

# Estimation of VaR and CVaR for a Portfolio of Stocks

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# 1 Introduction

There are two main measures of financial risk, namely, (i) **Value-at-Risk (VaR)** and (2) **Conditional Value-at-Risk (CVaR)**. The two measures are useful when looking at severe losses in extreme events in critical periods such as financial crises, etc.

## 1.1 Value-at-Risk (VaR)

Suppose an asset which has a return rate  $r_H$  over a horizon  $H$ . The Value at Risk at a given  $\alpha$  is denoted by  $\text{VaR}_\alpha(r_H)$  such that:

$$P(r_H \leq \text{VaR}_\alpha(r_H)) = \alpha$$

Simply, the VaR is just a quantile which satisfies the condition above.

## 1.2 Conditional Value-at-Risk (CVaR)

CVaR is also called **Tail VaR (TVaR)** or **Expected Shortfall**. At a risk level  $\alpha$ :

$$ES_\alpha(r_H) = \text{CVaR}_\alpha(r_H) = P(r_H \leq \text{VaR}_\alpha(r_H)) = \alpha$$

It is the opposite of the left-tail conditional expectation of  $-\text{VaR}_\alpha(r_H)$ . If  $|ES_\alpha(X)| < \infty$ , it can be proven that:

$$ES_\alpha(X) = \frac{1}{\alpha} \int_0^\alpha \text{VaR}_\theta d\theta > \text{VaR}_\alpha(X)$$

Consequently,

$$ES_\alpha(X) \geq \text{VaR}_\alpha(X)$$

## 1.3 Risk Measures of Stocks relative to Financial Crises

The interest of this empirical application is to see how the risk measures (VaR and CVaR) of stocks evolve over time and how these assets are affected by the major financial crises in history. VaR and CVaR can be impacted by a variety of factors, including market conditions, volatility, and correlation with other assets. Based on the general trends in the market, it's possible that the VaR and CVaR for many of the stocks could have increased during the COVID-19 pandemic, compared to their levels during the 2008 financial crisis. This is because the pandemic has introduced significant uncertainty and volatility into the market, which can increase the potential for losses in a portfolio. Additionally, the COVID-19 pandemic has had a more widespread impact on the economy and society than the 2008 financial crisis, which could lead to a broader and more significant impact on individual stocks. However, this is only a general hypothesis and may not apply to all of the stocks.

The actual VaR and CVaR of individual stocks will depend on many factors, including their specific business model, financial performance, and other relevant metrics. In general, if a stock valuation has increased significantly, especially during times of crisis, it is reasonable to expect a higher VaR and CVaR during these periods. This is because higher valuations typically imply greater levels of risk, as investors are willing to pay more for a given level of earnings or cash flow. In times of crisis, market volatility tends to increase and correlations among different asset classes can become more pronounced, leading to higher VaR and CVaR for many assets, including stocks. Additionally, stocks that have experienced significant increases in valuation may be more susceptible to corrections or pullbacks, which can lead to larger losses in a portfolio.

It's important to note that the relationship between valuation and risk is not always straightforward, and other factors, such as the underlying business fundamentals and market conditions, can also impact VaR and CVaR.

## 2 Portfolio Construction

For this empirical application, a theoretical equally-weighted portfolio is constructed from the following stocks:

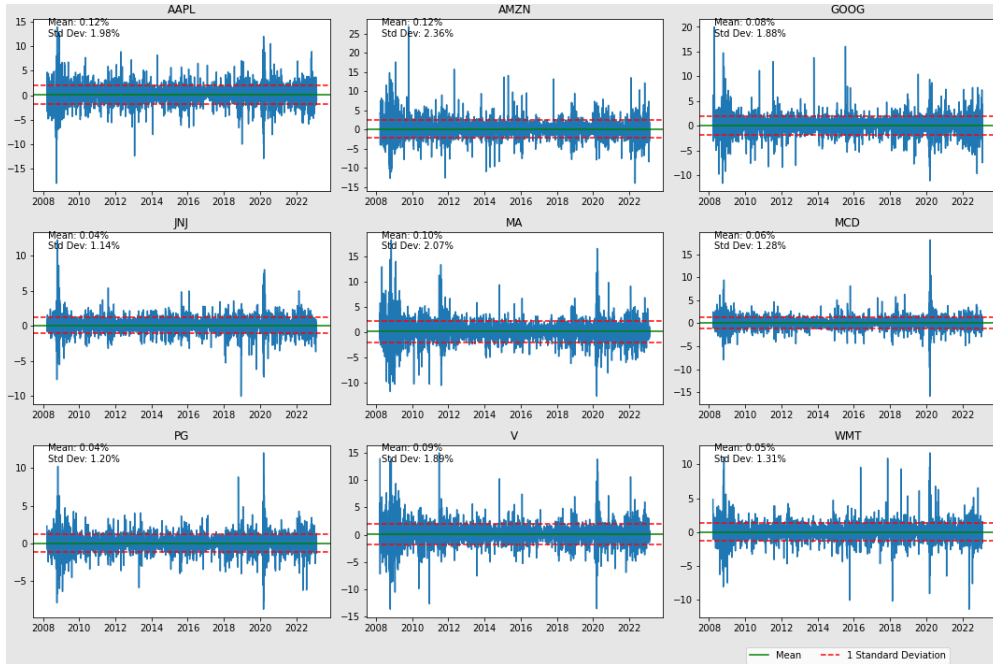
- **Amazon**, ticker symbol: **AMZN** - During the 2008 financial crisis, Amazon's stock price declined along with the broader market, but it rebounded relatively quickly and continued to perform well in the years that followed. In contrast, during the COVID-19 pandemic, Amazon's stock price has soared as demand for online shopping increased due to lockdowns and social distancing measures.
- **Apple Inc.**, ticker symbol: **AAPL** - Apple's stock price declined significantly during the 2008 financial crisis but it recovered and went on to become one of the most valuable companies in the world. During the COVID-19 pandemic, Apple's stock price has remained relatively stable and even saw significant gains as people relied more on technology and home entertainment.
- **Alphabet Inc.**, ticker symbol: **GOOGL** - Alphabet (parent company of Google) was not immune to the 2008 financial crisis, but it weathered the storm relatively well and continued to see strong growth in the years that followed. During the COVID-19 pandemic, Alphabet's stock price has seen gains as more people use online services and advertising revenue remains strong.
- **Procter & Gamble Co.**, ticker symbol: **PG** - Procter & Gamble is a consumer goods company that produces products like Tide laundry detergent, Pampers diapers, and Crest toothpaste. During the 2008 financial crisis, the company's stock price declined but it recovered relatively quickly. During the COVID-19 pandemic, Procter & Gamble's stock price has remained relatively stable as demand for household and personal care products has remained strong.
- **Walmart Inc.**, ticker symbol: **WMT** - Walmart is a discount retailer that tends to perform well during economic downturns, as consumers are more likely to focus on value and affordability. During the 2008 financial crisis, Walmart's stock price performed relatively well, and the company has also seen strong performance during the COVID-19 pandemic, as consumers have stocked up on household essentials.
- **Johnson & Johnson**, ticker symbol: **JNJ** - Johnson & Johnson is a healthcare company that has a diverse portfolio of products, including pharmaceuticals, medical devices, and consumer health products. The company's stock price held up relatively well during the 2008 financial crisis, and it has continued to perform well during the COVID-19 pandemic, as demand for healthcare products and services has remained strong.
- **McDonald's Corp.**, ticker symbol: **MCD** - McDonald's is a fast-food chain that tends to perform well during economic downturns, as consumers are more likely to opt for lower-priced dining options. During the 2008 financial crisis, McDonald's stock price held up relatively well, and the company has continued to perform well during the COVID-19

pandemic, as it has adapted to changing consumer preferences with the expansion of its delivery and drive-thru options.

- **Mastercard Inc. MA** - Mastercard's stock price was relatively unaffected by the 2008 financial crisis and has performed well in the years since. However, during the COVID-19 pandemic, Mastercard's stock price has been impacted by decreased consumer spending, particularly in the travel and entertainment sectors.
- **Visa Inc. V** - Visa, like Mastercard, saw strong growth in the years following the 2008 financial crisis. However, during the COVID-19 pandemic, the company's stock price has been impacted by decreased consumer spending and reduced transaction volume as a result of lockdowns and social distancing measures.

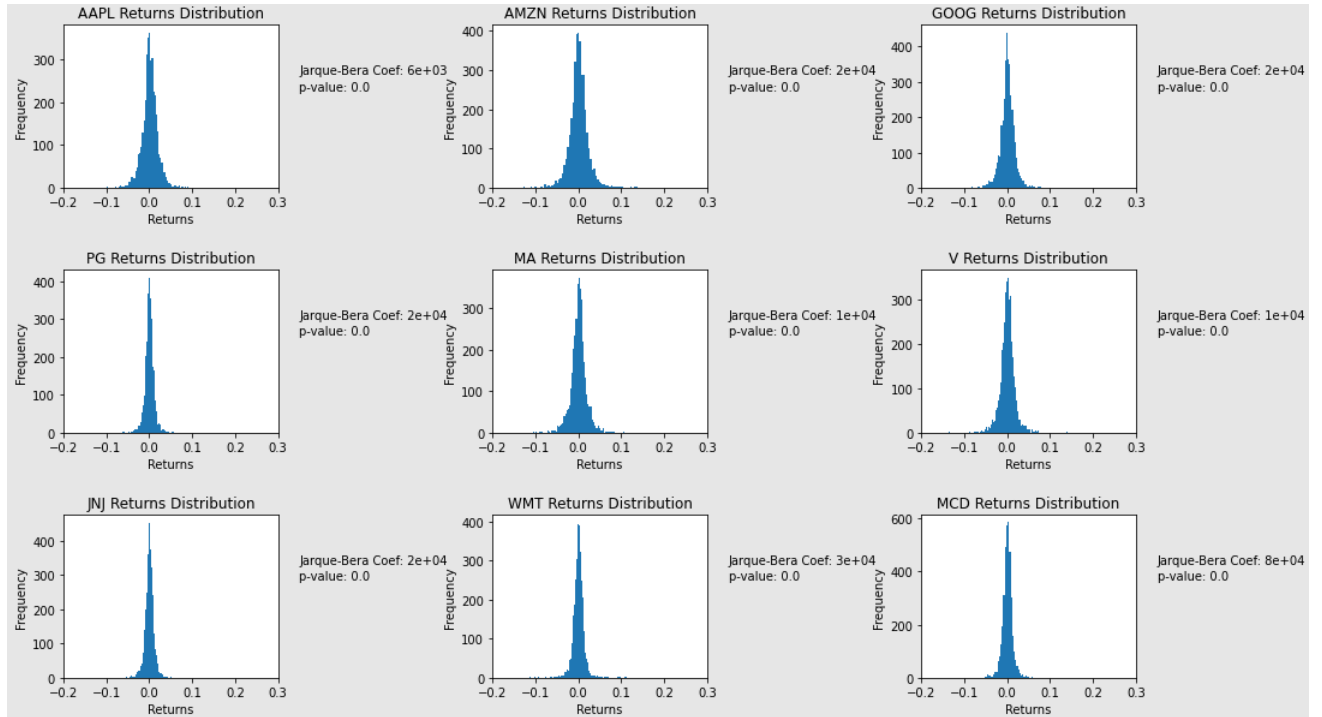
### 3 Exploratory Data Analysis

Figure 1 shows the plot of returns stock over time. All the stocks seem to be visually stationary around 0, although a unit root test is needed to confirm this and is out of scope of the analysis. High volatility is experienced by all the stocks around 2008. The same is experienced post 2020, an indication of the effects of the COVID-19 pandemic.



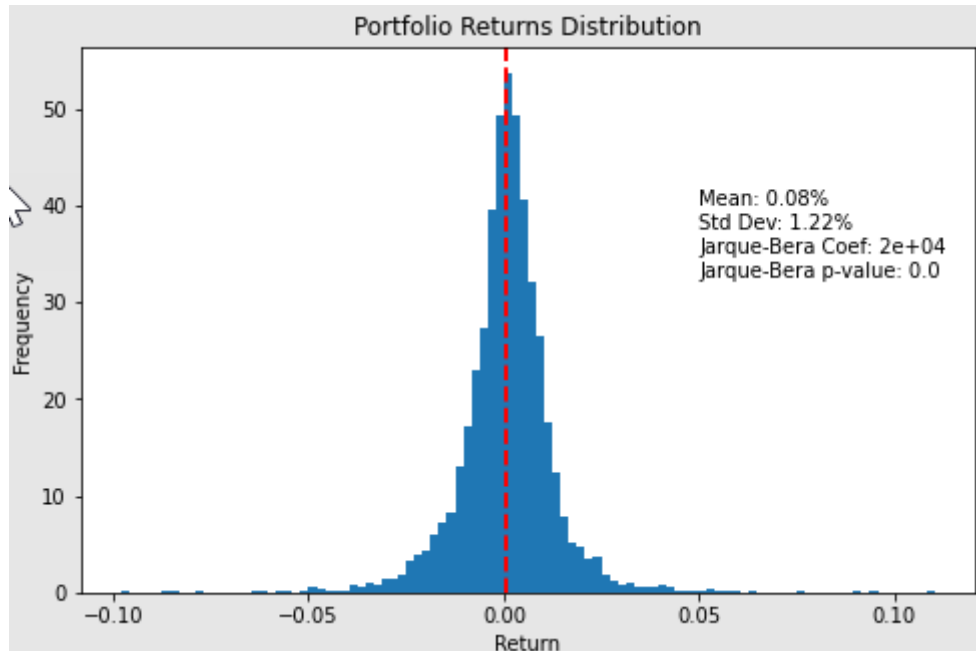
**Figure 1:** Returns Plot of the 9 Stocks

In Figure 2, histogram of each of the the stock was plotted and is tested for normality using the **Jarque-Bera Test**. For all stocks, the huge JB Coefficient and negligible  $p$ -values indicate that the distribution of the stock returns are not normally distributed, against a null hypothesis that a distribution follows Gaussian.



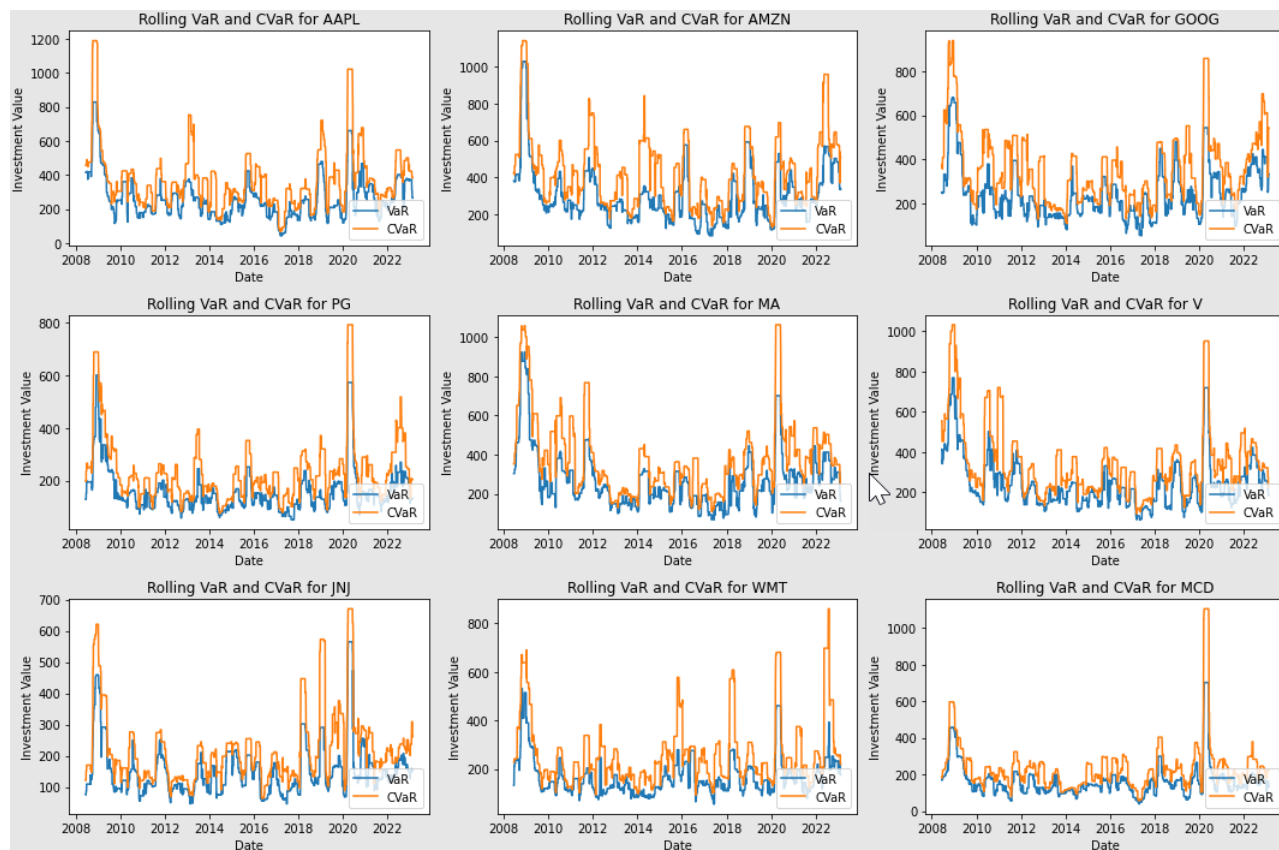
**Figure 2:** Histogram of the Returns of the 9 Stocks

Notice that in Figure 3, the mean return of the portfolio, as a weighted return, is a value along the range of returns of stocks in the portfolio. In fact, it is just an average of the returns due to the case that the portfolio is equally weighted. The same conclusion for the test of normality is derived from the JB coefficient and the  $p$ -value (i.e. portfolio distribution is not normally distributed). With this, an historical data can be tested to obtain the empirical VaR and CVaR.



**Figure 3:** Histogram of the Equally Weighted Portfolio

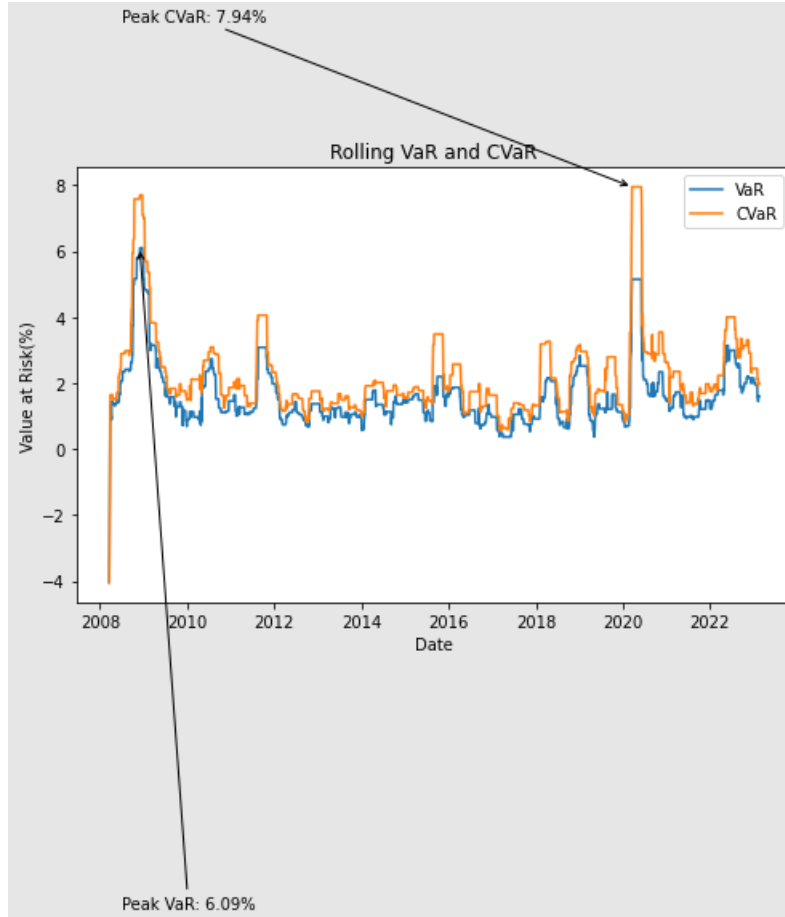
## 4 VaR and CVaR Computation



**Figure 4:** 60-Day Rolling VaR and CVaR of each Stock

In Figure 4, it can be inferred that the VaR / CVaR of McDonald's Corp is the most stable one in the portfolio, because the food industry is less likely to be affected by the cycles of the global economy. The VaR of Walmart Inc is the lowest generally when comparing with other stocks in the portfolio because Walmart is a large company with a diverse range of products and services, which can help to mitigate the impact of market fluctuations in any sector; Walmart has a reputation for being a low-cost retailer, which can make it an attractive investment for investors looking for stable returns.

Regarding the graph of AAPL, which is the most volatile one, it depicts that the global financial crisis lowered stock prices of AAPL causing huge losses. And Apple's stock price reached \$400 per share by mid-2011 after the 2010 iPad launch. Stock prices fell after Apple co-founder Steve Jobs' death in October 2011. Later, the iPhone 7 and 7 Plus helped Apple's stock price rise in 2016. When demand for digital products and services surged, AAPL's stock price rebounded from the COVID-19 pandemic in early 2020. Antitrust rules and probable App Store model changes lowered AAPL's stock price in late 2020 and early 2021. Strong financial performance and continuous growth in services and wearables sales helped Apple's stock price reach new highs in early 2022. The graph clearly shows the spikes are strongly correlated with the events of the company.



**Figure 5:** Rolling VaR and CVaR of the Portfolio

The equally weighted portfolio of the 9 stocks gives the 60-day rolling VaR and CVaR in Figure 5. One noticeable observation is that Peak VaR of 6.09% occurred during the 2008 Financial Crisis while the Peak CVaR of 7.94% occurred during the COVID-19 pandemic. However, the Portfolio VaR is lower during the COVID-19 pandemic. Indeed, VaR does not capture all the information about the risk and makes it only a threshold while CVaR takes an expected value in on the tail of the distribution beyond the VaR.

## 5 Conclusion

Generally, the CVaR for the stocks in the portfolio tends to be higher than VaR. CVaR (Conditional Value at Risk), also known as Expected Shortfall, is frequently regarded as a superior risk management indicator than VaR (Value at Risk) since it gives extra information regarding downside risk beyond VaR. VaR evaluates the highest probable loss over a specific horizon of time with a set level of certainty. VaR is important for establishing risk limits and capital requirements, but it only reveals the worst-case loss possibility. CVaR quantifies the projected loss beyond VaR in the worst-case scenario. It gives information beyond the VaR level on the severity of the loss. CVaR takes into account not just the possibility of a loss surpassing VaR, but also the amount of that loss. This means that CVaR takes into account both the chance and effect of severe occurrences, giving it a more informative risk metric.

Secondly, if the stock prices increase, it does not necessarily mean the risk will be lowered

according to the findings of this empirical application. The stock price of a corporation may rise despite no change in its fundamentals, such as earnings or sales growth. This can occur when investors get too enthusiastic about a company's future prospects and bid up the stock price above what is warranted by the company's actual performance. In this instance, the increase in stock price may raise the amount of risk for investors who purchase at inflated prices.

## 6 Python Codes

All python codes are done in Google Colaboratory and can be found in this [link](#).