

COM SCI 132 Week 5

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Reducing Variables

A program can have many variables, but a modern computer only has 16 registers. We must find some way to reduce the number of variables!

- Liveness Analysis \rightarrow Interference Graph

Consider the example:

```
a = 1
b = 10
c = 9 + a
d = a + c
e = c + d
f = b + 8
c = f + e
f = e + c
b = c + 5
return b + f
```

Notice that we have six variables. How do we compress this?

			a	b	c	d	e	f
		a = 1	*					
	a	b = 10		*				
	c b a	c = 9 + a			*			
	c b a	d = a + c				*		
	d c b	e = c + d	0					
e	b	f = b + 8			0	0	*	
f e		c = f + e		0				*
e	c	f = e + c			*			0
f	c	b = c + 5				0	*	
f	b	return b + f		*	0			
				0			0	

Above is a **liveness** graph.

- The middle section shows the code.
- The right section shows the "liveness" of each variable.
 - A variable becomes "alive" (denoted with *) when it is assigned a value.
 - A variable stays alive until the last time it is used before its next assignment or the end of the program.
 - A variable "dies" or can be deallocated after the last time it is used. (denoted with 0).
- The left section shows which variables are alive between each statement.
 - Note that these are written in between each line of code instead of on the same line.
 - The number of registers required is the maximum number of variables on a line plus 1. (In this case, 3 variables + 1 = 4 registers)
 - * The (+1) comes from the need to have an extra register to temporarily store a variable as it is being assigned.
 - * This is similar to why you need an extra variable to swap the values in two variables!
 - This rule ONLY works when the code is straight executing (e.g., no branches or loops).

Liveness with loops - Context-free Graph

Consider the following:

```

c = 0
a = 0
L1: b = a + 1
c = c + 2
a = b * 2
if (a < 10) goto L1
return c

```

statement			init		iter 1		iter 2		iter 3		iter 4		iter 5	
	def	use	in	out	in	out	in	out	in	out	in	out	in	out
a = 0	a	-	-	-	-	a	-	ac	c	ac	c	ac		
L1: b = a + 1	b	a	-	-	a	c	ac	bc	ac	bc	ac	bc		
c = c + 2	c	c	-	-	c	b	bc	b	bc	bc	bc	bc		
a = b * 2	a	b	-	-	b	a	b	ac	bc	ac	bc	ac		
if (a < 10) goto L1	-	a	-	-	a	ac	ac	ac	ac	ac	ac	ac		
return c	-	c	-	-	c	-	c	-	c	-	c	-		

- in and out refer to which variables are live going into the statement and which variables are live coming out of the statement.
- We can stop after iter 4 because iter 4 matches iter 3, signifying that it will no longer change.

Liveness Equations:

$$\begin{aligned}
 \text{in}[n] &= \text{use}[n] \cup (\text{out}[n] - \text{def}[n]) \\
 \text{out}[n] &= \bigcup_{s \in \text{succ}[n]} \text{in}[s]
 \end{aligned}$$

Number of Iterations Time Complexity:

- The number of iterations is $O(n^2)$.

- In the worst case scenario, every box is filled with every variable at the end of all the iterations. We have $2n$ boxes to fill in: n "in" boxes and n "out" boxes.
- In the worst case scenario, each iteration only adds one variable to one box. There are n variables.
- Therefore, the worst case scenario would be of order $2n \cdot n = n^2$.

Full Program Time Complexity:

- When there are n variables...
- $O(n^2)$ iterations
- $O(n)$ set unions per iteration
- $O(n)$ time to do a set union
- Total = $O(n)^4$ (polynomial time!)

Note that this is not the most efficient algorithm - this is a sloppy way to do the algorithm, in other words, the naive way. With more involved analyzing, this algorithm can be reduced to $O(n^2)$.