Binary Tree

Practicing Pointer & Recursive

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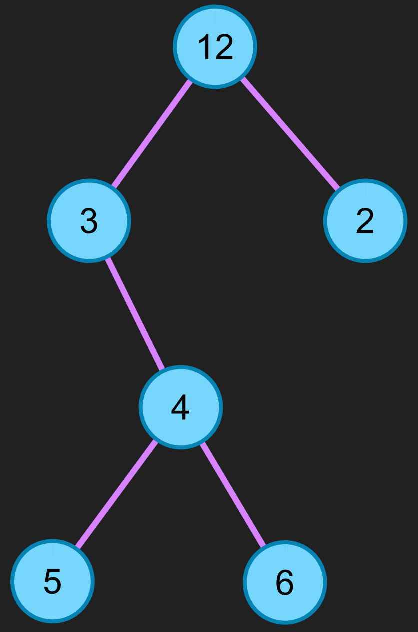
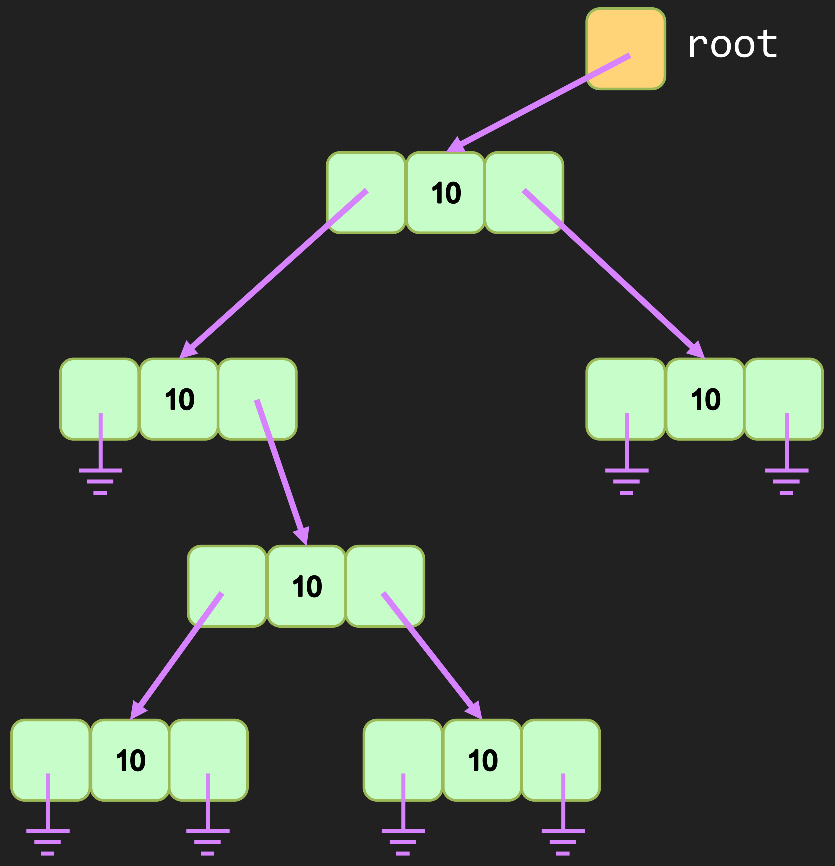
Overview

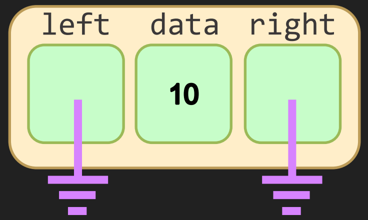
- This is a basic for the next data structure, Binary Search and AVL Tree
- Focus on using Node and Pointer
- Focus on using recursive programming
- Some applications using just Binary Tree
- There is no data structure in std that is Binary Tree

Binary Tree & Node



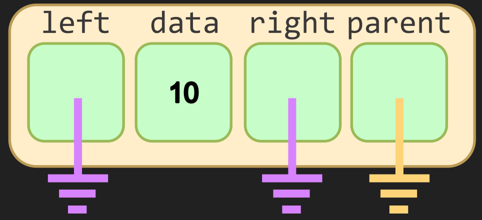


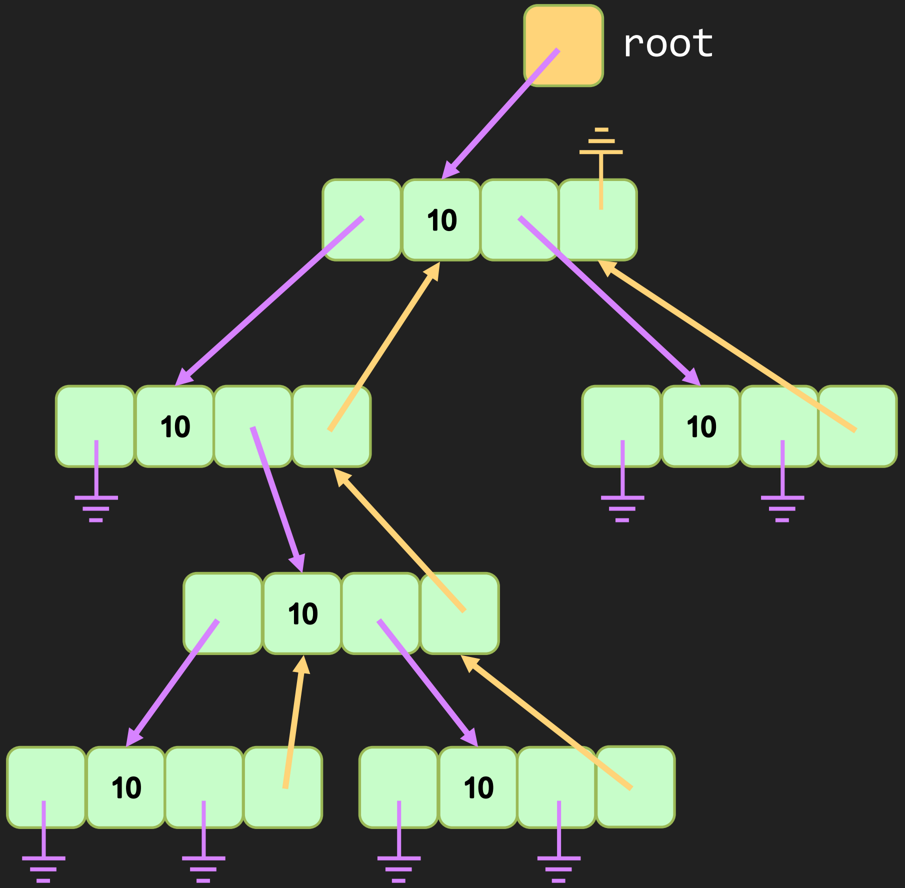
- A rooted tree where each node have at most two children
- Tree Node is very similar to a linked list node




```
class node {
  public:
    ValueT data;
    node  *left, *right;
    node() :
      data( ValueT() ), left( NULL ), right( NULL ) { }
    node(const ValueT& data, node* left, node* right) :
      data ( data ), left( left ), right( right ) { }
};
```

Node with parent link





- Sometime, we need a link to parent
- Root is the only node that parent is NULL

```
class node {
  public:
    ValueT data;
    node  *left, *right, *parent;
    node() :
      data( ValueT() ), left( NULL ), right( NULL ), parent( NULL ) { }
    node(const ValueT& data, node* left, node* right, node* parent) :
      data ( data ), left( left ), right( right ), parent( parent ) { }
};
```

Huffman Coding: Example Application of Tree

- David Huffman proposed this as his term project in Robert Fano's class (co-worker of Claude Shannon) which beats Shannon-Fano encoding
- Encoding = associate meaning to a representation
- ASCII Code
  - Fix length encoding
  - Each char = 8 bits

100 0001	101	65	41	A
100 0010	102	66	42	B
100 0011	103	67	43	C
100 0100	104	68	44	D
100 0101	105	69	45	E
100 0110	106	70	46	F
100 0111	107	71	47	G
100 1000	110	72	48	H
100 1001	111	73	49	I
100 1010	112	74	4A	J
100 1011	113	75	4B	K
100 1100	114	76	4C	L
100 1101	115	77	4D	M

Variable Length Encoding

Never gonna give you up

Never gonna let you down

Never gonna run around and desert you

16 different character

Fix-length needs 4 x 86 = 344 bits

Variable Length need 327 bits

n	e	o	u	r	a	v	g	d	y	t	w	s	p	l	i
14	11	9	7	7	6	5	5	5	4	3	2	2	2	2	2
0000	0001	0010	0011	0100	0101	0110	0111	1000	1001	1010	1011	1100	1101	1110	1111
11	010	011	0001	0011	0000	1011	1010	1000	00101	10011	100101	0010001	001001	100100	0010000

Encoding “Never”

Fix-length 00000001011000010100

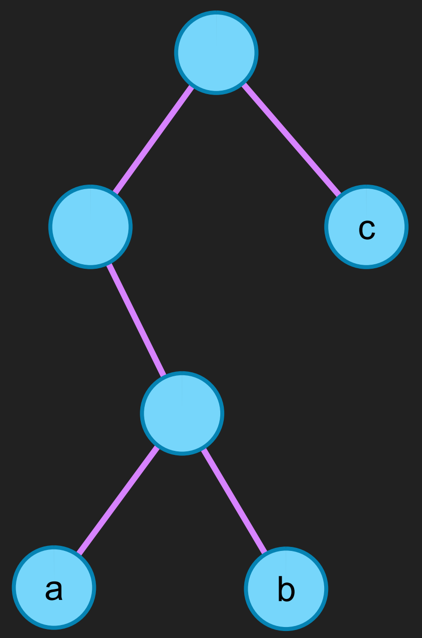
Variable Legnth 1101010110100011

Problem Statement

- Input: a string
- Output: encoding of each character in the string such that
  - The total length of encoding the string is minimum
  - The encoding of character is not ambiguous
    - Any character encoding is not a prefix of any other character

Tree Encoding

- Using a tree to represent encoding
- Each character is represented at leaf nodes
  - Leaf node is a node without children
- Encode by start at the root and walk toward leaf nodes
  - The path gives the encoding
  - Going to left child equal to 0
  - Going to right child equal to 1
- Guarantee to be non-ambiguous



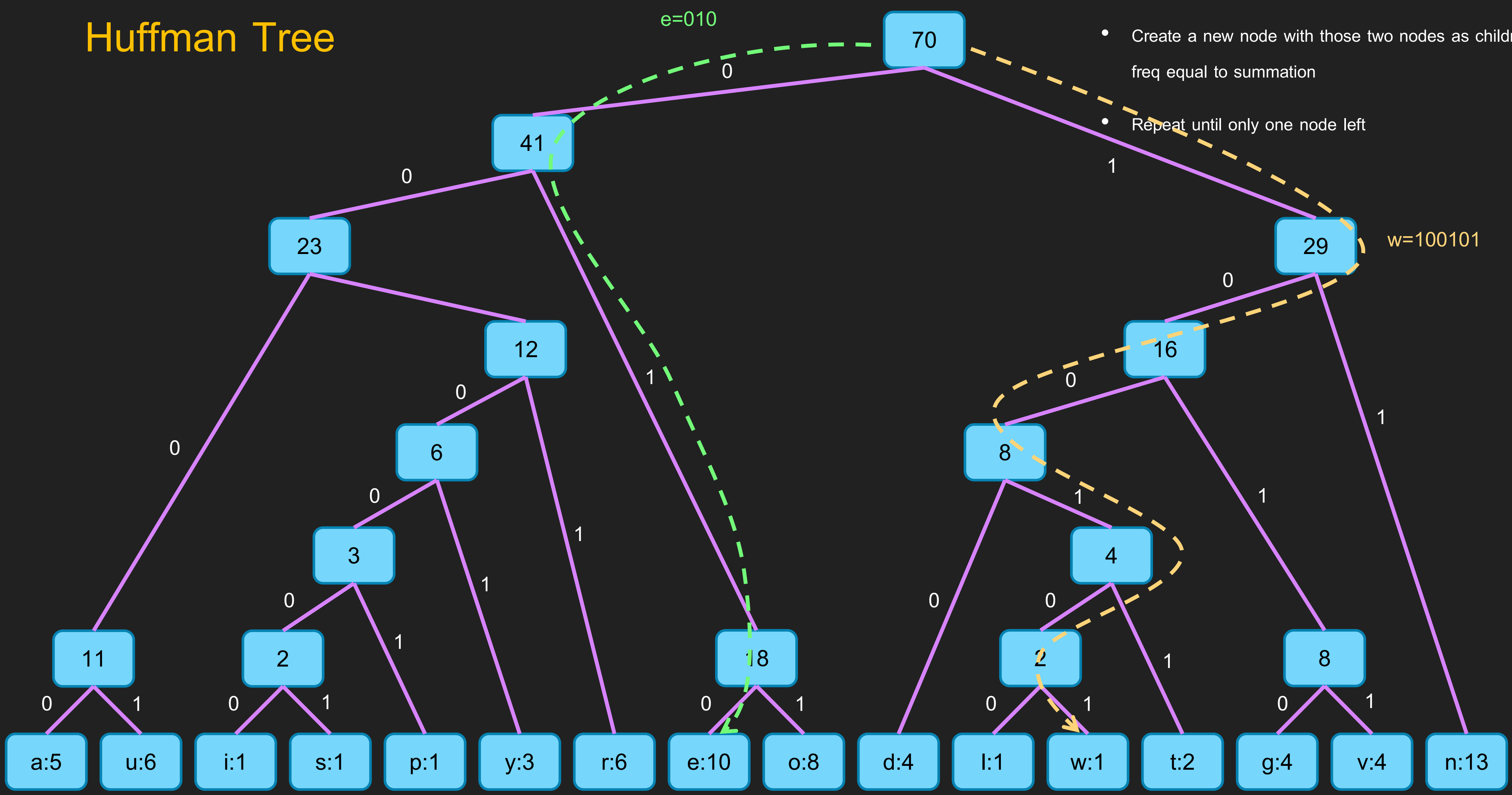
a = 010

b = 011

c = 1

Huffman Tree

- Find 2 min nodes
- Create a new node with those two nodes as children, set freq equal to summation
- Repeat until only one node left



Huffman Tree Node

- Instead of data, we have both character and frequency
- Since we have to pick two nodes with minimum freq, we overload operator< to do so and use priority_queue

Huffman Code : Node

```
class huffman_tree {
  protected:
    class huffman_node {
      public:
        char letter;
        int freq;
        huffman_node *left, *right;
        huffman_node() : letter('*'),freq(0),left(NULL),right(NULL) {}
        huffman_node(char letter,int freq,huffman_node *left,huffman_node *right) :
          letter(letter),freq(freq), left(left),right(right) {}

        bool is_leaf() { return left == NULL && right == NULL;  }
    };

    class node_comparator {
      public:
        bool operator()(const huffman_node *a, const huffman_node *b) {
          return a->freq > b->freq;
        }
    };
```

Huffman Code : Build Tree

```
class huffman_tree {
  protected:
    huffman_node *root;
    void build_tree(vector<huffman_node*> data) {
      priority_queue<huffman_node*,vector<huffman_node*>,node_comparator> pq;
      for (auto &x : data) pq.push(x);
      while (pq.size() > 1) {
        huffman_node *right = pq.top(); pq.pop();
        huffman_node *left = pq.top(); pq.pop();
        pq.push(new huffman_node('*',left->freq+right->freq,left,right));
      }
      root = pq.top();
    }
  public:
    huffman_tree(string s) {
      map<char,int> count;
      for (auto &c : s)
        count[c]++;
      vector<huffman_node*> nodes;
      for (auto &x : count)
        nodes.push_back(new huffman_node(x.first,x.second,NULL,NULL));
      build_tree(nodes);
    }
```

Recursive Programming

Calling itself

Recursive

- A function that call itself
- Must have some input, usually via function argument
- The function must check a condition for execution
  - Result in either terminating case where the function won't call itself
  - or recursion case where the function will call itself with different parameters

Terminating condition

```
// calculate sum 0..n
int recur1(int n) {
  if (n <= 0) {
    // terminating case
    return 0;
  } else {
    // recursion case
    return recur1(n-1) + n;
  }
}
```

Smaller parameter

Why recursion?

- Much simpler code
  - When the task is right
  - Recursion is natural for several mathematical model that is recursi
- Comparing to a normal loop, recursion has the same growth rate but recursion might takes more time because function call is costlier than a loop

More Example

```
void print_range1(int step,int goal) {
  if (step < goal) {
    std::cout << step << "";
    print_range1(step+1, goal);
  }
}
```

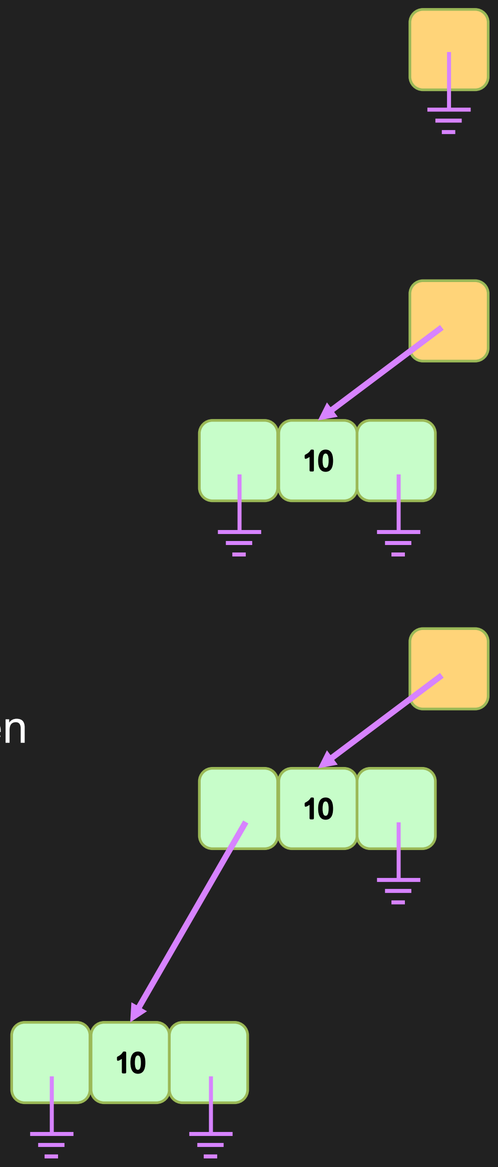
```
void print_range2(int step,int goal) {
  if (step < goal) {
    print_range2(step+1, goal);
    std::cout << step << "";
  }
}
```

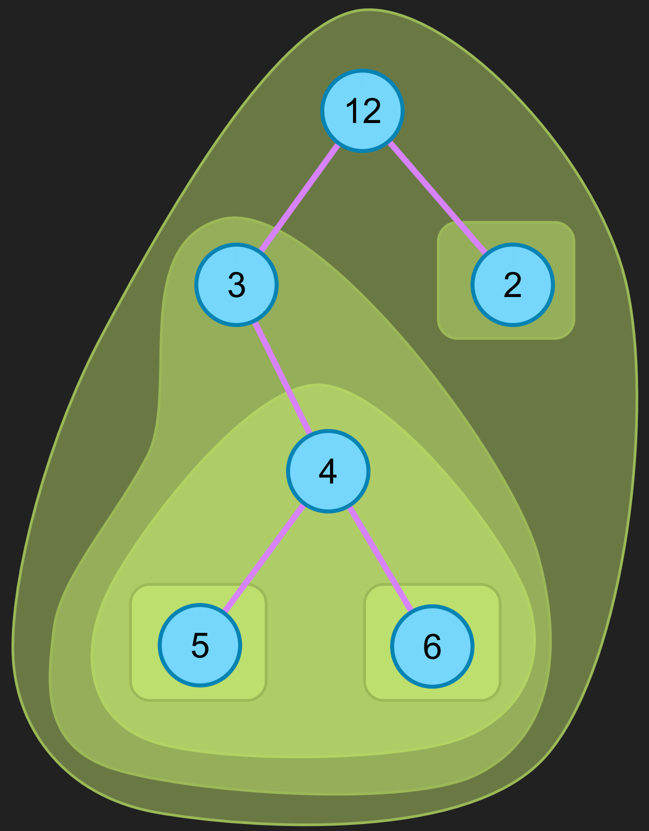
- Terminating Case do nothing
- Which is the output of print_range1(0,5) and print_range2(0,5)

0 1 2 3 4 5	0 1 2 3 4
5 4 3 2 1 0	4 3 2 1 0

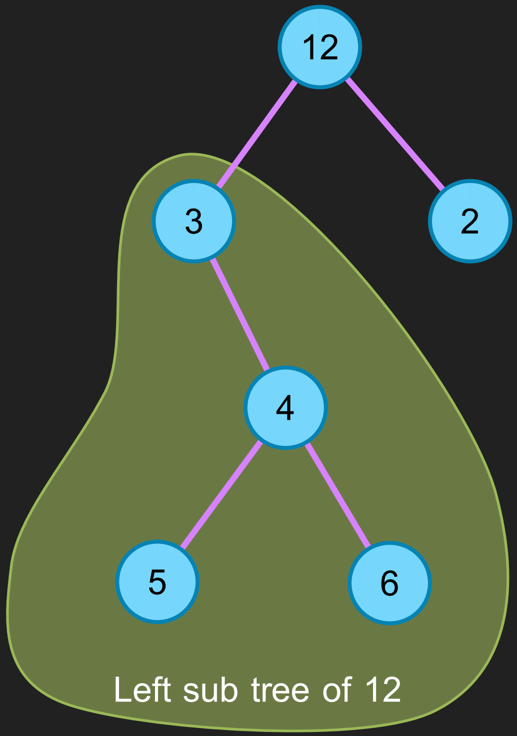
Binary Tree Recursive Definition

- A Binary Tree is
  - A tree with no nodes (root is NULL)
  - A tree with a root
    - both children of the root must be a binary tree
    - Each child is call left-subtree and right-subtree
- Since binary tree can be defined recursively, operation on a binary tree can be naturally written as a recursion





Subtree

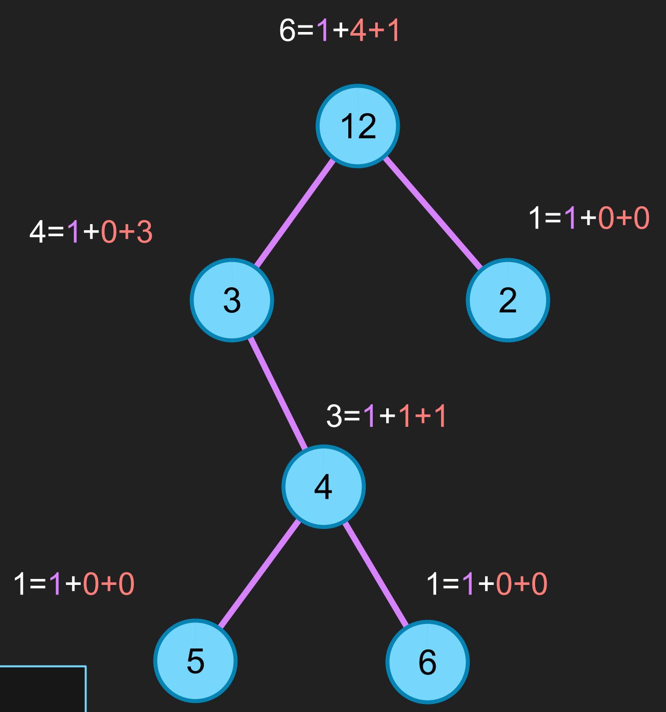


- For any node
  - its left (right) child and all of the child’s descendants is called left-subtree (right-subtree)

Tree Size by Recursion

- An empty tree has 0 node
- A tree with a root has 1 node (the root)
    - Plus the size of its two subtrees
- Easily written as recursive

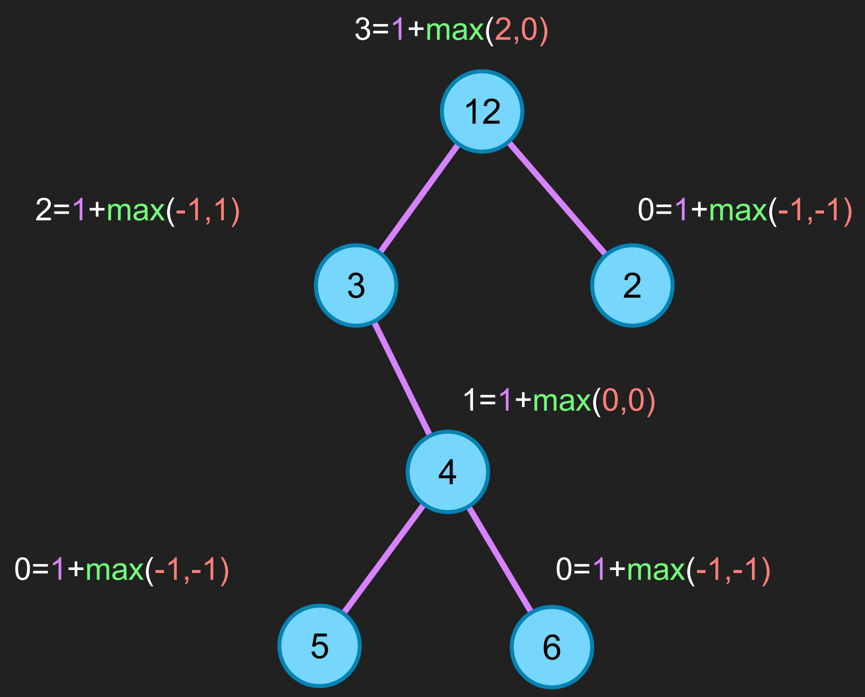
```
class node {
  public:
    int data;
    node *left, *right;
};

int get_size(node* n) {
  if (n == NULL) return 0;
  return 1 + get_size(n->left) + get_size(n->right);
}
```



Tree Height

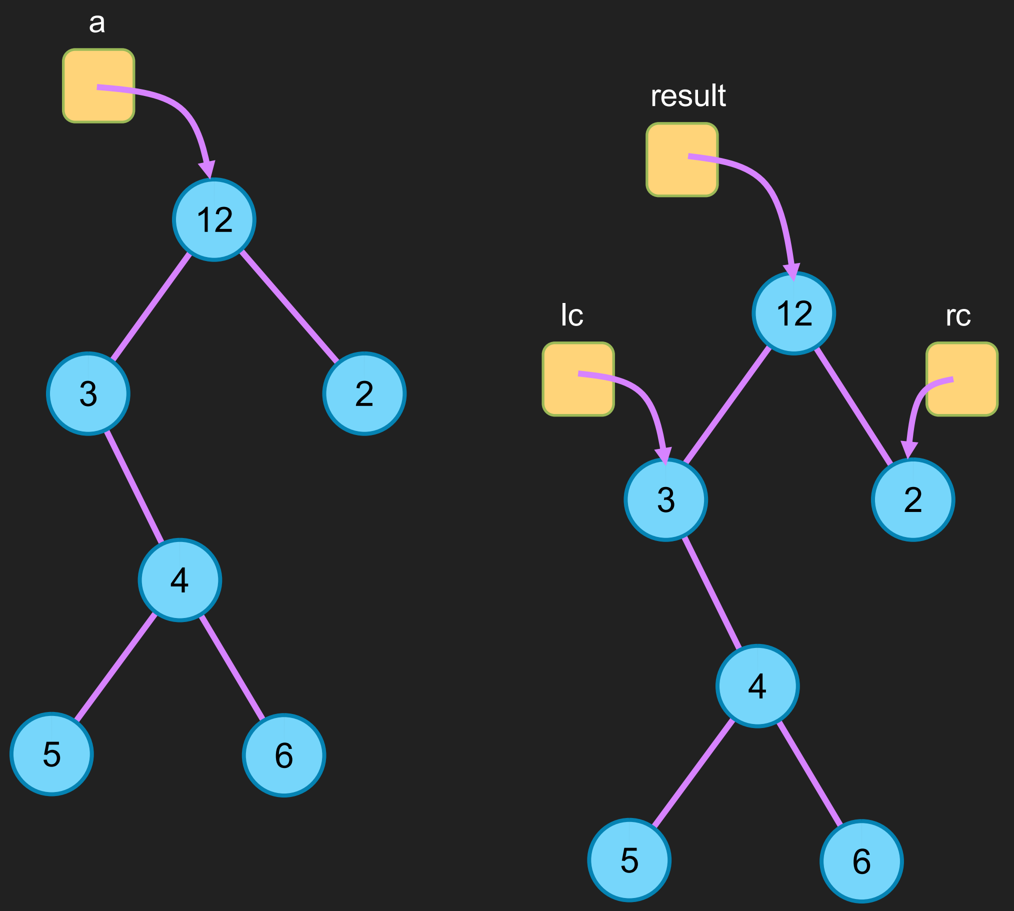
- Height of a tree is the number of link we have to go to reach it deepest children
- Empty tree has height -1
- Height of a tree is 1 + max of height of its children

```cpp
class node {
  public:
    int data;
    node *left, *right;
};
int get_height(node *n) {
  if (n == NULL) return -1;
  return 1 + std::max(get_height(n->left),
                      get_height(n->right));
}
```



Tree Copy

```
class node {
  public:
    int data;
    node *left, *right;
    node() : data(0),left(NULL),right(NULL);
    node(int data,node *left,node *right)
      : data(data),left(left),right(right);
};

node* copy(node *n) {
  if (n == NULL) return NULL;
  node *lc = copy(n->left);
  node *rc = copy(n->right);
  node *result = new node(n->data,lc,rc);
}
```



Walk over a tree

- Visiting all nodes (and maybe do something)

```
void preorder(node *n) {
  if (n == NULL) return NULL;
  std::cout << n->data << " ";
  preorder(n->left);
  preorder(n->right);
}
```

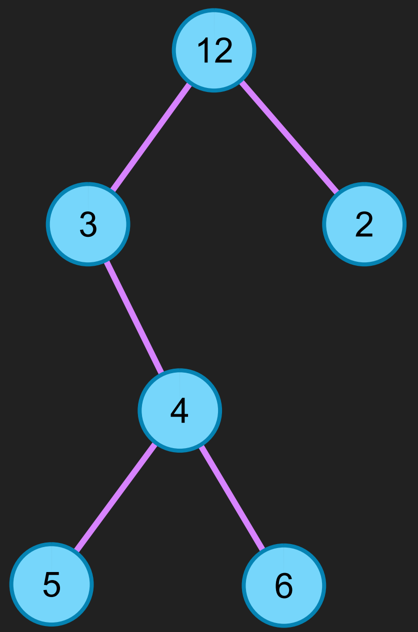
preorder traversal

```
void inorder(node *n) {
  if (n == NULL) return NULL;
  inorder(n->left);
  std::cout << n->data << " ";
  inorder(n->right);
}
```

inorder traversal

```
void postorder(node *n) {
  if (n == NULL) return NULL;
  postorder(n->left);
  postorder(n->right);
  std::cout << n->data << " ";
}
```

postorder traversal



What is the result of

- preorder(a);
- inorder(a);
- postorder(a);

Huffman Tree : Encoding

```
class huffman_tree {
  protected:
    class huffman_node { };
    class node_comparator {  };
    huffman_node *root;
  public:
    void print(huffman_node *n,string s) {
      if (n->is_leaf()) {
        cout << n->letter << ": " << s << endl;
      } else {
        print(n->left,s+"0");
        print(n->right,s+"1");
      }
    }

    void print() {
      print(root,"");
    }
};
```

- Recursive printing
- Use s to store path

Huffman Tree : Encoding

```
class huffman_tree {
  protected:
    class huffman_node { };
    class node_comparator {  };

    huffman_node *root;

    void delete_node(huffman_node *n) {
      if (n == NULL) return;
      delete_node(n->left);
      delete_node(n->right);
      delete n;
    }

  public:

    ~huffman_tree() {
      delete_node(root);
    }
};
```

- Recursive delete node
- Use postorder traversal
- Can we use inorder or preorder?