

# 02-620 Week 3

## Machine Learning for Scientists

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### Classification

The goal of classification is to find a functional mapping  $f : X \rightarrow Y$ , where  $Y$  is discrete-valued.

- SNPs for  $X$  and disease/healthy status for  $Y$
- Gene expression for  $X$  and disease/healthy status for  $Y$
- Pathology images for  $X$  and tumor/healthy for  $Y$
- Electronic medical records for  $X$  and diagnosis for  $Y$
- Genome sequence features for  $X$  and transcription factor binding site or not for  $Y$

### Training vs. Testing

- In training, the goal is to improve the model using input data and output pairs.
- In testing, the goal is to classify unseen new input data and provide the output.

### Different Types of Classifiers

- K-nearest neighbor
  - Non-parametric method: no model, no parameters, no learning (lazy)
- Naive Bayes
  - Parametric method, generative model: model  $P(Y, X|\theta)$  to obtain  $P(Y|X, \theta)$
- Logistic Regression
  - Parametric method, discriminative model: model  $P(Y|X, \theta)$

### K-nearest neighbors (KNN) classifier

- Given  $N$  training data points  $(x_1, y_1), \dots, (x_N, y_N)$ , kNN performs no explicit learning (i.e., no learnable parameters)
- **Inference:** A new data point  $x_i$ , is classified by majority vote among its  $k$ -nearest neighbors, defined as the  $k$  training points with the smallest Euclidean ( $l_2$ ) distances  $\|x_{i'} - x_i\|_2^2$

## How to select $k$

- Small  $k$ : classification is sensitive to noise
- Large  $k$ : too much smoothing. (If  $k = N$ , sample size, all test inputs will receive the same classification.)
- Select  $k$  that is not too small and not too large

## Computation Time

- **Learning:** No training or parameter learning - cheap!
- **Inference:** When a new data point  $x_{i'}$  arrives, kNN must compute the distance between  $x_{i'}$  and all  $N$  training samples, incurring an  $O(ND)$  computational cost - expensive!

## Naive Bayes Classifier

### Example: Predicting Cancer from genotype

Individual	Locus 1 $X_1$	Locus 2 $X_2$	Locus 3 $X_3$	Healthy/Cancer $Y$
1	0	0	1	1
2	1	0	2	1
3	0	2	0	1
4	2	0	0	0
5	2	1	2	0
6	1	2	1	0

Here, the input  $X$  represents the allele. 0 = AA (minor allele homozygous), 1 = AT (heterozygous), 2 = TT (major allele homozygous).  $Y$  represents healthy (0) or cancer (1). We want to

- learn a classifier,  $f : (X_1, X_2, X_3) \rightarrow Y$
- learn a probabilistic model for  $P(Y|X)$ , where  $Y$  is discrete

$P(Y|X)$  is given as

Combination	$X_1$	$X_2$	$X_3$	$P(Y = 1 X_1, X_2, X_3)$	$P(Y = 0 X_1, X_2, X_3)$
1	0	0	0	0.01	0.99
2	0	0	1	0.50	0.50
3	0	0	2	0.30	0.70
4	0	1	0	0.25	0.75
5	0	1	1	0.70	0.30
6	0	1	2	0.05	0.95
7	...	...	...	...	...

- How many probability parameters must be specified?
- How can this distribution be learned from data?
- Note that  $P(Y = 0|X_1, X_2, X_3) = 1 - P(Y = 1|X_1, X_2, X_3)$

### How many parameters are needed?

- Suppose  $X = [X_1, \dots, X_D]$  for  $D$  SNPs
  - $X_j$ 's: random variables taking values from  $\{0, 1, 2\}$
  - $Y$ : binary random variables
- To estimate  $P(Y|X_1, X_2, \dots, X_D)$
- If we have 30 SNPs in  $X$ :  $P(Y|X_1, X_2, \dots, X_{30})$