Fundamentals of Probability – Week 1

Problem 1

What is p?

$P = p^{53}(1-p)^{47}$

$$\frac{dP}{dp} = 53 * p^{52}(1-p)^{47} - 47 * p^{53}(1-p)^{46}$$

$$\frac{dP}{dp} = \left(\frac{53}{p} - \frac{47}{(1-p)^{47}}\right)(p^{53} * (1-p)^{47}) = 0$$

hence

$\underline{p = 0.53}$

What if you only tossed the coin once and got heads?

$\underline{p = 0.5}$

Is it reasonable to give a single answer?

The answers will be more accurate if uncertainty is considered. The less data we have, the more unsure will we be about p.

Problem 2

What is the probability that a driver has smoked cannabis in the last 72 hours if they have tested positive?

$A \rightarrow presence\ of\ cannabis$

$B \rightarrow positive\ test$

$$P(B|A) = 90\% = 0.9$$

$$P(A) = 20\% = 0.2$$

$$P(B|nA) = 10\% = 0.1$$

$$P(B) = P(B|A) * P(A) + P(B|nA) * P(nA)$$

We know that:

$$P(A|B) = \frac{P(B|A) * P(A)}{P(B)}$$

$$P(A|B) = \frac{0.9 * 0.2}{0.9 * 0.2 + 0.1 * 0.8} = 0.692 = 69.2\%$$

Hence, the probability that a driver has smoked cannabis in the past 72 hours is ***69.2%.***

What is the probability that someone smoked cannabis in the last 72 hours if they have not tested positive?

$A \rightarrow presence\ of\ cannabis$

$B \rightarrow positive\ test$

$$P(B|A) = 99.9\% = 0.999$$

$$P(A) = 20\% = 0.2$$

$$P(B|nA) = 20\% = 0.2$$

$$P(B) = P(B|A) * P(A) + P(B|nA) * P(nA)$$

We know that:

$$P(A|B) = \frac{P(B|A) * P(A)}{P(B)}$$

$$P(A|B) = \frac{0.999 * 0.2}{0.999 * 0.2 + 0.001 * 0.8} = 99.6\%$$

$$P(nA|nB) = 1 - P(A|B) = 0.004 = 0.4\%$$

Hence the probability that someone smoked cannabis in the last 72 hours if they have not tested positive is ***0.4%.***

Problem 3

What is the probability that a vote for the winner was cast by a viewer from Dover?

$$A \rightarrow cast\ of Dover$$

$$B \rightarrow vote\ for\ winner$$

$$P(B) = P(Dover) + P(Croydon) + P(Ruby)$$

$$P(B) = 0.51 * 0.16 + 0.38 * 0.88 + 0.61 * 0.46$$

$$P(B) = 0.0816 + 0.3344 + 0.2806$$

$$P(B) = 0.6966$$

$$P = \frac{P(B|A) * P(A)}{P(B)} = \frac{0.51 * 0.16}{0.6966} = \frac{0.0816}{0.6966} = 0.117$$

The probability that a vote for the winner was cast by a viewer from Dover is ***0.117***.

Problem 4

What are the chances that George knows what a neural network is?

- 50% certain George studied AI
  - 80% who study AI know what a neural network is
- 20% certain George majored in CS
  - 40% of the those who study CS know what a neural network is
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- 10% of rest of population know what neural network is

1. He could know if he did AI
    - You are 50% sure he studied AI and 80% if AI students know what a neural network.

$$0.5 * 0.8 = 0.4$$

2. He could know if he did AI
    - You are 20% sure he studied CS and 40% if CS students know what a neural network.

$$0.2 * 0.4 = 0.08$$

3. He could know randomly
    - 50% do AI, 20% do CS which means 30% don do either of those and 10% of the people who don't do AI or CS know what a neural network.

$$0.3 * 0.1 = 0.03$$

Hence

$$0.4 + 0.008 + 0.003 = 0.51$$

There is ***a 51%*** chance that George knows what a neural network is.

Problem 5

What is the probability that this is the biased one?

$$P(C) \rightarrow genuine\ coin$$

$$P(D) \rightarrow biased\ coin$$

$$P(C) = 0.999$$

$$P(D) = 0.001$$

$$probability\ if\ double\ headed = 1$$

$$P = \frac{\frac{1}{1000}}{\frac{999}{1000} * \left(\frac{1}{2}\right)^{10} + \frac{1}{1000} * 1^{10}} = 0.506$$

The probability that this coin is the biased one is ***0.506.***

Problem 6

What would you do?

- If I turn in
    - I get 3 years in jail if my friend turns in
    - I get 0 years in jail if my friend refuses to turn in
- If I don't turn in
    - I get 6 years in jail if my friend turns in
    - I get 1 year in jail if my friend refuses to turn in

- At the end of the day it depends on trust, but if I trust my friend then there is a higher probability that I would not turn him in, hoping that he would not turn me in, and share the guilt and stay in prison for 1 year.
- I would probably not turn myself in, hoping that my friend would not.