Online Appendix to Monopsony Power and Firm Organization

Álvaro Jáñez[a]*, Lukas Delgado-Prieto[b],

[a]Stockholm School of Economics

[b]University of Oslo

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*Corresponding author: Stockholm School of Economics, Sveavägen 65, 113 83 Stockholm, Sweden. E-mail: `alvaro.janez.garcia@hhs.se`

A Appendix: Derivations

A.1 Household problem

This section solves the household problem of Section 2. Each household type $o \in \{w, m\}$ chooses the measure of workers to supply to each firm n_{ijot}, the capital stock in the next period K_{ot+1} and consumption of each good c_{ijot} to maximize their utility:

$$\mathcal{U}_{ot} = \max_{\{n_{ijot}, c_{ijot}, K_{ot+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \left[\mathbf{C}_{ot} \; - \; \varphi_o \frac{\mathbf{N}_{ot}^{1+\frac{1}{\phi}}}{1+\frac{1}{\phi}} \right],$$

subject to the household's budget constraint:[1]

$$\mathbf{C}_{ot} + [K_{ot+1} - (1-\delta)K_{ot}] \;=\; \int_0^1 \sum_{i=1}^{M_j} w_{ijot} n_{ijot} \, dj \; + R_t K_{ot} + \kappa_o \Pi_t,$$

where we define the aggregate consumption and labor supply indexes as

$$\mathbf{C}_o := \int_0^1 \sum_{i=1}^{M_j} c_{ijo} \, dj$$

$$\mathbf{N_o} := \left[\int_0^1 \left(\frac{\mathbf{n_{jo}}}{B_{jo}} \right)^{\frac{\theta_o+1}{\theta_o}} dj \right]^{\frac{\theta_o}{\theta_o+1}} \qquad \mathbf{n_{jo}} := \left[\sum_{i=1}^{M_j} n_{ijot}^{\frac{\eta_o+1}{\eta_o}} \right]^{\frac{\eta_o}{\eta_o+1}}.$$

The Lagrangian of this maximization problem is:

$$\begin{aligned}\mathcal{L}(n_{ijot}, c_{ijot}, K_{ot+1}, \lambda) \;=\; & \sum_{t=0}^{\infty} \beta^t \Bigg[\mathbf{C}_{ot} \; - \; \varphi_o \frac{\mathbf{N}_{ot}^{1+\frac{1}{\phi}}}{1+\frac{1}{\phi}} \\ & + \; \lambda_t \Bigg(\int_0^1 \sum_{i=1}^{M_j} w_{ijot} n_{ijot} \, dj \; + R_t K_{ot} + \kappa_o \Pi_t \, dj - \mathbf{C}_{ot} - [K_{ot+1} - (1-\delta)K_{ot}] \Bigg) \Bigg].\end{aligned}$$

The first-order necessary conditions associated with this problem are:

$$\frac{\partial \mathcal{L}}{\partial c_{ijot}} = 0 \iff \lambda_t = \frac{\partial U_{ot}}{\partial \mathbf{C}_{ot}} \cdot \frac{\partial \mathbf{C}_{ot}}{\partial c_{ijot}} \iff \lambda_t = 1 \quad \forall ijot, \tag{A.1}$$

$$\frac{\partial \mathcal{L}}{\partial n_{ijot}} = 0 \iff \lambda_t w_{ijot} = \frac{\partial U_{ot}}{\partial \mathbf{N}_{ot}} \cdot \frac{\partial \mathbf{N}_{ot}}{\partial \mathbf{n}_{jot}} \cdot \frac{\partial \mathbf{n}_{jot}}{\partial n_{ijot}} \quad \forall ijot. \tag{A.2}$$

$$\frac{\partial \mathcal{L}}{\partial K_{ot+1}} = 0 \iff \lambda_t = \lambda_{t+1} \left[R_t + (1-\delta) \right] \quad \forall ot. \tag{A.3}$$

[1] For simplicity, we omit the non-negativity constraints associated with consumption and labor supply. In the unconstrained solution, we will observe that such values also satisfy the constrained solution because they are always greater than or equal to zero.

Substituting Equation (A.1) into Equation (A.3) yields the standard Euler Equation under linear utility:

$$1 = \beta \left[R_t + (1-\delta) \right]. \tag{A.4}$$

As a result, linear utility implies a perfectly elastic supply of capital by households. To distribute capital income across households, we assume that households split the aggregate equilibrium capital stock according to their respective population weights.

To obtain the labor supply to each firm, we substitute Equation (A.1) into Equation (A.2), which yields:

$$w_{ijot} = -\frac{\partial U_o}{\partial \mathbf{N}_{ot}} \cdot \frac{\partial \mathbf{N}_{ot}}{\partial \mathbf{n}_{jot}} \cdot \frac{\partial \mathbf{n}_{jot}}{\partial n_{ijot}}, \tag{A.5}$$

where each component of the last equation is equal to:

$$-\frac{\partial U_{ot}}{\partial \mathbf{N}_{ot}} = \phi_o \mathbf{N}_{ot}^{\frac{1}{\gamma}}, \tag{A.6}$$

$$\frac{\partial \mathbf{N}_{ot}}{\partial \mathbf{n}_{jot}} = \left(\frac{\mathbf{n}_{jot}}{\mathbf{N}_{ot}} \right)^{\frac{1}{\theta}} \cdot \frac{1}{B_{jo}^{\frac{1+\theta}{\theta}}}, \tag{A.7}$$

$$\frac{\partial \mathbf{n}_{jot}}{\partial n_{ijot}} = \left(\frac{n_{ijot}}{\mathbf{n}_{jot}} \right)^{\frac{1}{\eta}}. \tag{A.8}$$

Therefore, plugging the previous expressions into Equation (A.5):

$$w_{ijot} = \left(\frac{n_{ijot}}{\mathbf{n}_{jot}} \right)^{\frac{1}{\eta}} \cdot \left(\frac{\mathbf{n}_{jot}}{\mathbf{N}_{ot}} \right)^{\frac{1}{\theta}} \cdot \frac{1}{B_{jo}^{\frac{1+\theta}{\theta}}} \cdot \left(-\frac{\partial U_{ot}}{\partial \mathbf{N}_{ot}} \right). \tag{A.9}$$

To get the final expression of the firms' labor supply curve, we need to show that under optimality the aggregate wage is equal to the marginal disutility of aggregate labor supply. First, we define the market and aggregate wage indexes as follows:

$$\mathbf{w}_{jot} = \left[\sum_{i \in j} w_{ijot}^{1+\eta} \right]^{\frac{1}{1+\eta}}, \tag{A.10}$$

$$\mathbf{W}_{ot} = \left[\sum_{j=1}^{J} \left(B_{jo} \mathbf{w}_{jot} \right)^{1+\theta} \right]^{\frac{1}{1+\theta}}. \tag{A.11}$$

Substituting Equation (A.9) into Equation (A.10) implies:

$$\mathbf{w}_{jot} = \left(\frac{\mathbf{n}_{jot}}{\mathbf{N}_{ot}} \right)^{\frac{1}{\theta}} \cdot \frac{1}{B_{jo}^{\frac{1+\theta}{\theta}}} \cdot \left(-\frac{\partial U_{ot}}{\partial \mathbf{N}_{ot}} \right). \tag{A.12}$$

Then, substituting the last equation into Equation (A.11) yields the desired result:

$$\mathbf{W}_{ot} = -\frac{\partial U_{ot}}{\partial \mathbf{N}_{ot}}. \tag{A.13}$$

Moreover, we derive the expression for the aggregate labor supply disutility in Equation (6) by plugging Equation (A.6) into Equation (A.13) and rearranging:

$$\mathbf{N}_{ot} = \left(\frac{\mathbf{W}_{ot}}{\phi_o}\right)^{\gamma}.$$

Finally, we get the final expression for the firms' labor supply curve. Note that Equation (A.9) and Equation (A.13) imply the inverse firms' labor supply curve in Equation (7):

$$w_{ijot} = \frac{1}{B_{jo}^{\frac{1+\theta}{\theta}}} \cdot \left(\frac{n_{ijot}}{\mathbf{n}_{jot}}\right)^{\frac{1}{\eta}} \cdot \left(\frac{\mathbf{n}_{jot}}{\mathbf{N}_{ot}}\right)^{\frac{1}{\theta}} \cdot \mathbf{W}_{ot}.$$

Moreover, substituting Equation (A.13) into Equation (A.12) and rearranging imply:

$$\mathbf{n}_{ojt} = \left(\frac{\mathbf{w}_{ojt}}{\mathbf{W}_{ot}}\right)^{\theta} \cdot \mathbf{N}_{ot} B_{jo}^{1+\theta},$$

and substituting Equation (A.12) into Equation (A.9) and rearranging imply:

$$n_{ijot} = \left(\frac{w_{ijot}}{\mathbf{w}_{ojt}}\right)^{\eta} \cdot \mathbf{n}_{ojt}.$$

Hence, using the last two equations yields the expression for the firms' labor supply curve in Equation (7):

$$n_{ijot} = B_{jo}^{1+\theta} \cdot \left(\frac{w_{ijot}}{\mathbf{w}_{jot}}\right)^{\eta} \cdot \left(\frac{\mathbf{w}_{jot}}{\mathbf{W}_{ot}}\right)^{\theta} \cdot \mathbf{N}_{ot}.$$

A.2 Firm Problem

This section solves the firm problem of Section (2) for single-layer firms. Since we solve the organizational problem of choosing the number of layers numerically, we proceed to analytically solve the profit maximization problem given an arbitrary organizational structure.

A firm of organizational type $\ell \in \{1, 2\}$ chooses the measure of production workers n_{ijot} in each layer and the demand for capital k_{ijt} to maximize profits. When making these choices, firms take as given aggregate variables, $\mathbf{N}_{ot}$, and the employment policies of their local competitors, n^*_{-ijot}. Thus, the profit maximization problem is given by:

$$\pi(z, \ell) \; = \max_{n_{ijwt}, n_{ijmt}, k_{ijt}} \quad y(z, \ell) \; - \; R_t k_{ijt} \; - \sum_{o \in \mathcal{O}(\ell)} w_{ijot}\Big(n_{ijot}, n^*_{-ijot}, \mathbf{N}_{ot}, \mathbf{W}_{ot}\Big) n_{ijot},$$

subject to:

$$w_{ijot}\Big(n_{ijot}, n^*_{-ijot}, \mathbf{N}_{ot}, \mathbf{W}_{ot}\Big) = \left(\frac{1}{B_{jot}}\right)^{\frac{1+\theta_o}{\theta_o}} \left(\frac{n_{ijot}}{\mathbf{n}_{jot}(n_{ijot}, n^*_{-ijot})}\right)^{\frac{1}{\eta_o}} \left(\frac{\mathbf{n}_{jot}(n_{ijot}, n^*_{-ijot})}{\mathbf{N}_{ot}}\right)^{\frac{1}{\theta_o}} \mathbf{W}_{ot},$$

$$\mathbf{n}_{jot}(n_{ijot}, n^*_{-ijot}) = \left[n_{ijot}^{\frac{1+\eta_o}{\eta_o}} + \sum_{k\neq i} n^*_{kjot}{}^{\frac{1+\eta_o}{\eta_o}}\right]^{\frac{\eta_o}{1+\eta_o}},$$

$$w_{ijot} \geq \underline{w}, \quad \forall t, \forall o \in \mathcal{O}(\ell).$$

Where $\mathcal{O}(\ell)$ is the set of occupations of an organization of type ℓ.

Demand for capital. The FOC associated with capital implies that the demand for capital is then given by:

$$R_t \;=\; \frac{\partial y(z,\ell)}{\partial k_{ijt}} \quad \Longleftrightarrow \quad R_t \;=\; \frac{y(z,\ell)}{k_{ijt}}(1-\alpha)(1-\gamma).$$

Solving for capital implies:

$$\begin{aligned} k^*_{ijt} \;&=\; \left[\frac{(1-\alpha)(1-\gamma)}{R_t}\right]^{\frac{1}{\gamma+(1-\gamma)\alpha}} \cdot z_{ijt}^{\frac{1}{\gamma+(1-\gamma)\alpha}} \cdot n_w^{\frac{(1-\alpha)\gamma}{\gamma+(1-\gamma)\alpha}}, \quad \text{if } \ell = 1, \\ k^*_{ijt} \;&=\; \left[\frac{(1-\alpha)(1-\gamma)}{R_t}\right]^{\frac{1}{\gamma+(1-\gamma)\alpha}} \cdot z_{ijt}^{\frac{1}{\gamma+(1-\gamma)\alpha}} \cdot n_m^{\frac{(1-\alpha)\alpha}{\gamma+(1-\gamma)\alpha}} \cdot n_w^{\frac{(1-\alpha)\gamma}{\gamma+(1-\gamma)\alpha}}, \quad \text{if } \ell = 2. \end{aligned}$$

The closed-form solution for capital in terms of labor implies that we can rewrite firm profits after discounting capital costs as:

$$\begin{aligned} \tilde{y}(z,1) - R_t \;&=\; \tilde{z}_{ijt} n_w^{(1-\tilde{\alpha})}, \\ \tilde{y}(z,2) - R_t \;&=\; \tilde{z}_{ijt} n_m^{(1-\alpha)\tilde{\alpha}} n_w^{(1-\tilde{\alpha})}, \end{aligned}$$

where we define the following parameters:

$$\begin{aligned} \tilde{\alpha} \;&=\; \frac{\alpha}{\gamma + (1-\gamma)\alpha}, \\ \tilde{z}_{ijt} \;&=\; [1-(1-\gamma)(1-\alpha)] \left(\frac{(1-\gamma)(1-\alpha)}{R_t}\right)^{\frac{(1-\alpha)(1-\gamma)}{\gamma+(1-\gamma)\alpha}} \cdot z_{ijt}^{\frac{1}{\gamma+(1-\gamma)\alpha}}. \end{aligned}$$

The utility of these expressions is that we can proceed with the maximization problem just in terms of labor and then recover capital demand from the labor demand functions.

Demand for labor. The associated Lagrangian function after plugging in capital demand is given by:

$$\mathcal{L}\left(\{n_{ijot}\}_{o\in\mathcal{O}(\ell)},\mu_t\right) \;=\; \tilde{y}(z,\ell) \;-\; \sum_{o\in\mathcal{O}(\ell)} w_{ijot}\Big(n_{ijot},n^*_{-ijot},\mathbf{N}_{ot},\mathbf{W}_{ot}\Big)n_{ijot} \;+\; \nu\cdot\Big(w_{ijw}-\underline{w}\Big).$$

The system of Kuhn-Tucker conditions is given by:

$$\frac{\partial\mathcal{L}}{\partial n_{ijot}}=0 \iff \frac{\partial\tilde{y}(z,\ell)}{\partial n_{ijot}} \;+\; \nu \;=\; \frac{\partial w_{ijot}}{\partial n_{ijot}}\cdot n_{ijot} \;+\; w_{ijot}, \tag{A.14}$$

$$\nu\cdot\Big(w_{ijot}-\underline{w}\Big)=0, \tag{A.15}$$

$$\nu\geq 0, \tag{A.16}$$

$$w_{ijot}=\max\left(\underline{w}\;,\;\underbrace{\left(\frac{1}{B_{jo}}\right)^{\frac{1+\theta_o}{\theta_o}}\left(\frac{n_{ijot}}{\mathbf{n}_{jot}}\right)^{\frac{1}{\eta_w}}\left(\frac{\mathbf{n}_{jot}}{\mathbf{N}_{ot}}\right)^{\frac{1}{\theta_w}}\mathbf{W}_w}_{=\tilde{w}_{ijot}\text{ ,i.e., unconstrained labor supply curve}}\right), \tag{A.17}$$

$$\mathbf{n}_{jot}=\left[n_{ijot}^{\frac{1+\eta_w}{\eta_w}}+\sum_{k\neq i}n^*_{kjot}{}^{\frac{1+\eta_w}{\eta_w}}\right]^{\frac{\eta_w}{1+\eta_w}},\qquad \forall t,\forall o\in\mathcal{O}(\ell). \tag{A.18}$$

To solve the system of equations, we break the problem into three different cases.

Case I: The minimum wage is not binding. Suppose the case when the minimum wage is not binding $w^*_{ijot}>\underline{w}$. Then, Equation (A.15) implies that $\nu=0$, and Equation (A.14) is given by:

$$\begin{aligned}
\frac{\partial\tilde{y}(z,1)}{\partial n_{ijot}}\bigg|_{n^*_{ijot}} \;&=\; n^*_{ijot}\cdot\frac{\partial w_{ijot}}{\partial n_{ijot}}\bigg|_{n^*_{ijot}} \;+\; w^*_{ijot},\\
\frac{\partial\tilde{y}(z,1)}{\partial n_{ijot}}\bigg|_{n^*_{ijot}} \;&=\; \frac{w^*_{ijot}}{\varepsilon^*_{ijot}} \;+\; w^*_{ijot},\\
\Rightarrow\; w^*_{ijot} \;&=\; \mu^*_{ijot}\cdot\frac{\partial\tilde{y}(z,1)}{\partial n_{ijot}}\bigg|_{n^*_{ijot}}.
\end{aligned}$$

where ε_{ijot} is the structural elasticity of labor supply and μ_{ijot} represents the wage markdown:

$$\begin{aligned}
\varepsilon_{ijot} \;&=\; \left[\frac{\partial\log w_{ijot}}{\partial\log n_{ijot}}\right]^{-1},\\
\mu_{ijot} \;&=\; \frac{\varepsilon_{ijot}}{\varepsilon_{ijot}+1}\in[0,1].
\end{aligned}$$

In Appendix A.3 we show that the structural elasticity of labor supply has a closed-form solution given by:

$$\varepsilon_{ijot}(s_{ijot}) \;=\; \left[\frac{1}{\eta_o} + \left(\frac{1}{\theta_o} - \frac{1}{\eta_o}\right)\frac{\partial \log \mathbf{n}_{jot}}{\partial \log n_{ijot}}\right]^{-1} \;=\; \left[\frac{1}{\eta_o} + \left(\frac{1}{\theta_o} - \frac{1}{\eta_o}\right)s_{ijot}\right]^{-1}, \tag{A.19}$$

where s_{ijot} is the payroll share of firm i in market j:

$$s_{ijot} := \frac{w_{ijot}n_{ijot}}{\sum_{i\in j} w_{ijot}n_{ijot}}. \tag{A.20}$$

Case II: The minimum wage is binding, and the labor supply equals labor demand. Suppose the minimum wage is binding $w^*_{ijot} = \underline{w}$ and the labor supply equals labor demand, that is, Equation (A.17) satisfies:

$$w^*_{ijot} = \underline{w} \;=\; \left(\frac{1}{B_{jo}}\right)^{\frac{1+\theta_o}{\theta_o}} \left(\frac{n^*_{ijot}}{\mathbf{n}^*_{jot}}\right)^{\frac{1}{\eta_w}} \left(\frac{\mathbf{n}^*_{jot}}{\mathbf{N}_{ot}}\right)^{\frac{1}{\theta_w}} \mathbf{W}_{ot} \tag{A.21}$$

Then, the optimal level of employment is given by Equation (A.21). Moreover, the Lagrange multiplier associated with the inequality constraint may not be binding, i.e., $\nu \geq 0$. Thus, Equation (A.14) implies that the marginal revenue must be smaller than or equal to the marginal cost. In contrast, the marginal revenue must be greater than or equal to the minimum wage. We prove this by contradiction. Suppose $\underline{w} > \text{mrpl}(n^*_{ijot})$. Since the unconstrained labor supply curve is strictly increasing in labor, Equation (A.17) implies that $w(n_{ijot}) = \underline{w} \quad \forall n_{ijot} < n^*_{ijot}$. Thus, the marginal cost is also equal to the minimum wage within this employment range: $\text{mc}(n_{ijot}) = \underline{w} \quad \forall n_{ijot} < n^*_{ijot}$. Moreover, since the marginal revenue of labor is strictly decreasing in labor units, there exists a threshold $n'_{ijot} < n^*_{ijot}$ for which $\text{mrpl}(n'_{ijot}) = \underline{w}$ and $\text{mrpl}(n_{ijot}) < \underline{w} \quad \forall n_{ijot} \in (n'_{ijot}, n^*_{ijot}]$. However, this implies that n'_{ijot} is feasible and more profitable than n^*_{ijot} because any unit between them yields negative profits, i.e., their marginal cost is higher than their marginal revenue, which contradicts that n^*_{ijot} is optimal. Hence, it must be the case that $\underline{w} \leq \text{mrpl}(n^*_{ijot})$.

Overall, the previous results imply that:

$$w^*_{ijot} \;=\; \underline{w}, \tag{A.22}$$

$$\left.\frac{\partial \tilde{y}(z,1)}{\partial n_{ijot}}\right|_{n^*_{ijot}} \;\leq\; \left.\frac{\partial w_{ijot}}{\partial n_{ijot}}\right|_{n^*_{ijot}} + \underline{w}, \tag{A.23}$$

$$\left.\frac{\partial \tilde{y}(z,1)}{\partial n_{ijot}}\right|_{n^*_{ijot}} \;\geq\; \underline{w}. \tag{A.24}$$

Here, the markdown does not have a closed-form solution but is given by:

$$\mu_{ijot} \;=\; \frac{\underline{w}}{\frac{\partial y(z,1)}{\partial n_{ijot}}\Big|_{n^*_{ijot}}} \in [0,1]. \tag{A.25}$$

Case III: The minimum wage is binding, and the labor supply exceeds labor demand. Suppose that the minimum wage is binding $w^*_{ijot} = \underline{w}$ and the labor supply excess labor demand, that is, Equation (A.17) satisfies:

$$w^*_{ijot} = \underline{w} \;>\; \left(\frac{1}{B_{jo}}\right)^{\frac{1+\theta_o}{\theta_o}} \left(\frac{n^*_{ijot}}{\mathbf{n}^*_{jot}}\right)^{\frac{1}{\eta_w}} \left(\frac{\mathbf{n}^*_{jot}}{\mathbf{N}_{ot}}\right)^{\frac{1}{\theta_w}} \mathbf{W}_{ot}. \tag{A.26}$$

This implies that the marginal cost of an additional hire is the minimum wage. Graphically, the wage function is flat in a neighborhood of the optimal labor choice n^*_{ijot}, as shown in Figure A.3. To prove it mathematically, we also rely on the fact that the unconstrained labor supply curve is strictly increasing in labor. Thus, $\tilde{w}_{ijot}(\tilde{n}_{ijot}) = \underline{w}$ for $\tilde{n}_{ijot} > n^*_{ijot}$. Then, there exists $\bar{\varepsilon} > 0$ such that $\varepsilon \in [0, \bar{\varepsilon}]$ and $n'_{ijot} = n^*_{ijot} + \varepsilon < \tilde{n}_{ijot}$ for which the effective labor supply $w_{ijot}(n'_{ijot}) = \underline{w}$ (see Equation A.17). As a result, $\frac{\partial w_{ijot}}{\partial n_{ijot}} = 0$.

Moreover, this case also implies that $\nu = 0$. We prove this by contradiction. Suppose $\nu > 0$, then Equation (A.14) implies that $\frac{\partial \tilde{y}(z,1)}{\partial n_{ijot}}\Big|_{n^*_{ijot}} < \underline{w}$. Since we assume that the marginal productivity is strictly decreasing in labor, there exists $\varepsilon > 0$ such that $n''_{ijot} = n^*_{ijot} - \varepsilon$ and $\frac{\partial \tilde{y}(z,1)}{\partial n_{ijot}}\Big|_{n''_{ijot}} = \underline{w} > \frac{\partial \tilde{y}(z,1)}{\partial n_{ijot}}|_{n^*_{ijot}}$. Then, choosing n''_{ijot} and paying them $\underline{w}$ is feasible and more profitable because it raises revenue while keeping costs fixed. This is a contradiction because n^*_{ijot} is optimal. Hence, it must be $\nu = 0$.

Therefore, the aforementioned two results and Equation (A.14) imply that labor demand satisfies:

$$w^*_{ijot} \;=\; \underline{w} \;=\; \frac{\partial \tilde{y}(z,1)}{\partial n_{ijot}}\Big|_{n^*_{ijot}}, \qquad \mu_{ijot} = 1. \tag{A.27}$$

Where the markdown equals one by definition.

Figure A.1: Unbinding minimum wage in partial equilibrium analysis

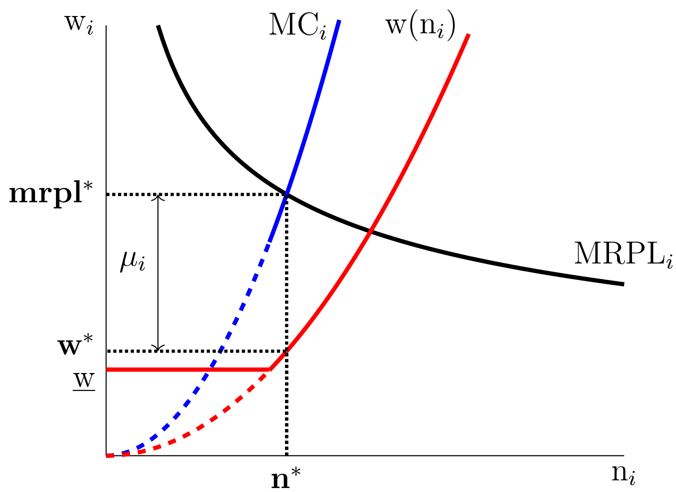


Figure A.2: Binding minimum wage on the labor supply curve

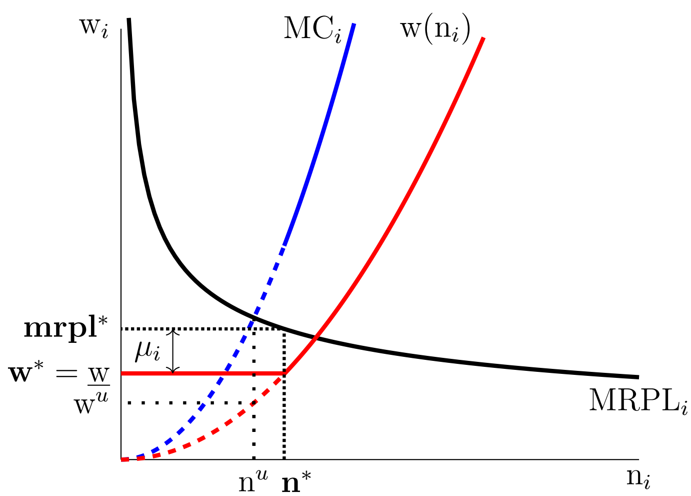
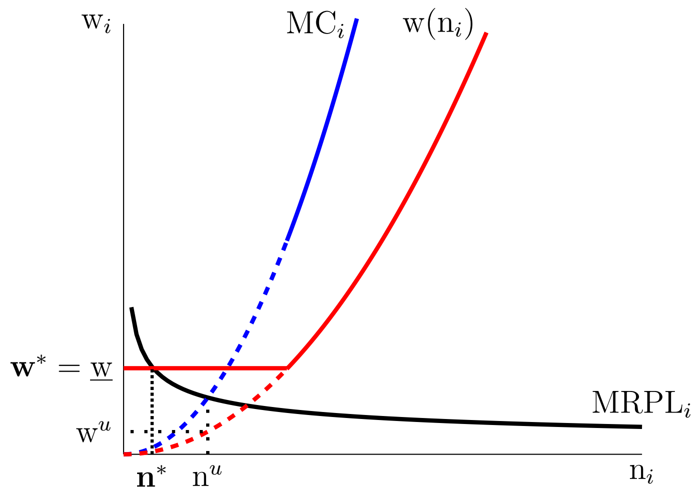


Figure A.3: Binding minimum wage off the labor supply curve



A.3 Structural Elasticity of Labor Supply

We now show that the structural elasticity has a closed-form solution. Consider the log transformation of the inverse labor supply curve in Equation (7):

$$\log\Big(w(n_{ijot}, n^*_{-ijot}, \mathbf{W}_{ot}, \mathbf{N}_{ot})\Big) = \frac{1}{\eta_o}\log(n_{ijot}) + \left(\frac{1}{\theta_o} - \frac{1}{\eta_o}\right)\log(\mathbf{n}_{jot})$$
$$- \frac{1}{\theta_o}\log(\mathbf{N}_{ot}) + \log(\mathbf{W}_{ot}) - \frac{1+\theta_o}{\theta_o}\log(B_{jo}).$$

Thus,

$$\frac{\partial\log\Big(w(n_{ijot}, n^*_{-ijot}, \mathbf{W}_{ot}, \mathbf{N}_{ot})\Big)}{\partial\log(n_{ijot})} = \frac{1}{\eta_o} - \left(\frac{1}{\theta_o} - \frac{1}{\eta_o}\right)\frac{\partial\log(\mathbf{n}_{jot})}{\partial\log(n_{ijot})}.$$

Note that the derivative of the economy-wide variables with respect to the firm's employment is zero because firms are atomistic with respect to the economy. Moreover, the definition of the market labor supply disutility index implies that:

$$\begin{aligned}
\frac{\partial\log(\mathbf{n}_{jot})}{\partial\log(n_{ijot})} &= \frac{\partial\mathbf{n}_{jot}}{\partial n_{ijot}} \cdot \frac{n_{ijot}}{\mathbf{n}_{jot}}, \\
&= n_{ijot}^{\frac{1+\eta_o}{\eta_o}} \cdot \mathbf{n}_{jot}^{-1} \cdot \left(\sum_{i\in j} n_{ijot}^{\frac{\eta_o+1}{\eta_o}}\right)^{-\frac{1}{\eta_o+1}}, \\
&= \left(\frac{n_{ijot}}{\mathbf{n}_{jot}}\right)^{\frac{1+\eta_o}{\eta_o}}.
\end{aligned}$$

Therefore,

$$\frac{\partial \log\Big(w(n_{ijot}, n^*_{-ijot}, \mathbf{W}_{ot}, \mathbf{N}_{ot})\Big)}{\partial \log(n_{ijot})} = \frac{1}{\eta_o} - \left(\frac{1}{\theta_o} - \frac{1}{\eta_o}\right) \cdot \left(\frac{n_{ijot}}{\mathbf{n}_{jot}}\right)^{\frac{1+\eta_o}{\eta_o}}.$$

Next, we show that the last fraction is equal to the payroll share of the firm i in market j. In particular, defining the payroll share and substituting the inverse labor supply curve:

$$\begin{aligned} s_{ijot} &= \frac{w_{ijot} n_{ijot}}{\sum_{i \in j} w_{ijot} n_{ijot}} = \frac{n_{ijot}^{\frac{1+\eta_o}{\eta_o}}}{\sum_{i \in j} n_{ijot}^{\frac{1+\eta_o}{\eta_o}}}, \\ &= \left(\frac{n_{ijot}}{\mathbf{n}_{jot}}\right)^{\frac{1+\eta_o}{\eta_o}}. \end{aligned}$$

Hence, the structural labor supply elasticity is given by:

$$\varepsilon_{ijot} := \left[\frac{\partial \log w(n_{ijot}, n^*_{-ijot}, \mathbf{W}_{ot}, \mathbf{N}_{ot})}{\partial \log n_{ijot}}\right]^{-1} = \left[\frac{1}{\eta_o} - \left(\frac{1}{\theta_o} - \frac{1}{\eta_o}\right) s_{ijot}\right]^{-1}.$$

We take the concept of "structural" from Berger et al. (2022). The motivation for this concept is twofold. First, it arises from a structural macroeconomic model with firm granularity and strategic interaction in labor demand. In these models, the structural elasticity is the welfare-relevant variable of the model because its distribution determines the distribution of wage markdowns. Second, it is useful to distinguish the concept from the more commonly estimated reduced-form labor supply elasticity. On the one hand, the structural elasticity is the labor supply elasticity faced by a firm that internalizes the employment responses of its competitors within the local labor market. It measures the percent change in the firm's labor supply due to an increase of one percent in the firm's wages, holding its competitors' employment constant. On the other hand, the reduced-form elasticity measures the percent change in the firm's labor supply due to an increase of one percent in the firm's wages. Thus, in our model, this variable includes the effect of the response of the firm's competitors on the firm's own wage. For example, when a firm receives an idiosyncratic positive shock and increases labor demand, Cournot competition implies that the firm's competitors best respond by decreasing labor demand, which also leads the shocked firm to best respond and increase its quantity of labor demand and so on.

B Appendix: Algorithm

The solution of the equilibrium consists of a fixed point in the wage vectors. We solve the problem for 20 different firm productivity grids and 1,000 markets. We explain the algorithm in three steps. First, we show how we deal with the organizational problem. Second, we describe the equilibrium solution in the absence of minimum wages to give some previous intuition. Third, we show the equilibrium solution with minimum wages.

B.1 Computing the Optimal Organization Structure

Calculating the maximum profits a firm could earn by opting for the off-equilibrium organizational structure is the main challenge when determining a firm's optimal organizational structure. This is because computing off-equilibrium profits in each iteration would be overly computationally expensive. To address this issue, we set an arbitrarily small tolerance level, φ^{org}, and then proceed as follows:

- When the distance between the initial and predicted wages exceeds φ^{org}, we assign off-equilibrium wages and employment to each firm as described below:

  - If the firm began the iteration as a multi-layer organization, we use its equilibrium wage and employment levels for production workers to calculate its profits as a single-layer organization.

  - If the firm began the iteration as a single-layer organization, we use the equilibrium wage and employment levels for production workers and managers of its nearest competitor to compute the firm's profits as a multi-layer organization. When there exist multi-layer firms within the same market, the nearest competitor is the multi-layer firm in the market located in the closest productivity bin. If no multi-layer firms are within the same market, we assign economy-wide minimum wage and employment levels for both occupations.

  With this information in hand, we compute the maximum profits of each firm for their off-equilibrium organizational structure. In parallel, the main algorithm provides the maximum profits of the on-equilibrium organization using the first-order conditions. Then, once we have maximum profits for both on- and off-equilibrium organizations, we solve the organizational problem in (9) for all firms.

- When the distance between the initial and predicted wages falls below the tolerance φ^{org}, it indicates that the algorithm is close to converging. In such cases, we compute the actual wage and employment levels for each firm's off-equilibrium organizational structure using a numerical solution. Specifically, we calculate off-equilibrium profits by numerically solving the profit maximization problem (10) for multi-layer firms and (11) for single-layer firms. With this information and the optimal profits for firms in equilibrium, we determine the optimal organizational structure as defined in the organizational problem (9). After computing the optimal organizational structure, we update wages within the same iteration k. If the firm is initially multi-layer and we find a deviation that makes being single-layer more profitable, we set managerial wages and employment to zero $w_{ijm}^{(k)} = 0$ and $n_{ijm}^{(k)} = 0$. For production workers, we set their wages and employment to the deviation values $w_{ijw}^{(k)} = w'$ and $n_{ijw}^{(k)} = n'$. If the firm is initially single-layer and we find a deviation that makes being multi-layer more profitable, we set both wages and employment to their deviation values $w_{ijo}^{(k)} = w'$ and $n_{ijo}^{(k)} = n'$ $\forall o \in \{w, m\}$.

The numerical solution of the organizational structure in the algorithm aims to correct any potential misassignments of optimal organizational structures to firms, as we rely on information from their competitors. Given that this step is computationally intensive, we perform it only once. Following a single implementation, we continue with the previous method that utilizes information from competitors until convergence.[2]

Overall, this numerical solution for the optimal organization structure of firms performs well. After convergence, we observe an optimal deviation from the equilibrium organizational choice for 0.34 percent of firms. This error arises because an optimal deviation implies that only one firm deviates from equilibrium while its local market competitors maintain constant labor demand. However, in practice, our algorithm solution sometimes implies that more than one firm deviates, for instance, in markets with multiple firms. Therefore, implementing such an optimal deviation for those firms becomes impractical.

[2]Furthermore, we note that implementing the algorithm more than once does not significantly improve its accuracy in correcting the mistakes from assigning the outcomes of competitors' firms.

B.2 Algorithm without Minimum Wages

The idea of the algorithm is that whenever a firm faces an excess of labor demand for one occupation, then we smoothly increase its wage. In contrast, we smoothly decrease it whenever it faces an excess of labor supply. Thus, the algorithm always converges as long as the labor supply curve firms face is strictly increasing and the marginal revenue is strictly decreasing in employment. Since we solve the model in steady state, where variables are constant over time, we omit the time subscript to ease notation.

We initialize the algorithm by guessing a vector of wages $\left\{w_{ijo}^{(0)}\right\}_{\forall ijo}$. Consider iteration k:

1. **Compute the labor supply**.

   Note that firms' wages $w_{ijo}^{(k)}$ are enough to get $\mathbf{w}_{jo}^{(k)}$, $\mathbf{W}_{o}^{(k)}$, and $\mathbf{N}_{o}^{(k)}$ from Equations (6) and (8). Then, we compute the labor supply to each firm $n_{ijo}^{s,(k)}$ by substituting the previous variables into Equation (7).

2. **Compute organizational choice.**

   Market clearing implies that labor supply is equal to labor demand in equilibrium. Thus, labor supply is equal to labor demand at equilibrium wages $n_{ijo}^{d,(k)} = n_{ijo}^{s,(k)}$. Then, we use wages $\left\{w_{ijo}^{(k)}\right\}_{\forall ijo}$ and labor demands $\left\{n_{ijo}^{d,(k)}\right\}_{\forall ijo}$ to compute profits for the on-equilibrium organizational structure. With this information, we compute the optimal organizational choice $\ell^{(k)}$ for all firms based on the method that we described in the previous subsection. Note that wages and employment change when we update the firm's optimal organizational structure, e.g., we set the wage and employment of managers to zero for single-layer firms: $w_{ijm}^{(k)} = 0$ and $n_{ijm}^{s,(k)} = n_{ijm}^{d,(k)} = 0$ if $\ell^{(k)} = 1$.

3. **Compute markdowns**.

   We compute the payroll market share $s_{ijo}^{(k)}$ using wages $w_{ijo}^{(k)}$ and labor demand $n_{ijo}^{d,(k)}$ of all firms in market j. Then, we compute the firm's structural elasticity $\varepsilon_{ijo}^{(k)}$ and markdown $\mu_{ijo}^{(k)}$ from Equations (12)-(13).

4. **Compute wages from the labor demand FOCs.**

   For each occupation, we use the optimal organization $\ell^{(k)}$ and labor demand $n_{ijo}^{d,(k)}$ of each firm to compute its marginal revenue product of labor:

$$\text{mrpl}_{ijo}^{(k)} = \left.\frac{\partial \tilde{y}(z, \ell^{(k)})}{\partial n_{ijo}}\right|_{n_{ijo}^{d,(k)}}$$

Then, we update the occupation-specific wages of all firms as:

$$w_{ijo}^{\prime\,(k)} = \begin{cases} \mu_{ijo}^{(k)} \cdot \text{mrpl}_{ijo}^{(k)}, & \text{if } n_{ijo}^{d,(k)} > 0, \\ 0, & \text{if } n_{ijo}^{d,(k)} = 0. \end{cases}$$

5. **Iteration**.

   Iterate over (1) to (4) until convergence of wages. Whenever $\max\left\{\text{abs}(w_{ijo}^{(k)} - w_{ijo}^{\prime\,(k)})\right\} >$ tol, we update wages with the following criterion:

$$w_{ijo}^{(k+1)} = \rho w_{ijo}^{(k)} + (1-\rho) w_{ijo}^{\prime\,(k)} \quad \text{for} \quad \rho \in (0,1).$$

B.3 Algorithm with Minimum Wages

We solve the equilibrium with minimum wages using a shadow wage approach similar to Berger et al. (2023). This approach is useful to deal with non-market-clearing wages and considers that workers at constrained firms perceive a lower wage whenever an excess of labor supply exists at the minimum wage. In particular, the shadow or perceived wage is the wage for which the firm's labor supply equals its labor demand at the minimum wage, implying that the excess labor supply at the minimum wage is reallocated towards other firms.

We initialize the algorithm by guessing a vector of wages $\left\{w_{ijo}^{(0)}\right\}_{\forall ijo}$ such that the minimum wage is not binding for any of the firms in the first iteration. Thus, we set the initial vector of shadow wages $\left\{\tilde{w}_{ijo}^{(0)}\right\}_{\forall ijo}$ equal to the initial vector of wages. Consider iteration k:

1. **Compute the labor supply**.

   We use shadow wages to compute $\tilde{\mathbf{w}}_{jo}^{(k)}$, $\tilde{\mathbf{W}}_{o}^{(k)}$, and $\tilde{\mathbf{N}}_{o}^{(k)}$ from Equations (6) and (8). Then, we compute the labor supply to each firm $n_{ijo}^{s,(k)}$ by substituting the previous variables into Equation (7). Note that this equation always holds in terms of shadow wages by definition. Nevertheless, this is not the case for actual wages, as the minimum wage may be binding.

2. **Compute organizational choice.**

   Using the shadow wage approach implies market clearing even with minimum wages. Thus, labor supply is equal to labor demand at the shadow wage $n_{ijo}^{d,(k)} = n_{ijo}^{s,(k)}$. Then, we use wages $\left\{w_{ijo}^{(k)}\right\}_{\forall ijo}$ and labor demands $\left\{n_{ijo}^{d,(k)}\right\}_{\forall ijo}$ to compute profits for the on-equilibrium organizational structure. With this information, we compute the optimal organizational choice $\ell^{(k)}$ for all firms based on the method that we described in the previous subsection. Note that wages and employment change when we update the firm's optimal organizational structure, e.g., we set the wage and employment of managers to zero for single-layer firms: $w_{ijm}^{(k)} = 0$ and $n_{ijm}^{s,(k)} = n_{ijm}^{d,(k)} = 0$ if $\ell^{(k)} = 1$.

3. **Compute markdowns**.

   For each occupation, we use the optimal organization $\ell^{(k)}$ and labor demand $n_{ijo}^{d,(k)}$ of each firm to compute its marginal revenue product of labor $\text{mrpl}_{ijo}^{(k)} = \frac{\partial \tilde{y}(z,\ell^{(k)})}{\partial n_{ijo}}\Big|_{n_{ijo}^{d,(k)}}$. Then, we use this marginal product $\text{mrpl}_{ijo}^{(k)}$ and initial wages $w_{ijo}^{(k)}$ to compute:

   (a) *Minimum wage is not binding*: Whenever the firm's wage and marginal product are both above the minimum wage, we compute the payroll market share $s_{ijo}^{(k)}$ using wages $w_{ijo}^{(k)}$ and labor demand $n_{ijo}^{d,(k)}$ of all firms in market j. Then, we compute the firm's structural elasticity $\varepsilon_{ijo}^{(k)}$ and markdown $\mu_{ijo}^{(k)}$ from Equations (12)-(13).

   (b) *Minimum wage is binding, and the labor supply equals the labor demand at the minimum wage*: Whenever the minimum wage is binding and the firm's marginal product is above the minimum wage, we compute the firm's markdown from Equation (15).

   (c) *Minimum wage is binding, and the labor supply exceeds the labor demand at the minimum wage*: Whenever the minimum wage is binding and the firm's marginal product is lower than or equal to the minimum wage, we set $\mu_{ijo}^{(k)} = 1$.

4. **Compute wages.**

   For all firms, we update the occupation-specific wages as follows:

$$
w_{ijo}^{\prime(k)} = \begin{cases} \max\left\{\underline{w}\ ,\ \mu_{ijo}^{(k)} \cdot \text{mrpl}_{ijo}^{(k)}\right\}, & \text{if} \quad n_{ijo}^{(k)} > 0, \\ 0, & \text{if} \quad n_{ijo}^{(k)} = 0. \end{cases}
$$

We use these updated wages to construct $\mathbf{w}'^{(k)}_{jo}$ and $\mathbf{W}'^{(k)}_{o}$ using Equation (8).

5. **Compute the labor demand implied by minimum wages.**

   Here, we guarantee that the labor demand of constrained firms that face excess labor supply at the minimum wage is given by the inverse labor demand evaluated at the minimum wage. Particularly, we use the marginal product of labor $\text{mrpl}^{(k)}_{ijo}$ and updated wages $w'^{(k)}_{ijo}$ to compute:

   (a) *Minimum wage is not binding*: Whenever the firm's marginal product and wage are both above the minimum wage, the firm's labor demand coincides with the firm's labor supply $n'^{\,d,(k)}_{ijo} = n^{s,(k)}_{ijo}$.

   (b) *Minimum wage is binding, and the labor supply equals the labor demand at the minimum wage*: Whenever the minimum wage is binding and the firm's marginal product is above the minimum wage, the firm's labor demand coincides with the firm's labor supply $n'^{\,d,(k)}_{ijo} = n^{s,(k)}_{ijo}$.

   (c) *Minimum wage is binding, and the labor supply exceeds the labor demand at the minimum wage*: Whenever the minimum wage is binding and the firm's marginal product is lower than or equal to the minimum wage, we construct the labor demand of the firm from the employment level for which the minimum wage equals the marginal product of labor:

$$\underline{w} \;=\; \left.\frac{\partial \tilde{y}(z,\ell^{(k)})}{\partial n_{ijo}}\right|_{n^{d,(k)}_{ijo}} .$$

6. **Update shadow wages.**

   The shadow wage is the wage that implies market clearing for all firms. That is, it does not coincide with the actual wage only for firms that face an excess of labor supply when they pay the minimum wage. We first use the updated employment levels $n'^{\,d,(k)}_{ijo}$

to update the market and aggregate employment levels from their definition:

$$\mathbf{n}'^{\,d,(k)}_{jo} := \left[\sum_{i=1}^{M_j} \left(n'^{\,d,(k)}_{ijo} \right)^{\frac{\eta_o+1}{\eta_o}} \right]^{\frac{\eta_o}{\eta_o+1}},$$

$$\mathbf{N}'^{\,d,(k)}_{o} := \left[\int_0^1 \left(\frac{\mathbf{n}'^{\,d,(k)}_{jot}}{B_{jo}} \right)^{\frac{\theta_o+1}{\theta_o}} dj \right]^{\frac{\theta_o}{\theta_o+1}}.$$

Then, we update shadow wages:

$$\tilde{w}^{(k+1)}_{ijo} = \left(\frac{1}{B_{jo}} \right)^{\frac{1+\theta_o}{\theta_o}} \left(\frac{n'^{\,d,(k)}_{ijo}}{\mathbf{n}'^{\,d,(k)}_{jo}} \right)^{\frac{1}{\eta_o}} \left(\frac{\mathbf{n}'^{\,d,(k)}_{jo}}{\mathbf{N}'^{\,d,(k)}_{o}} \right)^{\frac{1}{\theta_o}} \mathbf{W}'^{\,(k)}_{o}$$

7. **Iteration**.
   Iterate over (1) to (6) until convergence of wages. Whenever $\max \left\{ \text{abs}(w^{(k)}_{ijo} - w'^{\,(k)}_{ijo}) \right\} >$ tol, we update wages with the following criterion:

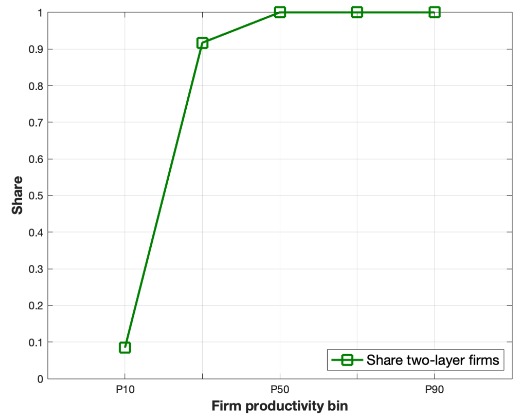
$$\begin{aligned} \text{Unconstrained firms:} \quad & w^{(k+1)}_{ijo} = \rho w^{(k)}_{ijo} + (1-\rho) w'^{\,(k)}_{ijo} \quad \text{for} \quad \rho \in (0,1), \\ \text{Any constrained firm:} \quad & w^{(k+1)}_{ijo} = \underline{w}. \end{aligned}$$

   Iteration in actual wages implies the iteration in shadow wages almost surely.

C Appendix: Market Characterization

This section discusses the interaction between firm organization and monopsony power in equilibrium. The top panels of Figure C.1 display the share of two-layer firms and the average employment level of production workers (blue) and managers (green) across firms. Both outcomes increase with firm productivity, as these firms have more incentives to produce at large scales, and hiring managers enable them to manage a larger workforce. The bottom left panel shows average (log) wages across firms for each occupation. Wages are close to the minimum wage for low-productivity firms, especially for production workers. Furthermore, managers earn substantially higher wages than production workers. This occurs because the aggregate disutility from labor supply is higher for managers and because the ratio of production workers to managers is high, which drives up the marginal productivity of managers relative to that of production workers. Moreover, the model captures the fact that wage dispersion between both occupations increases with firm size. This result stems

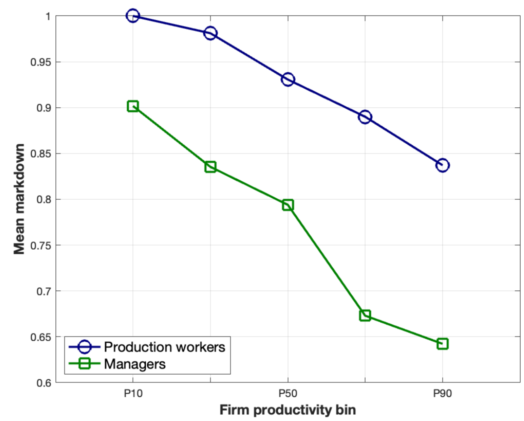
Figure C.1: Illustration of general equilibrium outcomes



(a) Two-layer firms

(b) Employment

(c) Wages



(d) Markdown

Note: Figures constructed from the model under the estimated parameters in Table 1.

from the fact that the ratio of production workers to managers increases with firm size and, therefore, so does the ratio of their marginal productivity.

Finally, the bottom right panel displays average wage markdowns, i.e., the ratio between wages and the marginal product of labor, across firms and occupations. Monopsony power is stronger over managers than over production workers for three reasons. First, minimum wages are more likely to be binding for production workers, limiting firms' ability to depress their wages below their marginal productivity. Second, managers exhibit lower firm-substitutability, making them less responsive to labor demand changes. Third, market concentration is also higher for managers. If both occupations had the same degree of firm-substitutability, firms would exert wider markdowns over managerial wages because the labor supply elasticity of managers would be closer to the across-market elasticity than that of production workers.

D Appendix: Data

This section provides a detailed description of the data, the occupation and the market definition, as well as the methodology to measure market concentration using the Herfindahl-Hirschman Index (HHI).

D.1 Quadros de Pessoal

Our primary data source is *Quadros de Pessoal* (QP), an annual census of private sector employees conducted by the Portuguese Ministry of Employment each October. This census provides matched employer-employee data on all firms based in Portugal with at least one worker, except those related to public administration and non-market services. The database incorporates unique time-invariant identifiers for each firm, establishment, and worker entering the report, which allows tracking them over time. Our sample covers the period from 2010 to 2016 for all results except for estimating the firm-substitutability parameters, which covers from 2004 to 2016 because we require more observations.

Regarding the occupational definition, the Portuguese law obliges firms to assign their workers to an occupational category based on tasks performed and skills required so that each category considers the level of the worker within the firm's hierarchy in terms of increasing responsibility and task complexity. We follow a hierarchical classification similar to Caliendo et al. (2020). In particular, we partition professional categories into two layers. We assign top executives, intermediary executives, supervisors, and team leaders to the management layer. In addition, we assign higher-skilled professionals, skilled professionals, skilled professionals, semi-skilled professionals, and non-skilled professionals to the bottom layer. To distinguish between managers and other occupations, the critical difference is that managers are responsible for a team of production workers. Table D.1 provides further information about the categories of the occupational classification, which is based on *Decreto-Lei nº. 121/78 de 2 de Junho, Ministério do Trabalho.*

We classify labor markets based on three observable characteristics of the job: geography, industry, and occupation. This classification stems from the fact that workers are more attached to their current labor market because of imperfect geographical mobility and imperfect substitutability of skills across jobs and sectors (Neal, 1995; Kambourov and Manovskii, 2009; Sullivan, 2010; Monte et al., 2018). In particular, we define two broad occupations, i.e., managers and production workers, and define a local labor market for each occupation as the intersection of the geography (Municipality) and industry (2-digit NACE). These broad categories represent a persistent occupational state. Figure D.1 shows that most workers remain within the same category after changing employer. We use the municipality or *concelho* administrative division as the benchmark geographic unit, which splits the country into 278 areas of an average of 320 square kilometers. We use the 2-digit NACE classification of industries as a baseline measure. This includes 78 different economic sectors such as *Manufacture of food products* or *Accommodation and food service activities*. Given that our model does not distinguish between across-industry and across-region mobility, we use these baseline definitions because worker transitions are similar in both cases. In particular, the unconditional across-municipality and across-industry annual transition probabilities are 9.8 percent.

Regarding the sample selection, we exclude workers younger than 18 or older than 64, those working outside of continental Portugal, and those working in agriculture, forestry, fishing, or mining industries. We also exclude apprentices, workers with missing information on earnings or occupation, and workers with misreported identifiers. Most workers with missing earnings include unpaid family members and owners of the firm. In addition, workers with misreported identifiers (e.g., duplicated) account for about 2% of the sample. Finally, we drop chief executive officers because their market is not local, which is a core feature of the theory in this paper. This selection results in 3,243,966 workers and 12,073,646 worker-year observations.

Table D.1: Classification of occupations

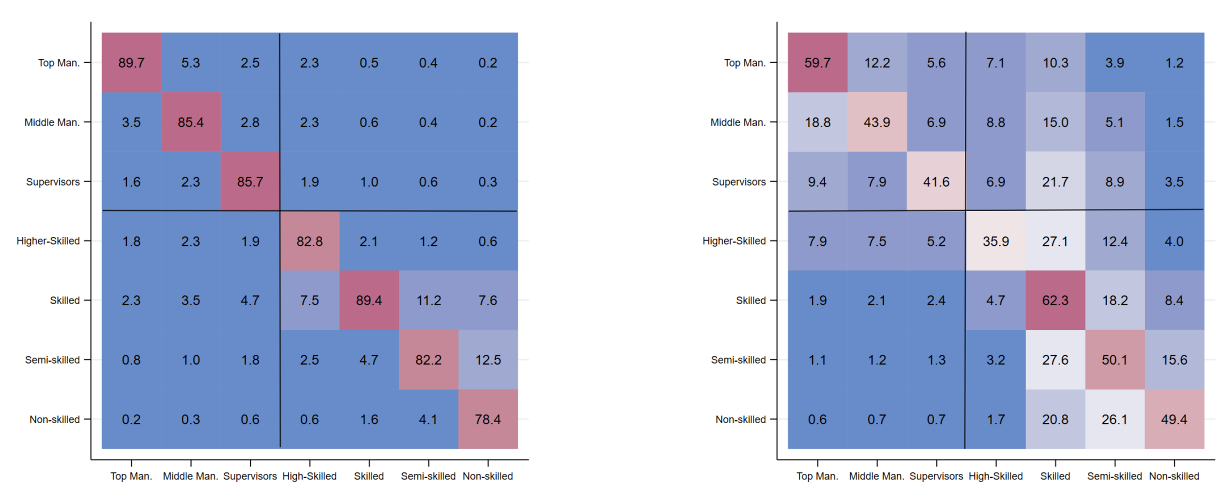
Level	Tasks	Skills
Top Management	Definition of the firm general policy or consulting on the organization of the firm; strategic planning; creation or adaptation of technical, scientific and administrative methods or processes	Knowledge of management and coordination of firms fundamental activities; knowledge of management and coordination of the fundamental activities in the field to which the individual is assigned and that requires the study and research of high responsibility and technical level problems
Middle Management	Organization and adaptation of the guidelines established by the superiors and directly linked with the executive work	Technical and professional qualifications directed to executive, research, and management work
Supervisors	Orientation of teams, as directed by the superiors, but requiring the knowledge of action processes	Complete professional qualification with a specialization
Higher-skilled Professionals	Tasks requiring a high technical value and defined in general terms by the superiors	Complete professional qualification with a specialization adding to theoretical and applied knowledge
Skilled Professionals	Complex or delicate tasks, usually not repetitive, and defined by the superiors	Complete professional qualification implying theoretical and applied knowledge
Semi-skilled Professionals	Well defined tasks, mainly manual or mechanical (no intellectual work) with low complexity, usually routine and sometimes repetitive	Professional qualification in a limited field or practical and elementary professional knowledge
Non-skilled Professionals	Simple tasks and totally determined	Practical knowledge and easily acquired in a short time

Sources: (i) *Decreto-Lei nº. 121/78 de 2 de Junho, Ministério do Trabalho*, (ii) Caliendo et al. (2020).

Table D.2: Share of occupations

Level	Share (%)	Share Hierarchy (%)	Mean Wage
Managers	19.2	100	1,696
Top Management	8	41.8	2,108
Middle Management	6	31.2	1,481
Supervisors and Team Leaders	5.2	27	1,308
Workers	80.8	100	717
Higher-skilled Professionals	8	9.9	1,192
Skilled Professionals	40.1	49.6	729
Semi-skilled Professionals	21.6	26.8	599
Non-skilled Professionals	11.1	13.7	562

Figure D.1: Transition probabilities across occupations



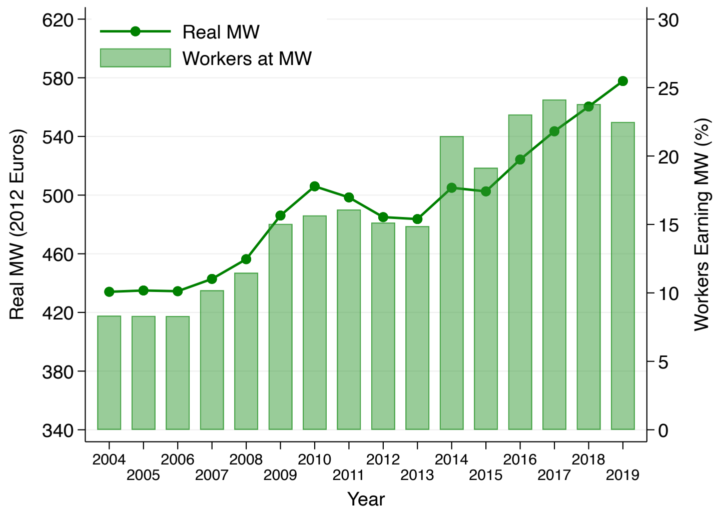
(a) Unconditional

(b) Conditional

Source: Elaboration based on QP.

Note: The Figures display the transition probabilities of changing sub-occupation. The vertical axis represents the sub-occupation before the transition, and the horizontal axis the sub-occupation afterward. The left panel shows the unconditional transition probability, whereas the right panel shows the transition probability conditional on changing firms. The black lines delimit the quadrants of moving across or within the two broad occupation categories (managers and production workers), where the top left and right bottom quadrants represent within-occupation transitions.

Figure D.2: Evolution of minimum wage and share of minimum wage earners



Note: The Figure displays on the left axis the real minimum wage (MW) in Continental Portugal, while the right axis shows the share of workers bound by the MW. We focus on full-time workers (with over 150 monthly working hours), aged between 18 and 65, and exclude supplementary and bonus payments from their wages to define whether they are bound by the MW.

D.2 Measuring Market Payroll Concentration

Our baseline measure of market concentration is the HHI. Given the employment n_{ijot} and wage w_{ijot} level at firm i in a local labor market j for occupation o in period t, we define the HHI in the market as:

$$\mathrm{HHI}_{jot} := \sum_{i=1}^{M_{jot}} s_{ijot}^2 = \frac{1}{M_{jot}} + \sum_{i=1}^{M_{jot}} \left(s_{ijot} - \frac{1}{M_{jot}} \right)^2, \tag{A.28}$$

$$s_{ijot} := \frac{w_{ijot} n_{ijot}}{\sum_{i=1}^{M_{jot}} w_{ijot} n_{ijot}}. \tag{A.29}$$

Here, M_{jot} is the number of establishments in market j that hire workers in occupation o, and s_{ijot} stands for the payroll share of the firm i. The HHI equals the average payroll market share weighted by the payroll share itself. The index ranges from $\frac{1}{M}$ to 1, where a low value reflects low concentration or many firms having similar payroll shares. Note that this index gives more weight to larger establishments, especially penalizing markets where a few firms have a large share of the market payroll. The rightmost equality of Equation (A.28) shows that the HHI has an economically meaningful decomposition into two concentration sources. The first element involves the *number of establishments* in each market. All else being constant, increasing the number of establishments lowers the average establishment size in the market. The second element entails the *dispersion level of payroll shares* across establishments relative to the case in which they hold identical shares. All else being constant, increasing the dispersion in payroll shares leads to greater payroll concentration.

E Appendix: Quantification of the Model

E.1 Targeted Moments

Table E.1 reports additional details for the quantification of the model parameters. In particular, it shows the model fit of each parameter to either its calibration target or its most associated moment in the SMM estimation. The estimation brings about a close fit to the data, as it obtains an average absolute deviation of 5 percent between the model and data moments.

E.2 Firm Substitutability Parameters

This section reports the detailed regression output of the IV regression for the structural labor supply elasticities (η_o, θ_o). Regarding the within-market elasticities, Table E.2 shows the estimates from the IV regression of Equation (17). In addition, Table E.3 reports the estimates from the IV regression of Equation (18), which we use to estimate the across-market elasticities.

Table E.1: Targeted moments

Parameter Value	**Moment**	**Model**	**Data**
Panel II: Endogenous calibration			
α	Labor share of 62%	0.60	0.62
γ	Capital share of 31%	0.29	0.31
(η_w, η_m)	Within-market labor supply elasticity	(7.82 , 2.32)	(7.75 , 2.32)
Panel III: SMM Estimation			
A: Preferences			
φ_w	Average firm size	5.33	5.28
φ_m	Ratio managers/production workers	0.23	0.24
B: Firm Organization			
$\bar{z}_w$	Log mean wage of prod. workers	6.76	6.87
$\bar{z}_m/\bar{z}_w$	Wage gap managers vs prod. workers	0.62	0.62
σ_z	Weighted mean HHI prod. workers	0.21	0.20
C: Market Characteristics			
B_{ijw}	Share workers in markets $M_j \leq 10$	0.12	0.12
mass m_j	Share single-firm markets	0.29	0.29
ζ_0	Mean number of firms	17.23	17.23
ζ_1	Variance number of firms	66.47	66.47
D: Firm Substitutability			
(θ_w, θ_m)	Across-municipality labor supply elasticity	(2.51 , 1.00)	(2.51 , 1.04)

Note: The Table reports the model fit of endogenously calibrated or estimated parameters. Moreover, we show the calibrated firm distribution with their respective moment description and fit.

Table E.2: Regression output for the within-market elasticity

	Production Workers	Managers
Log employment	0.13***	0.43***
	(0.00)	(0.00)
Market-Year FE	Yes	Yes
Observations	444,587	221,535
Implied Elasticity $(1/\beta)$	7.82	2.32
Inferred within-market substitutability (η_o)	7.82	2.32

Note: The Table reports the estimates from regressing equation (17) by IV for each occupation. Standard errors in parentheses.

Table E.3: Regression output for the across-market firm elasticity

	(1) Production Workers	(2) Managers
Log employment	0.40***	1.00***
	(0.06)	(0.24)
Municipality FE	Yes	Yes
Observations	1,854	1,945
Implied Elasticity ($1/\beta$)	2.50	1.00
Inferred across-market substitutability (θ_o)	2.51	1.04

Note: The Table reports the estimates of the IV regression of Equation (18). The baseline period of the instrument is 2004, but we run the regression between 2009 and 2016 to exploit variation from the Great Recession. Standard errors in parentheses.

Table E.4: Robustness checks for the across-market firm elasticity of production workers

	Baseline	Excluding highest-skilled	Excluding lowest-skilled	Stayers
Log employment	0.40***	0.35***	0.48***	0.43***
	(0.06)	(0.07)	(0.08)	(0.06)
Implied Elasticity ($1/\beta$)	2.50	2.86	2.08	2.32

Note: The Table reports the estimates of the IV regression of Equation (18). The baseline period of the instrument is 2004, but we run the regression between 2009 and 2016 to exploit variation from the Great Recession. Standard errors in parentheses. For the skill-based restrictions, the highest-skilled column excludes higher-skilled professionals and the lowest-skilled column excludes non-skilled professionals, both as defined in table D.1. The stayers specification focuses on the wages of workers who have been in the same firm for at least three years.

E.3 Discussion on the Estimation of Labor Supply Elasticities

Given the importance of the firm-substitutability parameters in estimating wage markdowns, we provide additional evidence on mobility measures that support our findings and compare our estimates with the literature.

Our results align with estimates from recent papers that also use general equilibrium models of oligopsony. In particular, recall that our estimates are $\theta_w = 2.5$, $\theta_m = 1.0$, $\eta_w = 7.8$, and $\eta_m = 2.3$. Table E.5 shows that our estimates for both the within- and across-market

Table E.5: Estimates of firm-substitutability parameters in the literature

Paper	**Data**	**Across-Market**	**Within-Market**
Lamadon et al. (2022)	U.S.	4.9	0.7
Berger et al. (2022)	U.S.	0.4	10.9
Deb et al. (2024)	U.S	[1.9 , 2]	[2.4 , 2.5]
Ahlfeldt et al. (2022)	Germany	-	5.5
Azkarate-Askasua and Zerecero (2024)	France	0.4	$[1.2, 4.1]$

Note: This table reports the within-market and across-market elasticities in recent papers from the oligopsony general equilibrium literature.

substitutability parameters are within the range in the literature, namely, $\theta \in [0.4\,,\,2]$ and $\eta \in [2.4\,,\,10.9]$.

In addition, our estimates show that production workers are significantly more responsive to exogenous labor demand changes than managers. This result is consistent with the literature quantifying labor supply elasticities across worker types, which shows that elasticities decline with educational attainment (Diamond, 2016), are lowest among top earners (Langella and Manning, 2021), are smaller for workers in non-routine cognitive than in routine or manual tasks (Bachmann et al., 2022), are lower for high-wage workers (Bils et al., 2025), and decrease with tenure in higher education labor markets (Goolsbee and Syverson, 2023).

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