Time Series Forecasting

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Loading the dataset:

```
data("AirPassengers")
AP.ts<- AirPassengers
```

The data set contains the AirPassengers travel rate from the year 1949 to 1960

Checking the dataset

```
class(AP.ts)

## [1] "ts"

start(AP.ts)

## [1] 1949    1

end(AP.ts)

## [1] 1960   12

frequency(AP.ts)

## [1] 12

summary(AP.ts)

##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##   104.0   180.0   265.5   280.3   360.5   622.0
```

It starts from year 1949 (January); It ends at 1960 (December) and the Frequency interval is 12

Time period of the dataset

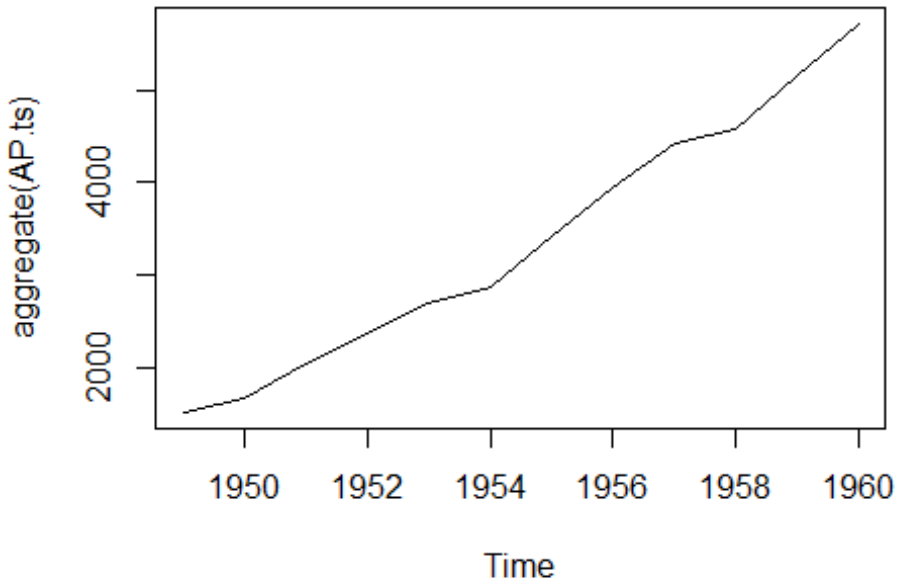
```
cycle(AP.ts)

##      Jan Feb Mar Apr May Jun Jul Aug Sep Oct Nov Dec
## 1949   1   2   3   4   5   6   7   8   9  10  11  12
## 1950   1   2   3   4   5   6   7   8   9  10  11  12
## 1951   1   2   3   4   5   6   7   8   9  10  11  12
## 1952   1   2   3   4   5   6   7   8   9  10  11  12
## 1953   1   2   3   4   5   6   7   8   9  10  11  12
## 1954   1   2   3   4   5   6   7   8   9  10  11  12
## 1955   1   2   3   4   5   6   7   8   9  10  11  12
## 1956   1   2   3   4   5   6   7   8   9  10  11  12
```

```
## 1957   1   2   3   4   5   6   7   8   9  10  11  12
## 1958   1   2   3   4   5   6   7   8   9  10  11  12
## 1959   1   2   3   4   5   6   7   8   9  10  11  12
## 1960   1   2   3   4   5   6   7   8   9  10  11  12
```

```
aggregate(AP.ts) #adds up all of the months for each year
```

```
## Time Series:
## Start = 1949
## End = 1960
## Frequency = 1
##  [1] 1520 1676 2042 2364 2700 2867 3408 3939 4421 4572 5140 5714
```

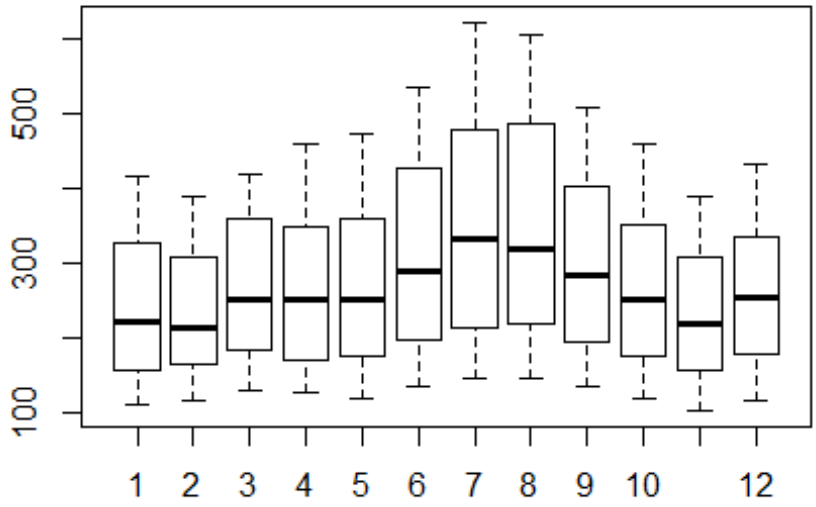
```
plot(aggregate(AP.ts))
```



It has trend.

Box Plot

```
boxplot(AP.ts~cycle(AP.ts))
```




```
aggregate(AP.ts, FUN=mean)
```

```
## Time Series:
## Start = 1949
## End = 1960
## Frequency = 1
##  [1] 126.6667 139.6667 170.1667 197.0000 225.0000 238.9167 284.0000
##  [8] 328.2500 368.4167 381.0000 428.3333 476.1667
```

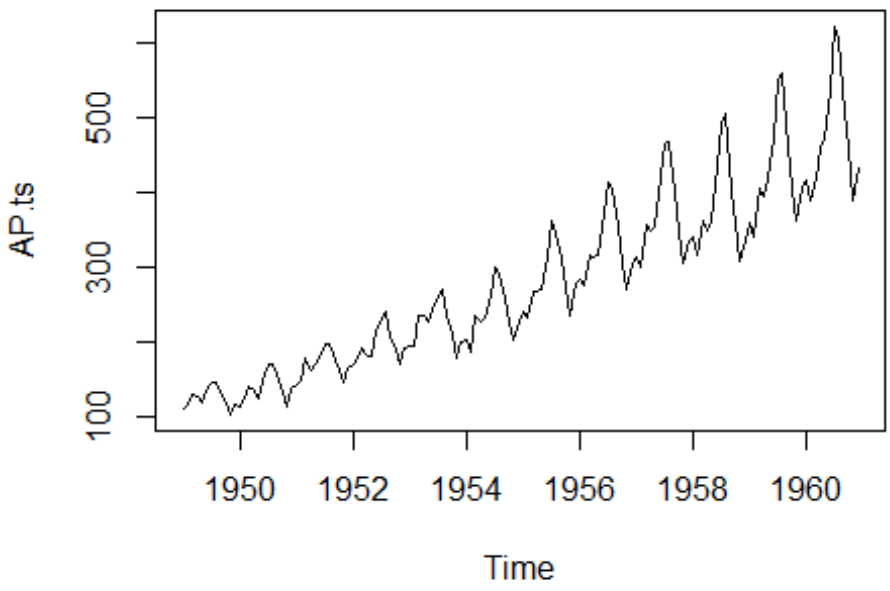
Average frequesncy per month for each year; average monthly travel increased from 126 in 1949 to 476 in 1960

July and August have the highest travel volume

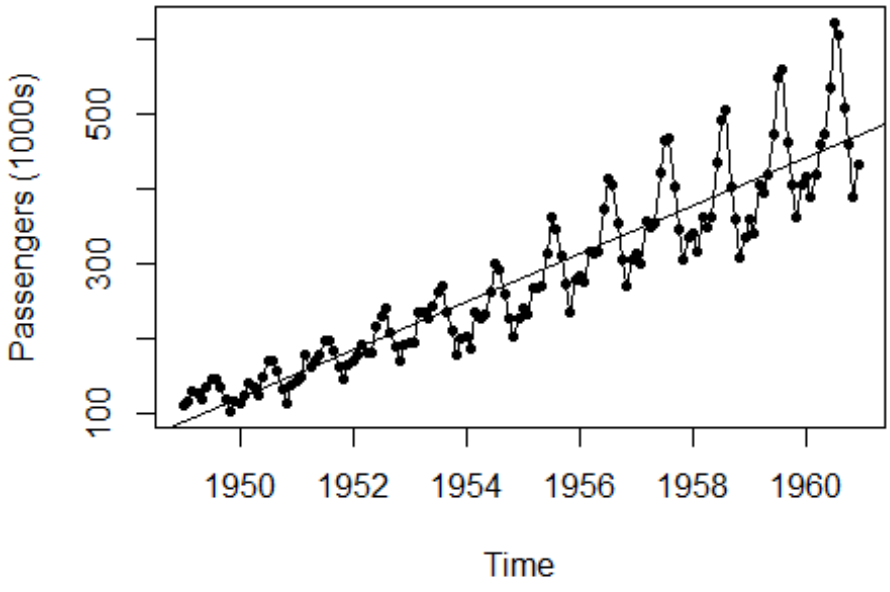
There is growth in the airline passenges numbers, it increases steadily. There is trend and seasonality. We can infer that passengers might prefer to travel during summer holiday, again there might be a downfall, and the number again increases during Christmas holiday

Time series plot:

```
plot(AP.ts)
```




```
plot(AP.ts, ylab="Passengers (1000s)", type="o", pch=20)
abline(reg=lm(AirPassengers~time(AirPassengers)))
```



We can see from in the time series there seems to be seasonal variation in the number of AirPassenger Travel Rate.

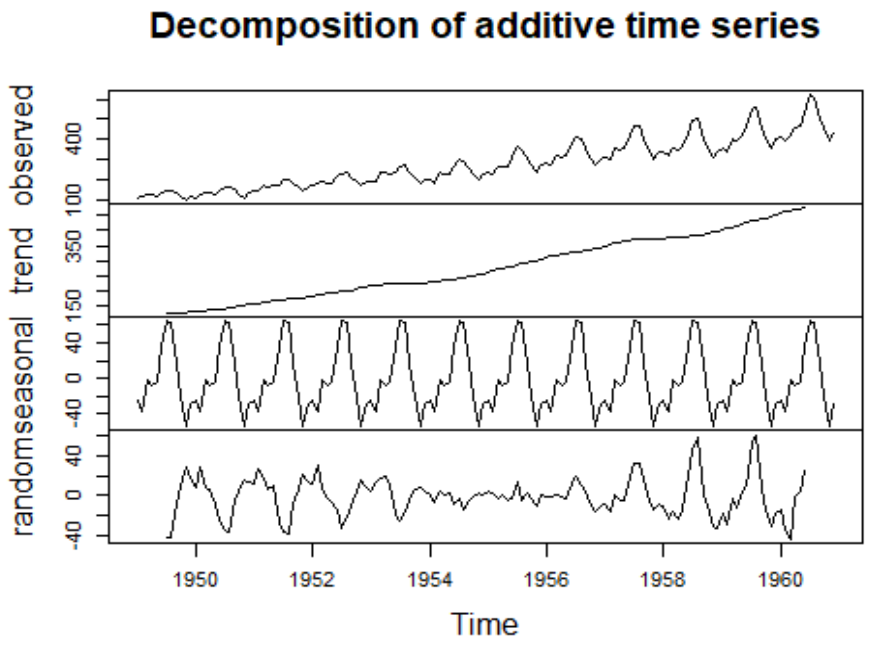
Decomposing the data

```
decompose(AP.ts)
```

```
## $x
##      Jan Feb Mar Apr May Jun Jul Aug Sep Oct Nov Dec
## 1949 112 118 132 129 121 135 148 148 136 119 104 118
## 1950 115 126 141 135 125 149 170 170 158 133 114 140
## 1951 145 150 178 163 172 178 199 199 184 162 146 166
## 1952 171 180 193 181 183 218 230 242 209 191 172 194
## 1953 196 196 236 235 229 243 264 272 237 211 180 201
## 1954 204 188 235 227 234 264 302 293 259 229 203 229
## 1955 242 233 267 269 270 315 364 347 312 274 237 278
## 1956 284 277 317 313 318 374 413 405 355 306 271 306
## 1957 315 301 356 348 355 422 465 467 404 347 305 336
## 1958 340 318 362 348 363 435 491 505 404 359 310 337
## 1959 360 342 406 396 420 472 548 559 463 407 362 405
## 1960 417 391 419 461 472 535 622 606 508 461 390 432
##
## $seasonal
##             Jan        Feb        Mar        Apr        May        Jun
## 1949 -24.748737 -36.188131  -2.241162  -8.036616  -4.506313  35.402778
## 1950 -24.748737 -36.188131  -2.241162  -8.036616  -4.506313  35.402778
## 1951 -24.748737 -36.188131  -2.241162  -8.036616  -4.506313  35.402778
## 1952 -24.748737 -36.188131  -2.241162  -8.036616  -4.506313  35.402778
## 1953 -24.748737 -36.188131  -2.241162  -8.036616  -4.506313  35.402778
## 1954 -24.748737 -36.188131  -2.241162  -8.036616  -4.506313  35.402778
## 1955 -24.748737 -36.188131  -2.241162  -8.036616  -4.506313  35.402778
## 1956 -24.748737 -36.188131  -2.241162  -8.036616  -4.506313  35.402778
## 1957 -24.748737 -36.188131  -2.241162  -8.036616  -4.506313  35.402778
## 1958 -24.748737 -36.188131  -2.241162  -8.036616  -4.506313  35.402778
## 1959 -24.748737 -36.188131  -2.241162  -8.036616  -4.506313  35.402778
## 1960 -24.748737 -36.188131  -2.241162  -8.036616  -4.506313  35.402778
##             Jul        Aug        Sep        Oct        Nov        Dec
## 1949  63.830808  62.823232  16.520202 -20.642677 -53.593434 -28.619949
## 1950  63.830808  62.823232  16.520202 -20.642677 -53.593434 -28.619949
## 1951  63.830808  62.823232  16.520202 -20.642677 -53.593434 -28.619949
## 1952  63.830808  62.823232  16.520202 -20.642677 -53.593434 -28.619949
## 1953  63.830808  62.823232  16.520202 -20.642677 -53.593434 -28.619949
## 1954  63.830808  62.823232  16.520202 -20.642677 -53.593434 -28.619949
## 1955  63.830808  62.823232  16.520202 -20.642677 -53.593434 -28.619949
## 1956  63.830808  62.823232  16.520202 -20.642677 -53.593434 -28.619949
## 1957  63.830808  62.823232  16.520202 -20.642677 -53.593434 -28.619949
## 1958  63.830808  62.823232  16.520202 -20.642677 -53.593434 -28.619949
## 1959  63.830808  62.823232  16.520202 -20.642677 -53.593434 -28.619949
## 1960  63.830808  62.823232  16.520202 -20.642677 -53.593434 -28.619949
##
```

```
## $trend
##           Jan      Feb      Mar      Apr      May      Jun      Jul
## 1949       NA       NA       NA       NA       NA       NA 126.7917
## 1950 131.2500 133.0833 134.9167 136.4167 137.4167 138.7500 140.9167
## 1951 157.1250 159.5417 161.8333 164.1250 166.6667 169.0833 171.2500
## 1952 183.1250 186.2083 189.0417 191.2917 193.5833 195.8333 198.0417
## 1953 215.8333 218.5000 220.9167 222.9167 224.0833 224.7083 225.3333
## 1954 228.0000 230.4583 232.2500 233.9167 235.6250 237.7500 240.5000
## 1955 261.8333 266.6667 271.1250 275.2083 278.5000 281.9583 285.7500
## 1956 309.9583 314.4167 318.6250 321.7500 324.5000 327.0833 329.5417
## 1957 348.2500 353.0000 357.6250 361.3750 364.5000 367.1667 369.4583
## 1958 375.2500 377.9167 379.5000 380.0000 380.7083 380.9583 381.8333
## 1959 402.5417 407.1667 411.8750 416.3333 420.5000 425.5000 430.7083
## 1960 456.3333 461.3750 465.2083 469.3333 472.7500 475.0417       NA
##           Aug      Sep      Oct      Nov      Dec
## 1949 127.2500 127.9583 128.5833 129.0000 129.7500
## 1950 143.1667 145.7083 148.4167 151.5417 154.7083
## 1951 173.5833 175.4583 176.8333 178.0417 180.1667
## 1952 199.7500 202.2083 206.2500 210.4167 213.3750
## 1953 225.3333 224.9583 224.5833 224.4583 225.5417
## 1954 243.9583 247.1667 250.2500 253.5000 257.1250
## 1955 289.3333 293.2500 297.1667 301.0000 305.4583
## 1956 331.8333 334.4583 337.5417 340.5417 344.0833
## 1957 371.2083 372.1667 372.4167 372.7500 373.6250
## 1958 383.6667 386.5000 390.3333 394.7083 398.6250
## 1959 435.1250 437.7083 440.9583 445.8333 450.6250
## 1960       NA       NA       NA       NA       NA
##
## $random
##              Jan         Feb         Mar         Apr         May
## 1949          NA          NA          NA          NA          NA
## 1950   8.4987374  29.1047980   8.3244949   6.6199495  -7.9103535
## 1951  12.6237374  26.6464646  18.4078283   6.9116162   9.8396465
## 1952  12.6237374  29.9797980   6.1994949  -2.2550505  -6.0770202
## 1953   4.9154040  13.6881313  17.3244949  20.1199495   9.4229798
## 1954   0.7487374  -6.2702020   4.9911616   1.1199495   2.8813131
## 1955   4.9154040   2.5214646  -1.8838384   1.8282828  -3.9936869
## 1956  -1.2095960  -1.2285354   0.6161616  -0.7133838  -1.9936869
## 1957  -8.5012626 -15.8118687   0.6161616  -5.3383838  -4.9936869
## 1958 -10.5012626 -23.7285354 -15.2588384 -23.9633838 -13.2020202
## 1959 -17.7929293 -28.9785354  -3.6338384 -12.2967172   4.0063131
## 1960 -14.5845960 -34.1868687 -43.9671717  -0.2967172   3.7563131
##              Jun         Jul         Aug         Sep         Oct
## 1949          NA -42.6224747 -42.0732323  -8.4785354  11.0593434
## 1950 -25.1527778 -34.7474747 -35.9898990  -4.2285354   5.2260101
## 1951 -26.4861111 -36.0808081 -37.4065657  -7.9785354   5.8093434
## 1952 -13.2361111 -31.8724747 -20.5732323  -9.7285354   5.3926768
## 1953 -17.1111111 -25.1641414 -16.1565657  -4.4785354   7.0593434
## 1954  -9.1527778  -2.3308081 -13.7815657  -4.6868687  -0.6073232
## 1955  -2.3611111  14.4191919  -5.1565657   2.2297980  -2.5239899
```

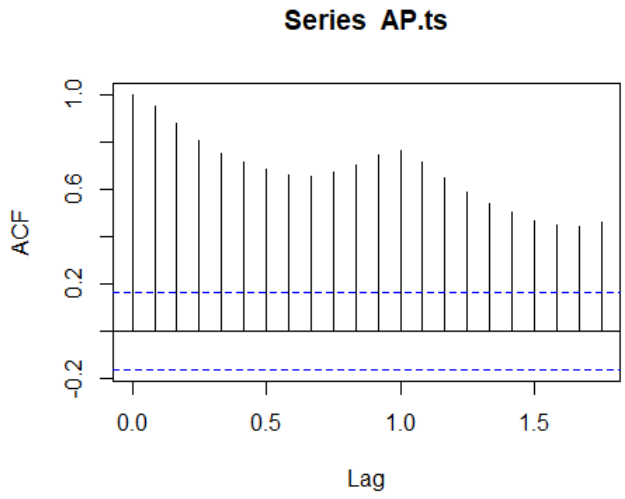
```
## 1956  11.5138889  19.6275253  10.3434343   4.0214646 -10.8989899
## 1957  19.4305556  31.7108586  32.9684343  15.3131313  -4.7739899
## 1958  18.6388889  45.3358586  58.5101010   0.9797980 -10.6906566
## 1959  11.0972222  53.4608586  61.0517677   8.7714646 -13.3156566
## 1960  24.5555556          NA          NA          NA          NA
##              Nov         Dec
## 1949  28.5934343  16.8699495
## 1950  16.0517677  13.9116162
## 1951  21.5517677  14.4532828
## 1952  15.1767677   9.2449495
## 1953   9.1351010   4.0782828
## 1954   3.0934343   0.4949495
## 1955 -10.4065657   1.1616162
## 1956 -15.9482323  -9.4633838
## 1957 -14.1565657  -9.0050505
## 1958 -31.1148990 -33.0050505
## 1959 -30.2398990 -17.0050505
## 1960          NA          NA
##
## $figure
##  [1] -24.748737 -36.188131  -2.241162  -8.036616  -4.506313  35.402778
##  [7]  63.830808  62.823232  16.520202 -20.642677 -53.593434 -28.619949
##
## $type
## [1] "additive"
##
## attr(,"class")
## [1] "decomposed.ts"

plot(decompose(AP.ts))
```



The plot above shows the original time series (top), the estimated trend component (second from top), the estimated seasonal component (third from top), and the estimated irregular component (bottom).

Autocorrelation

```
acf(AP.ts)
```



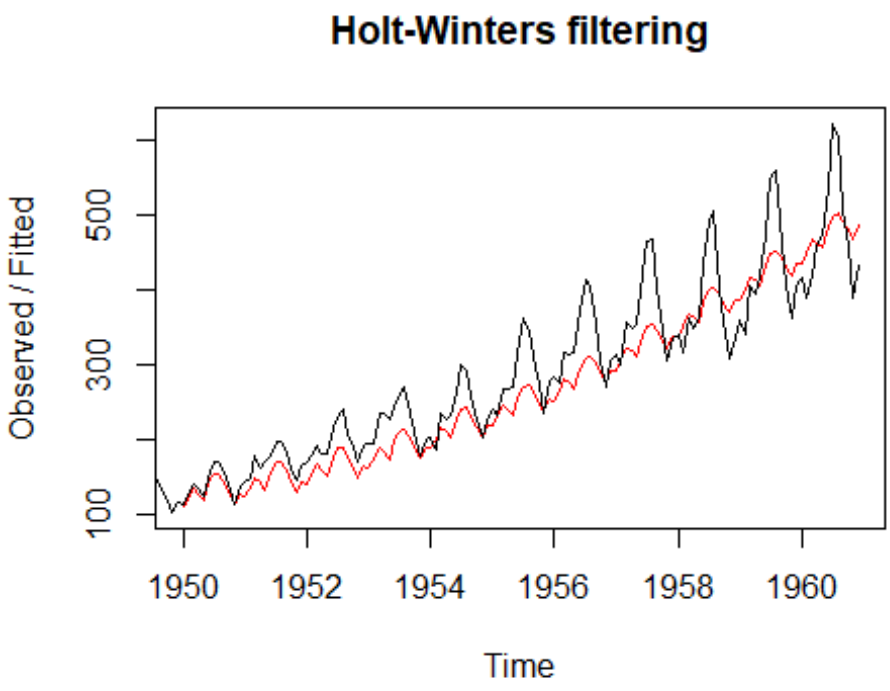
We can see that there is lot of persistence from one period to another.

It is corellated over time. There is seasonality, we can a high peak at one year, air travel today is much correlated with air travel one year ago

Blue line is the confidence interval where anything outside the boundary has significant relation.

Holt Winters Method

```
plot(HoltWinters(AP.ts, alpha=0.001, beta=1, gamma=0))
```



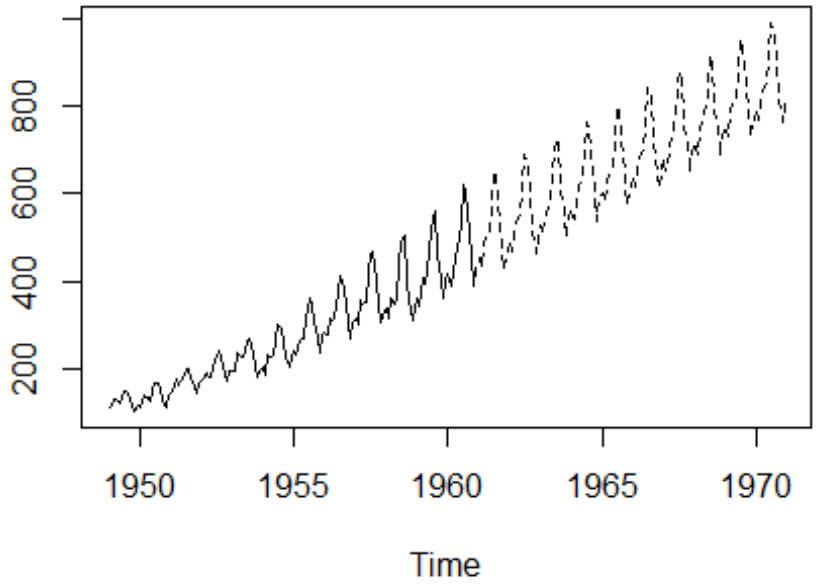
Black graph is the original data and red graph is the Holt-Winters It Fits well.

```
AR.hw <- HoltWinters(AP.ts, seasonal="additive")

summary(AR.hw)

##              Length Class  Mode
## fitted       528    mts    numeric
## x            144    ts     numeric
## alpha          1    -none- numeric
## beta           1    -none- numeric
## gamma          1    -none- numeric
## coefficients  14    -none- numeric
## seasonal       1    -none- character
## SSE            1    -none- numeric
## call           3    -none- call
```

Predictions

We are predicting ahead by 10 years

```
AR.predict<-predict(AR.hw, n.ahead=10*12)
ts.plot(AP.ts, AR.predict, lty=1:2)
```



Predicted values of AirTravel Rates.

```
AR.predict
```

```
##           Jan      Feb      Mar      Apr      May      Jun      Jul
## 1961 453.4977 429.3906 467.0361 503.2574 512.3395 571.8880 652.6095
## 1962 491.0292 466.9221 504.5676 540.7889 549.8710 609.4195 690.1411
## 1963 528.5608 504.4536 542.0991 578.3204 587.4026 646.9510 727.6726
## 1964 566.0923 541.9851 579.6306 615.8520 624.9341 684.4825 765.2041
## 1965 603.6238 579.5166 617.1621 653.3835 662.4656 722.0140 802.7356
## 1966 641.1553 617.0482 654.6936 690.9150 699.9971 759.5456 840.2671
## 1967 678.6868 654.5797 692.2252 728.4465 737.5286 797.0771 877.7986
## 1968 716.2184 692.1112 729.7567 765.9780 775.0602 834.6086 915.3302
## 1969 753.7499 729.6427 767.2882 803.5096 812.5917 872.1401 952.8617
## 1970 791.2814 767.1742 804.8197 841.0411 850.1232 909.6716 990.3932
##           Aug      Sep      Oct      Nov      Dec
## 1961 637.4623 539.7548 490.7250 424.4593 469.5315
## 1962 674.9938 577.2863 528.2565 461.9908 507.0630
## 1963 712.5253 614.8178 565.7880 499.5223 544.5946
## 1964 750.0568 652.3493 603.3195 537.0538 582.1261
## 1965 787.5883 689.8808 640.8511 574.5853 619.6576
## 1966 825.1199 727.4124 678.3826 612.1169 657.1891
## 1967 862.6514 764.9439 715.9141 649.6484 694.7206
## 1968 900.1829 802.4754 753.4456 687.1799 732.2522
```

```
## 1969 937.7144 840.0069 790.9771 724.7114 769.7837
## 1970 975.2459 877.5384 828.5087 762.2429 807.3152
```

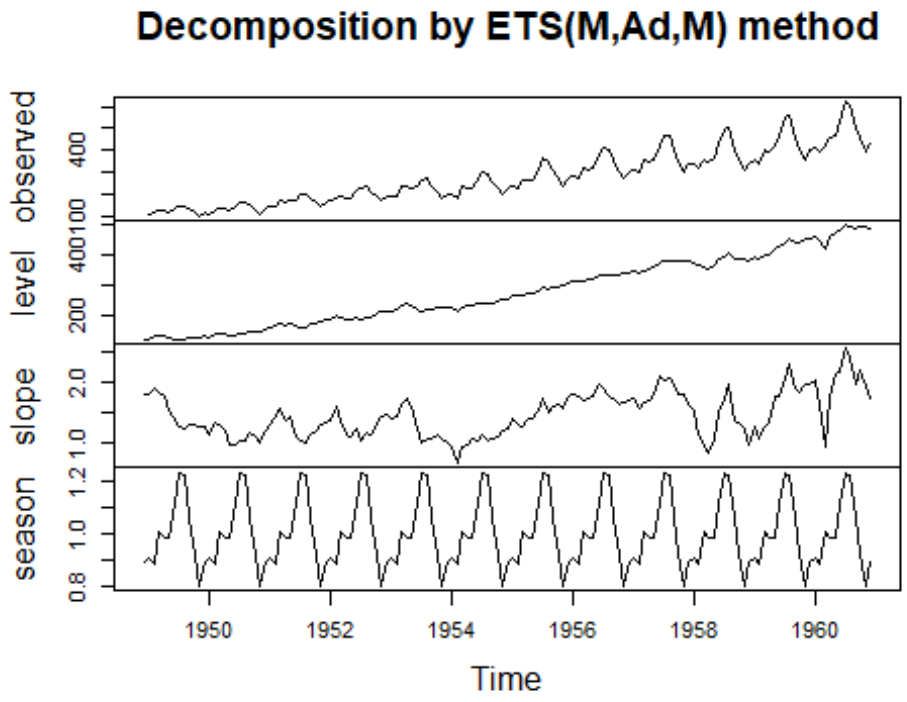
ETS- Error Trend and Seasonality.

```
library(forecast)

## Warning: package 'forecast' was built under R version 3.4.3

ets_forecast <- ets(AP.ts)
ets_forecast

## ETS(M,Ad,M)
##
## Call:
##  ets(y = AP.ts)
##
##   Smoothing parameters:
##     alpha = 0.7322
##     beta  = 0.0188
##     gamma = 1e-04
##     phi   = 0.98
##
##   Initial states:
##     l = 120.9759
##     b = 1.8015
##     s=0.8929 0.7984 0.9211 1.0604 1.2228 1.2324
##            1.1107 0.9807 0.9807 1.0106 0.8843 0.9051
##
##   sigma:  0.0368
##
##      AIC     AICc      BIC
## 1395.092 1400.564 1448.548
```

We can see that alpha = 0.73 which is close to one which means that more weight is place on the most recent observations.

ETS(M,Ad,M) - Error type-"M"=Multiplicative, Tread-additive, Seasonality- Multiplicative

```
plot(ets_forecast)
```




```
attributes(ets)

## NULL

attributes(ets_forecast)

## $names
##  [1] "loglik"     "aic"        "bic"        "aicc"       "mse"
##  [6] "amse"       "fit"        "residuals"  "fitted"     "states"
## [11] "par"        "m"          "method"     "series"     "components"
## [16] "call"       "initstate"  "sigma2"     "x"
##
## $class
## [1] "ets"

ets_forecast$mse #MSE gives an overall idea of the error occurred during fore
casting.

## [1] 113.8122
```

Forecast

To make a forecast of Air Passengers Travel for the year 1961 to 1970 using forecast

```
forecast.ets(ets_forecast, h=120)

##          Point Forecast    Lo 80     Hi 80      Lo 95     Hi 95
## Jan 1961       441.7479 420.9284  462.5675 409.907146  473.5887
## Feb 1961       433.0931 407.5924  458.5938 394.093140  472.0931
```

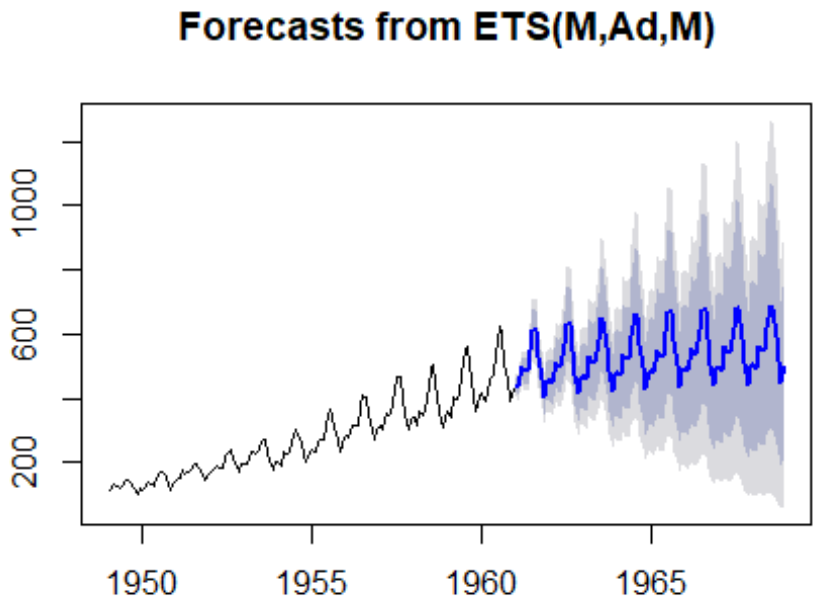
```
## Mar 1961       496.6067 462.3205  530.8930 444.170469  549.0430
## Apr 1961       483.5263 445.6985  521.3541 425.673651  541.3790
## May 1961       485.1026 443.0128  527.1925 420.731744  549.4735
## Jun 1961       551.1085 498.8558  603.3612 471.194877  631.0221
## Jul 1961       613.3810 550.5136  676.2484 517.233630  709.5284
## Aug 1961       610.4359 543.3613  677.5105 507.854183  713.0177
## Sep 1961       530.9494 468.8133  593.0855 435.920410  625.9783
## Oct 1961       462.5032 405.1625  519.8439 374.808139  550.1982
## Nov 1961       402.0130 349.4447  454.5813 321.616682  482.4093
## Dec 1961       450.8391 388.8923  512.7858 356.099616  545.5785
## Jan 1962       458.2429 392.2920  524.1938 357.379753  559.1060
## Feb 1962       448.8867 381.4061  516.3673 345.684075  552.0894
## Mar 1962       514.2943 433.7353  594.8534 391.089888  637.4988
## Apr 1962       500.3479 418.8586  581.8372 375.720767  624.9751
## May 1962       501.5883 416.8132  586.3634 371.935926  631.2406
## Jun 1962       569.4049 469.7070  669.1029 416.930082  721.8798
## Jul 1962       633.2762 518.5862  747.9663 457.872908  808.6795
## Aug 1962       629.7813 511.9731  747.5894 449.609269  809.9533
## Sep 1962       547.3908 441.7638  653.0177 385.848311  708.9332
## Oct 1962       476.4983 381.7623  571.2344 331.612057  621.3846
## Nov 1962       413.9010 329.2070  498.5950 284.372743  543.4293
## Dec 1962       463.8685 366.2764  561.4606 314.614301  613.1227
## Jan 1963       471.1867 369.3577  573.0157 315.452704  626.9207
## Feb 1963       461.2801 358.9695  563.5908 304.809512  617.7507
## Mar 1963       528.1740 408.0425  648.3056 344.448655  711.8994
## Apr 1963       513.5480 393.8590  633.2371 330.499497  696.5966
## May 1963       514.5247 391.7350  637.3145 326.734025  702.3155
## Jun 1963       583.7623 441.2073  726.3174 365.743175  801.7815
## Jul 1963       648.8882 486.8445  810.9319 401.063756  896.7126
## Aug 1963       644.9618 480.3533  809.5703 393.214886  896.7087
## Sep 1963       560.2925 414.2285  706.3565 336.906896  783.6782
## Oct 1963       487.4805 357.7437  617.2173 289.065233  685.8958
## Nov 1963       423.2297 308.2984  538.1611 247.457369  599.0021
## Dec 1963       474.0929 342.7906  605.3952 273.283306  674.9025
## Jan 1964       481.3439 345.4460  617.2417 273.506071  689.1817
## Feb 1964       471.0054 335.5054  606.5053 263.776101  678.2347
## Mar 1964       539.0656 381.1121  697.0191 297.496553  780.6346
## Apr 1964       523.9063 367.6125  680.2002 284.875478  762.9372
## May 1964       524.6762 365.3767  683.9756 281.048717  768.3036
## Jun 1964       595.0288 411.2327  778.8249 313.936895  876.1207
## Jul 1964       661.1391 453.4498  868.8284 343.505707  978.7725
## Aug 1964       656.8741 447.0864  866.6619 336.031466  977.7168
## Sep 1964       570.4167 385.2656  755.5678 287.252499  853.5809
## Oct 1964       496.0984 332.4910  659.7058 245.882416  746.3144
## Nov 1964       430.5501 286.3286  574.7716 209.982305  651.1178
## Dec 1964       482.1161 318.1311  646.1011 231.322690  732.9095
## Jan 1965       489.3143 320.3609  658.2677 230.922380  747.7063
## Feb 1965       478.6369 310.9132  646.3607 222.125577  735.1483
## Mar 1965       547.6123 352.9155  742.3091 249.849260  845.3753
## Apr 1965       532.0346 340.1614  723.9079 238.589800  825.4794
```

```
## May 1965       532.6421 337.8396  727.4445 234.717462  830.5667
## Jun 1965       603.8697 379.9535  827.7858 261.419496  946.3199
## Jul 1965       670.7525 418.6426  922.8625 285.183663 1056.3214
## Aug 1965       666.2219 412.4536  919.9902 278.116815 1054.3270
## Sep 1965       578.3613 355.1497  801.5729 236.988591  919.7339
## Oct 1965       502.8609 306.2644  699.4574 202.192556  803.5293
## Nov 1965       436.2944 263.5390  609.0499 172.087800  700.5011
## Dec 1965       488.4120 292.5821  684.2419 188.915969  787.9080
## Jan 1966       495.5688 294.4017  696.7360 187.910177  803.2275
## Feb 1966       484.6255 285.4938  683.7572 180.079849  789.1711
## Mar 1966       554.3190 323.8044  784.8337 201.777336  906.8607
## Apr 1966       538.4130 311.8523  764.9737 191.918316  884.9077
## May 1966       538.8931 309.4738  768.3123 188.026645  889.7595
## Jun 1966       610.8073 347.7687  873.8459 208.524513 1013.0901
## Jul 1966       678.2963 382.8664  973.7262 226.475300 1130.1174
## Aug 1966       673.5572 376.8946  970.2198 219.850941 1127.2635
## Sep 1966       584.5955 324.2606  844.9304 186.447602  982.7434
## Oct 1966       508.1676 279.3921  736.9431 158.285602  858.0496
## Nov 1966       440.8021 240.2115  641.3927 134.025252  747.5790
## Dec 1966       493.3525 266.4555  720.2495 146.343447  840.3615
## Jan 1967       500.4769 267.8808  733.0729 144.751922  856.2018
## Feb 1967       489.3248 259.5487  719.1009 137.912534  840.7371
## Mar 1967       559.5819 294.1181  825.0457 153.590087  965.5737
## Apr 1967       543.4182 283.0094  803.8270 145.157339  941.6791
## May 1967       543.7983 280.5980  806.9986 141.268244  946.3284
## Jun 1967       616.2513 315.0327  917.4699 155.577212 1076.9254
## Jul 1967       684.2161 346.5073 1021.9248 167.735093 1200.6971
## Aug 1967       679.3134 340.7849 1017.8418 161.578804 1197.0479
## Sep 1967       589.4876 292.9173  886.0578 135.922565 1043.0525
## Oct 1967       512.3318 252.1451  772.5185 114.410569  910.2530
## Nov 1967       444.3394 216.5762  672.1026  96.005587  792.6732
## Dec 1967       497.2293 240.0029  754.4558 103.835409  890.6233
## Jan 1968       504.3282 241.0476  767.6089 101.675275  906.9812
## Feb 1968       493.0124 233.3156  752.7093  95.840346  890.1845
## Mar 1968       563.7118 264.1219  863.3016 105.528590 1021.8949
## Apr 1968       547.3459 253.8838  840.8079  98.534408  996.1573
## May 1968       547.6475 251.4571  843.8379  94.663381 1000.6317
## Jun 1968       620.5233 282.0158  959.0308 102.820746 1138.2259
## Jul 1968       688.8614 309.8574 1067.8654 109.224806 1268.4980
## Aug 1968       683.8303 304.4068 1063.2538 103.552200 1264.1084
## Sep 1968       593.3264 261.3583  925.2946  85.625022 1101.0279
## Oct 1968       515.5995 224.7250  806.4741  70.745322  960.4538
## Nov 1968       447.1151 192.8028  701.4275  58.177934  836.0523
## Dec 1968       500.2716 213.4090  787.1342  61.553062  938.9901
## Jan 1969       507.3505 214.0840  800.6170  58.838079  955.8629
## Feb 1969       495.9062 206.9672  784.8451  54.012123  937.8002
## Mar 1969       566.9525 234.0074  899.8977  57.756882 1076.1482
## Apr 1969       550.4280 224.6561  876.1998  52.202932 1048.6530
## May 1969       550.6681 222.2263  879.1098  48.359782 1052.9764
## Jun 1969       623.8756 248.9106  998.8407  50.416087 1197.3352
```

```
## Jul 1969       692.5066 273.1239 1111.8893  51.116148 1333.8971
## Aug 1969       687.3748 267.9597 1106.7899  45.934785 1328.8149
## Sep 1969       596.3389 229.7511  962.9267  35.691186 1156.9866
## Oct 1969       518.1638 197.2728  839.0548  27.403310 1008.9243
## Nov 1969       449.2933 169.0097  729.5769  20.636494  877.9501
## Dec 1969       502.6589 186.8019  818.5159  19.597235  985.7206
## Jan 1970       509.7221 187.1156  832.3286  16.338053 1003.1062
## Feb 1970       498.1769 180.6223  815.7315  12.519049  983.8348
## Mar 1970       569.4956 203.9059  935.0854  10.374366 1128.6169
## Apr 1970       552.8465 195.4494  910.2437   6.254825 1099.4383
## May 1970       553.0383 193.0247  913.0520   2.444916 1103.6318
## Jun 1970       626.5063 215.8472 1037.1654  -1.542632 1254.5552
## Jul 1970       695.3671 236.4463 1154.2880  -6.491765 1397.2260
## Aug 1970       690.1563 231.5770 1148.7356 -11.180266 1391.4928
## Sep 1970       598.7028 198.2071  999.1985 -13.802511 1211.2082
## Oct 1970       520.1760 169.8818  870.4702 -15.552787 1055.9048
## Nov 1970       451.0025 145.2749  756.7302 -16.567536  918.5726
## Dec 1970       504.5323 160.2656  848.7990 -21.978232 1031.0428
```

The forecast ets() function gives the forecast for a year, a 80% prediction interval for the forecast, and a 95% prediction interval for the forecast. For example, the forecasted Air pasenger travel rate for the year 1970 is about 504,with a 95% prediction interval of (1031).

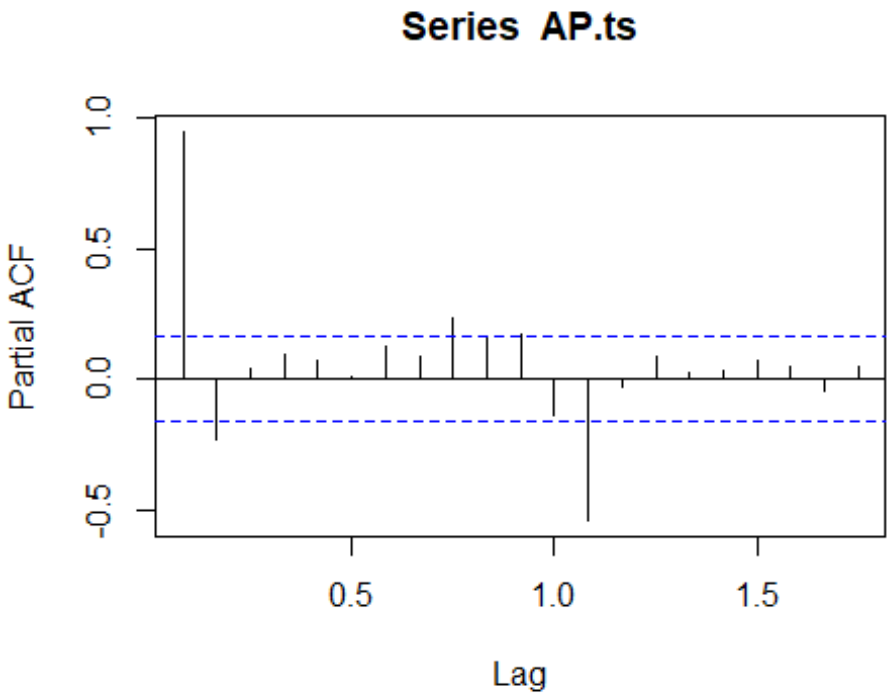
```
forecast_ets <- forecast.ets(ets_forecast, h=96)
plot(forecast_ets)
```



Here the forecasts are plotted as a blue line, the 80% prediction interval as an dark grey shaded area, and the 95% prediction interval as a light grey shaded area.

Partial autocorrelation

```
pacf(AP.ts)
```



Regression on time dimension

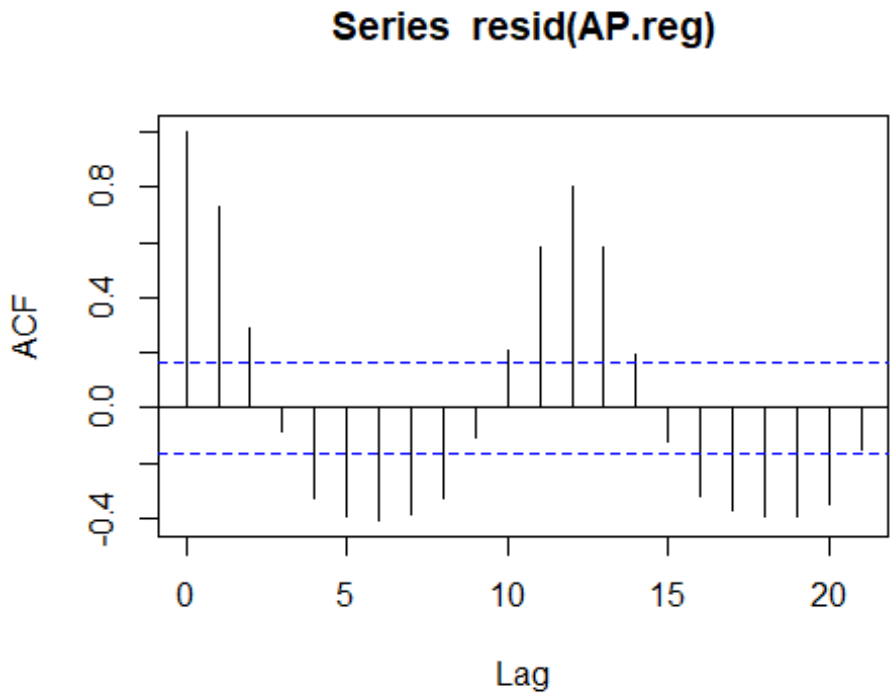
```
AP.reg<-lm(AP.ts~time(AP.ts))
summary(AP.reg)

##
## Call:
## lm(formula = AP.ts ~ time(AP.ts))
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -93.858 -30.727  -5.757  24.489 164.999
##
## Coefficients:
##                Estimate Std. Error t value Pr(>|t|)
## (Intercept) -62055.907   2166.077  -28.65   <2e-16 ***
## time(AP.ts)     31.886      1.108   28.78   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 46.06 on 142 degrees of freedom
```

```
## Multiple R-squared:  0.8536, Adjusted R-squared:  0.8526
## F-statistic: 828.2 on 1 and 142 DF,  p-value: < 2.2e-16
```

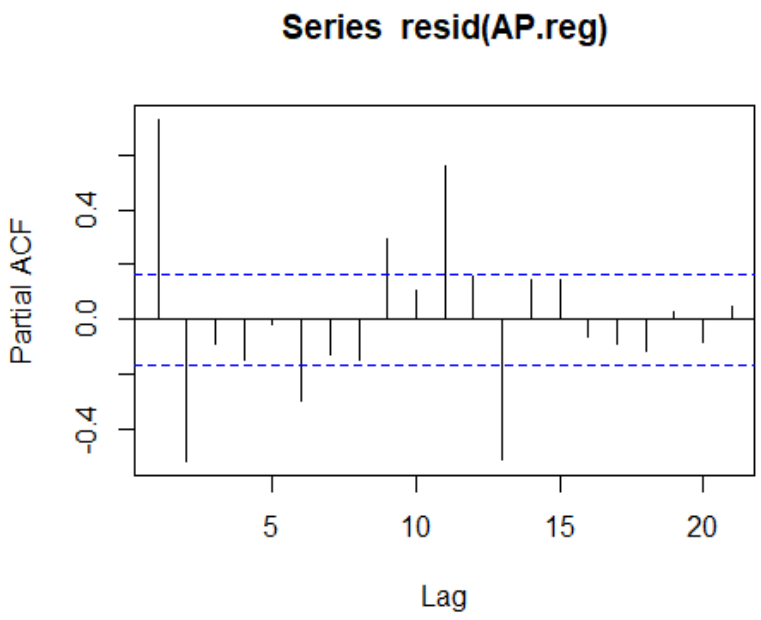
```
confint(AP.reg)
```

```
##                     2.5 %       97.5 %
## (Intercept) -66337.83205 -57773.98251
## time(AP.ts)     29.69592     34.07649
```

```
acf(resid(AP.reg))
```




```
pacf(resid(AP.reg))
```



ARIMA (Autoregressive integrated moving average)

MA(3)

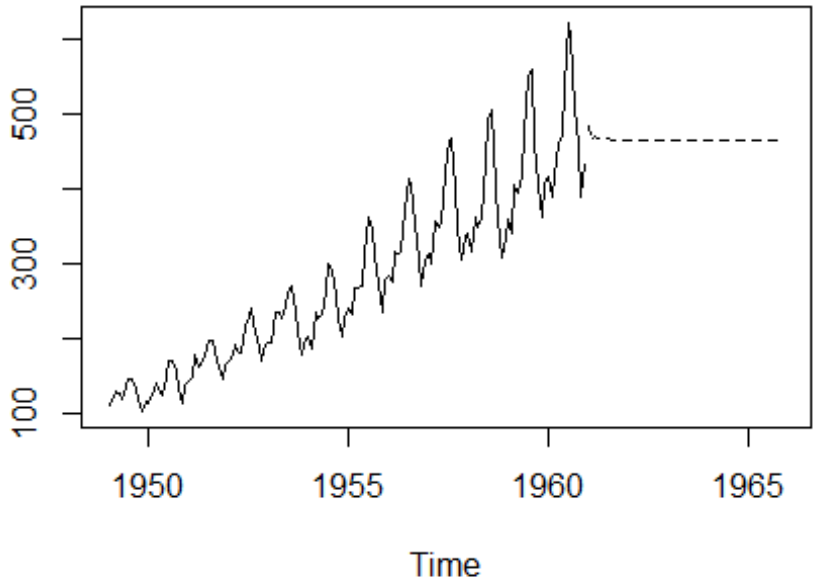
```
AP.ma <- arima(AP.ts, order=c(0,0,3)) #estimates moving average of three time periods of "white noise"

ar=auto.arima(AP.ts)
summary(ar)

## Series: AP.ts
## ARIMA(2,1,1)(0,1,0)[12]
##
## Coefficients:
##          ar1     ar2      ma1
##       0.5960  0.2143  -0.9819
## s.e.  0.0888  0.0880   0.0292
##
## sigma^2 estimated as 132.3:  log likelihood=-504.92
## AIC=1017.85   AICc=1018.17   BIC=1029.35
##
## Training set error measures:
##                    ME     RMSE      MAE       MPE     MAPE      MASE
## Training set 1.3423 10.84619 7.86754 0.420698 2.800458 0.245628
##                      ACF1
## Training set -0.00124847
```

ARMA(1,1)

```
AP.arma <- arima(AP.ts, order=c(1,0,1)) #1 prior period of AP, 1 prior period of white noise
```

ARIMA(2,1,2) This model is composed of 2 prior periods of AP, 2 prior periods of white noise and a first-order difference

```
AP.arima <- arima(AP.ts, order=c(2,1,2))
AP.predict<-predict(AP.arima, n.ahead=60)
ts.plot(AP.ts,AP.predict$pred, lty=1:2)
```



GARCH

Generalized autoregressive conditioned heteroskedastic

```
library(tseries)

## Warning: package 'tseries' was built under R version 3.4.3

garch(AP.ts, grad="numerical")

##
##  ***** ESTIMATION WITH NUMERICAL GRADIENT *****
##
##
##      I     INITIAL X(I)        D(I)
##
##      1     1.295273e+04     1.000e+00
##      2     5.000000e-02     1.000e+00
```

```
##      3     5.000000e-02     1.000e+00
##
##     IT   NF      F         RELDF    PRELDF    RELDX   STPPAR   D*STEP   NP
RELDF
##      0    1  1.024e+03
##      1    2  8.735e+02  1.47e-01  1.66e+00  3.8e-05  1.7e+03  1.0e+00  1.4
2e+03
##      2    4  8.718e+02  2.02e-03  1.19e-03  1.7e-06  2.1e+00  5.0e-02  5.0
2e-01
##      3    6  8.691e+02  3.01e-03  2.99e-03  3.3e-06  1.9e+00  1.0e-01  9.5
4e-03
##      4    8  8.688e+02  4.44e-04  4.44e-04  6.5e-07  9.7e+00  2.0e-02  2.6
2e-03
##      5   10  8.681e+02  7.29e-04  7.29e-04  1.3e-06  2.0e+00  4.0e-02  1.6
4e-03
##      6   12  8.680e+02  1.20e-04  1.20e-04  2.6e-07  1.6e+01  8.0e-03  1.0
3e-03
##      7   14  8.678e+02  2.15e-04  2.15e-04  5.3e-07  2.7e+00  1.6e-02  1.0
8e-03
##      8   16  8.678e+02  3.89e-05  3.89e-05  1.1e-07  3.0e+01  3.2e-03  9.7
3e-04
##      9   19  8.678e+02  7.64e-07  7.64e-07  2.1e-09  1.4e+03  6.4e-05  1.0
1e-03
##     10   21  8.678e+02  1.53e-06  1.53e-06  4.3e-09  1.8e+02  1.3e-04  1.0
3e-03
##     11   23  8.678e+02  3.05e-07  3.05e-07  8.5e-10  3.5e+03  2.6e-05  1.0
2e-03
##     12   26  8.678e+02  2.44e-06  2.44e-06  6.8e-09  1.1e+02  2.0e-04  1.0
2e-03
##     13   29  8.678e+02  4.87e-08  4.87e-08  1.4e-10  2.2e+04  4.1e-06  1.0
2e-03
##     14   31  8.678e+02  9.74e-08  9.74e-08  2.7e-10  2.7e+03  8.2e-06  1.0
2e-03
##     15   33  8.678e+02  1.95e-08  1.95e-08  5.5e-11  5.4e+04  1.6e-06  1.0
2e-03
##     16   35  8.678e+02  3.90e-08  3.90e-08  1.1e-10  6.8e+03  3.3e-06  1.0
2e-03
##     17   37  8.678e+02  7.79e-09  7.79e-09  2.2e-11  1.4e+05  6.6e-07  1.0
2e-03
##     18   39  8.678e+02  1.56e-08  1.56e-08  4.4e-11  1.7e+04  1.3e-06  1.0
2e-03
##     19   41  8.678e+02  3.12e-09  3.12e-09  8.8e-12  3.4e+05  2.6e-07  1.0
2e-03
##     20   43  8.678e+02  6.23e-09  6.23e-09  1.8e-11  4.3e+04  5.2e-07  1.0
3e-03
##     21   45  8.678e+02  1.25e-09  1.25e-09  3.5e-12  8.6e+05  1.0e-07  1.0
4e-03
##     22   47  8.678e+02  2.49e-10  2.49e-10  7.0e-13  4.2e+06  2.1e-08  1.0
1e-03
##     23   49  8.678e+02  4.99e-11  4.99e-11  1.4e-13  1.5e+07  4.2e-09  4.2
```

```
7e-04
##     24   52  8.678e+02  3.99e-10  3.99e-10  1.1e-12  1.5e+04  3.4e-08  5.1
5e-03
##     25   55  8.678e+02  7.98e-12  7.98e-12  2.2e-14  6.5e+06  6.7e-10  4.6
3e-03
##     26   57  8.678e+02  1.60e-12  1.60e-12  4.5e-15  2.0e+00  1.3e-10 -1.1
6e-02
##     27   60  8.678e+02  1.28e-11  1.28e-11  3.6e-14  1.5e+00  1.1e-09 -9.2
1e-03
##     28   62  8.678e+02 -1.15e+07  2.55e-12  7.2e-15  1.5e+00  2.1e-10 -9.1
6e-03
##
##  ***** FALSE CONVERGENCE *****
##
##  FUNCTION     8.677961e+02   RELDX        7.174e-15
##  FUNC. EVALS      62         GRAD. EVALS     112
##  PRELDF       2.553e-12      NPRELDF     -9.155e-03
##
##      I      FINAL X(I)        D(I)          G(I)
##
##      1    1.295273e+04     1.000e+00     2.297e-04
##      2    9.153174e-01     1.000e+00     5.170e+00
##      3    5.534449e-11     1.000e+00     8.928e+00

##
## Call:
## garch(x = AP.ts, grad = "numerical")
##
## Coefficient(s):
##        a0         a1         b1
## 1.295e+04  9.153e-01  5.534e-11
```