E)\, O_p^\dagger(0) \rangle \qquad \text{where } O_p \propto \epsilon_{jkl}\, u_j u_k d_l$$

create proton at $\tau_E = 0$, and destroy @ τ_E.

$$= Z_p e^{-m_p \tau_E} + Z_{p'} e^{-m_{p'} \tau_E} + \cdots$$

ground state proton mass

excited proton

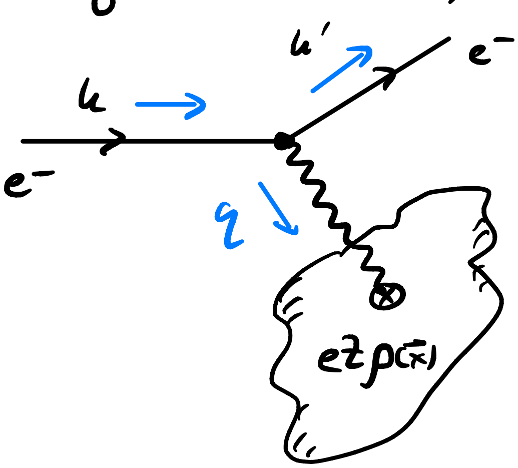


Deep Inelastic Scattering

If QCD is so complicated, how did we accept the theory as an explanation for hadrons? Consider the high-energy limit, $Q^2 \to \infty$, then the coupling is small, and QCD exhibits asymptotic freedom. That is, at high-energies quarks and gluons "behave" as "free" particles, before they hadronize.

There are many types experiments that show this, we will focus on a particular process called Deep Inelastic Scattering (DIS).

Consider electron scattering off a static classical charge distribution,



The electrostatic potential is

$$A^\mu(x) = (\varphi(q\vec{x}), \vec{0})$$

$$\vec{\nabla}^2 \varphi = -eZ\rho$$

$$\int d^3\vec{x}\, \rho(\vec{x}) = 1$$

Fourier transform, $\tilde{A}^\mu(q)$

$$\tilde{A}^0(q) = 2\pi\,\delta(q^0)\int d^3\vec{x}\; e^{i\vec{q}\cdot\vec{x}}\,\varphi(\vec{x})$$

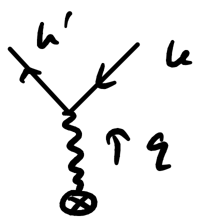
$$= 2\pi\,\delta(q^0)\left(\frac{-1}{|\vec{q}|^2}\int d^3\vec{x}\; e^{i\vec{q}\cdot\vec{x}}\,\vec{\nabla}^2\varphi(\vec{x})\right)$$

$$= 2\pi\,\delta(q^0)\underbrace{\left[\frac{eZ}{|\vec{q}|}\int d^3\vec{x}\; e^{i\vec{q}\cdot\vec{x}}\,\rho(\vec{x})\right]}_{\equiv F(\vec{q})}$$

"form-factor"

$Q^2 = -q^2$

Scattering amplitude is

$iM =$ 

$$= ie\,\bar{u}(k')\gamma^\mu u(k)\,\tilde{A}_\mu(q)$$

$$= 2\pi i e\;\delta(E-E')\,\frac{Ze}{|\vec{q}|^2}\,F(\vec{q})$$

So cross-section is

$$\frac{d\sigma}{d\Omega} = \frac{Z\alpha^2}{4E^2\sin^4\frac{\theta}{2}}\,|F(\vec{q})|^2 \quad ; \quad |\vec{q}|^2 = 2E^2(1-\cos\theta) = 4E^2\sin^2\frac{\theta}{2}$$

$$= \frac{Z\alpha^2}{Q^4}\,|F(\vec{q})|^2$$

can learn about charge distribution

experimentally measure

Let's Taylor expand the form-factor,

$$F(\vec{q}) = \int d^3\vec{x}\left[1 - i\vec{q}\cdot\vec{x} - \frac{(\vec{q}\cdot\vec{x})^2}{2} + \mathcal{O}(q^3)\right]\rho(\vec{x})$$

→ assume spherically symmetric

$$= 1 - \frac{|\vec{q}|^2}{6}\int d^3\vec{x}\, |\vec{x}|^2 \rho(\vec{x}) + \mathcal{O}(q^4)$$

$$\equiv 1 - \frac{|\vec{q}|^2}{6}\underline{\langle|\vec{x}|^2\rangle} + \mathcal{O}(q^4)$$

→ mean squared charge radius

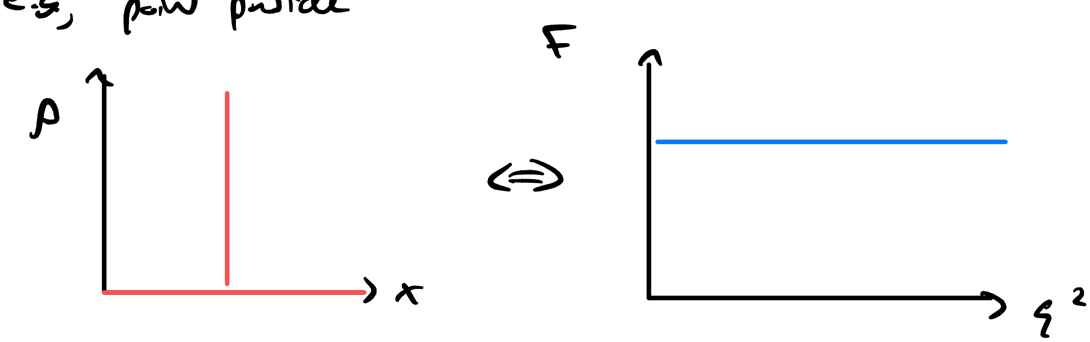
$$\Rightarrow \boxed{\langle r^2\rangle \equiv \langle|\vec{x}|^2\rangle = 6\left.\frac{dF}{dq^2}\right|_{q^2=0}} \qquad q^2 = -|\vec{q}|^2$$

For example, the proton is observed to have (to lowest approximation)

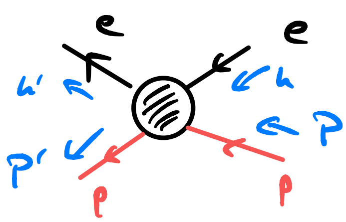
$$F(\vec{q}) = \frac{1}{\left(1 - \frac{q^2}{\Lambda^2}\right)^2} \Rightarrow \rho(\vec{x}) \propto e^{-\Lambda|\vec{x}|}$$

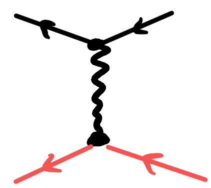
where $\Lambda \simeq 1\ \text{fm}^{-1} = 1\ \text{GeV}$.

e.g., point particle

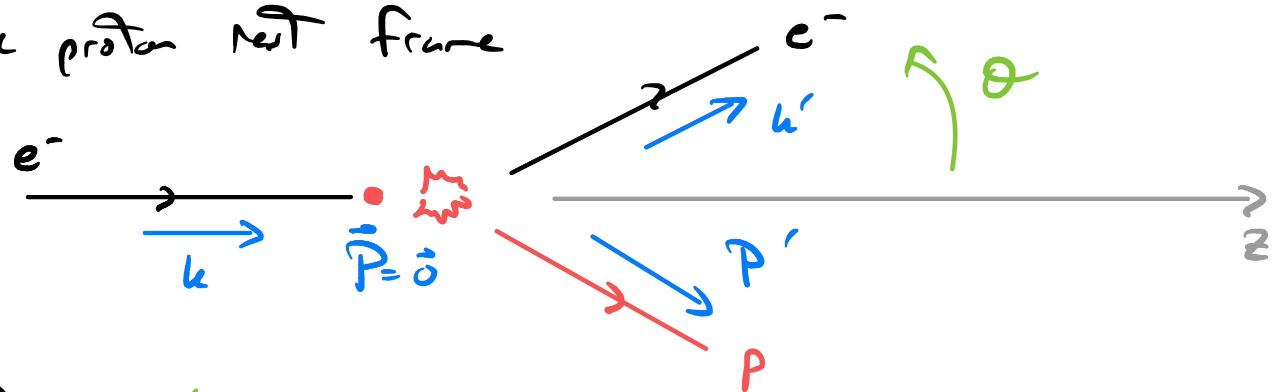


In a realistic experiment, the proton recoils.
Consider elastic $e^-p \to e^-p$ scattering, where the proton is a point particle. To leading order in α, we have

$i\mathcal{M}$ = 

=  + $\mathcal{O}(\alpha^2)$

In the proton rest frame



Cross-section is (exercise)

$$\boxed{\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E^2 \sin^4\frac{\theta}{2}} \underbrace{\frac{E'}{E}}_{\text{recoil factor}} \left(\cos^2\frac{\theta}{2} + \frac{Q^2}{2m_p^2} \sin^2\frac{\theta}{2} \right)}$$

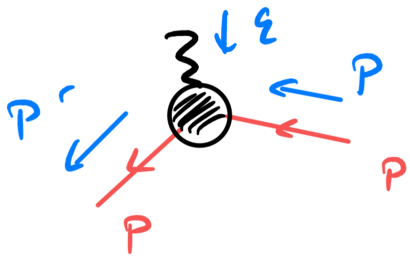
kinematics: $k = (E, 0, 0, E)$ where $E \gg m_e$

$k' = (E', E'\sin\theta, 0, E'\cos\theta)$ $E' \gg m_e$

$P = (m_p, 0, 0, 0)$

and $q^2 = (k-k')^2 = -2EE'(1-\cos\theta) = -Q^2$

For a realistic proton, we need to account for it being a composite particle. As in QED, can parametrize in terms of Form-Factors,



$$\Rightarrow \Gamma^\mu = F_1(Q^2)\gamma^\mu + \frac{i\sigma^{\mu\nu}}{2m_p} q_\nu F_2(Q^2)$$

In e-μ scattering of QED, $F_1(Q_1^2) - F_1(Q_2^2)$

$$\simeq -\frac{\alpha}{4\pi}\log\left(\frac{Q_1^2}{Q_2^2}\right)$$

For ep scattering, it is observed that

a relatively mild change

$$F_1(Q^2) \simeq \frac{1}{\left(1+\frac{Q^2}{\Lambda^2}\right)^2}$$

with $\Lambda^2 \simeq 0.71\ \mathrm{GeV}^2$

This includes non-perturbative QCD effects.

Can show, in proton rest frame

$$\boxed{\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E^2\sin^4\frac{\theta}{2}}\frac{E'}{E}\left[\left(F_1^2 + \frac{Q^2}{4m_p^2}F_2^2\right)\cos^2\frac{\theta}{2} + \frac{Q^2}{2m_p^2}(F_1+F_2)^2\sin^2\frac{\theta}{2}\right]}$$

Convenient to define electric and magnetic form-factors

$$G_E \equiv F_1 - \tau F_2 \qquad , \quad \tau = \frac{Q^2}{4m_p}$$

$$G_M \equiv F_1 + F_2$$

So,

Rosenbluth formula

$$\boxed{\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E^2\sin^4\frac{\theta}{2}} \frac{E'}{E}\left[\frac{G_E^2 + \tau G_M^2}{1+\tau}\cos^2\frac{\theta}{2} + 2\tau G_M^2 \sin^2\frac{\theta}{2}\right]}$$

Experimentally, for proton $G_E(0) = 1$, $G_M(0) = 2.79$

neutron $G_E(0) = 0$, $G_M(0) = -1.91$

Recall for point particle, Dirac showed

$$g = 2G_M(0) = 2(1 + F_2(0)) = 2(1 + \mathcal{O}(\alpha))$$

For the proton $F_2(0) = G_M(0) - 1 = 1.79 \neq \mathcal{O}(\alpha)$

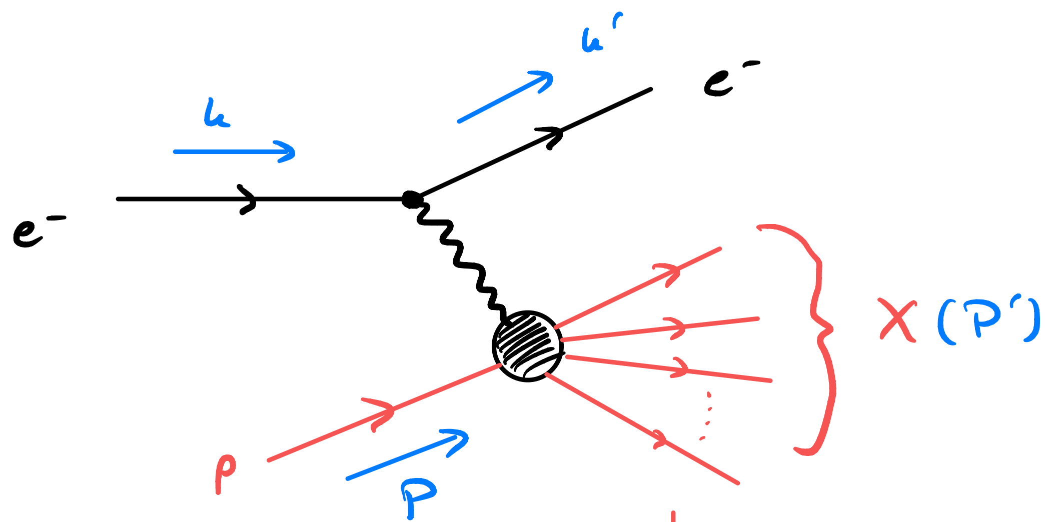
neutron $F_2(0) = G_M(0) - 1 = \underbrace{-2.91 \neq \mathcal{O}(\alpha)}$

proton & neutron are not elementary fermions!

At higher virtualities $Q^2 \gg m_p^2$, then we can blow the proton apart in an inelastic collision

$e^- p \to e^- X$

↳ A bunch of hadrons



↳ A number of hadrons, $p, n, \pi, K, \ldots$

In this process, we do not measure X (inclusive process). Can think of this as a high-energy photon probing the structure of the proton.

Let's define a few more useful kinematic variables,

$$\nu = E - E' = \frac{P \cdot q}{m_p}$$ energy loss

For DIS, $Q^2 \gg m_p^2 \Rightarrow \nu \gg m_p$

↳ $\Rightarrow \lambda \ll \frac{1}{m_p} \ll r_p$

"photon probes internal structure"

"Bjorken x"

$$\boxed{x = \frac{Q^2}{2m_p\nu} = \frac{Q^2}{2p\cdot q}}$$

Since $p'^2 = M_x^2 \geq m_p^2$ $\longrightarrow$ = for elastic scattering

$$= (q+p)^2$$

$$= -Q^2 + 2p\cdot q + m_p^2$$

So,
$$x = \frac{Q^2}{Q^2 + M_x^2 - m_p^2} \Rightarrow \boxed{0 \leq x \leq 1}$$

$x = 0$ if $Q^2 = 0$
$x = 1$ if $M_x^2 = m_p^2$

It is convenient to parametrize DIS by x & Q^2.

Another useful variable,

$$y = \frac{E-E'}{E}$$ relative energy loss

$$= \frac{\nu}{E} = \frac{p\cdot q}{p\cdot k}$$

Can show that the cross-section for DIS is of the form

$$\frac{d\sigma}{d\Omega\, dE'} = \frac{\alpha^2}{4\pi m_p Q^4}\frac{E'}{E} L^{\mu\nu} W_{\mu\nu}$$

↳ More d.o.f. since $M_X > m_p$

where $L^{\mu\nu} = \frac{1}{2} tr[\not{k}'\gamma^\mu \not{k}\gamma^\nu]$ lepton tensor

&

$$W^{\mu\nu} = F_1\left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) + F_2\left(p^\mu - \frac{p\cdot q}{q^2}q^\mu\right)\left(p^\nu - \frac{p\cdot q}{q^2}q^\nu\right)$$

↳ Ward identity enforced

Hadronic tensor (parametrize)

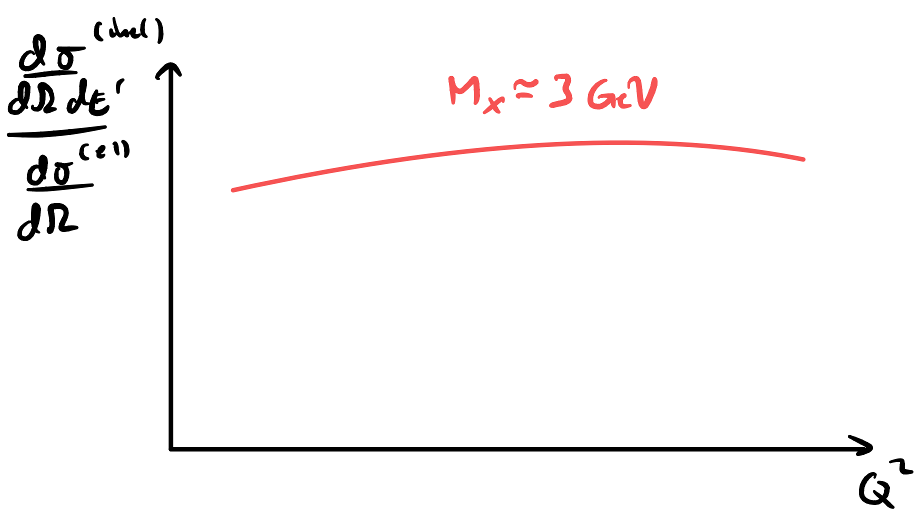
& $F_{1,2} = F_{1,2}(x, Q^2)$ have been measured.

↳ structure functions (not form-factors)

We find,

$$\frac{d\sigma}{d\Omega\, dE'} = \frac{\alpha^2}{4E^2 \sin^4\frac{\theta}{2}}\left[\frac{1}{\nu}F_2(x,Q^2)\cos^2\frac{\theta}{2} + \frac{2}{m_p}F_1(x,Q^2)\sin^2\frac{\theta}{2}\right]$$

It has been experimentally observed that



Result looks approx constant ⇒ looks like scattering off point-like constituents.

Useful to reformulate cross-section as

$$\frac{d^2\sigma}{dx\,dQ^2} = \frac{4\pi\alpha^2}{Q^2}\left[(1-y)\frac{F_2(x,Q^2)}{x} + y^2 F_1(x,Q^2)\right]$$

Can measure separately due to y-dependence

From y-dependence, $0 \leq y \leq 1$, find

as $y \sim 0 \Rightarrow \dfrac{d^2\sigma}{dx\,dQ^2} \propto \dfrac{F_2(x,Q^2)}{x}$

as $y \sim 1 \Rightarrow \dfrac{d^2\sigma}{dx\,dQ^2} \propto F_1(x,Q^2)$

Some features of F's,

- For sufficiently large Q^2 (deep probe)

$$F_1(x,Q^2) \simeq F_1(x)$$
$$F_2(x,Q^2) \simeq F_2(x)$$

Q independence!

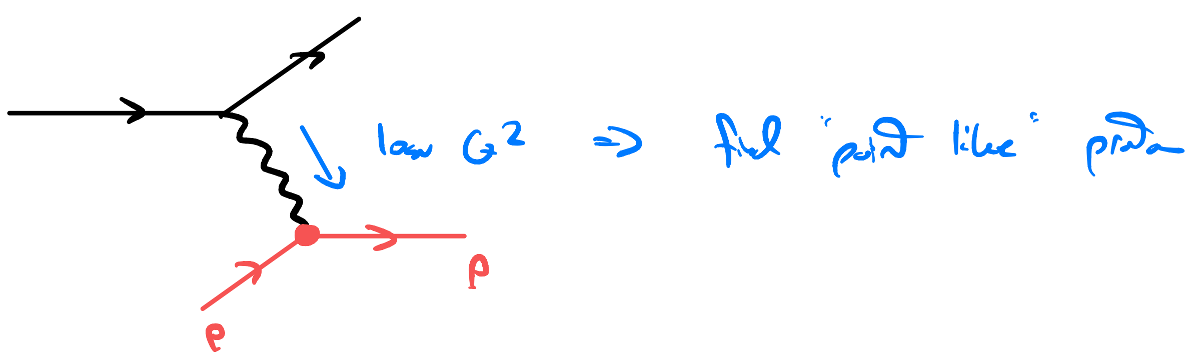
- For sufficiently large Q^2

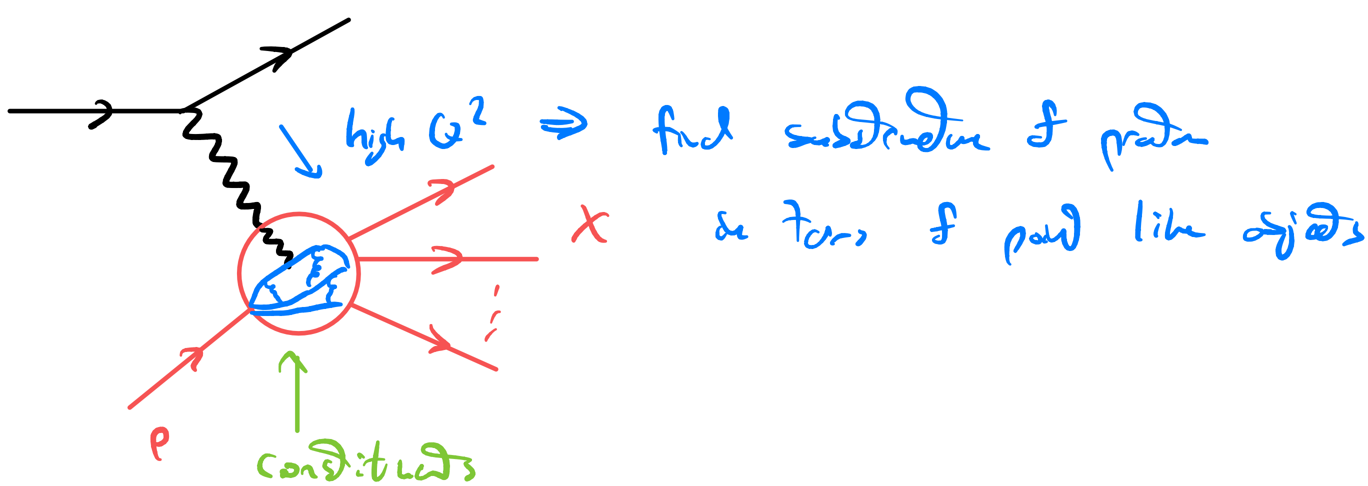
$$\boxed{2F_1(x) = \frac{F_2(x)}{x}}$$

This is the Callan-Gross relation

The Parton Model

From the data, $d^2\sigma/dx\,dQ^2$ looks like a cross-section for a point particle for large Q^2. Since F_1, F_2 are Q^2 independent for DIS, it suggests that the proton is made up of point-like constituents. Feynman called these Partons. We now know that these are constituent quarks & gluons in QCD. The idea of DIS is illustrated as



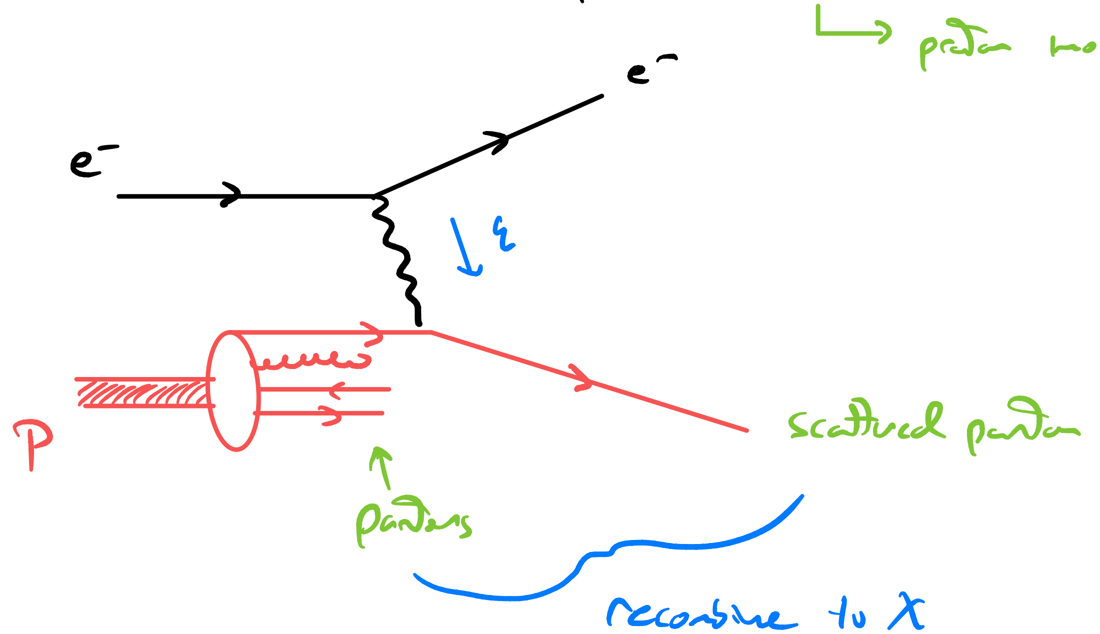


DIS consists of elastic scattering off individual "parts", followed by "hadronization".

Each parton carries a fraction ξ of the protons momentum,

$$p^\mu_{\text{parton}} = \xi\, p^\mu$$

↳ proton momentum



Recall that $x = \dfrac{Q^2}{2p\cdot q}$, $0 \leq x \leq 1$

for elastic scattering, $M_X = m_p \Rightarrow x = 1 = \dfrac{Q^2}{2p\cdot q}$

For a parton, $p_{\text{parton}} = \xi p$

So, x_f (fraction for proton) is

$$x_f = \frac{Q^2}{2p_{\text{parton}}\cdot q} = \frac{Q^2}{2\xi p\cdot q} = \frac{1}{\xi}\, x$$

But, partons scatter elastically, $\Rightarrow \; x_f = 1$

$$\Rightarrow \quad x_f = \frac{1}{\xi} x = 1 \quad \Rightarrow \quad \boxed{x = \frac{Q^2}{2p\cdot q} = \xi}$$

So, in the parton model, Bjorken's x <u>is</u> the parton's momentum fraction.

Can show that

Q_f frac $Q_f e$ charge

$$\left.\frac{d\sigma}{dQ^2}\right|_{\text{parton}} = \frac{4\pi\alpha^2}{Q^4} Q_f^2 \left[(1-y) + \frac{y^2}{2}\right]$$

For a single parton.

Therefore, integrating over all parton distributions,

$$\Rightarrow \boxed{\frac{d^2\sigma}{dx\,dQ^2} = \frac{4\pi\alpha^2}{Q^4}\left[(1-y) + \frac{y^2}{2}\right]\sum_f f(x)\, Q_f^2}$$

$f(x)$ is called a <u>parton distribution function</u> (PDF)

$$f(x)\,dx = \#\text{ of } f\text{-type quarks in proton with momentum fractions between } x \text{ and } x+dx$$

$f(x) = \{ u(x), d(x), s(x), \ldots, \bar{u}(x), \bar{d}(x), \bar{s}(x), \ldots \}$

Can also have gluon distribution $g(x)$.

Compare Parton model with experimental observable,

$$\frac{d^2\sigma}{dx\,dQ^2} = \frac{4\pi\alpha^2}{Q^2}\left[(1-y)\frac{F_2(x,Q^2)}{x} + y^2 F_1(x,Q^2)\right]$$

$$\simeq \frac{4\pi\alpha^2}{Q^2}\left[(1-y)+\frac{y^2}{2}\right] 2F_1(x,Q^2)$$

↳ assumes Callan–Gross

$$2F_1 = \frac{F_2}{x}$$

parton model,

$$\frac{d^2\sigma}{dx\,d\Omega} = \frac{4\pi\alpha^2}{Q^2}\left[(1-y)+\frac{y^2}{2}\right]\sum_f f(x)\,Q_f$$

Find that the Parton model predicts

$$\boxed{\underbrace{2F_1(x,Q^2) = \frac{F_2(x,Q^2)}{x}}_{\text{Callan-Gross}} = \underbrace{\sum_f Q_f^2 f(x)}_{\text{independent of } Q^2\,!}}$$

For proton,

$$\frac{F_2}{x} = \frac{4}{9}\left[u(x) - \bar{u}(x)\right] + \frac{1}{9}\left[d(x) + \bar{d}(x) + s(x) + \bar{s}(x)\right]$$

So, can measure cross-section for $ep \to eX$,
can measure then $F_1(x, Q^2)$ & $F_2(x, Q^2)$.
Similarly, can measure eD scattering, to
infer neutron structure functions,

$$F_{1,2}^{eD} \simeq \frac{F_{1,2}^{ep} + F_{1,2}^{en}}{2}$$

For neutron, using isospin symmetry, find

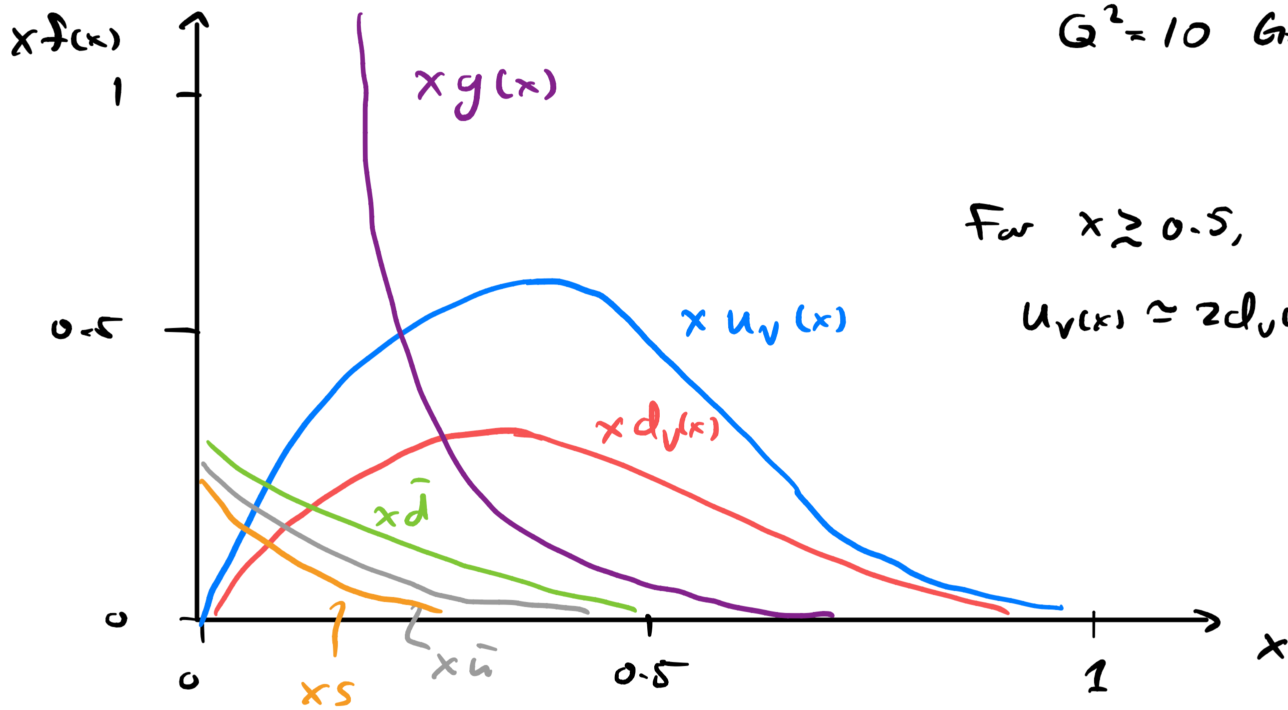
$$\frac{F_2^{en}}{x} \simeq \frac{4}{9}\left[d(x) + \bar{d}(x)\right] + \frac{1}{9}\left[u(x) + \bar{u}(x) + s(x) + \bar{s}(x)\right]$$

isospin: $u \leftrightarrow d$, $\bar{d} \leftrightarrow \bar{u}$, $s \leftrightarrow s$, $\bar{s} \leftrightarrow \bar{s}$

If one ignores $s + \bar{s}$, can use F_2^{ep} and F_2^{en}
together to determine $u(x) + \bar{u}(x)$ & $d(x) + \bar{d}(x)$.

To obtain further PDFs, one uses (anti)-neutrino DIS,
results from $p\overset{(-)}{p}$ scattering (e.g., Drell-Yan $p\overset{(-)}{p} \to \mu^+\mu^- + X$)

Find (roughly) the following distributions,



$Q^2 = 10$ GeV^2

For $x \gtrsim 0.5$,

$u_v(x) \simeq 2 d_v(x)$

Here, "valence" PDF are $u_v(x) \equiv u(x) - \bar{u}(x)$

$d_v(x) \equiv d(x) - \bar{d}(x)$

Crudely, proton is made up of u_v, u_v, d_v "valence" quarks, with sea quarks & gluons

Note the sum rules (for proton)

$$\int_0^1 dx \; u_v(x) = 2, \quad \int_0^1 dx \; d_v(x) = 1, \quad \int_0^1 dx \; s_v(x) = 0$$

& momentum sum rule

$$\sum_j \int_0^1 dx \; x f_j(x) = 1$$

$\hookrightarrow$ sum over partons

Find that

$$\int_0^1 dx\,(u_v + d_v)\,x\,dx \simeq 0.38$$

$$\int_0^1 dx\, g(x)\, x \simeq 0.5$$

rest in sea quarks

$\Rightarrow$ Most momentum in gluons!