02-680 Module 7

Essentials of Mathematics and Statistics

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Linear Systems of Equations

We can define a **linear system** of n linear equations on m variables as follows:

$$\begin{array}{ccccccccc} C_{11}x_1 & + & C_{12}x_2 & + & \cdots & + & C_{1m}x_m & = & b_1 \\ C_{21}x_1 & + & C_{22}x_2 & + & \cdots & + & C_{2m}x_m & = & b_2 \\ \vdots & & \vdots & & \ddots & & \vdots & & \vdots \\ C_{n1}x_1 & + & C_{n2}x_2 & + & \cdots & + & C_{nm}x_m & = & b_n \end{array}$$

As an example:

$$\begin{cases} 3z_1 + 2z_2 = -1 \\ z_1 - 5z_2 = 3 \end{cases}$$

$$\begin{bmatrix} 3 & 2 \\ 1 & -5 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$$

If we want to find x (or in our example z), we can say it is

$$x = C^{-1}b$$

where C^{-1} is the inverse matrix of C. It turns out this matrix may not always exist.

- The inverse is the matrix such that $CC^{-1} = C^{-1}C = I_n$ (thus the first condition to the inverse existing is if C is square).