

EC ENGR 102 Week 1

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Introduction

- Society relies on being able to:
 1. Represent information (Signals)
 2. Communicate, process, and operate on that information (Systems)
- Technology is a reflection of our ability to do these things.

The Signals and Systems perspective is basically:

- Information is represented as "signals", and information changes, or is processed, through "systems".

Examples of signals and systems

- My voice is a **signal**, and my cell phone (**a system**) records it, transforms it into a transmittable form, communicates it to a cell phone tower(s), eventually reaching the person I'm speaking to who hears it... in almost real time.
- YouTube videos are a **signal**, and our computer or phone (**a system**) plays them, adjusting their resolution based on our WiFi speed, etc.
- Moving a computer mouse or typing on a keyboard is a **signal**, and our computer then uses circuits (**a system**) to translate this information to show you an updated computer screen.
- Note that signals and systems **are not** limited to digital signals.
 - Any physical or abstract quantity that can be measured is a **signal**.
 - * The federal deficit is a **signal**...
 - Anything that changes a signal is a **system**
 - * Policies passed by Congress, the interaction of national and global economies, etc. are **systems**.
 - This is a general abstraction.

The goal of the signals and systems abstraction is to decompose a problem into components with the following block diagram.

$$\text{Input signal } x(t) \longrightarrow \boxed{\text{System}} \longrightarrow \text{Output signal } y(t)$$

This abstraction enables systems can be combined together to form composite systems.

A Diversity of Signals and Systems

How do we (rigorously) represent signals and systems, considering their variety?

- The short answer: depending on the application, there can be several ways to represent signals; **how we represent signals, and what we aim to do with them, determines the types of tools we need to analyze them.**
- In traditional signal processing, signals are 1-D, and do not have a noise model. Ex. Radio, communications, control systems, circuit analysis.
- In statistical signal processing, signals can be multi-dimensional, and incorporate noise models. Ex. Communications over noisy channels, information theory, noisy control.
- In machine learning: signals can be multi-dimensional, and incorporate noise models. Ex. AI, neural networks and deep learning, prediction systems, unsupervised learning.
- This class will focus on traditional signal processing.

Signals in ECE 102

What is a signal?

- A signal is a *function* of one or more variables
- What is a function?
 - We ought be familiar with functions from mathematics: denoted by $f(\cdot)$, it typically accepts some input, x and return some output, y . We write this as:

$$y = f(x)$$

- We usually denote this function as: $f : \mathbb{R} \rightarrow \mathbb{R}$, indicating that f is a function mapping a real number (the first \mathbb{R}) to another real number (the second \mathbb{R}).

The Time Domain

Signals usually have to do with *time* domain representations

- That is, signals are usually functions that accept an input time t , and return the value of the signal at that time. For example, a signal could be represented $x(t)$, which denotes the value of the signal at time t .

Music

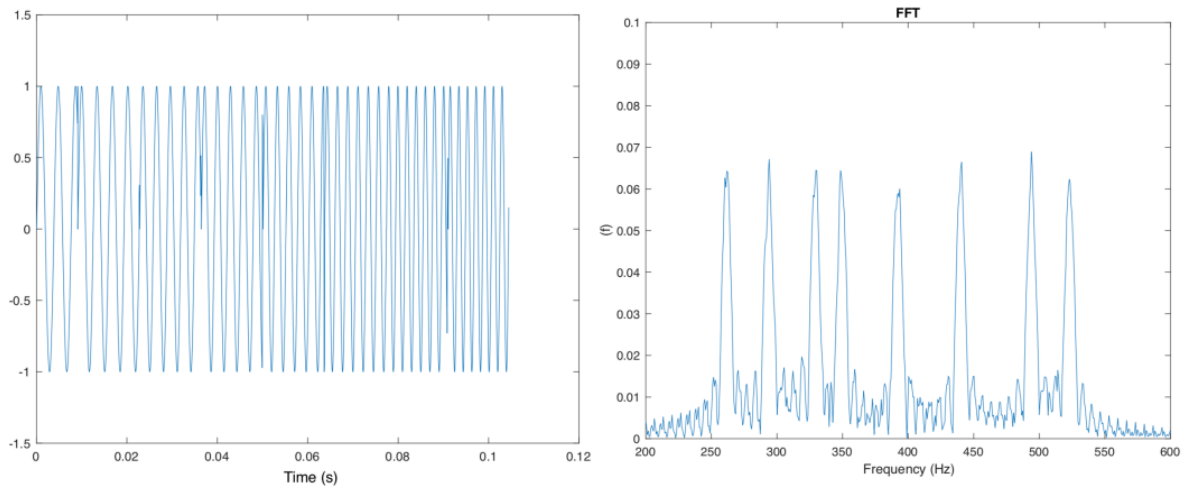
- Suppose you recorded your voice or a musical instrument. Your computer (system) recorded a sound (signal), which can be mathematically modeled as a sine wave.
- When you hear it, played out from your speaker, the sound wave (signal) vibrates bones located inside your ear (system), which vibrates hair cells present, which transmits the neural signal to your brain.
- Music is essentially a combined sine wave.

Aside: One of the great secrets of the Universe

- "Every signal has a spectrum and is determined by its spectrum. You can analyze the signal either in the time (or spatial) domain or in the frequency domain. I think this qualifies as a Major Secret of the Universe." -Prof. Brad Osgood, Stanford University.

- A "spectrum" is basically a graph where frequency is on the x-axis, and power on the y-axis. Instead of a Concert A being represented as a sine wave at 440 Hz., it can be represented by its spectrum, which is a graph with a peak at the 440Hz. frequency. (Basically, run FFT on the signal to get the spectrum.)

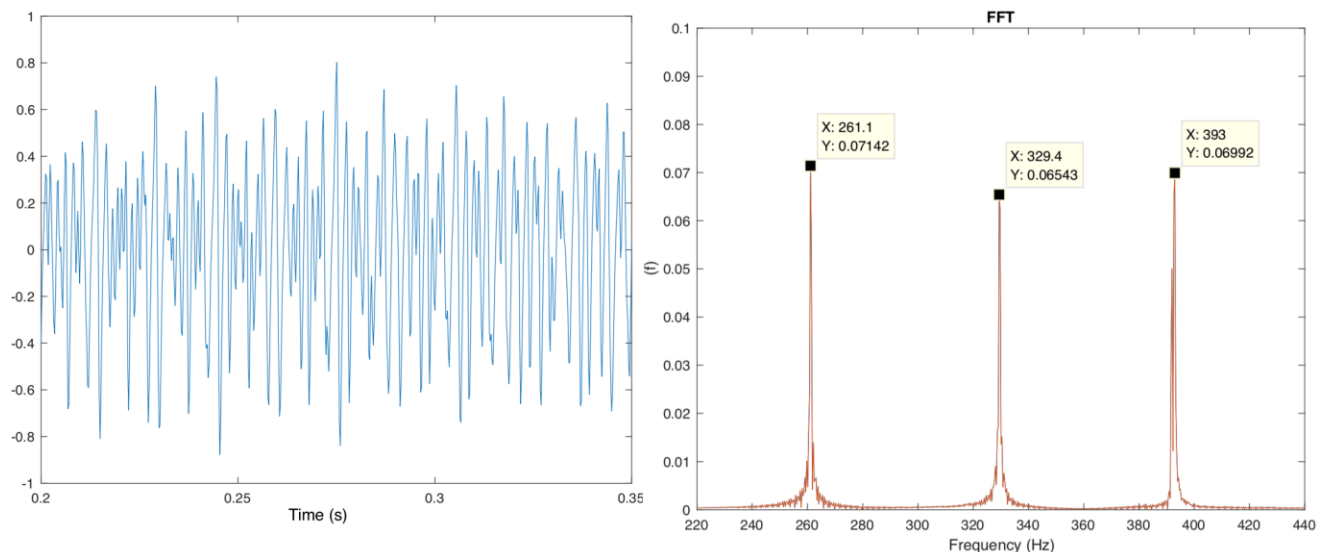
Example:



Left: Sine wave of sound signal. Right: Spectrum of sound signal (FFT)

Notice that the spectrum generated does not display time! By observing the peaks on the left sine wave, we can tell it is an ascending scale, but the spectrum does not show that!

Example:



This is the waveform and spectrum of the C-Major Triad. The transform (a function) can bring context to a seemingly unintelligible waveform.

- We can run functions in the spectrum domain! Say for example we want to recover the C from the triad waveform. This would be incredibly difficult to do in the waveform graph, but easy in the spectrum graph. All we have to do is to set a filter. For example, our filter can multiply all the frequencies above 300Hz by zero. Then, convert the remaining frequencies back to a waveform.

Sine waves are important

- When we talk about music, we talk about sine waves with frequencies of units **Hertz**.
- Radio frequencies, wireless communication, music, etc. It would not be an exaggeration to say that none of this technology would exist without the math that we'll learn in ECE 102.
- Any signal can be formed by adding sine waves.
- **The Bottom Line:** once we understand the mathematics of how to create things with sine waves (frequency domain or spectrum), we can do very powerful operations. This is the basis for many technologies that we may (sometimes) take for granted.

Applications of Signals and Systems

- The design of analog circuits
- Magnetic resonance imaging (MRI)
- Traditional control systems
- Mixing music

Signals

Discrete vs. Continuous Signals

- A continuous signal is one that is defined for every point in time.
- A discrete signal is one that is defined at individual points
- Continuous signals can be graphed as lines, discrete signals are graphed as points.

Signal Operations

1. Amplitude Scaling

Suppose we have a signal $x(t) = \alpha \sin(t)$.

- If we choose a value of α that is between 0 and 1, the signal gets smaller. (*attenuation*)
- If α is greater than 1, the signal is amplified. (*amplification*)
- If α is negative, the signal is flipped. (*inversion*)

2. Time Scaling

Suppose we have a signal $x(t) = \sin(\alpha t)$.

- If the value of α is between 0 and 1, the signal's period increases. (*Time expansion*)
- If the value of α is greater than 1, the signal's period decreases. (*Time compression*)
- If the value of α is less than 0, the time of the signal is reversed. (*Time reversal*)

3. Time Shift

Suppose we have a signal $x(t) = \sin(t - \alpha)$

- If α is positive (e.g., you are subtracting a positive number), the signal is delayed. (*Delayed signal*)
- If α is negative (e.g., you are subtracting a negative number), the signal is advanced. (*Advanced signal*)

Combining Operations

Suppose we have a function that is zero everywhere, except between -1 and 1, where the function evaluates to 1. Let $x(t)$ represent this function. **What is $x(2(t-1))$?**

The naive method (**Incorrect**):

- First, apply the time delay. Now the signal is 1 between $x = 0$ and $x = 2$.
- Next, apply the compression by a factor of 2. Now the signal goes from $x = 0$ to $x = 1$.
- The result is a signal that is zero everywhere, but is 1 between $x = 0$ and $x = 1$.

The above answer is incorrect. Why?

- We can test this with $x(1)$, which should evaluate to 1.
- To make $x(2(t-1))$ equivalent to $x(1)$, we set $t = 1.5$. In our proposed answer, $t = 1.5$ results in 0, where it should result in 1, matching the original.

The correct method:

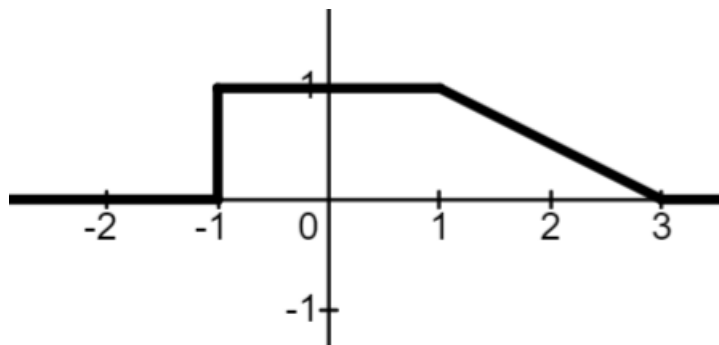
- Do reverse PEMDAS -> SADMEP.
- First, apply the time compression. (Instead of the parenthesis, time delay first) Now the signal begins at $t = -0.5$ and ends at $t = 0.5$.
- Now, we apply the time delay. Now the signal begins at $t = 0.5$ and ends at $t = 1.5$.
- This gives the correct result.

Alternatively, simply distribute the 2 into the parenthesis and separate all the terms. That would also fix the problem.

Even and Odd Decomposition

- An even function is a $f(x)$ such that $f(-x) = f(x)$.
- An odd function is a $f(x)$ such that $f(-x) = -f(x)$.

Suppose we have the following function:



Let's make an assertion that $x(t) = x_e(t) + x_o(t)$, where x_e is some even function and x_o is some odd function. How do we find the functions?

Our goal is to define $x_e(t)$ and $x_o(t)$ in terms of $x(t)$.

First, let's rearrange the equation

$$x_e(t) = x(t) - x_o(t)$$

$$x_o(t) = x(t) - x_e(t)$$

Following the rules of odd functions, we can do

$$x_o(t) = -(x(-t) - x_e(-t))$$

Simplifying the even function, we get

$$x_o(t) = -(-x(-t) - x_e(t))$$

$$x_o(t) = x(-t) + x_e(t)$$

Plugging this for $x_o(t)$ in the first equation, we get

$$x_e(t) = x(t) + x(-t) - x_e(t)$$

$$\boxed{x_e(t) = \frac{1}{2} (x(t) + x(-t))}$$

The odd function can be found by rearranging the original even function, then plugging it into the odd function.

Periodic Signals

- The concept of periodic signals is very important in this class. Colloquially, these are signals that repeat after a given interval, T_0 .
- The formal definition of a periodic signal is as follows:
 - A continuous time signal is periodic if and only if there exists a $T_0 > 0$ such that

$$x(t + T_0) = x(t)$$

for all t . T_0 is called the period of $x(t)$.

Periodic signal properties

Suppose $x(t)$ is periodic.

$$\begin{aligned} x(t + T_0) &= x(t) \\ x(t + 2 \cdot T_0) &= x(t) \\ &= x(t + T_0 + T_0) \end{aligned}$$

Define $\tau = t + T_0$. By substitution,

$$\begin{aligned} x(t + 2 \cdot T_0) &= x(\tau) \\ &= x(t + T_0) \end{aligned}$$

Therefore, $x(t + nT_0) = x(t)$ is true, where $x(t)$ is periodic, for all integers n .

The smallest T_0 for which $x(t + T_0) = x(t)$ is called the **"fundamental period"**.

Sinusoids

The most basic signal in this class is the sine or cosine wave. We'll use them *extensively* so it's worth reviewing their properties. By the end of this class, you'll be proficient at manipulating sinusoids. A cosine function is defined by:

$$\begin{aligned} x(t) &= A \cos(\omega t - \theta) \\ &= A \cos(2\pi f t - \theta) \end{aligned}$$

- $\omega = 2\pi f$, and represents *angular frequency*.
- f represents frequency, and is measured in Hertz. $f = \frac{1}{T_0}$