02-680 Module 20

Essentials of Mathematics and Statistics

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Hypothesis Testing

Hypothesis Testing is

- A technique to evaluate if a model fit matches our assumptions about the data.
- Allows us to assign a numerical value (e.g., p-value) to assess this match.

There are two main types of hypothesis:

- **Null Hypothesis (H_0)**: The assumption that there is no effect or no difference.
- **Alternate Hypothesis (H_1)**: The assumption that there is an effect or a difference.

Hypothesis Test Outcome

The test helps us decide to:

- Reject H_0 $\rightarrow$Evidence supports H_1.
- Retain H_0 $\rightarrow$Not enough evidence to support H_1.

Example context: Testing whether a drug impacts cholesterol:

- H_0: Cholesterol stays the same. (No effect.)
- H_1: Cholesterol level changes.

Errors

- Type I Error (False Positive): Rejecting the null hypothesis when H_0 is actually true.
- Type II Error (False Negative): Retaining the null hypothesis when H_0 is actually false.

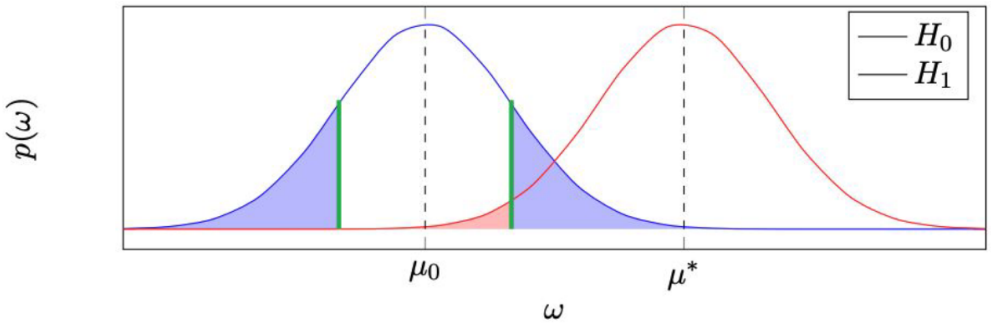
	Hypothesis Test Result	
Truth	Retain H_0	Reject H_0
H_0	Correct	Type I Error
H_1	Type II Error	Correct

We say the Type I Errpr Rate is $p(\text{reject } H_0 \mid H_0 \text{ is true})$, and Type II Error Rate is $p(\text{retain } H_0, H_1 \text{ is true})$, and Statistical Power is $1 -$ Type II Error Rate. The last point means that the higher power tests have a stronger ability to detect signals for H_1.

Example

Let's look at it visually, first for what we call a two-sided test, that is

$$H_0 \ : \ \mu = x \text{ and } H_1 \ : \ \mu \neq x$$



In the figure above, when we pick some boundary around our desired x (the green lines) we will have some probability of Type I Error (blue shaded regions) and Type II Error (red shaded region).