FRIEDRICH-LOEFFLER-INSTITUT

FLI

Bundesforschungsinstitut für Tiergesundheit

Federal Research Institute for Animal Health

Livestock Trade Networks

Social network analysis in one health epidemiology

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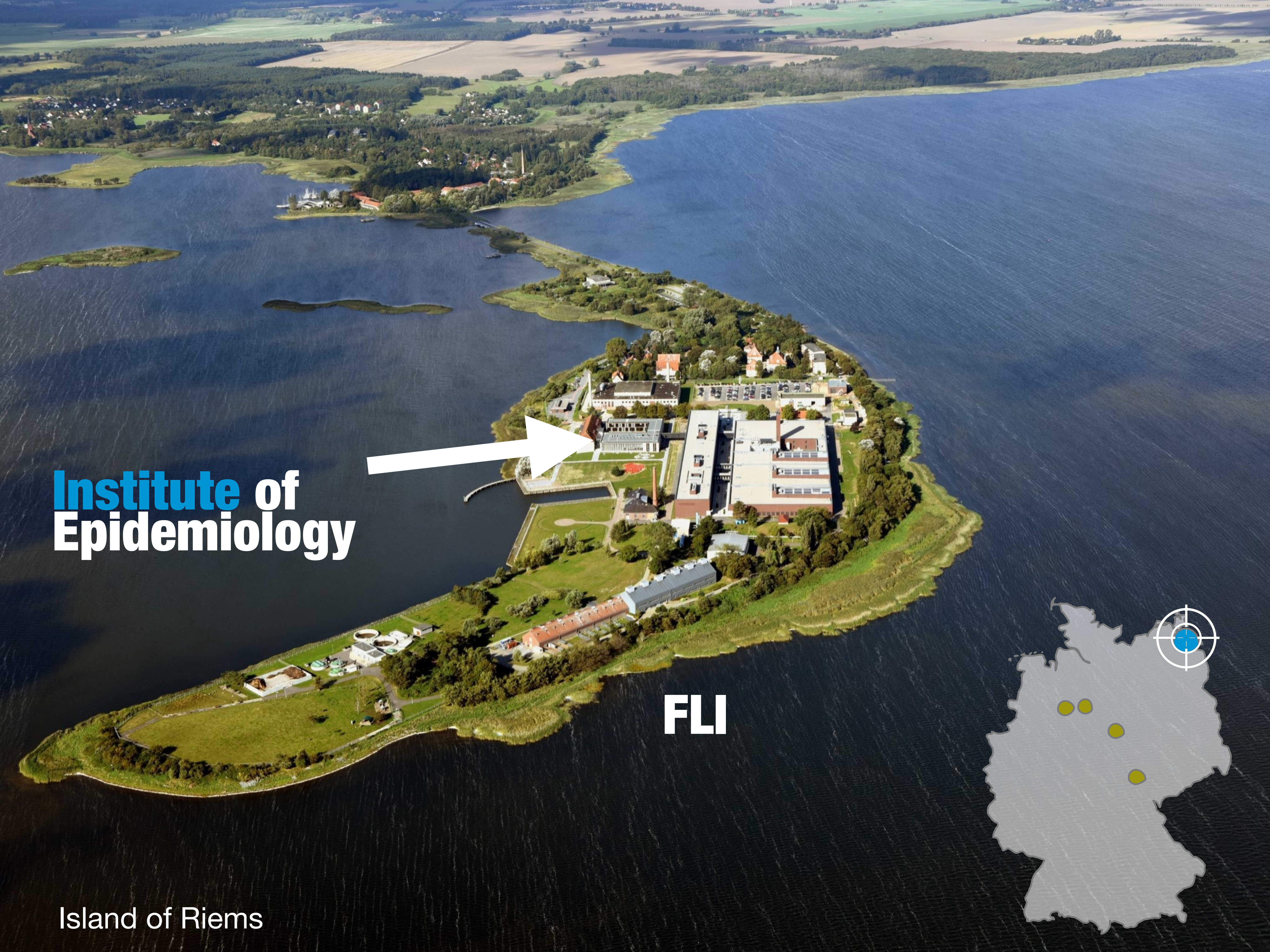
Thomas Selhorst
Igor M Sokolov


FLI
Island of Riems


Institute of
Epidemiology
FLI
Island of Riems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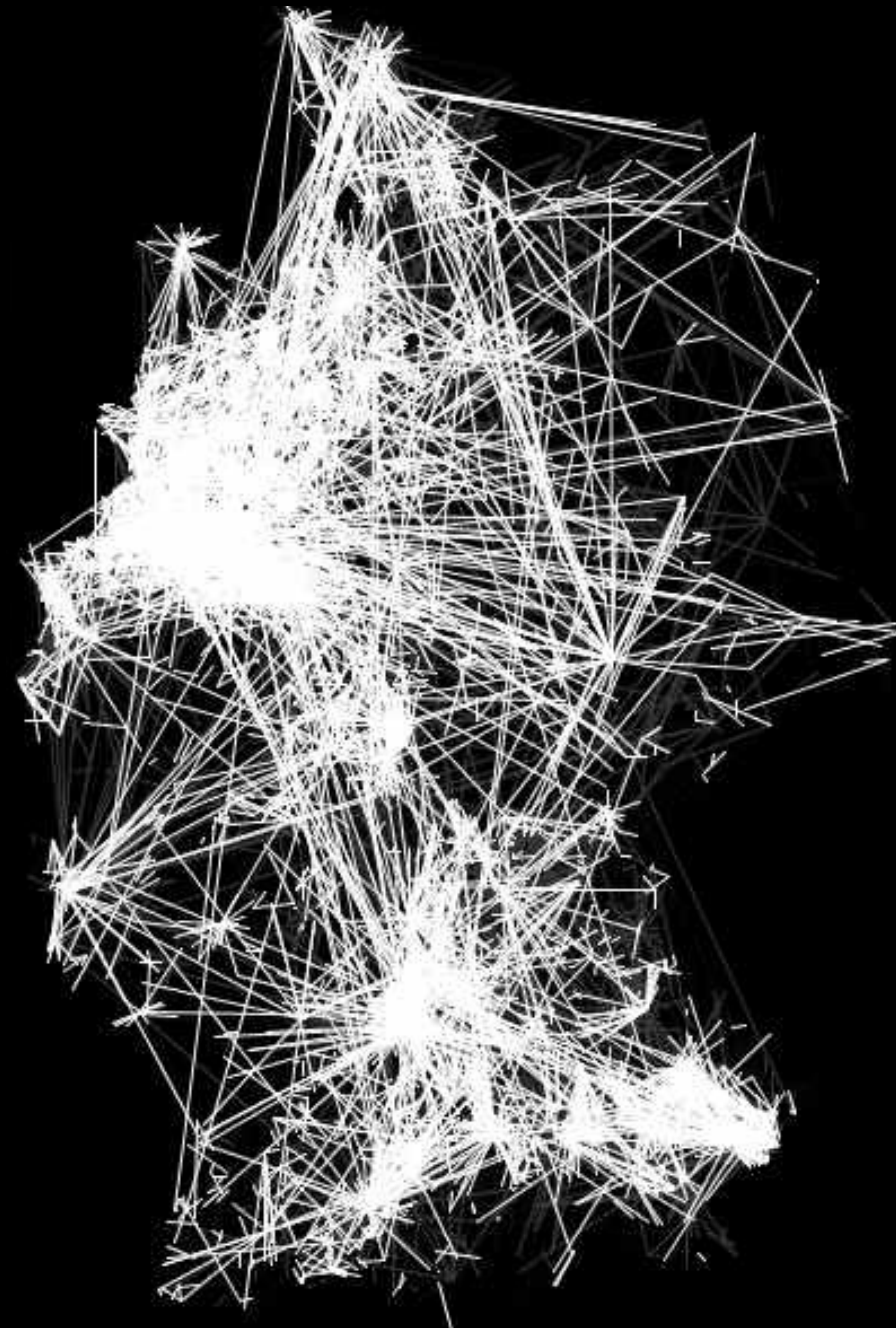
network dataset

livestock pig trade

farm to farm



classical network view:

main questions

classical network view:

main questions

Where can a disease spread in the network?

classical network view:

main questions

Where can a disease spread in the network?

What is a good strategy for targeted vaccination?

temporal network dataset

livestock pig trade

farm to farm

temporal

network dataset

livestock pig trade

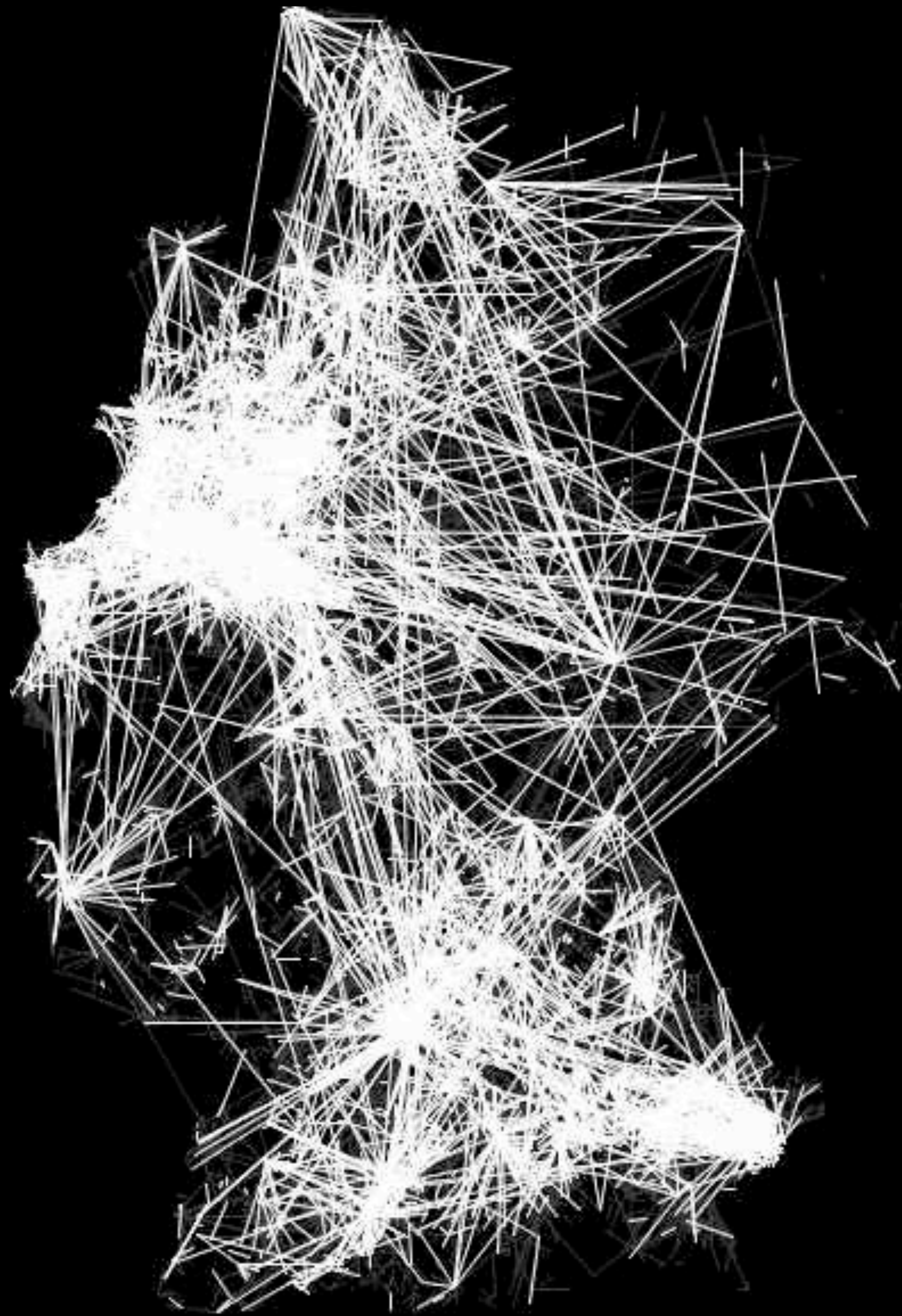
farm to farm

temporal

network dataset

livestock pig trade

farm to farm



Graph. An ordered pair $G = (V, E)$ comprising a set V of nodes together with a set E of edges which are 2-element subsets of V.

Graph. An ordered pair G = (V, E) comprising a set V of nodes together with a set E of edges which are 2-element subsets of V.

this is not a graph!

temporal network view:

main questions

temporal network view:

main questions

Where can a disease spread in the network?

temporal network view:

main questions

Where can a disease spread in the network?

What is a good strategy for targeted vaccination?

temporal network view:

main questions

Where can a disease spread in the network?

What is a good strategy for targeted vaccination?

How much time does it take to traverse?

temporal network view:
main questions

Where can a disease spread in the network?

What is a good strategy for targeted vaccination?

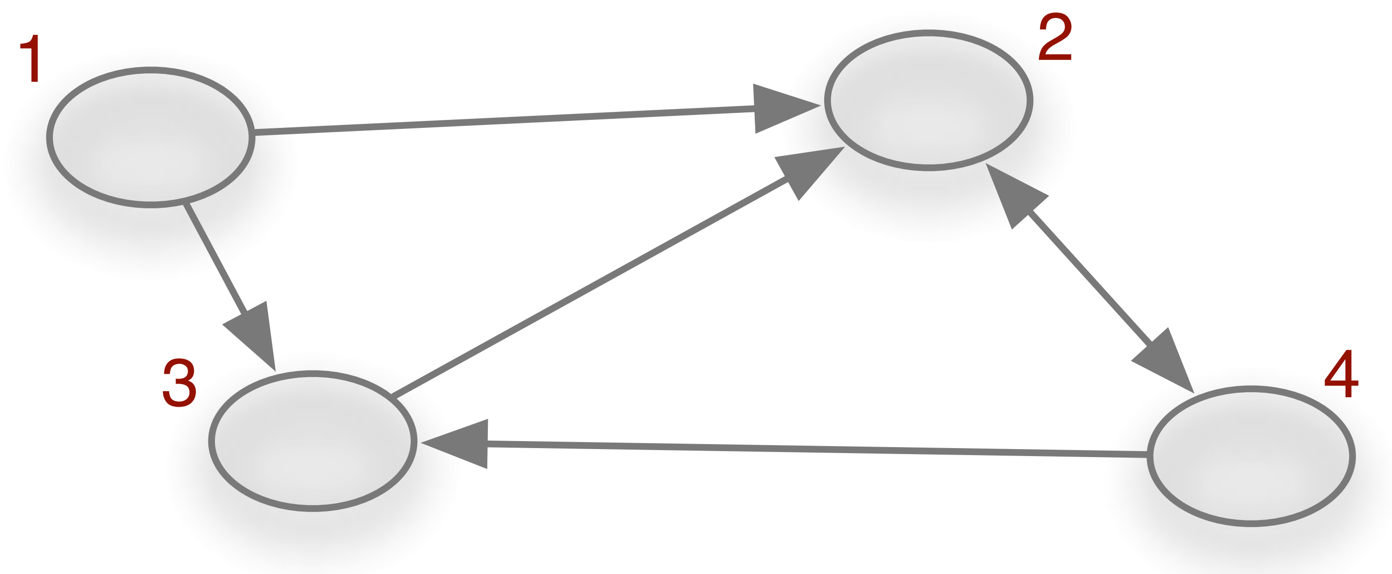
How much time does it take to traverse?

Can I pretend it’s NOT time dependent?

static network

representation

representation: Adjacency matrix **A**

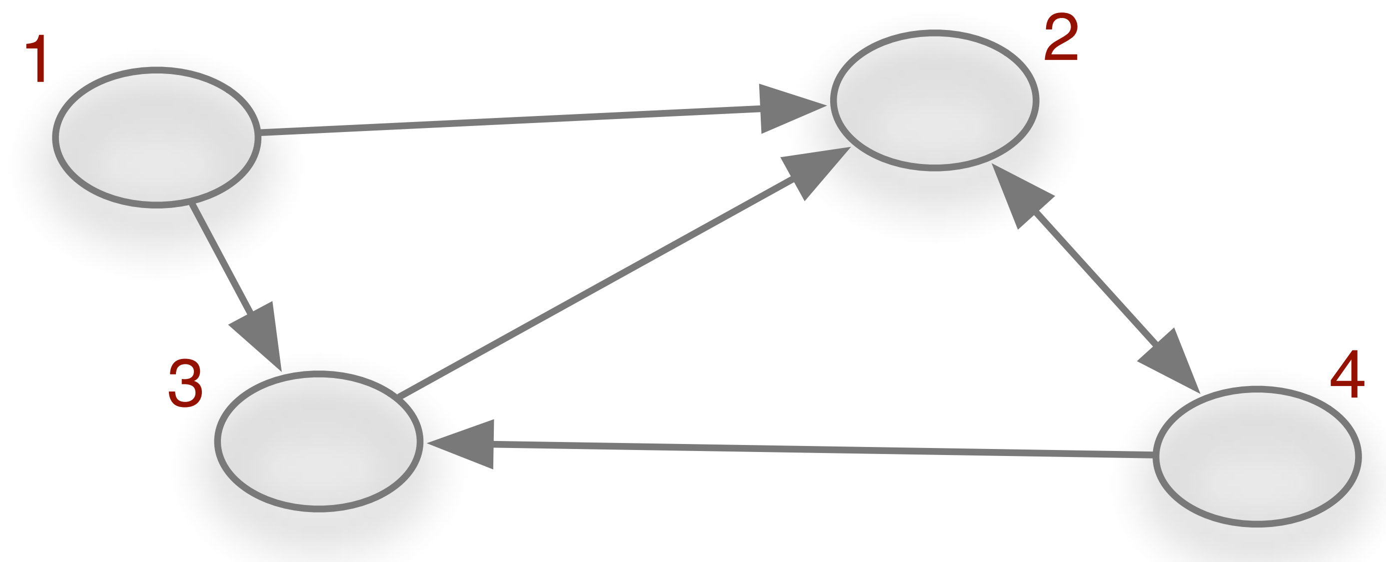


$$\mathbf{A} = \begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{pmatrix}$$

static network

representation

representation: Adjacency matrix **A**



$$\mathbf{A} = \begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{pmatrix}$$

the big picture

temporal network

representation

adjacency matrix sequence

$$\mathbf{A}_4 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\mathbf{A}_3 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\mathbf{A}_2 = \begin{pmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix}$$

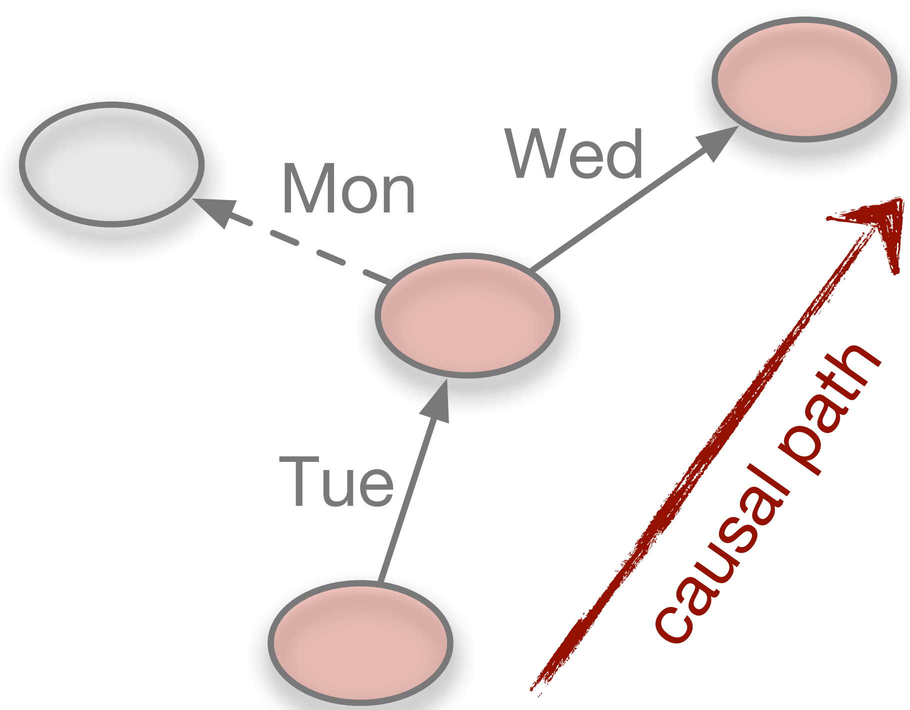
$$\mathbf{A}_1 = \begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

time

causal
paths

chronologic sequence of edges

fundamental difference to static: **causal** paths



static & temporal networks

common ground?

static & temporal networks

common ground?

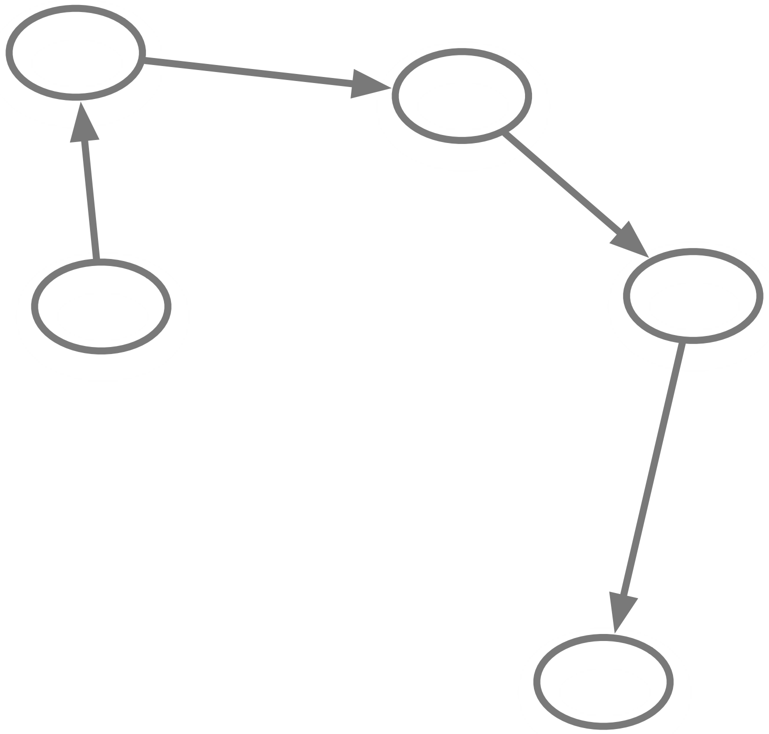
path structure!

static & temporal networks

common ground?

path structure!

some network

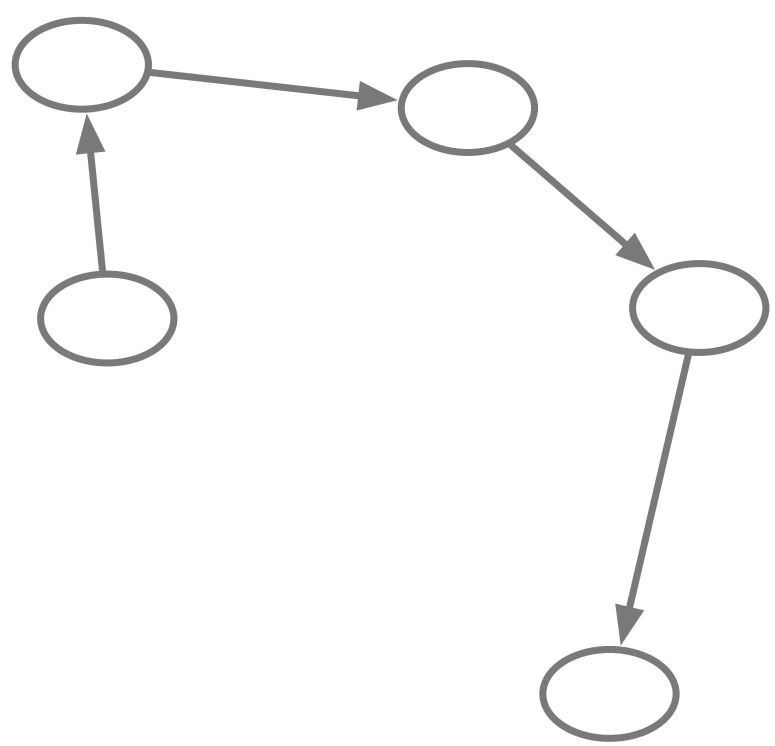


static & temporal networks

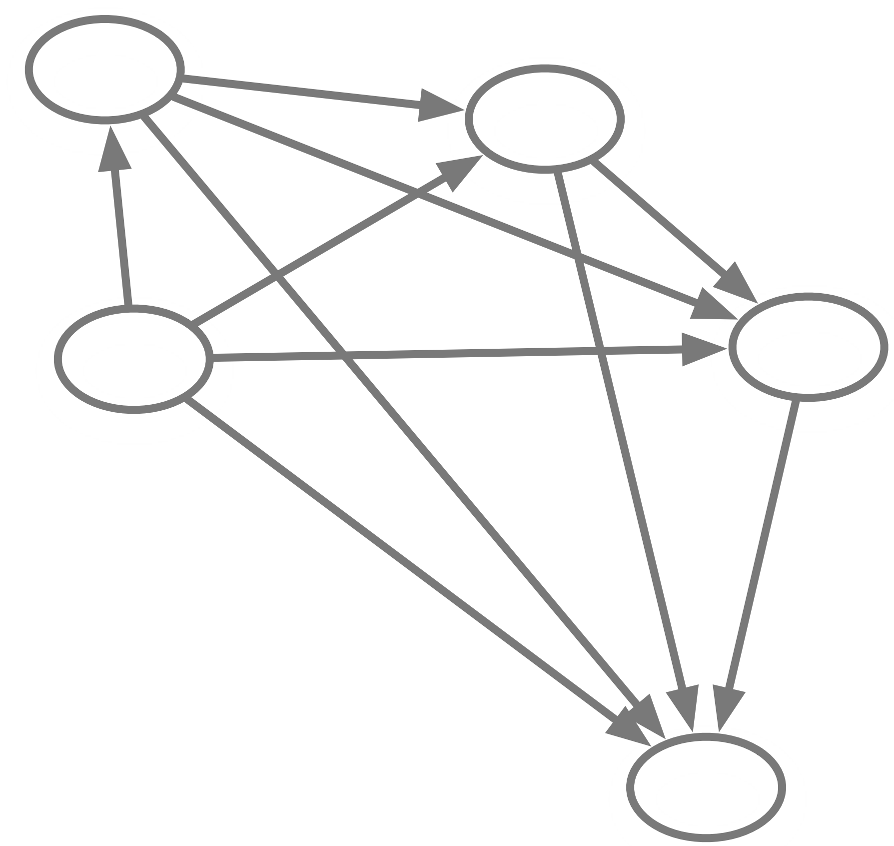
common ground?

path structure!

some network



its accessibility-graph



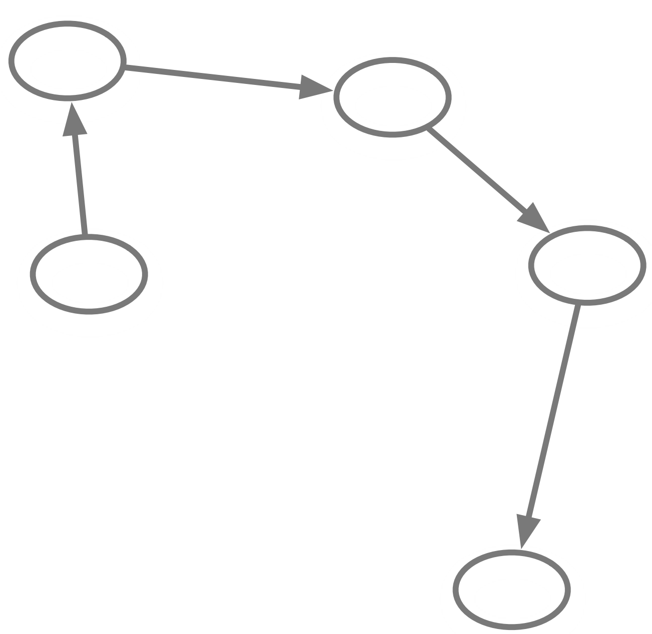
accessibility matrices &

accessibility graphs

adjacency matrix **A**:
number of paths of length 1

square of adjacency matrix $\mathbf{A}^2$:
number of paths of length 2

Example (static):



accessibility matrices &

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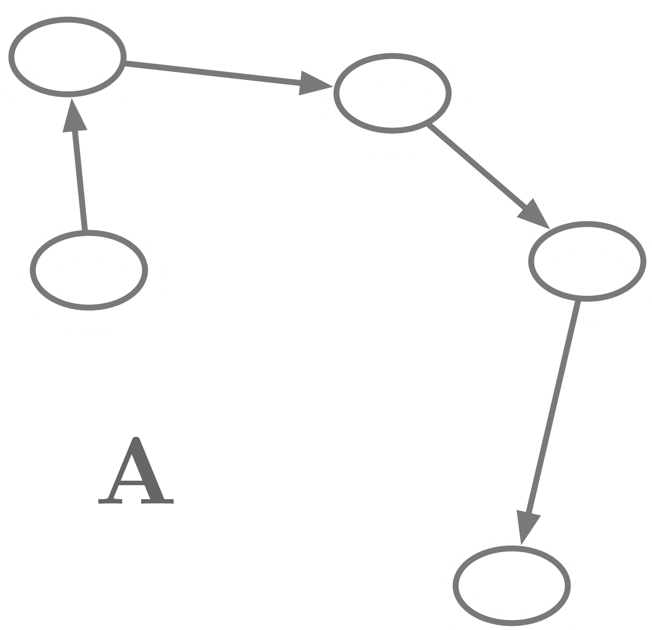

$\mathbf{A}$

accessibility matrices &
accessibility graphs

adjacency matrix **A**:
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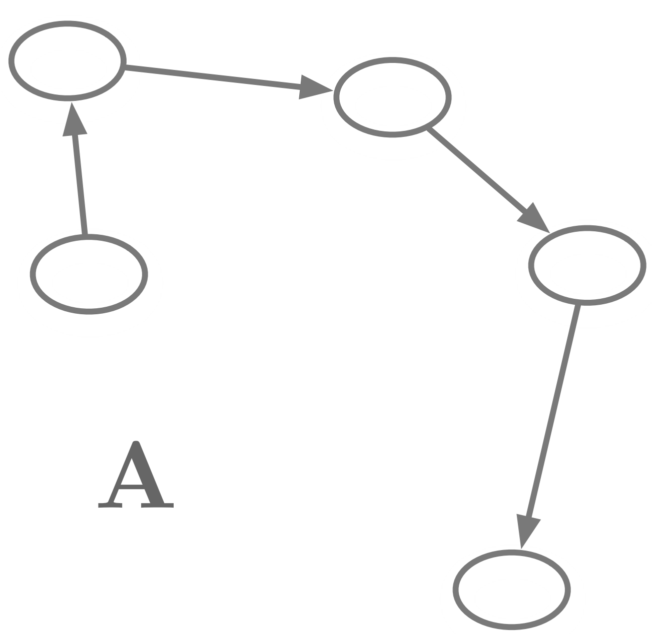
Example (static):



$\mathbf{A}$

$\mathbf{A}^2$

accessibility matrices & accessibility graphs

adjacency matrix **A**:
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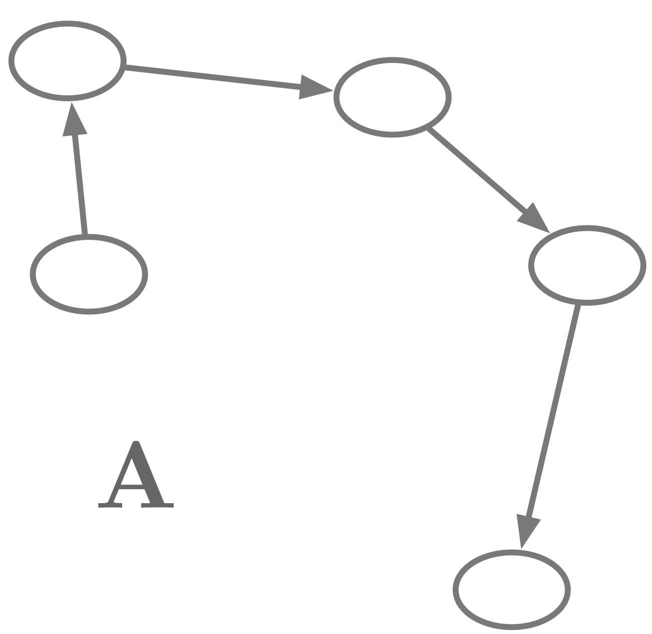
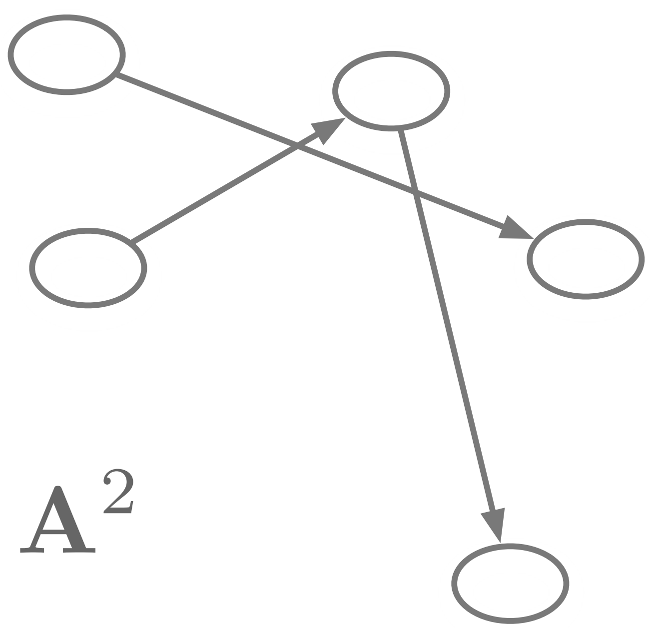
Example (static):



$\mathbf{A}$



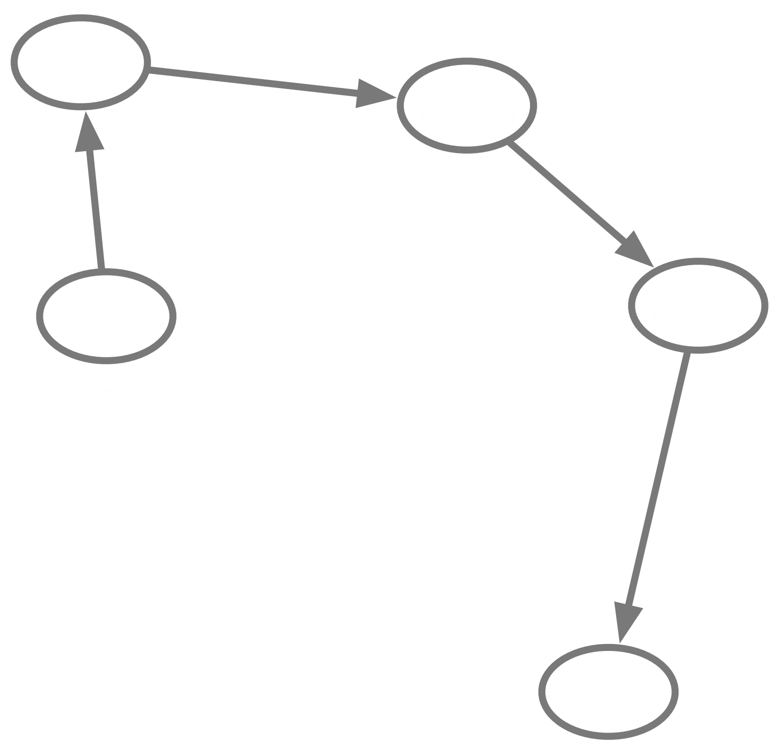
$\mathbf{A}^2$

accessibility matrices &

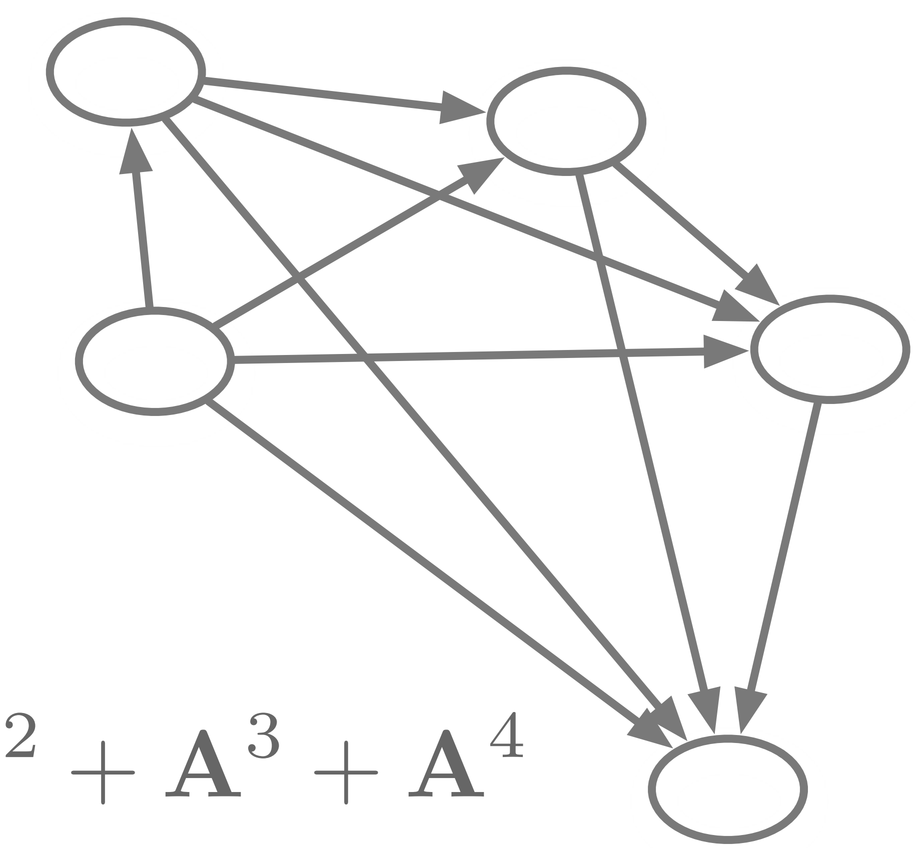
accessibility graphs

initial graph

$\mathbf{A}$



accessibility-graph



$$\mathbf{P} = \mathbf{A} + \mathbf{A}^2 + \mathbf{A}^3 + \mathbf{A}^4$$

accessibility matrices of

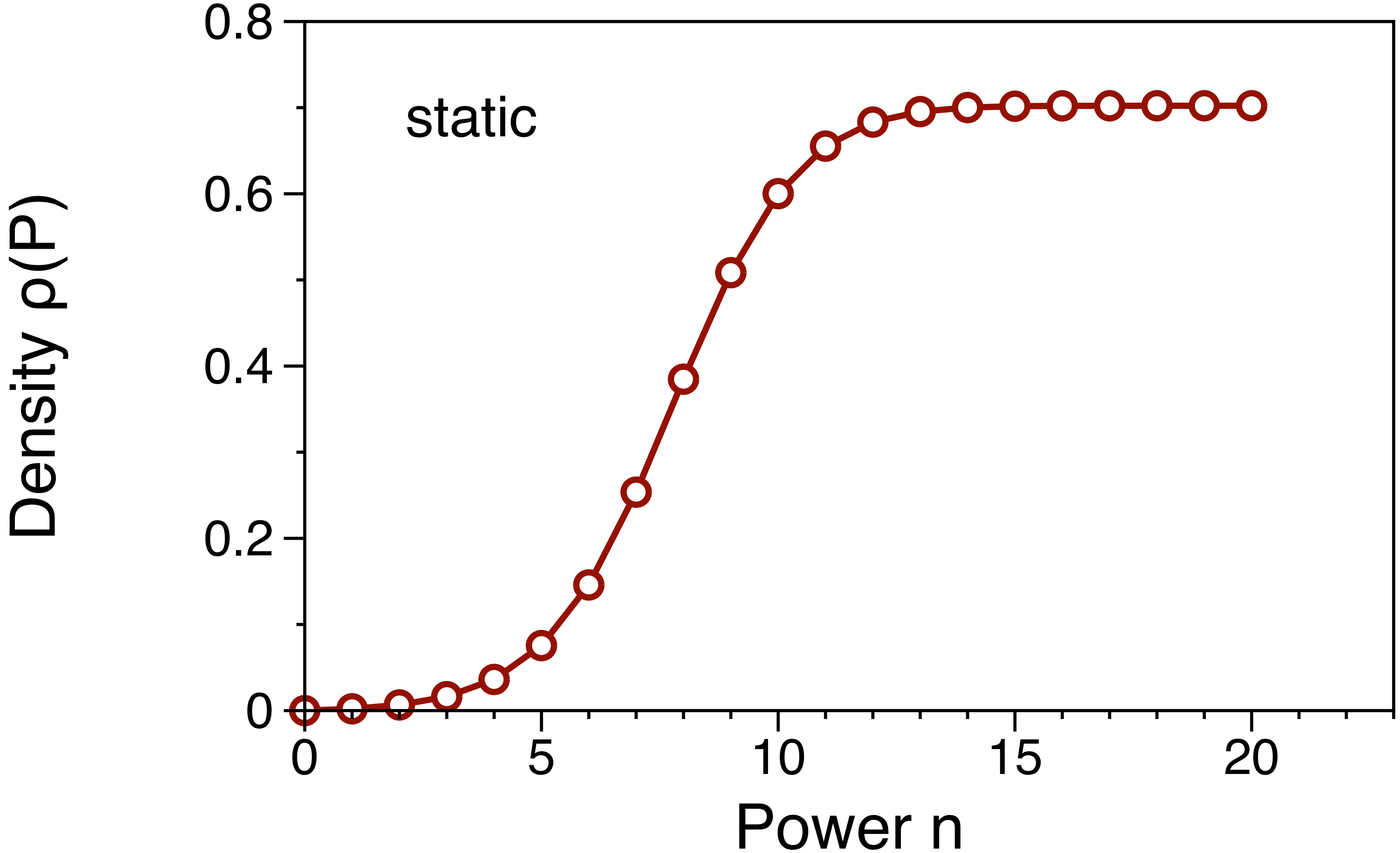
static networks

example:
Random DiGraph
1000 nodes
2000 edges

$$\mathbf{P} = \mathbf{A} + \mathbf{A}^2 + \mathbf{A}^3 + \cdots + \mathbf{A}^n$$

accessibility matrices of
static networks

example:
Random DiGraph
1000 nodes
2000 edges



$$\mathbf{P} = \mathbf{A} + \mathbf{A}^2 + \mathbf{A}^3 + \cdots + \mathbf{A}^n$$

accessibility matrices of

static networks

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Random DiGraph
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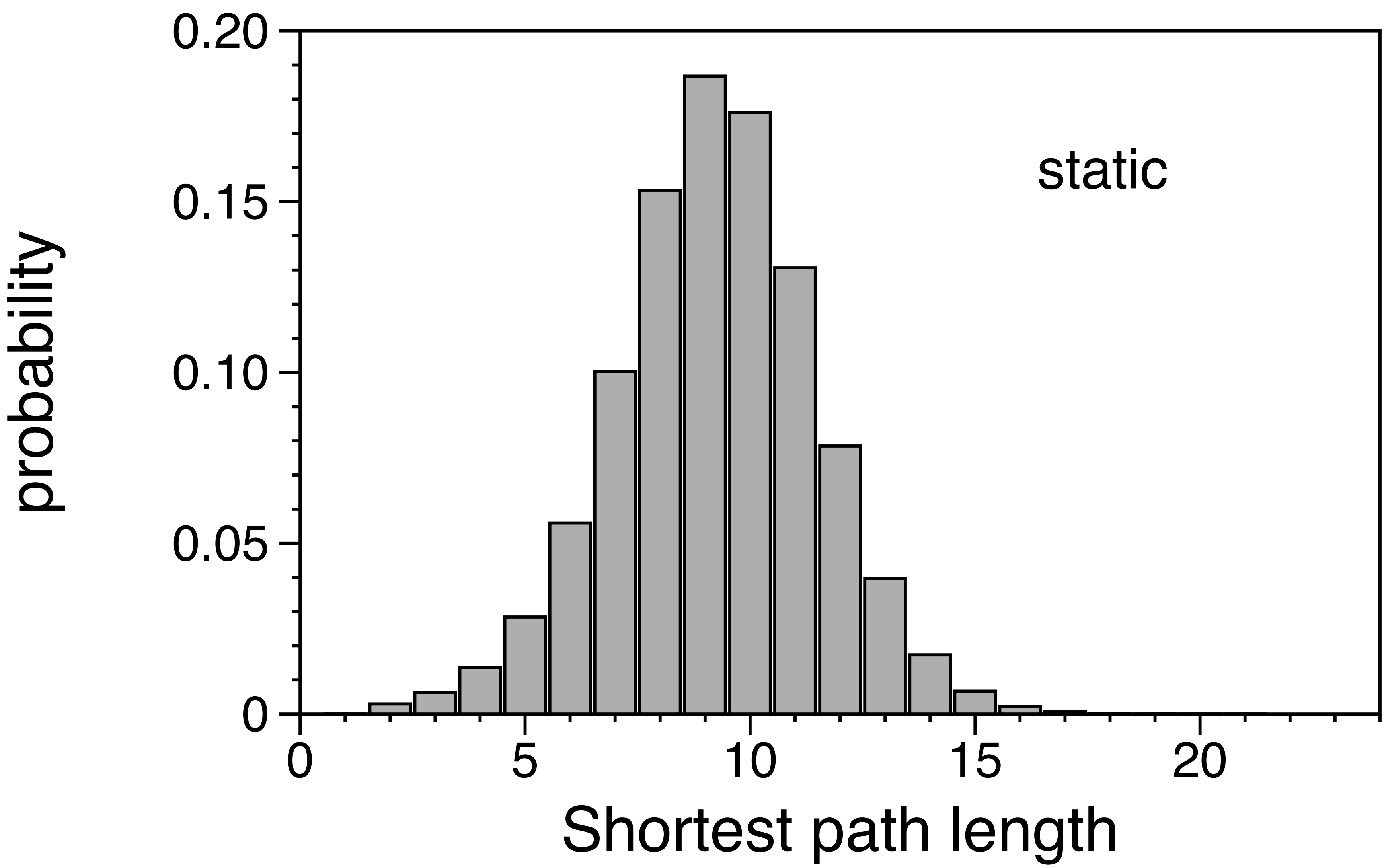
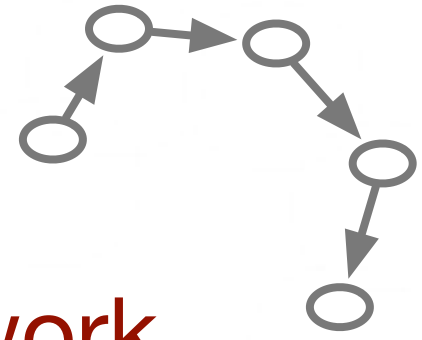


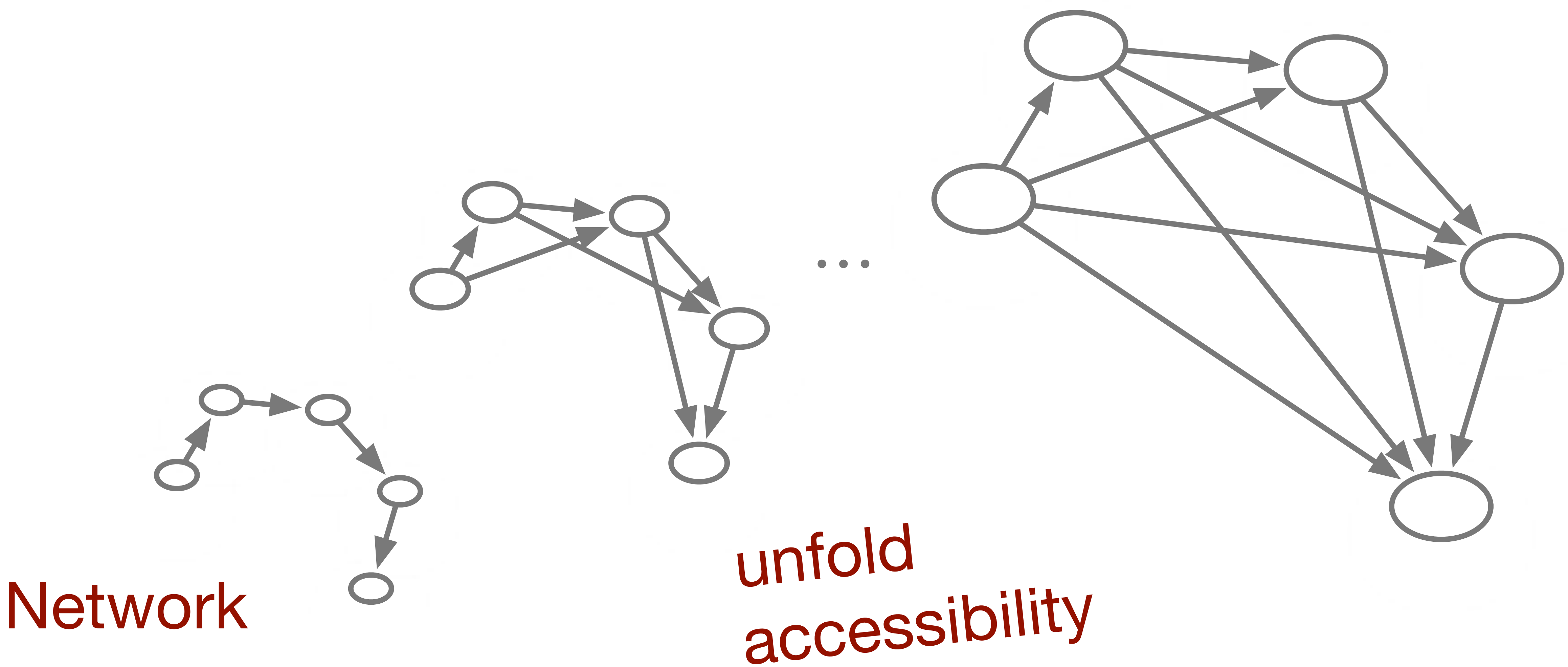
recap:
the idea



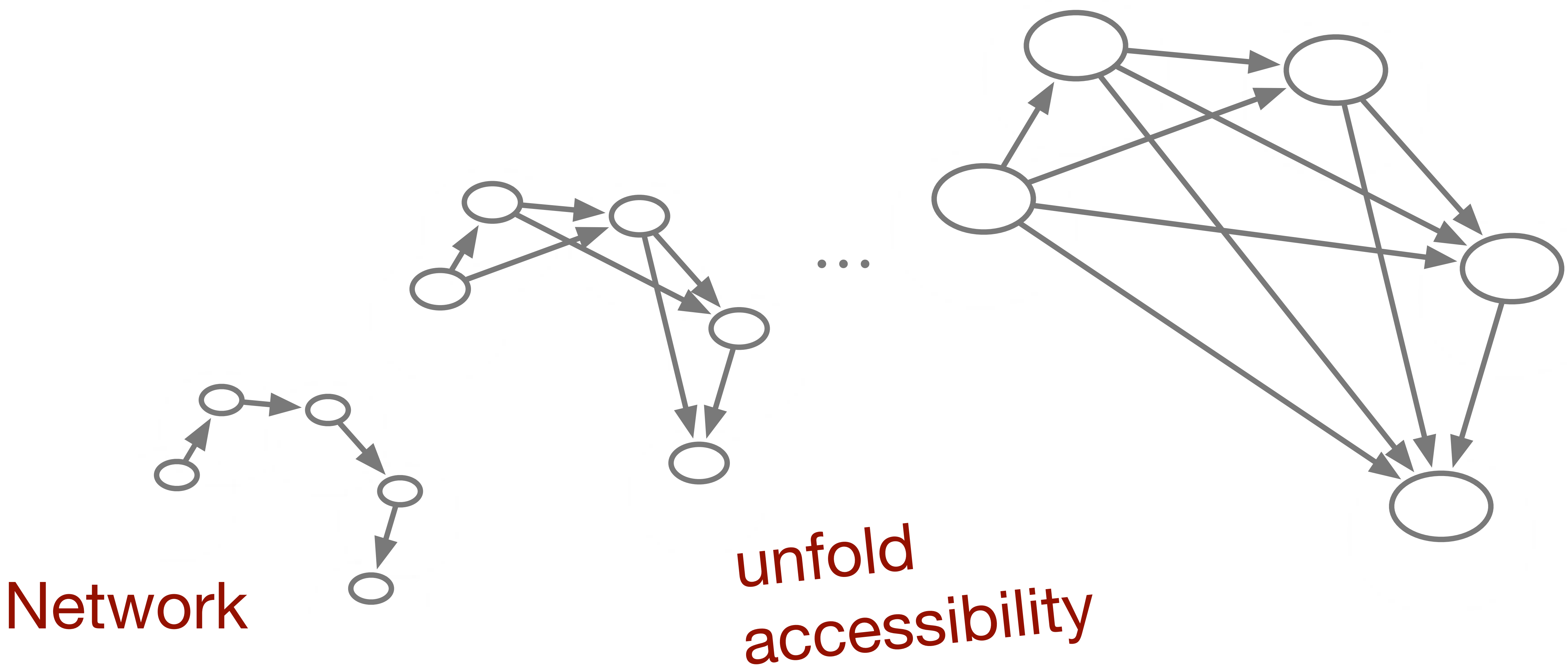
Network

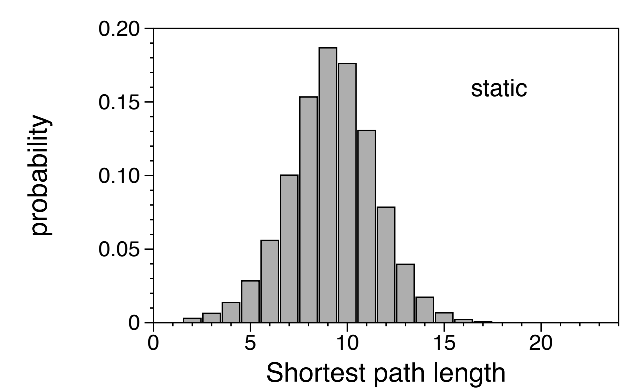
recap:

the idea



recap:
the idea





accessibility of

temporal networks

accessibility of **temporal** networks

static $$\mathbf{P} = \mathbf{A} + \mathbf{A}^2 + \mathbf{A}^3 + \cdots$$

accessibility of
temporal networks

static $\mathbf{P} = \mathbf{A} + \mathbf{A}^2 + \mathbf{A}^3 + \cdots$

temporal $\mathbf{A} \rightarrow \mathbf{A}_1, \mathbf{A}_2, \mathbf{A}_3, \ldots$

accessibility of

temporal networks

static $$\mathbf{P} = \mathbf{A} + \mathbf{A}^2 + \mathbf{A}^3 + \cdots$$

temporal $$\mathbf{A} \rightarrow \mathbf{A}_1, \mathbf{A}_2, \mathbf{A}_3, \ldots$$

replace $$\mathbf{A}^2 \rightarrow \mathbf{A}_1 \mathbf{A}_2$$

accessibility of
temporal networks

static $\mathbf{P} = \mathbf{A} + \mathbf{A}^2 + \mathbf{A}^3 + \cdots$

temporal $\mathbf{A} \rightarrow \mathbf{A}_1, \mathbf{A}_2, \mathbf{A}_3, \ldots$

replace $\mathbf{A}^2 \rightarrow \mathbf{A}_1 \mathbf{A}_2$ but also $\mathbf{A}^2 \rightarrow \mathbf{A}_1 \mathbf{A}_3$

accessibility of

temporal networks

$$\mathcal{P}_t = \mathbf{I} + \underbrace{\mathbf{A}_1 + \mathbf{A}_2 + \mathbf{A}_3 + \cdots + \mathbf{A}_t +}_{\mathbf{A}}$$

$$+ \underbrace{\mathbf{A}_1\mathbf{A}_2 + \mathbf{A}_2\mathbf{A}_3 + \mathbf{A}_1\mathbf{A}_3 + \mathbf{A}_1\mathbf{A}_2\mathbf{A}_3 + \cdots}_{\mathcal{O}(\mathbf{A}_i^2)}$$

$$= \prod_{i=1}^{t} (\mathbf{I} + \mathbf{A}_i)$$

Lentz et al., Phys. Rev. Lett, 2013
Grindrod et al., Phys. Rev. E, 2011

accessibility of
temporal networks

$$\mathcal{P}_t = \mathbf{I} + \underbrace{\mathbf{A}_1 + \mathbf{A}_2 + \mathbf{A}_3 + \cdots + \mathbf{A}_t +}_{\mathbf{A}}$$

$$+ \underbrace{\mathbf{A}_1\mathbf{A}_2 + \mathbf{A}_2\mathbf{A}_3 + \mathbf{A}_1\mathbf{A}_3 + \mathbf{A}_1\mathbf{A}_2\mathbf{A}_3 + \cdots}_{\mathcal{O}(\mathbf{A}_i^2)}$$

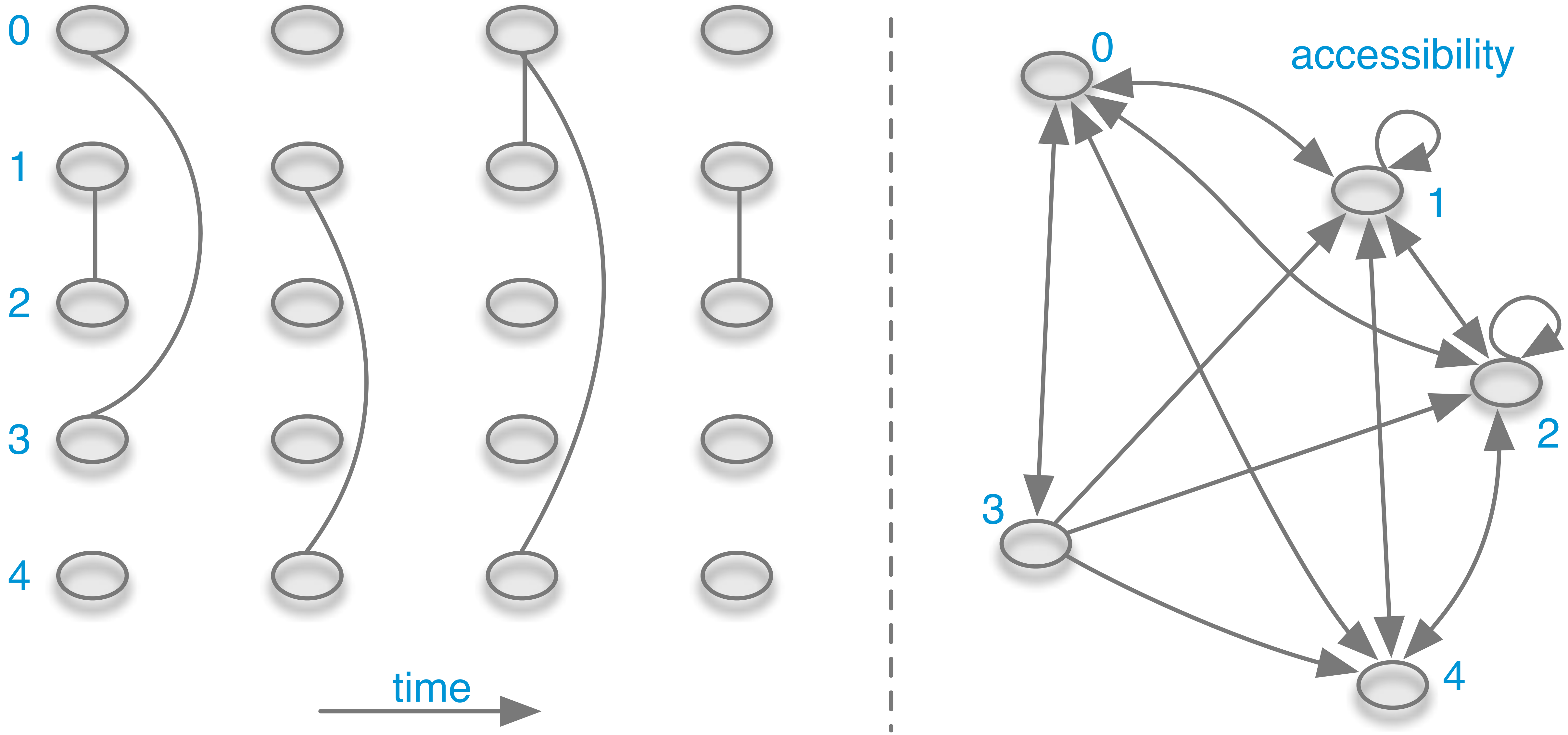
$$= \prod_{i=1}^{t} (\mathbf{I} + \mathbf{A}_i)$$

easy to implement

Lentz et al., Phys. Rev. Lett, 2013
Grindrod et al., Phys. Rev. E, 2011

Compute **temporal** accessibility graph

Tutorial.



implementation.

implementation.

$$\mathcal{P}_n = \bigwedge_{i=1}^{n} (\mathbf{I} \vee \mathbf{A}_i)$$

Boolean matrices

implementation.

Sparsity

`dense matrix` `sparse matrix`

$$\mathcal{P}_n = \bigwedge_{i=1}^{n} (\mathbf{I} \vee \mathbf{A}_i)$$

Boolean matrices

implementation.

Partitioning. For each node

$$i(t) = \left[\prod_{t'=0}^{t} \left(\mathbf{I} + \mathbf{A}_{t'} \right) \right] i(0)$$

Sparsity

`dense matrix` `sparse matrix`

$$\mathcal{P}_n = \bigwedge_{i=1}^{n} \left(\mathbf{I} \vee \mathbf{A}_i \right)$$

Boolean matrices

unfolding accessibility

software-tool

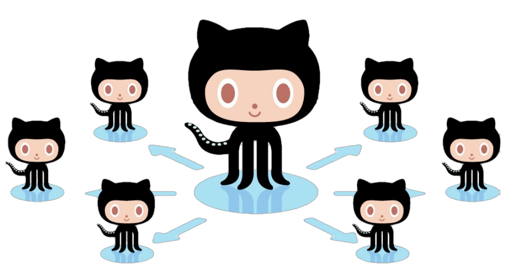
easy-usage object based tool

github repositories:

`hartmutlentz/TemporalNetworkAccessibility`

`andreaskoher/TemporalNetworkEpidemics`

Download



Language

python

Uses

example

usage

example

usage

Input:

Edges.txt

0	1	0
2	1	0
3	1	0
0	4	21
5	6	47
5	6	73
7	8	105
9	10	121
11	12	124
13	12	126
7	14	168
15	16	171
17	12	172
18	19	179
18	20	179
21	19	179
22	23	184
24	25	185
26	27	192
17	28	193

example

usage

```python
from AdjacencyMatrixSequence import AdjMatrixSequence
```

example

usage

```
from AdjacencyMatrixSequence import AdjMatrixSequence

At = AdjMatrixSequence(„Edges.txt", directed=True)
```

example

usage

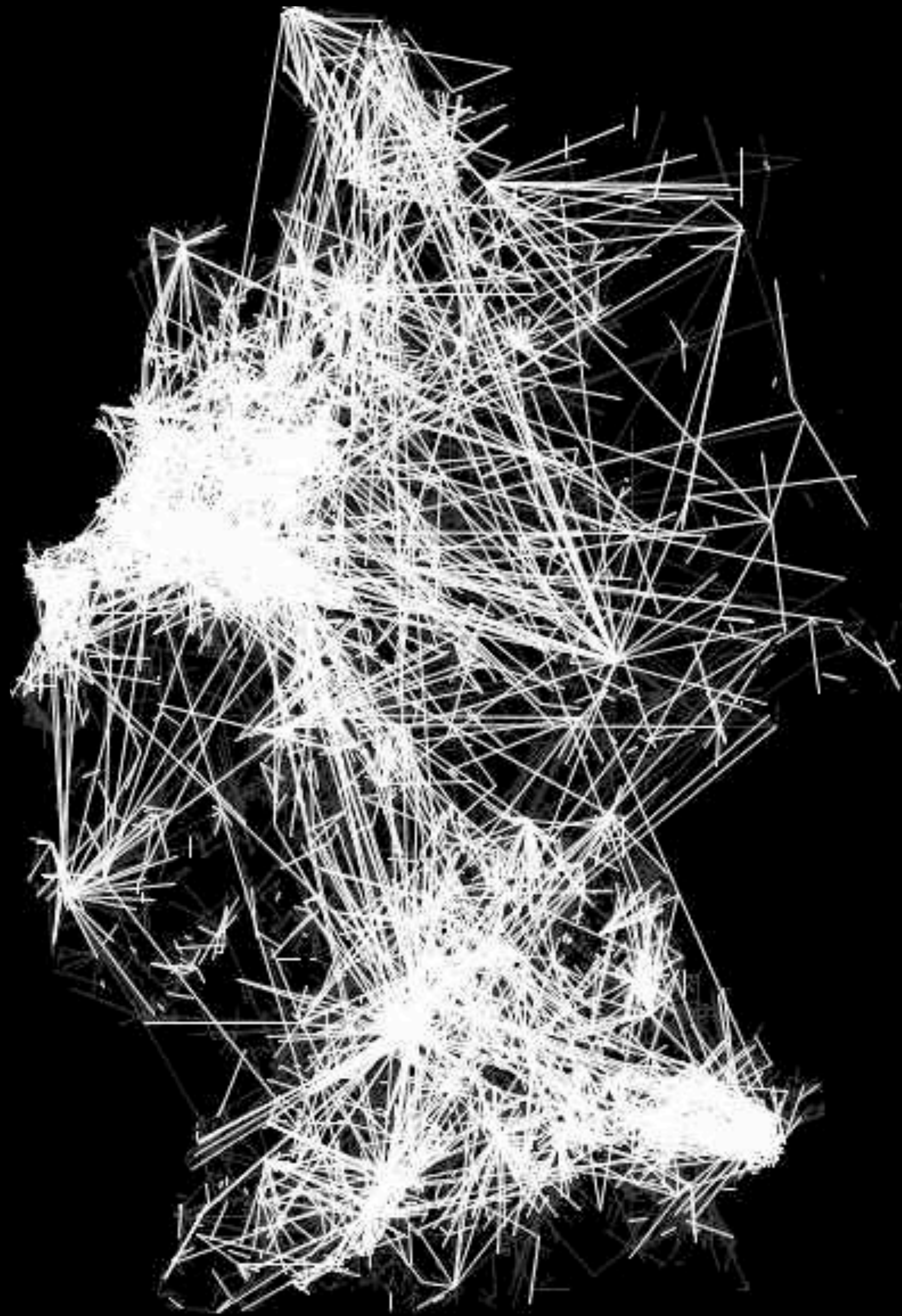
```
from AdjacencyMatrixSequence import AdjMatrixSequence

At = AdjMatrixSequence(„Edges.txt", directed=True)

c = At.unfold_accessibility()
h = np.gradient(c)
```

application.

livestock network data-set

livestock pig trade

livestock network data-set

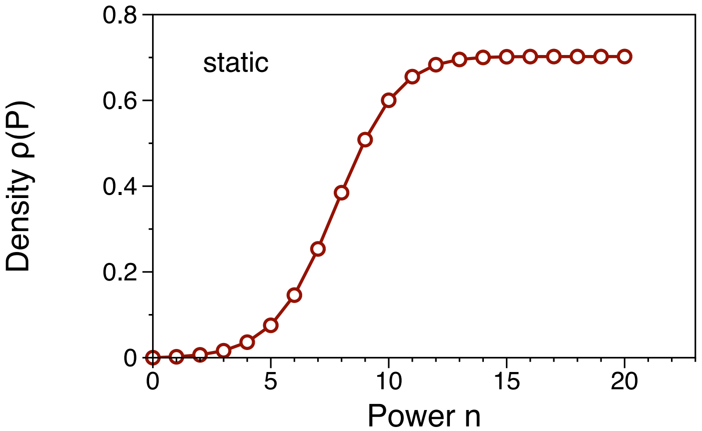


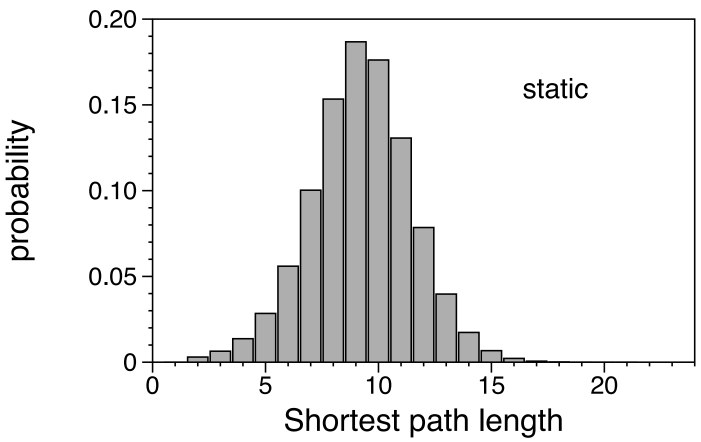
livestock pig trade

characteristic time scales

results I

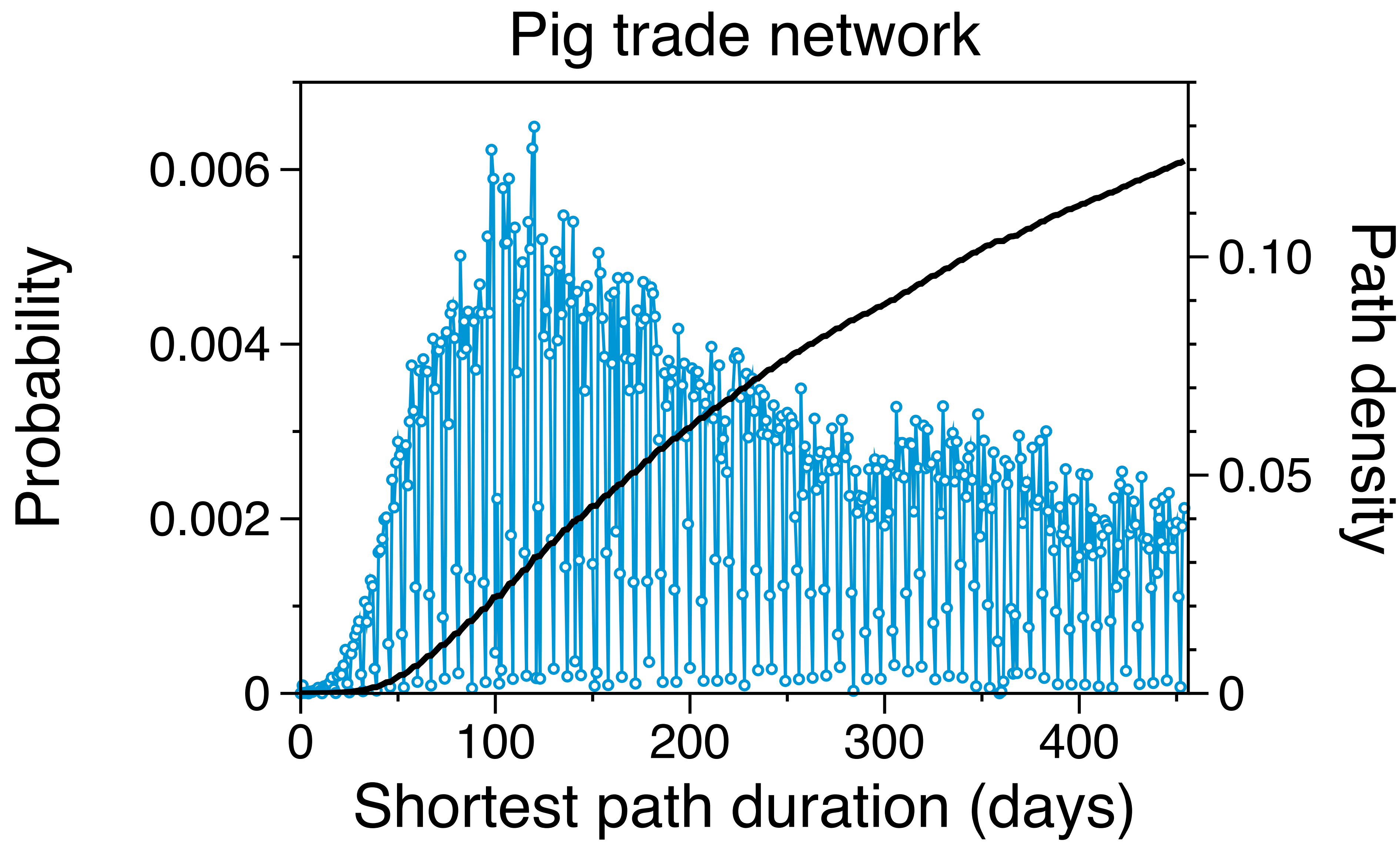
static random network



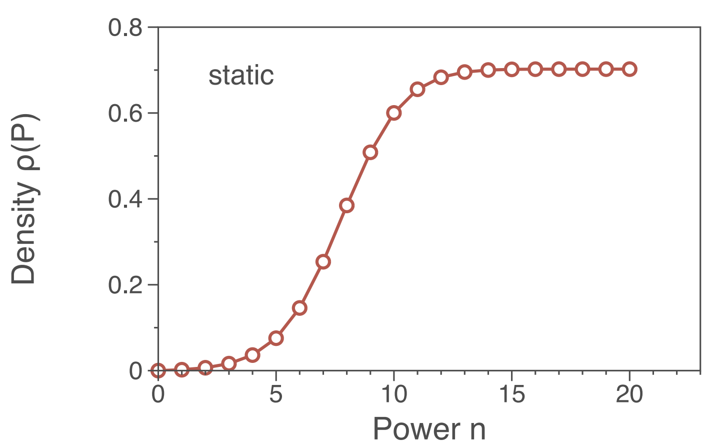


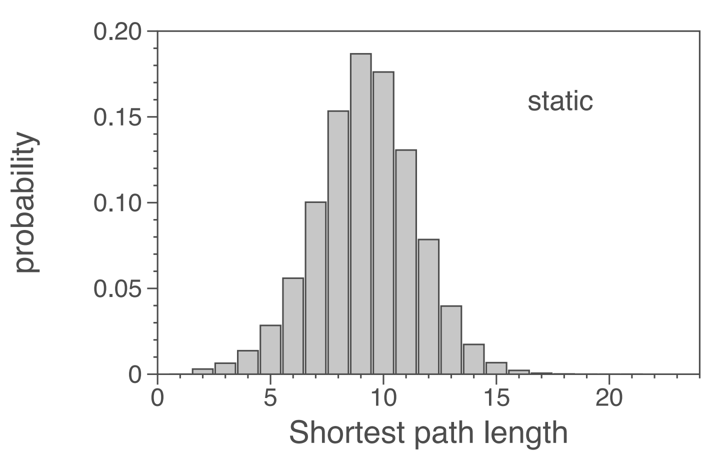
characteristic time scales

results I



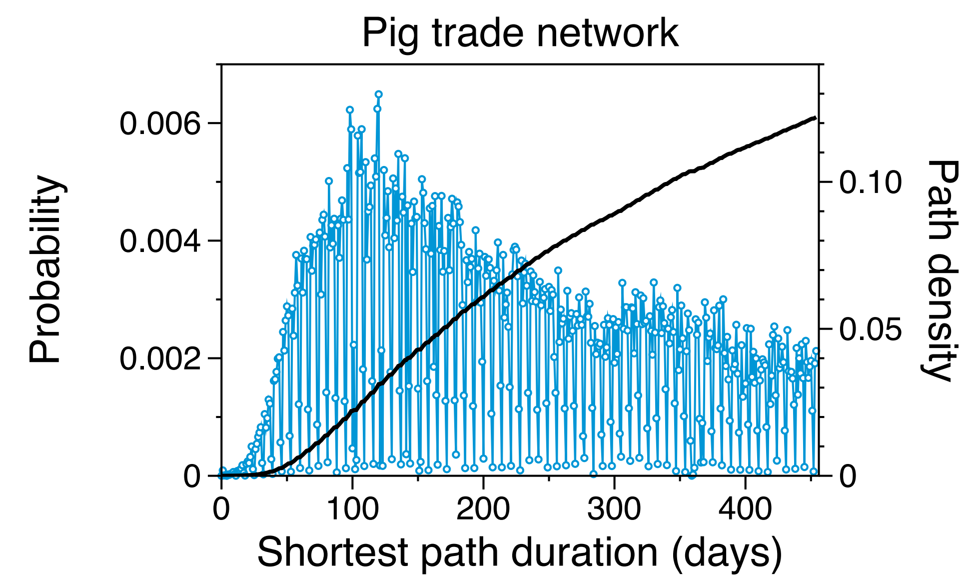
static random network

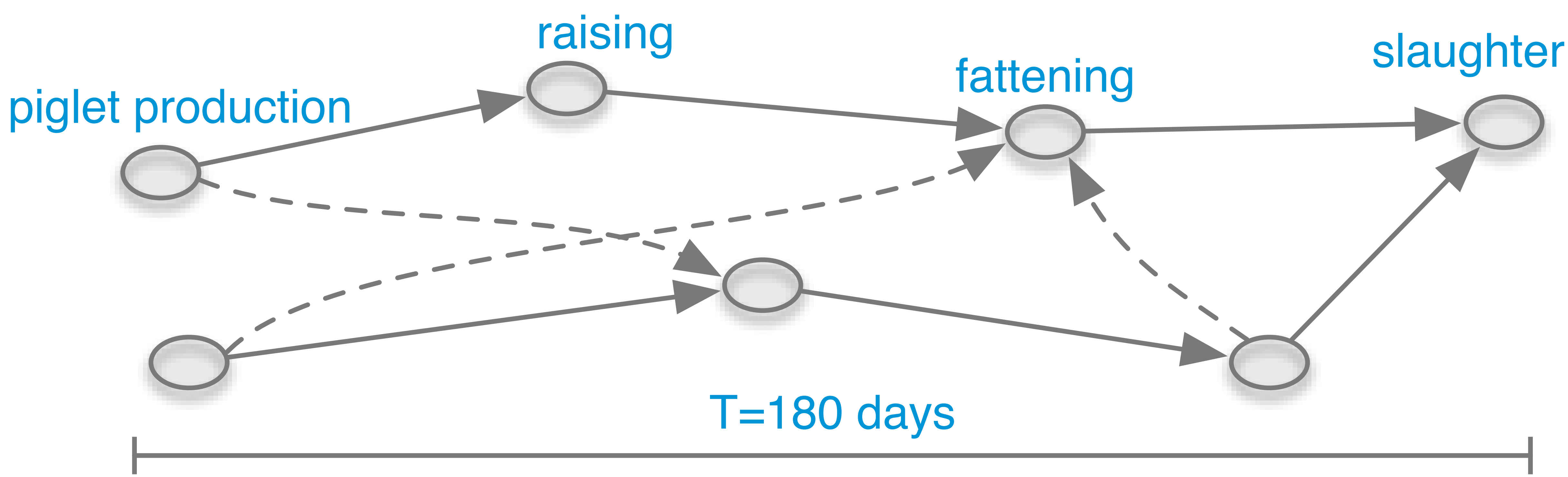




characteristic time scales

results I

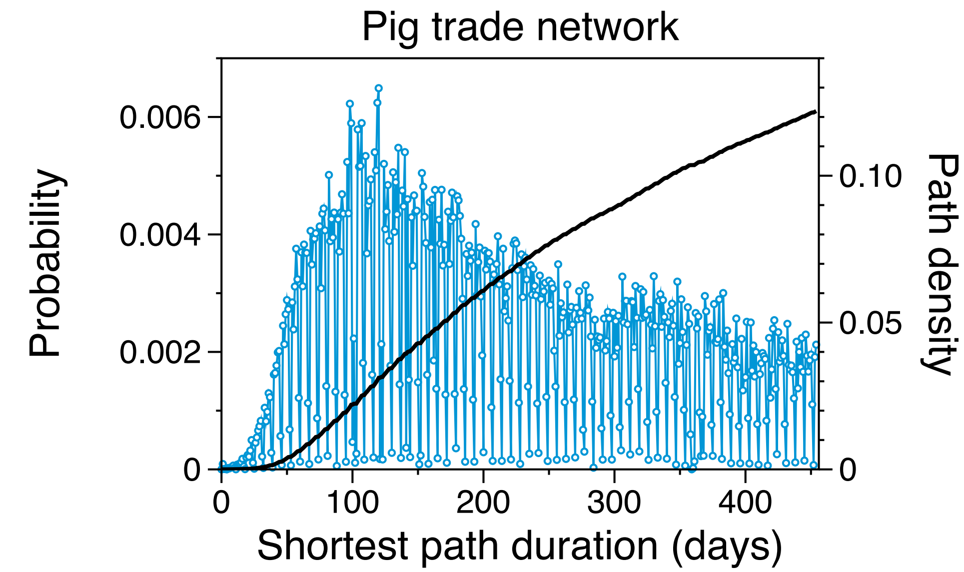




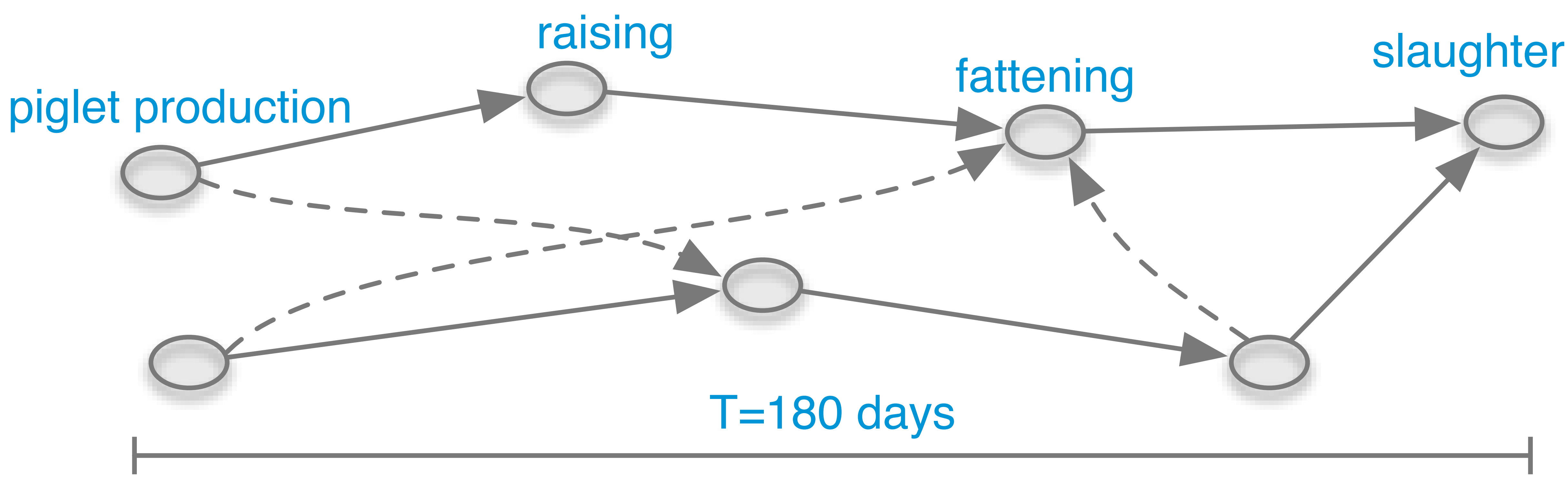
Lentz et al., PLOS ONE, 2016

characteristic time scales

results I



Pork production chain



Lentz et al., PLOS ONE, 2016

causal fidelity

results II

causal fidelity

results II

comparison:
static vs. temporal accessibility-graph

causal fidelity

results II

comparison:
static vs. temporal accessibility-graph

number of paths:

static representation:	1.4 billion
temporal representation:	1.0 billion

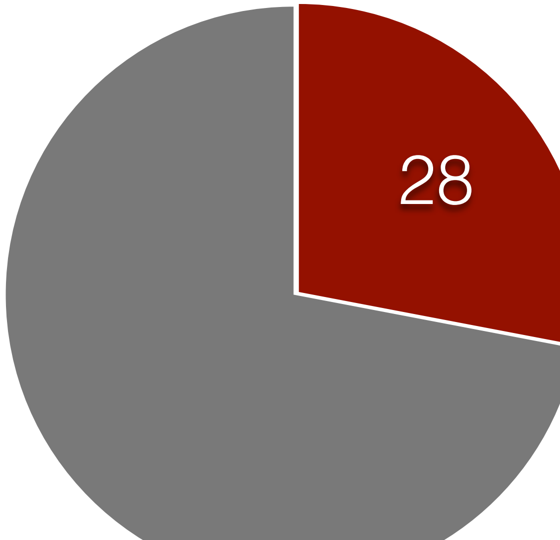
causal fidelity
results II

comparison:
static vs. temporal accessibility-graph

number of paths:

static representation:	1.4 billion
temporal representation:	1.0 billion

28 out of 100 paths in the aggregated network do actually not exist!



causal fidelity

results II

comparison:

static vs. temporal accessibility-graph

Definition: causal fidelity

$$\boxed{c = \frac{\rho(\mathcal{P}_T)}{\rho(\mathbf{P}_T)}}$$

causal fidelity

results II

comparison:

static vs. temporal accessibility-graph

Definition: causal fidelity

$$\boxed{c = \frac{\rho(\mathcal{P}_T)}{\rho(\mathbf{P}_T)}}$$

temporal (real) path density

static path density

causal fidelity

results II

comparison:

static vs. temporal accessibility-graph

Definition: causal fidelity

temporal (real) path density

$$\boxed{c = \frac{\rho(\mathcal{P}_T)}{\rho(\mathbf{P}_T)}}$$

static path density

pig trade net

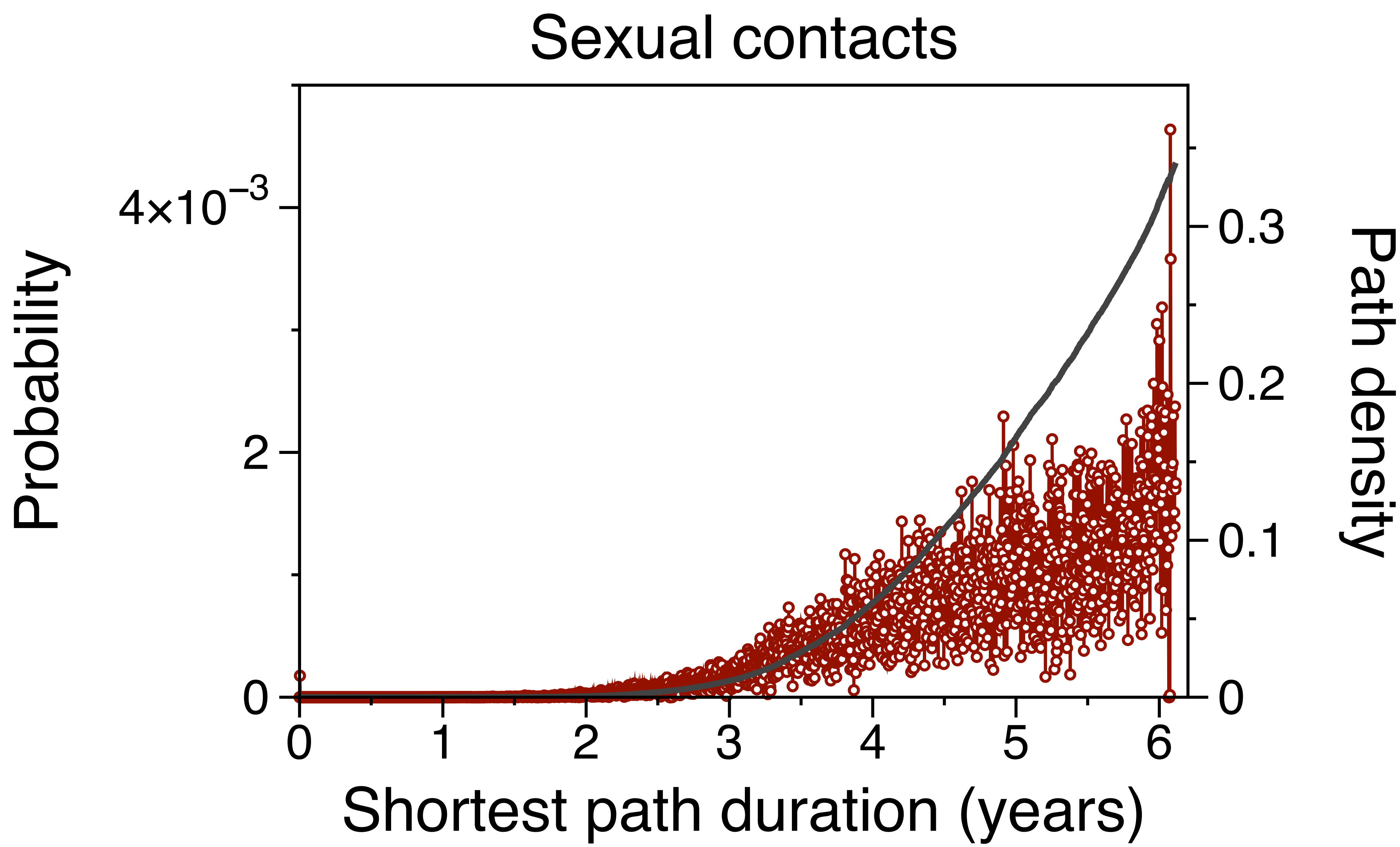
$c = 0.72$

further datasets.

sexual contact network
Rocha et al., PNAS 2010

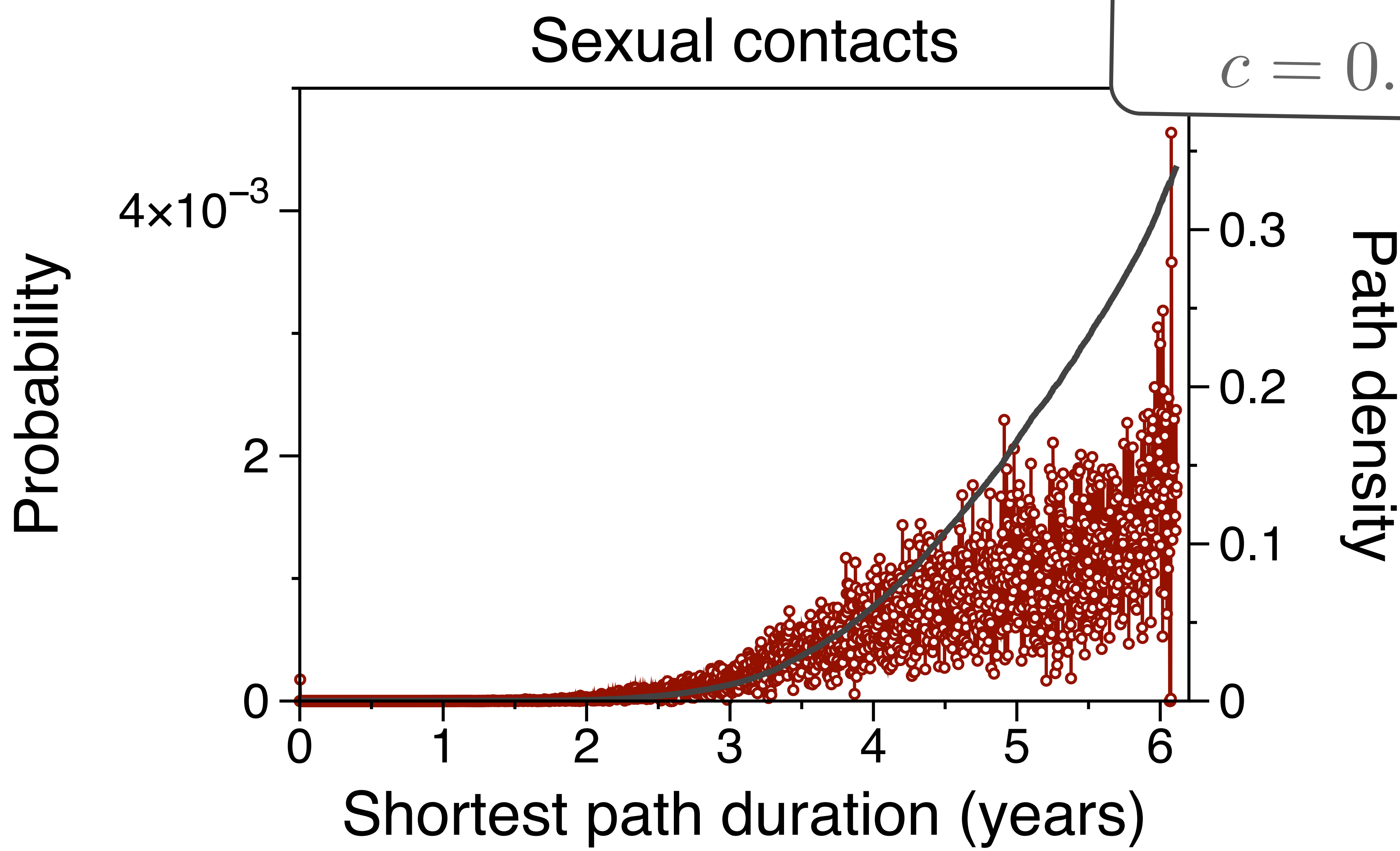
face-to-face conference contacts
sociopatterns.org

sexual contact network
results III



sexual contact network

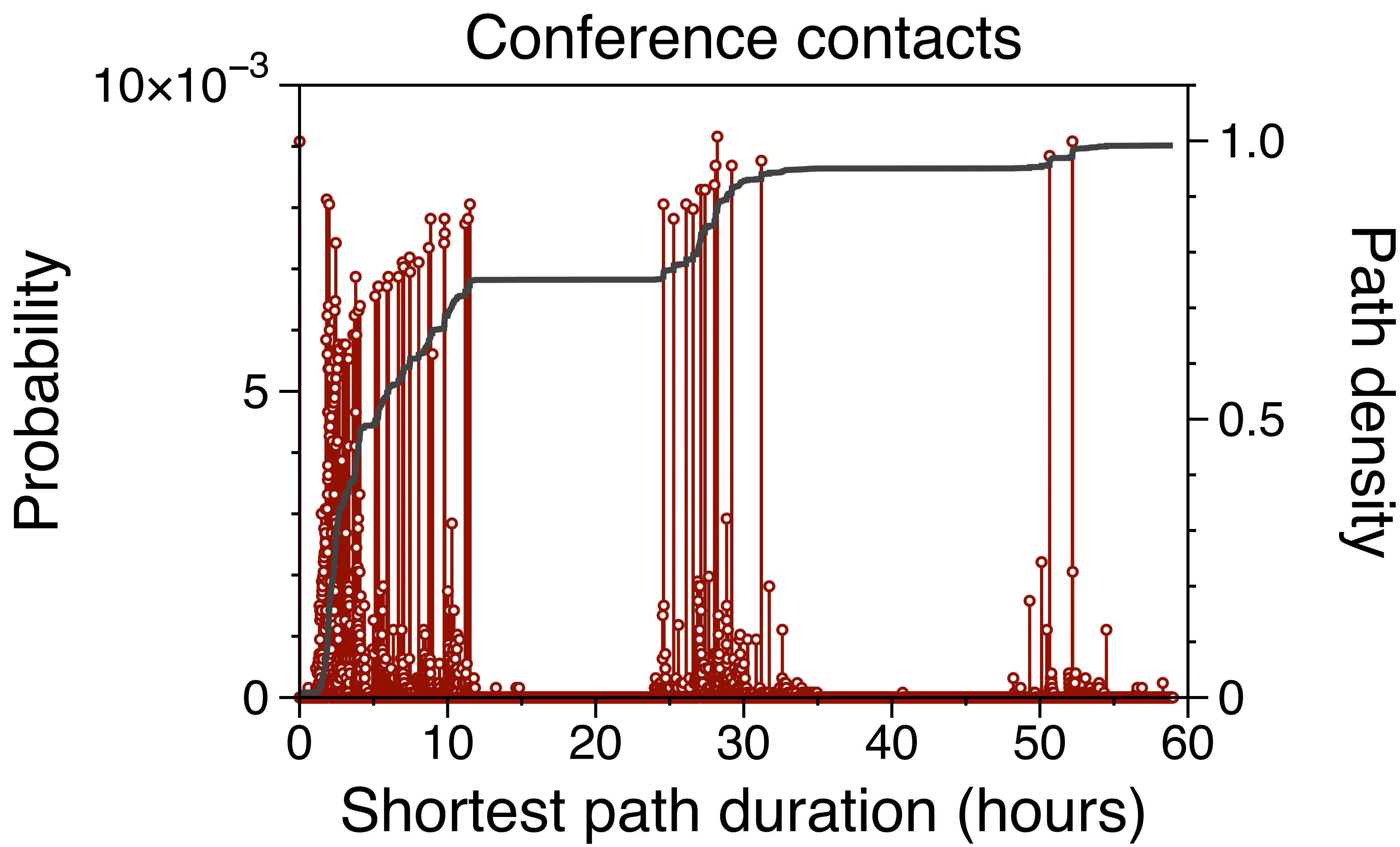
results III



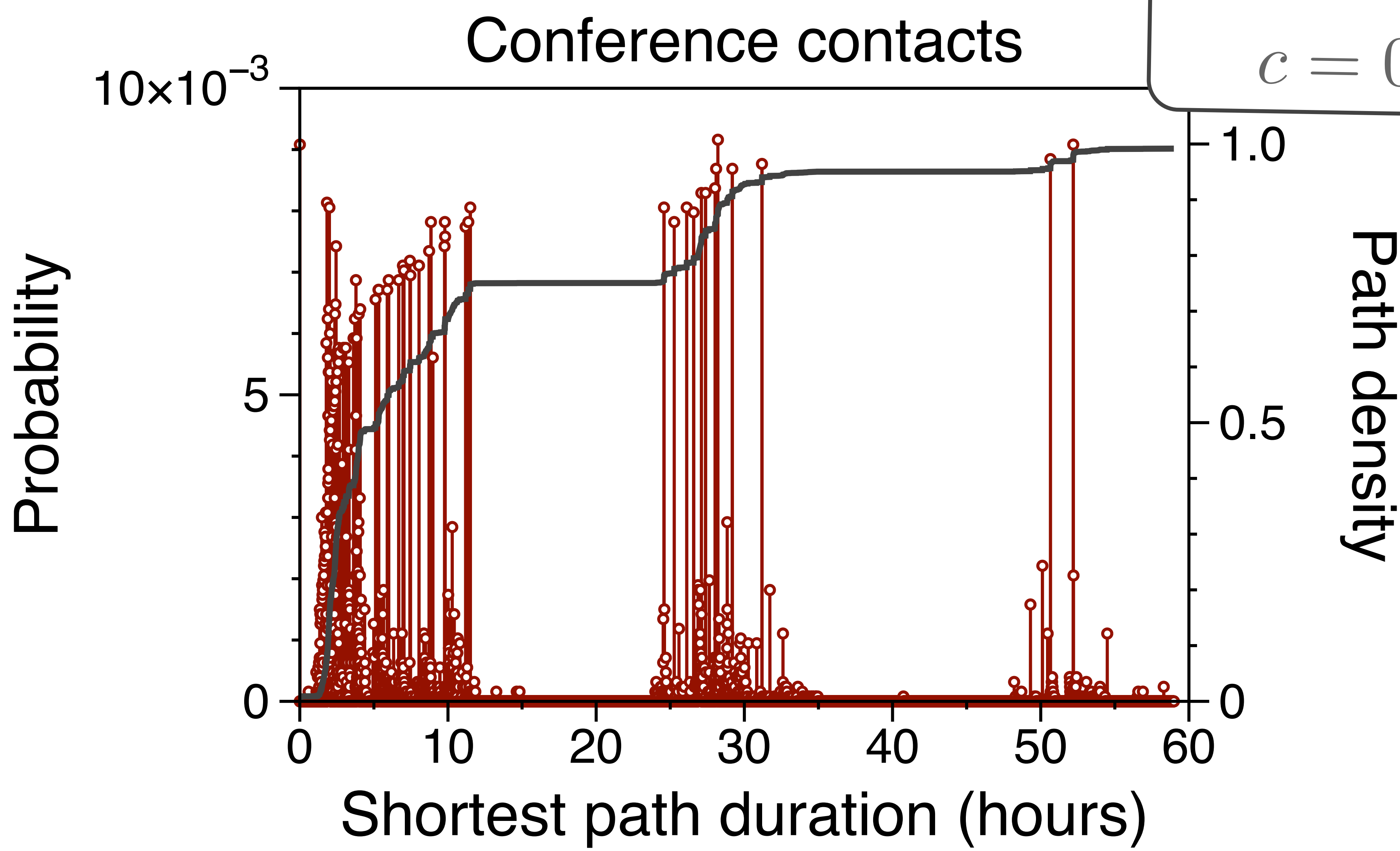
causal fidelity

$c = 0.38$

conference contacts
results III



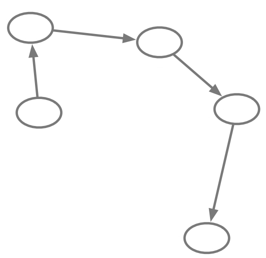
conference contacts
results III



causal fidelity

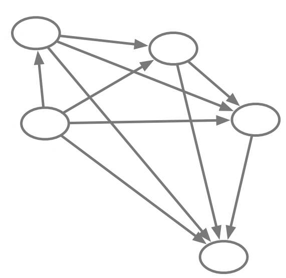
$c = 0.99$

recap.

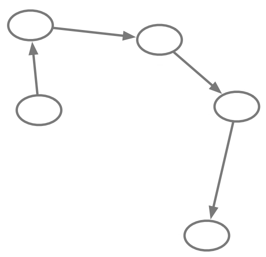


network

perspective:
accessibility

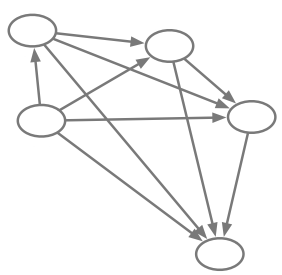


recap.



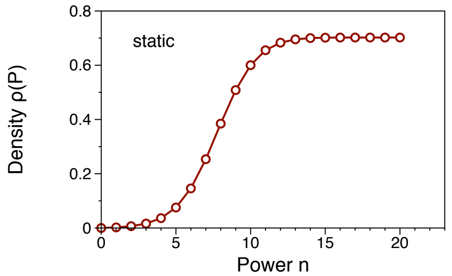
network

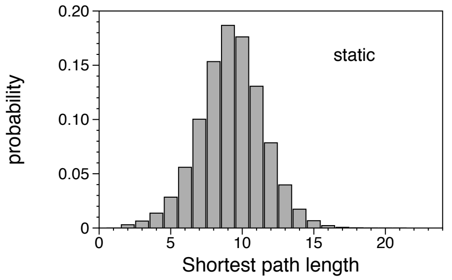
perspective:
accessibility



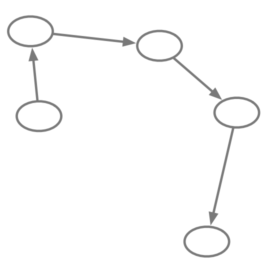
new

path-lengths



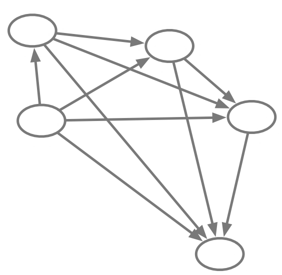


recap.



network

perspective:
accessibility

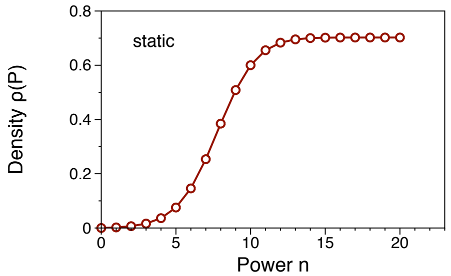


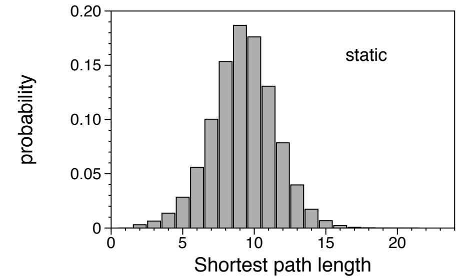
new

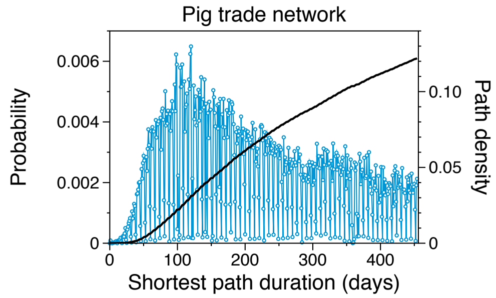
path-lengths

new:
temporal
accessibility

$$\mathcal{P}_n = \prod_{i=1}^{n} (\mathbf{I} + \mathbf{A}_i)$$

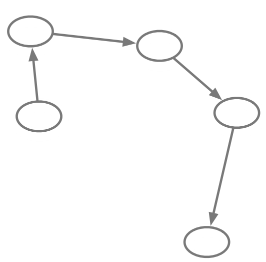




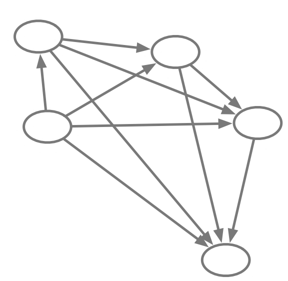


recap.

network



perspective:
accessibility



new

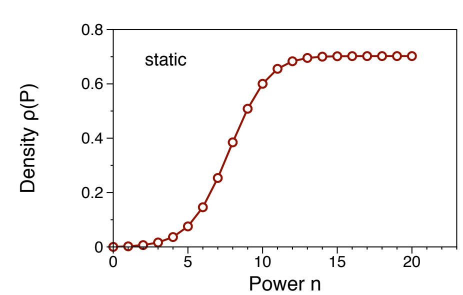
path-lengths

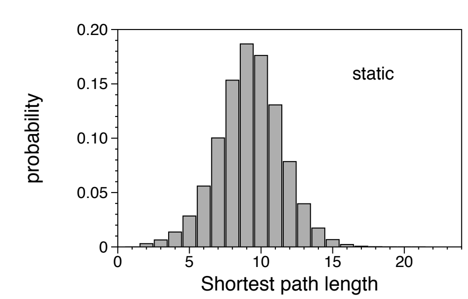
new:
causal fidelity

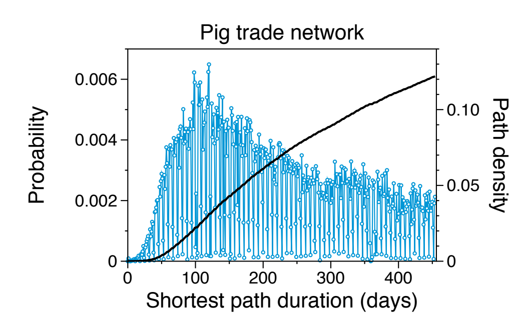
$$c = \frac{\rho(\mathcal{P}_T)}{\rho(\mathbf{P}_T)}$$

new:
temporal accessibility

$$\mathcal{P}_n = \prod_{i=1}^{n} (\mathbf{I} + \mathbf{A}_i)$$



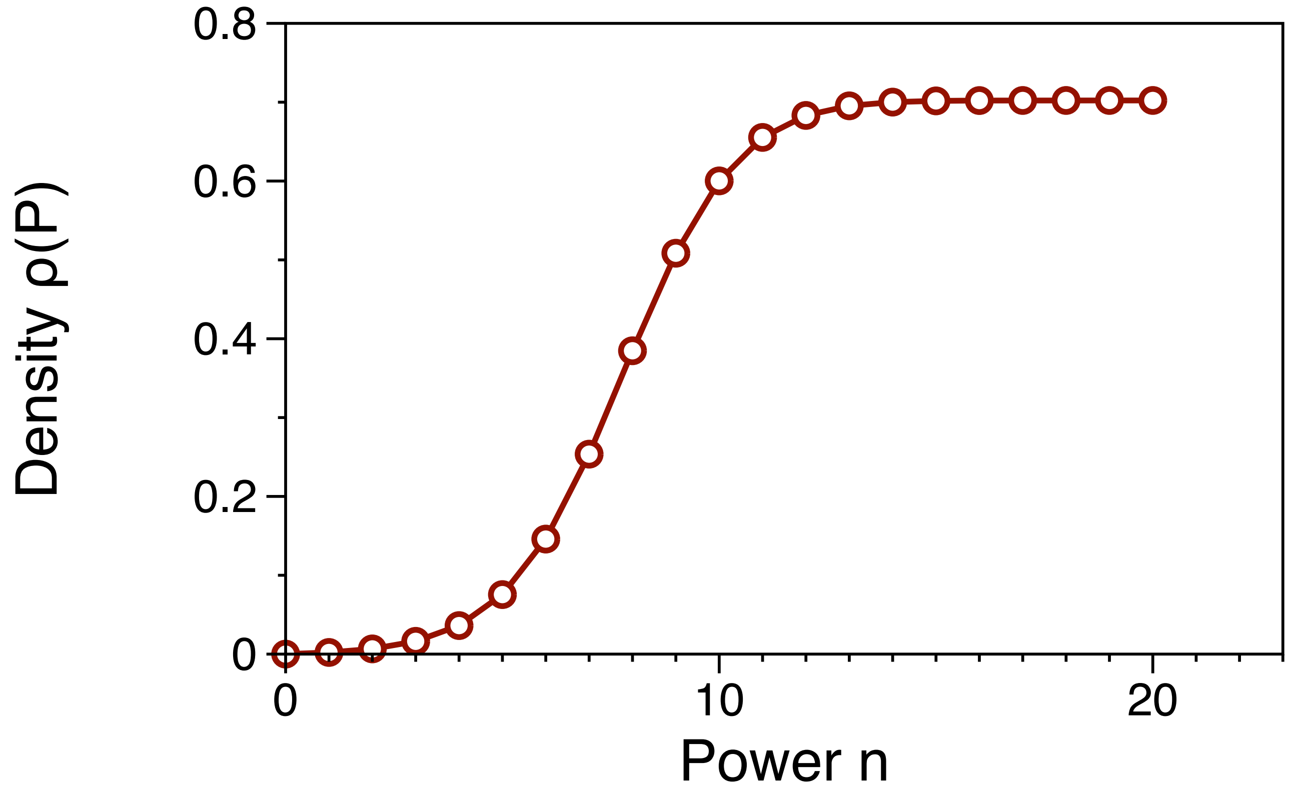




paths vs.
epidemic models

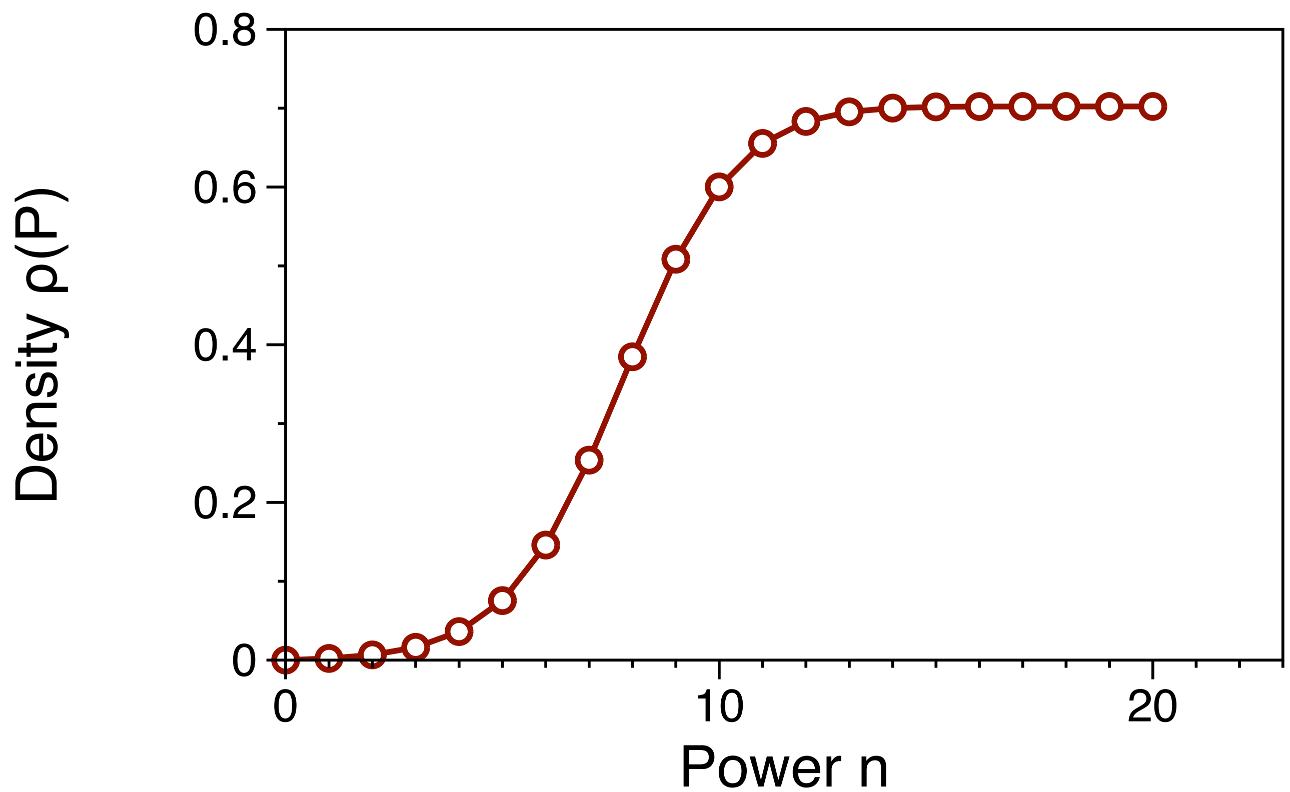
so far...

so far...



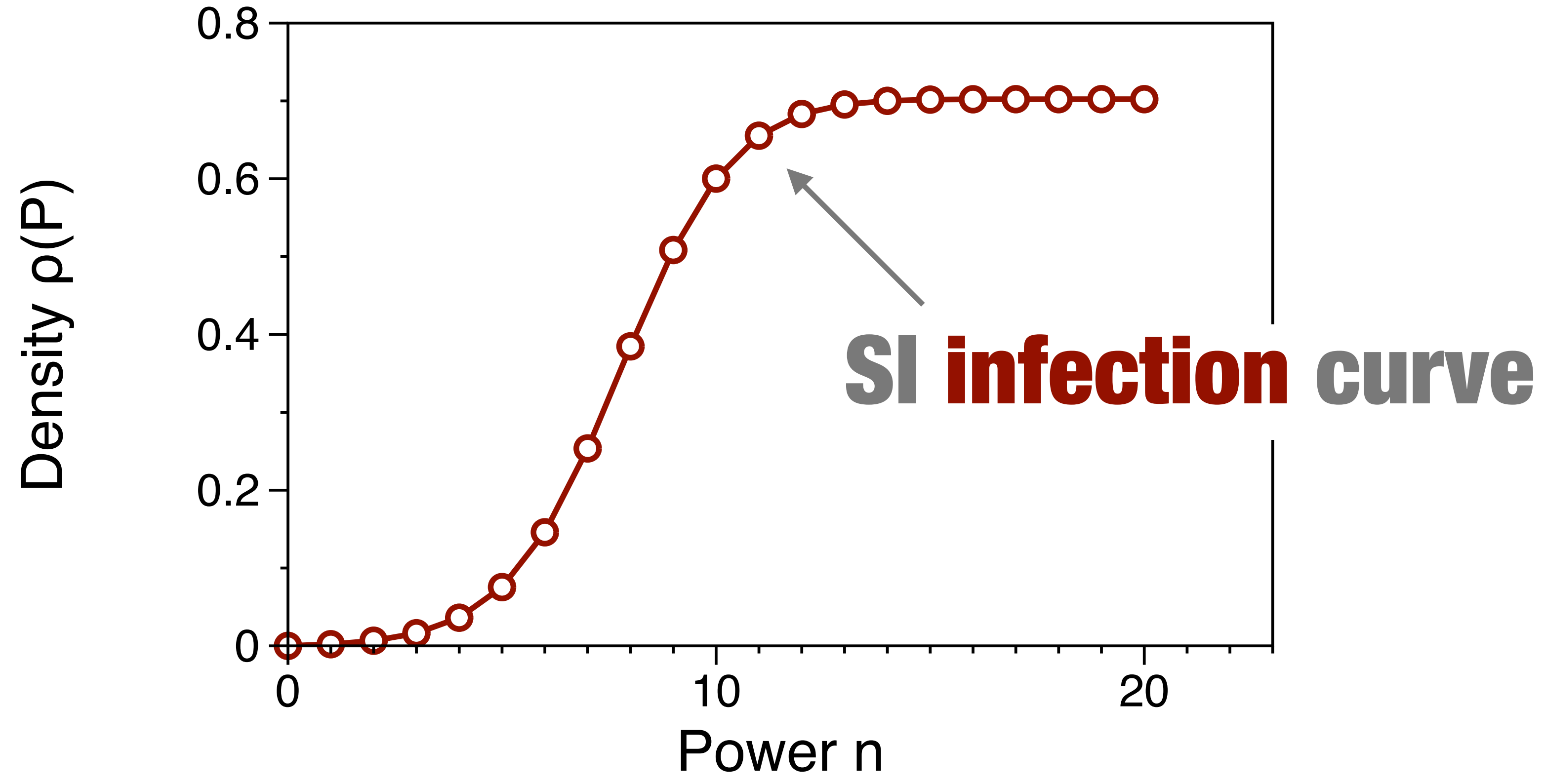
so far...

density of infectious paths



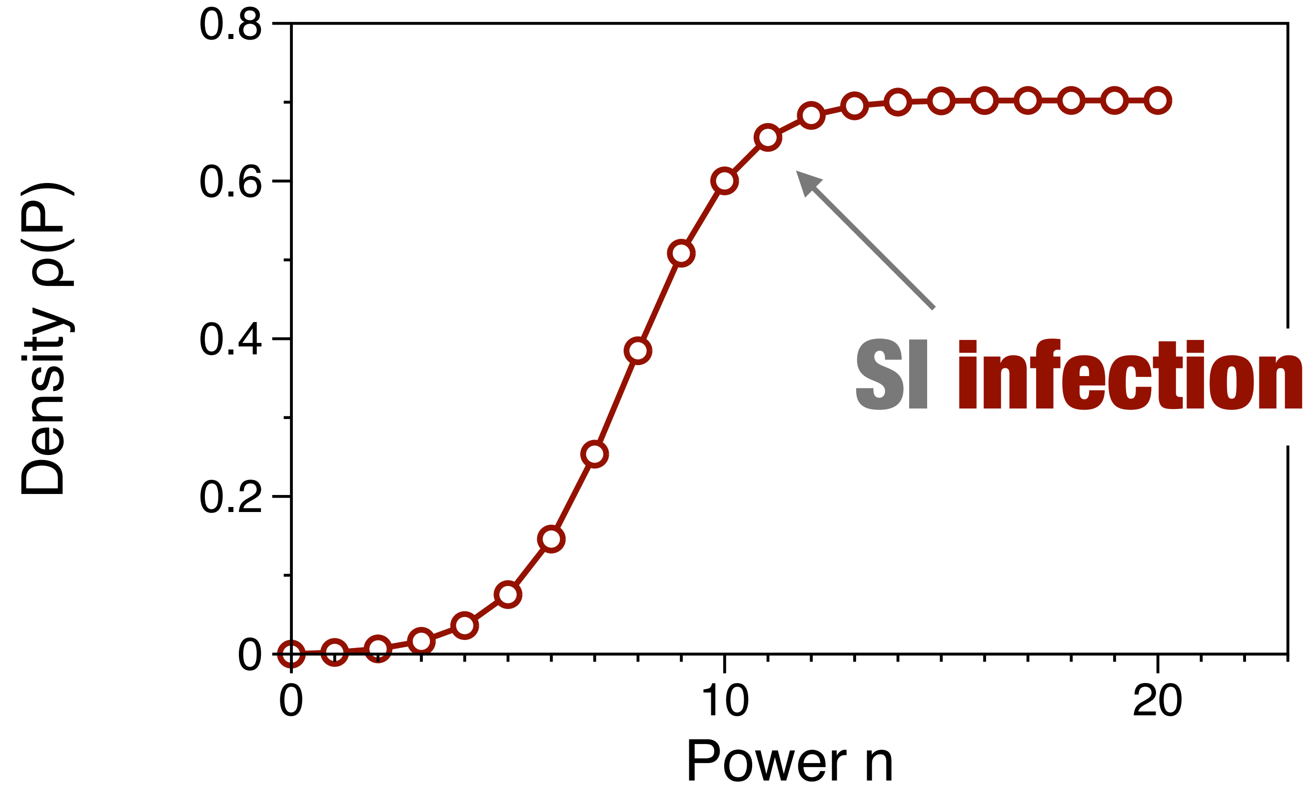
so far...

density of
infectious paths



so far...

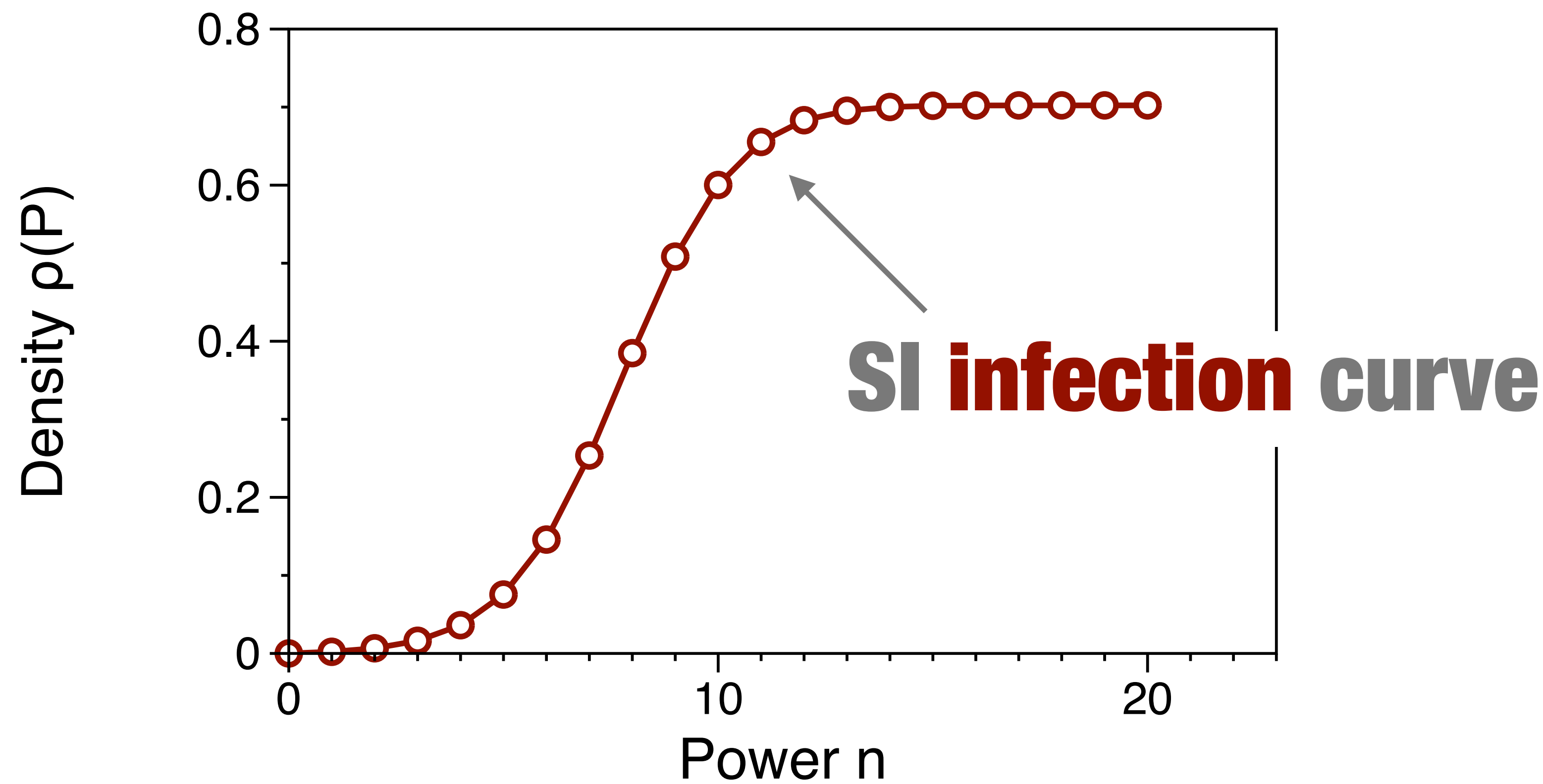
density of **infectious** paths



SI type process with infectious period $\tau = \infty$

so far...

density of **infectious** paths



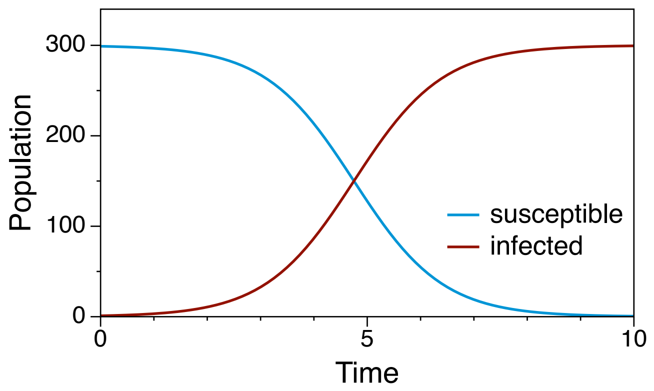
SI type process with infectious period $\tau = \infty$

What if infectious period τ is finite?

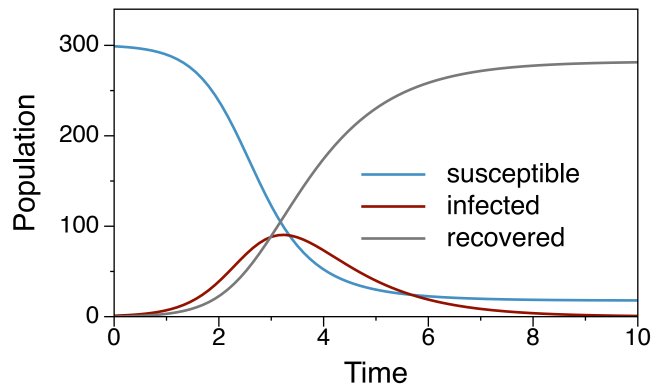
→ **SIR type** process!

classical models:
SI vs. SIR

SI model

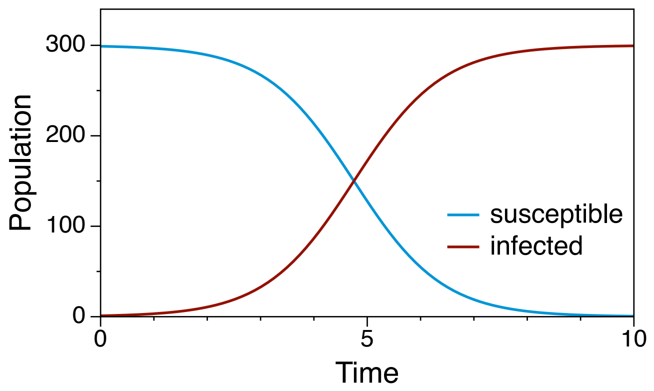


SIR model

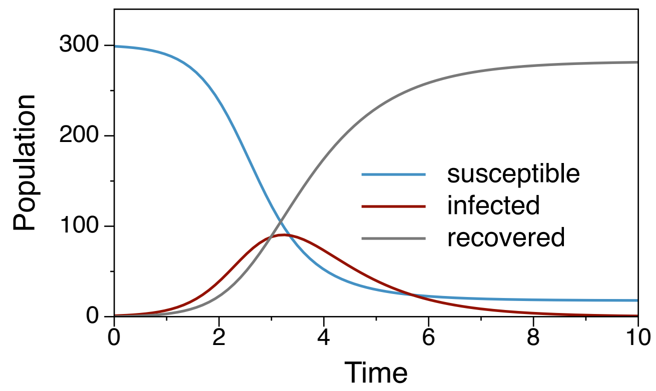


classical models:
SI vs. SIR

SI model



SIR model



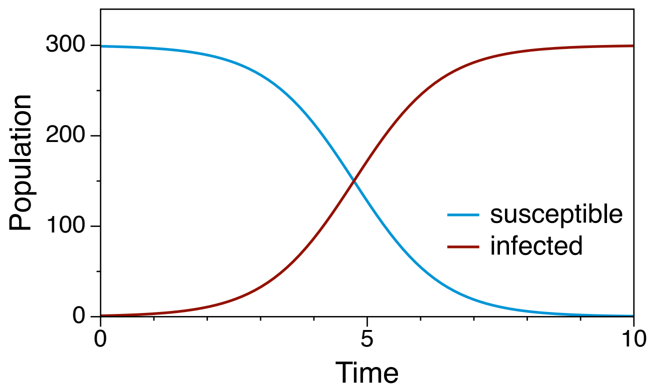
model

$$\frac{dS}{dt} = -\beta S I$$

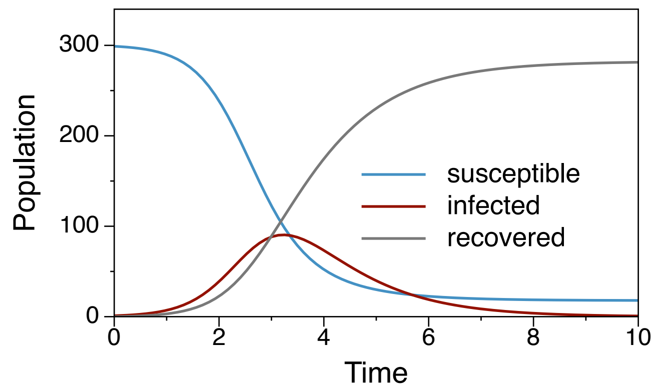
$$\frac{dI}{dt} = \beta S I$$

classical models:
SI vs. SIR

SI model



SIR model



model

$$\frac{dS}{dt} = -\beta SI$$

$$\frac{dI}{dt} = \beta SI$$

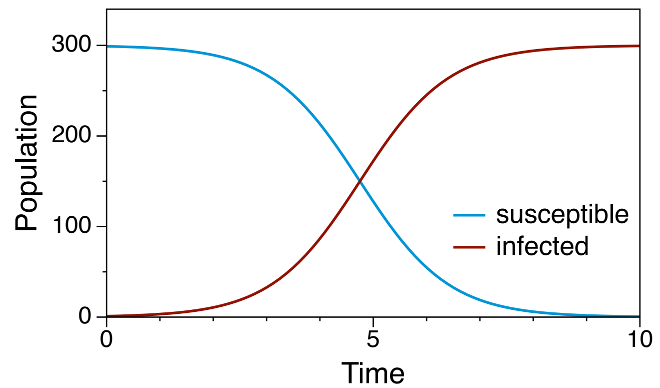
incidence

$$\frac{dI}{dt} = \beta(N - I)I$$

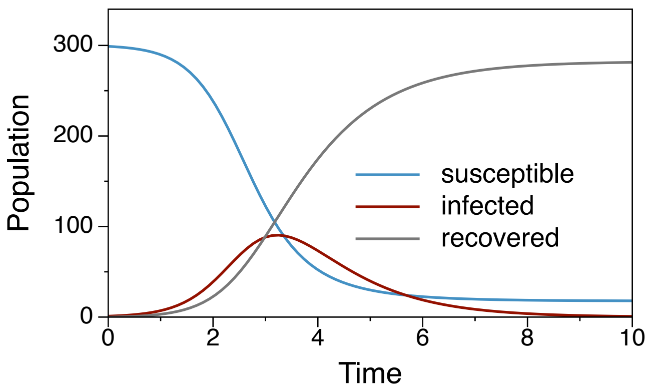
classical models:

SI vs. SIR

SI model



SIR model



model

$$\frac{dS}{dt} = -\beta S I$$

$$\frac{dI}{dt} = \beta S I$$

incidence

$$\frac{dI}{dt} = \beta (N - I) I$$

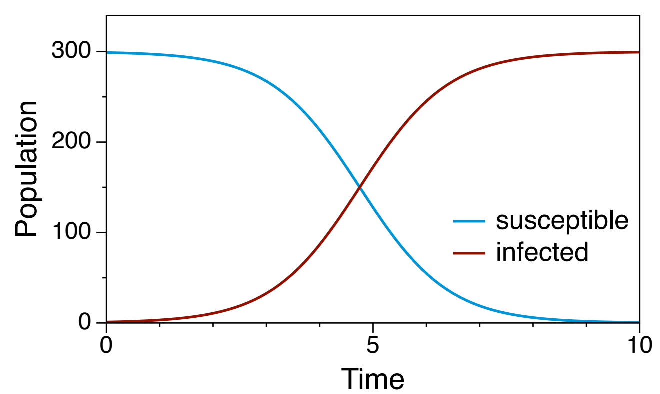
solution

$$I(t) = \frac{I_0 N}{I_0 + (N - I_0) \mathrm{e}^{-\beta N t}}$$

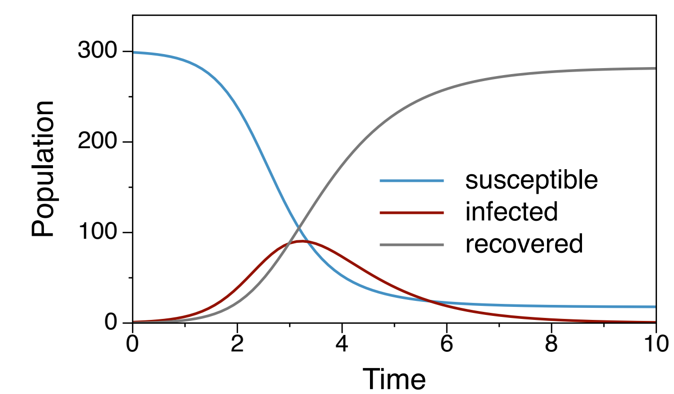
classical models:

SI vs. SIR

SI model



SIR model



model

dS

temporal **network** case

$$\langle I(t) \rangle = \rho(\mathcal{P}(t))$$

$$\beta = 1$$

dt

incidence

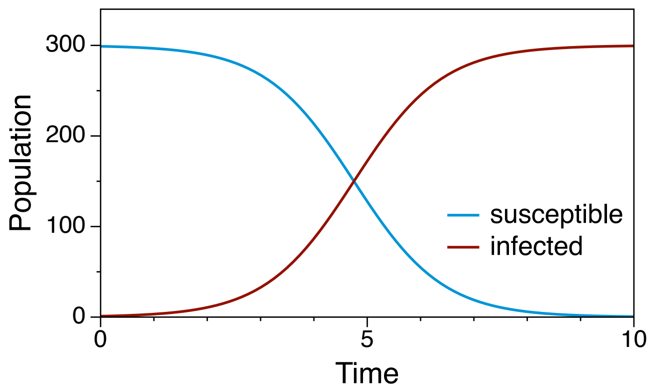
solution

$$I(t) = \frac{I_0 N}{I_0 + (N - I_0)\mathrm{e}^{-\beta N t}}$$

classical models:

SI vs. SIR

SI model



model

dS

temporal network case

$$\langle I(t) \rangle = \rho(\mathcal{P}(t))$$

$$\beta = 1$$

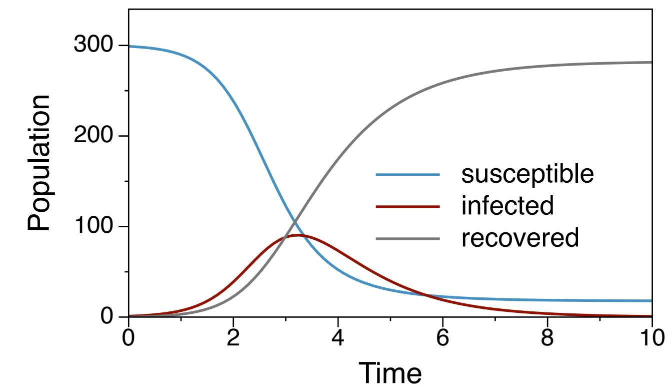
incidence

dt

solution

$$I(t) = \frac{I_0 N}{I_0 + (N - I_0)\mathrm{e}^{-\beta N t}}$$

SIR model



model

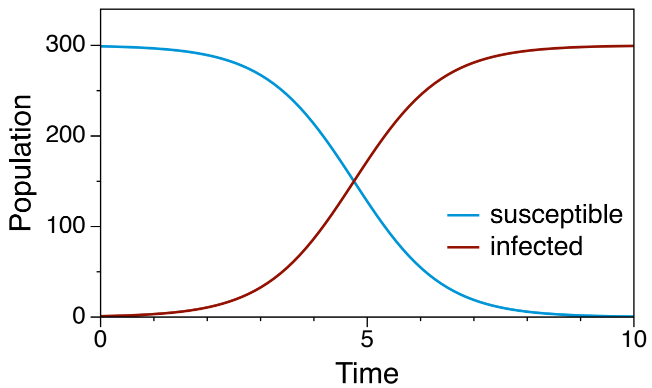
$$\frac{dS}{dt} = -\beta S I$$

$$\frac{dI}{dt} = \beta S I - \gamma I$$

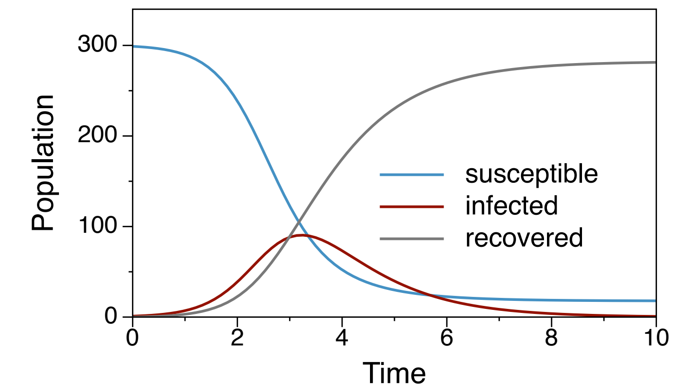
$$\frac{dR}{dt} = \gamma I$$

classical models:
SI vs. SIR

SI model



SIR model



model

dS

temporal network case

$$\langle I(t) \rangle = \rho(\mathcal{P}(t))$$

$$\beta = 1$$

incidence

dt

solution

$$I(t) = \frac{I_0 N}{I_0 + (N - I_0)\mathrm{e}^{-\beta N t}}$$

model

$$\frac{dS}{dt} = -\beta S I$$

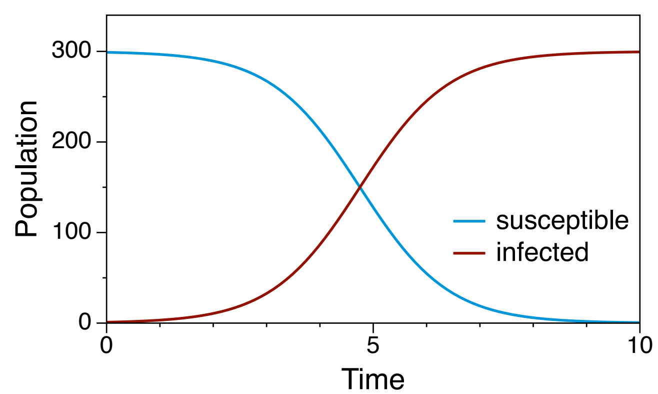
$$\frac{dI}{dt} = \beta S I - \gamma I$$

$$\frac{dR}{dt} = \gamma I$$

no explicit solution!

classical models:
SI vs. SIR

SI model



model

dS

dt

temporal network case

$$\langle I(t)\rangle = \rho(\mathcal{P}(t))$$

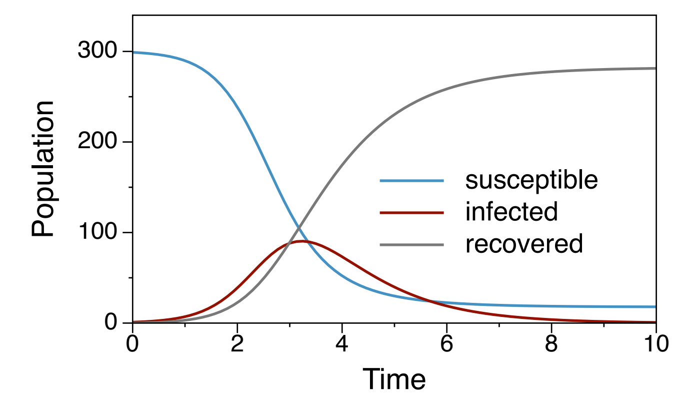
$$\beta = 1$$

incidence

solution

$$I(t) = \frac{I_0 N}{I_0 + (N - I_0)\mathrm{e}^{-\beta N t}}$$

SIR model



model

$$\frac{dS}{dt} = -\beta S I$$

"incidence"

$$\boxed{\frac{dI}{dt} = \beta S I - \gamma I}$$

$$\frac{dR}{dt} = \gamma I$$

no explicit solution!

incidence formulation

from SI to SIR

incidence formulation

from SI to SIR

1. Starting point.

SI model

$$\mathcal{P}(t) = \prod_{t'=1}^{t} (\mathbf{I} + \mathbf{A}(t'))$$

incidence formulation

from SI to SIR

1. Starting point.

SI model

$$\mathcal{P}(t) = \prod_{t'=1}^{t} (\mathbf{I} + \mathbf{A}(t'))$$

2. Rewrite.

SI model as incidence

$$\mathcal{I}(t) = \mathbf{A}(t) \sum_{t'=1}^{t-1} \mathcal{I}(t'-1)$$

$$\mathcal{P}(t) = \sum_{t'=0}^{t} \mathcal{I}(t')$$

incidence formulation
from SI to SIR

1. Starting point.

SI model

$$\mathcal{P}(t) = \prod_{t'=1}^{t} (\mathbf{I} + \mathbf{A}(t'))$$

2. Rewrite.

SI model as incidence

$$\mathcal{I}(t) = \mathbf{A}(t) \sum_{t'=1}^{t-1} \mathcal{I}(t'-1)$$

$$\mathcal{P}(t) = \sum_{t'=0}^{t} \mathcal{I}(t')$$

3. Finite memory.

$$\mathcal{I}(t) = \mathbf{A}(t) \sum_{t'=1}^{\tau} \mathcal{I}(t-t')$$

incidence formulation

from SI to SIR

1. Starting point.

SI model

$$\mathcal{P}(t) = \prod_{t'=1}^{t} (\mathbf{I} + \mathbf{A}(t'))$$

2. Rewrite.

SI model as incidence

$$\mathcal{I}(t) = \mathbf{A}(t) \sum_{t'=1}^{t-1} \mathcal{I}(t'-1)$$

$$\mathcal{P}(t) = \sum_{t'=0}^{t} \mathcal{I}(t')$$

3. Finite memory.

$$\mathcal{I}(t) = \mathbf{A}(t) \sum_{t'=1}^{\tau} \mathcal{I}(t-t')$$

4. Recovered state.

subtract recovered paths

incidence formalism

SIR model

$$\mathcal{I}(t) = \mathbf{A}(t) \sum_{t'=1}^{\tau} \mathcal{I}(t-t') - \sum_{t'=1}^{t} \mathcal{I}(t-t')$$

new infectious paths

contacts within infectious period

path was not infectious before

Koher et al., PLoS ONE, 2016

incidence formalism

SIR model

$$\underbrace{\mathcal{I}(t)}_{\text{new infectious paths}} = \underbrace{\mathbf{A}(t) \sum_{t'=1}^{\tau} \mathcal{I}(t-t')}_{\text{contacts within infectious period}} - \underbrace{\sum_{t'=1}^{t} \mathcal{I}(t-t')}_{\text{path was not infectious before}}$$

new infectious paths

contacts within infectious period

path was not infectious before

$$\mathcal{I}(t) = \mathbf{A}(t) \bigvee_{t'=1}^{\tau} \mathcal{I}(t-t') \wedge \neg \left[\bigvee_{t'=1}^{t} \mathcal{I}(t-t') \right]$$

Boolean formulation

Koher et al., PLoS ONE, 2016

application.

pig trade network

sexual contact network
Rocha et al., PNAS 2010

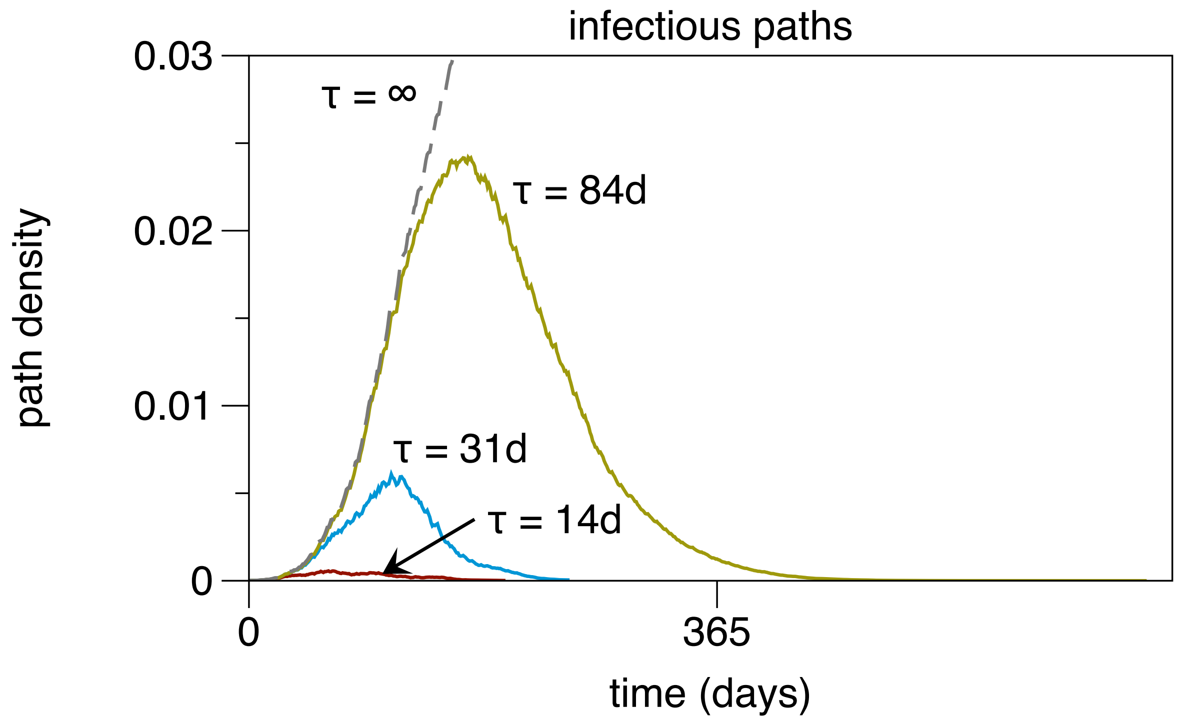
face-to-face conference contacts
sociopatterns.org

pig trade network

results

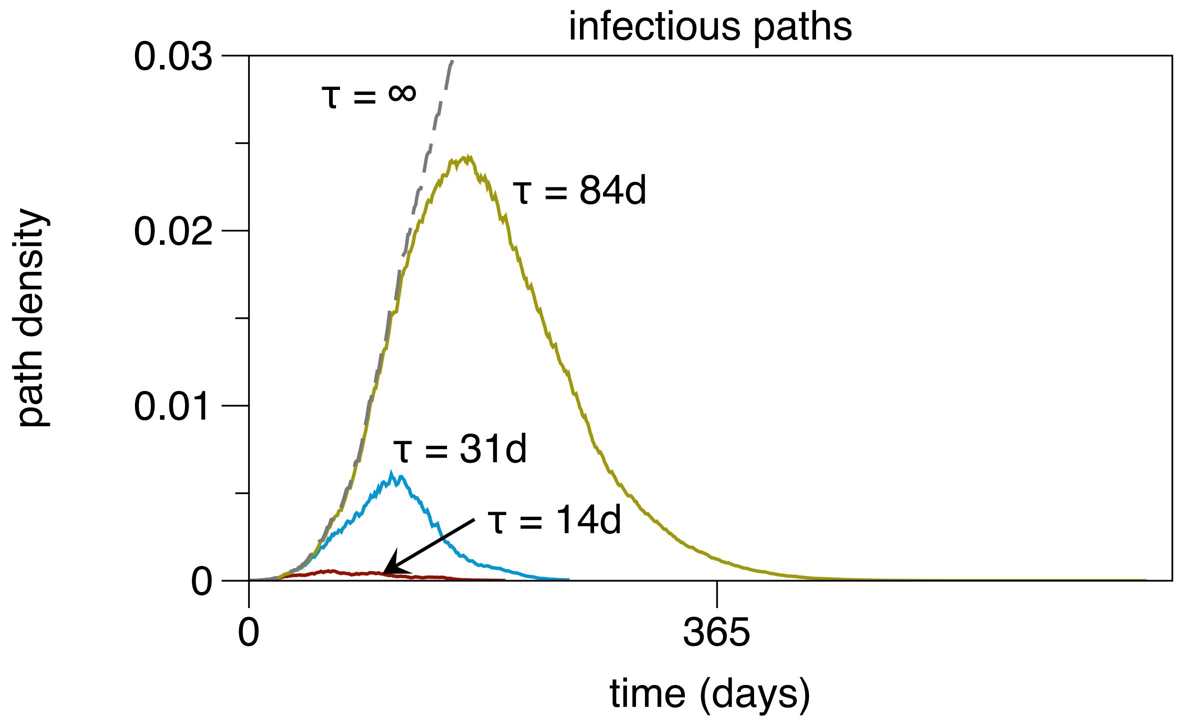
pig trade network

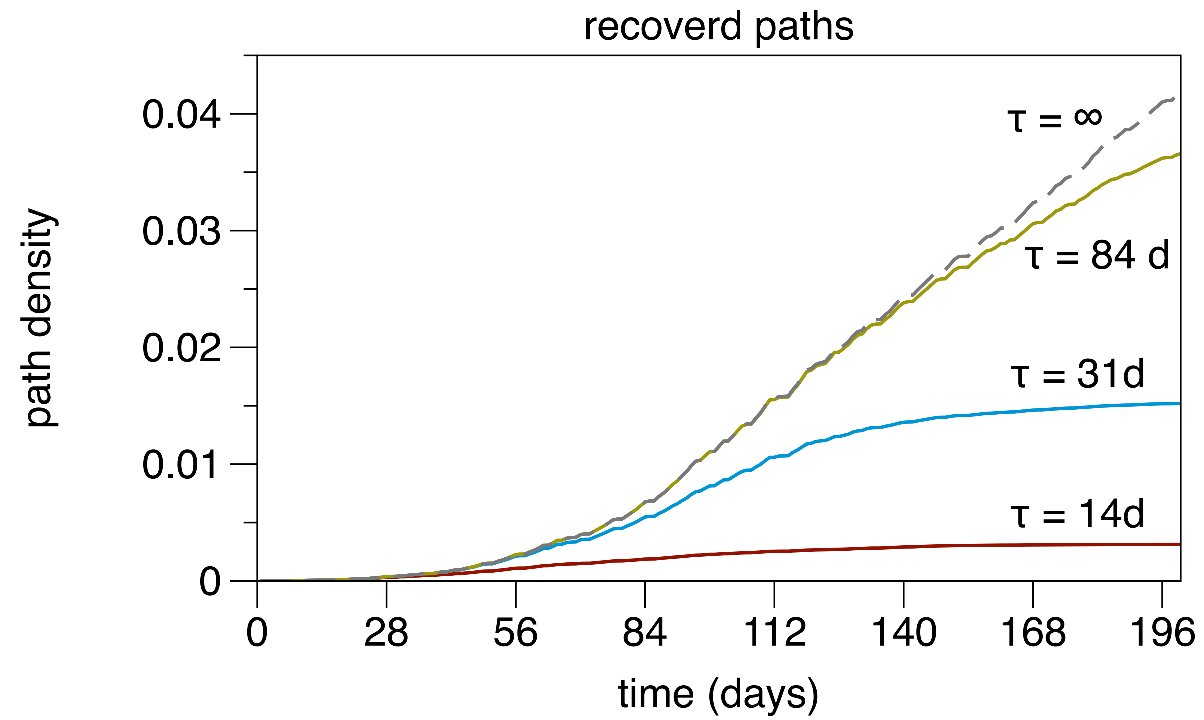
results



pig trade network

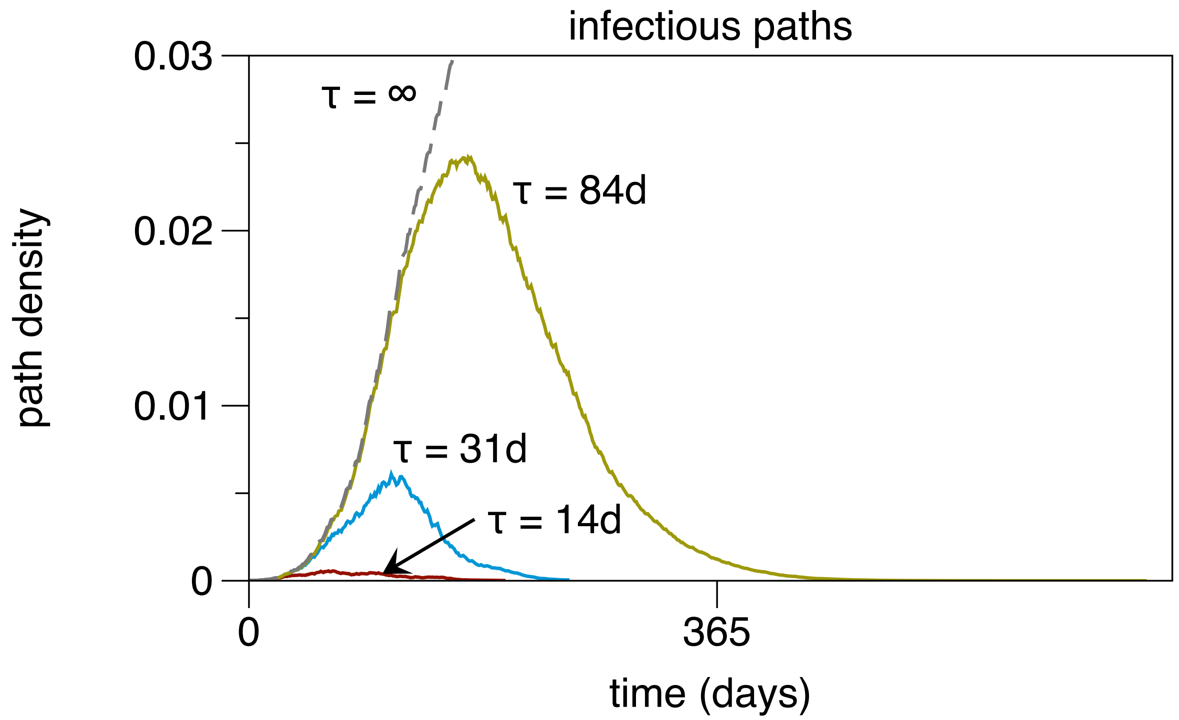
results

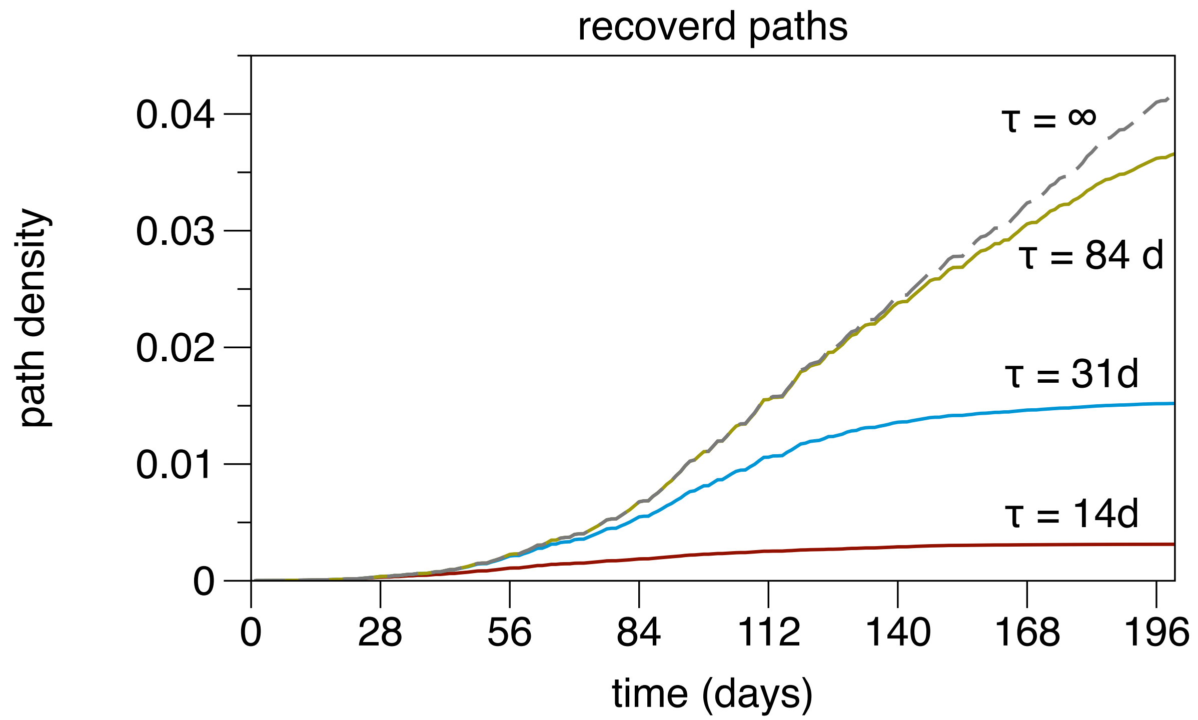


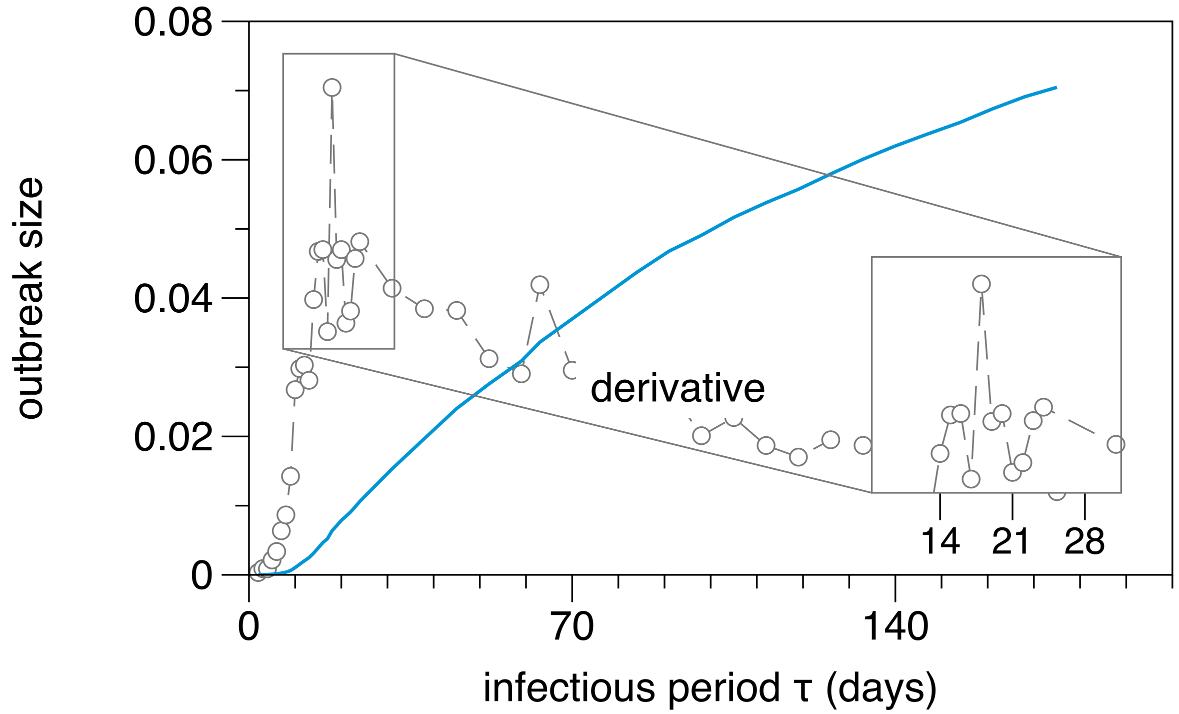


pig trade network

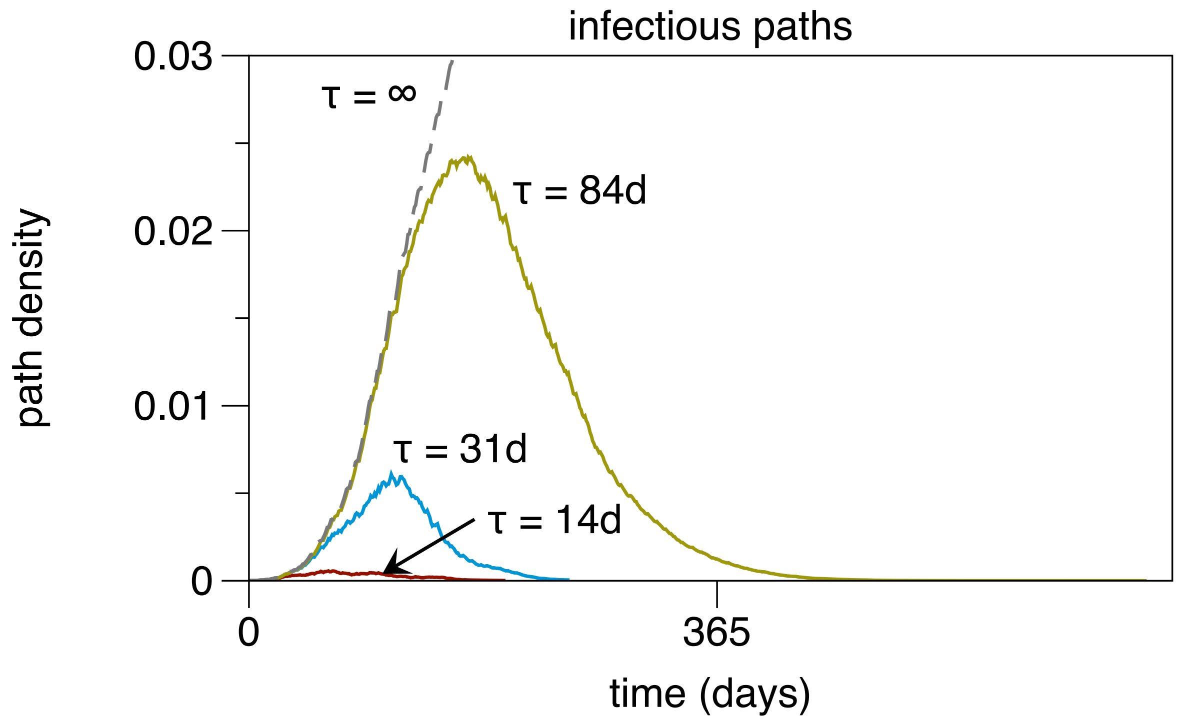
results

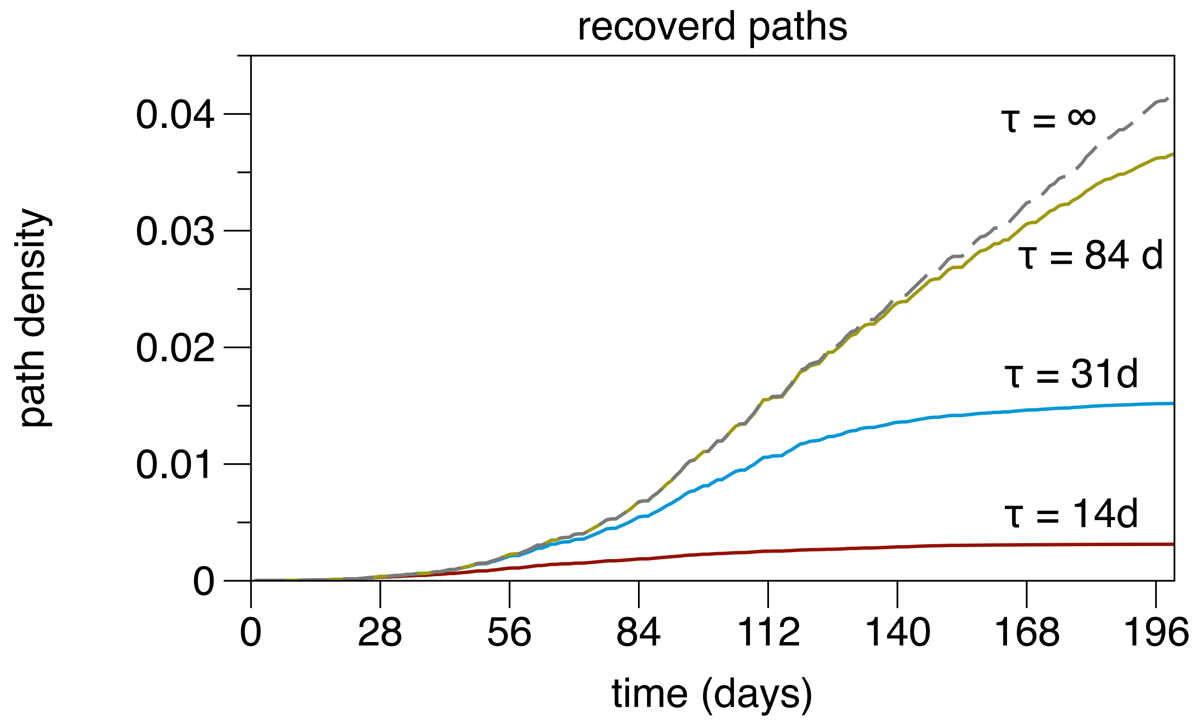


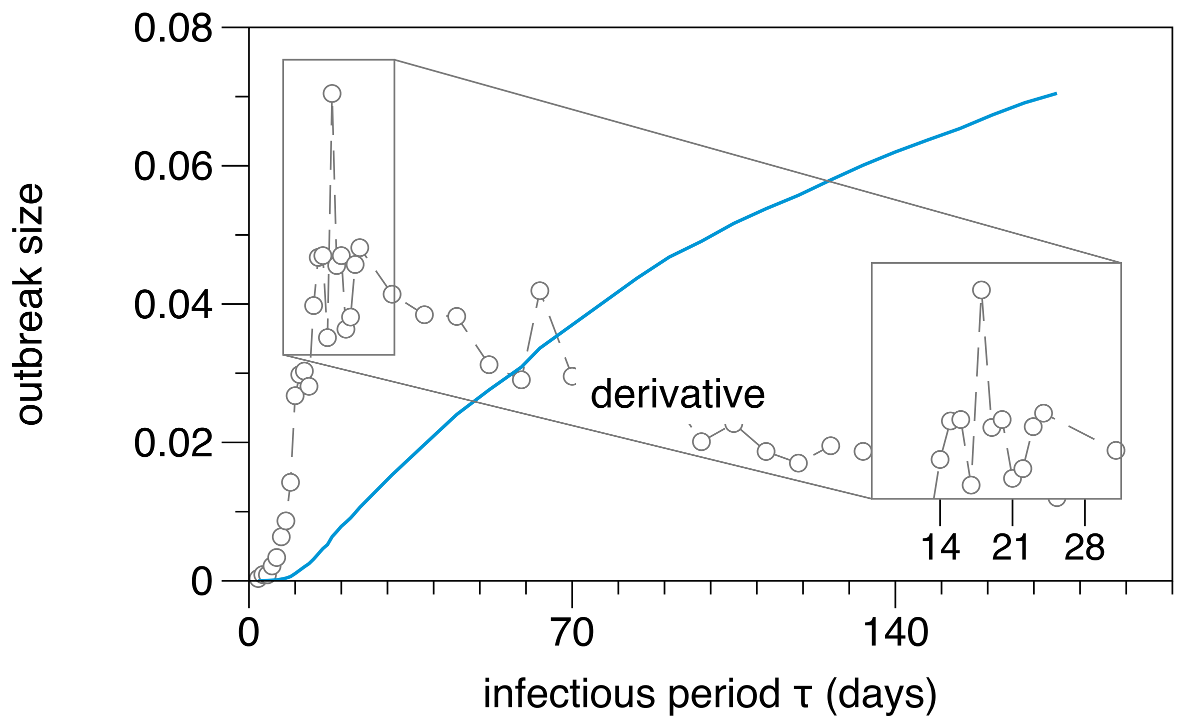




pig trade network
results





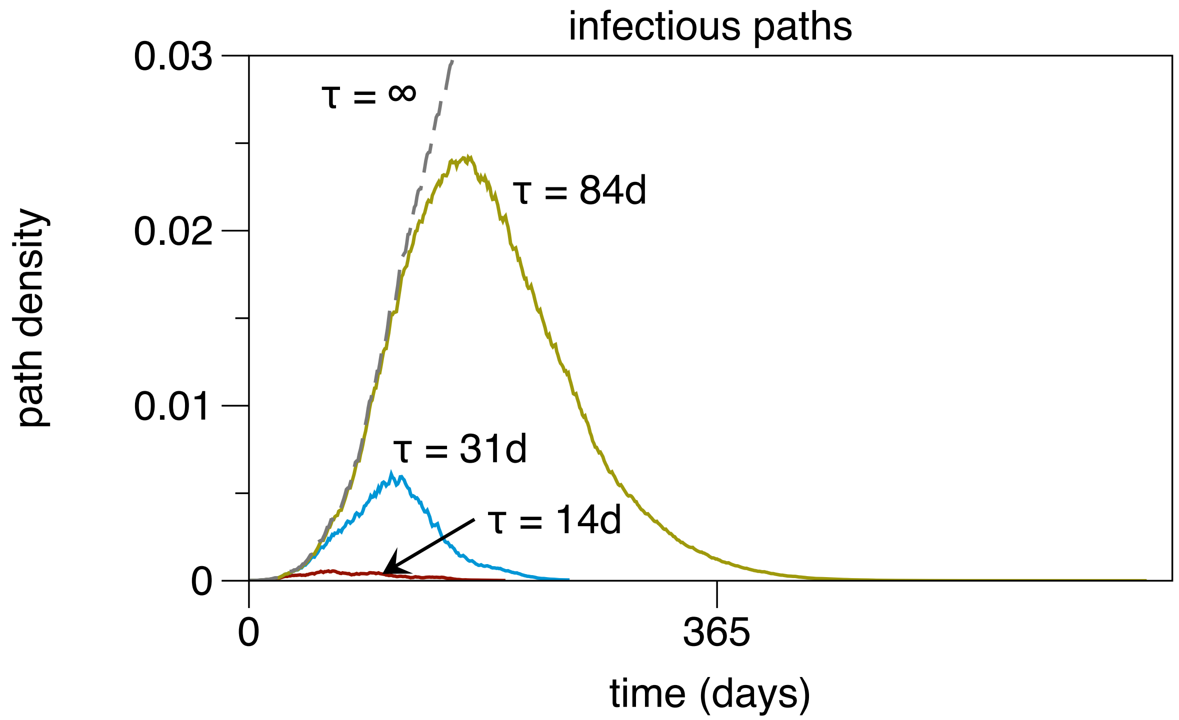


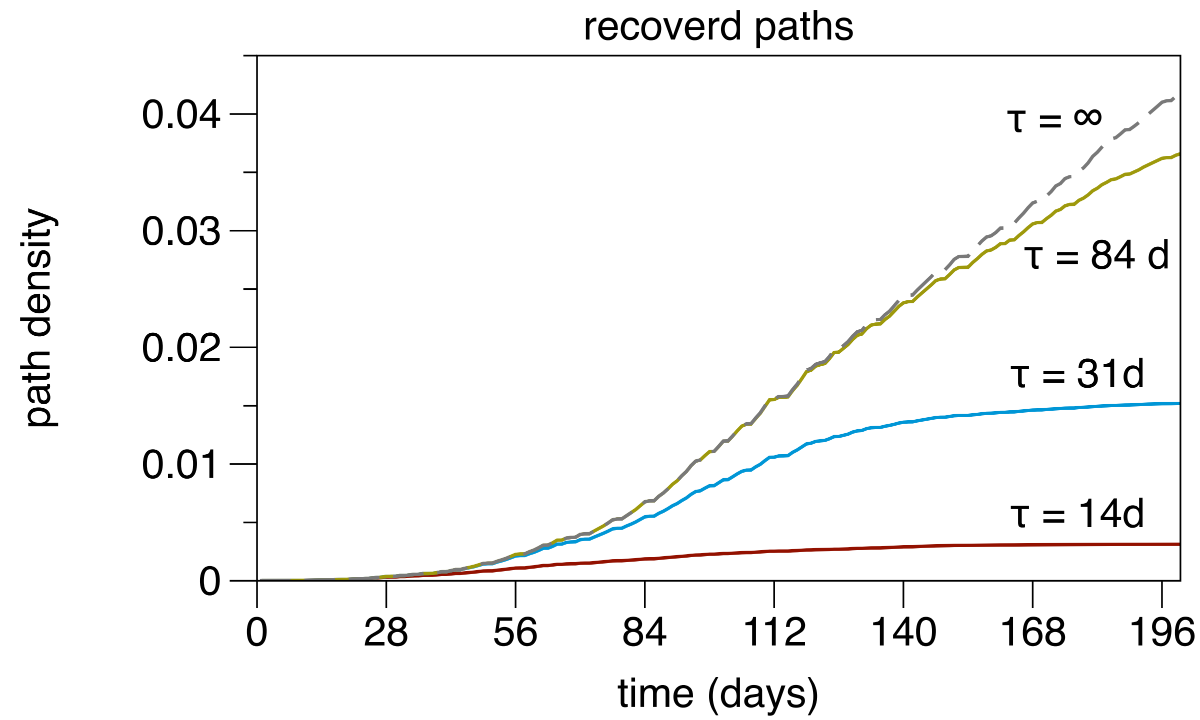
critical infectious period

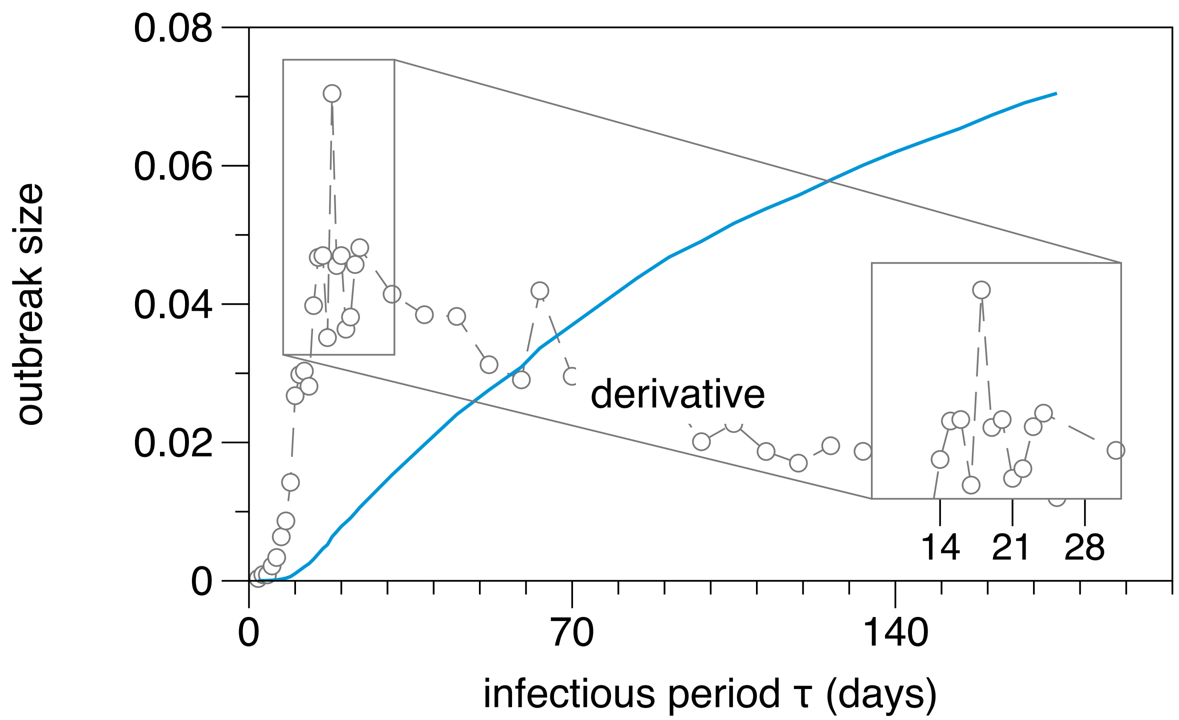
$\tau_c \sim 20\ d$

pig trade network

results







critical infectious period

$\tau_c \sim 20\ d$

example: classical swine fever

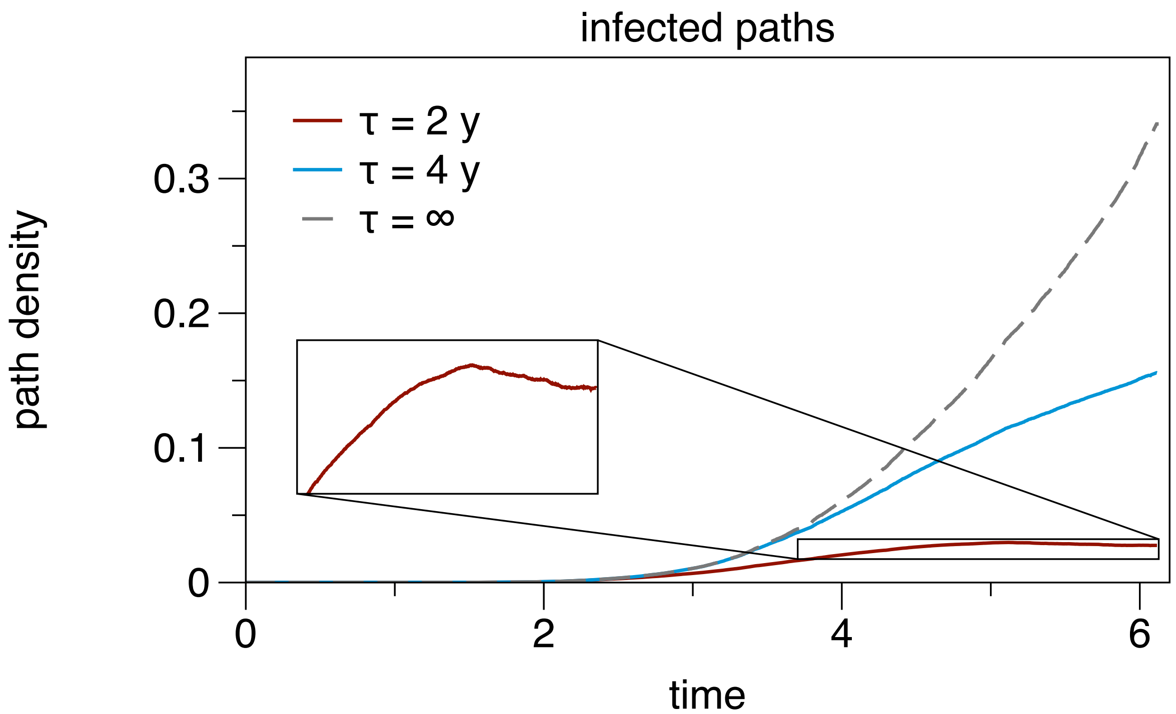
$\tau \sim 28\ d$

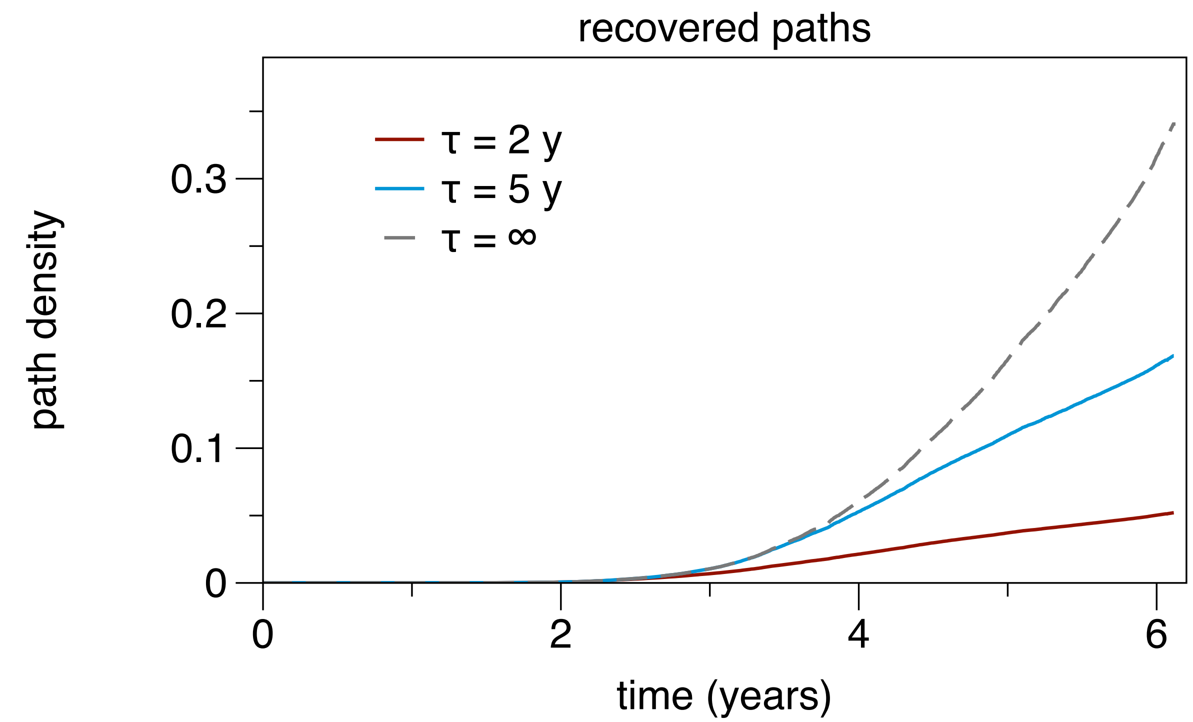
further datasets.

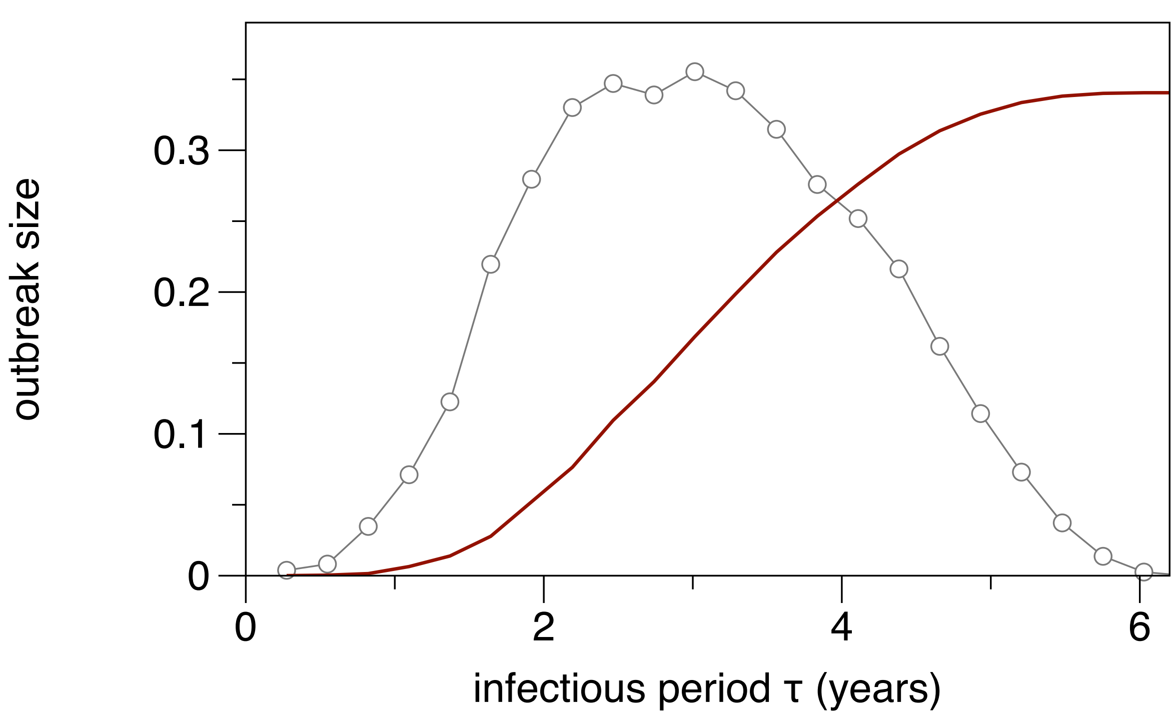
sexual contact network
Rocha et al., PNAS 2010

face-to-face conference contacts
sociopatterns.org

sexual contacts
results





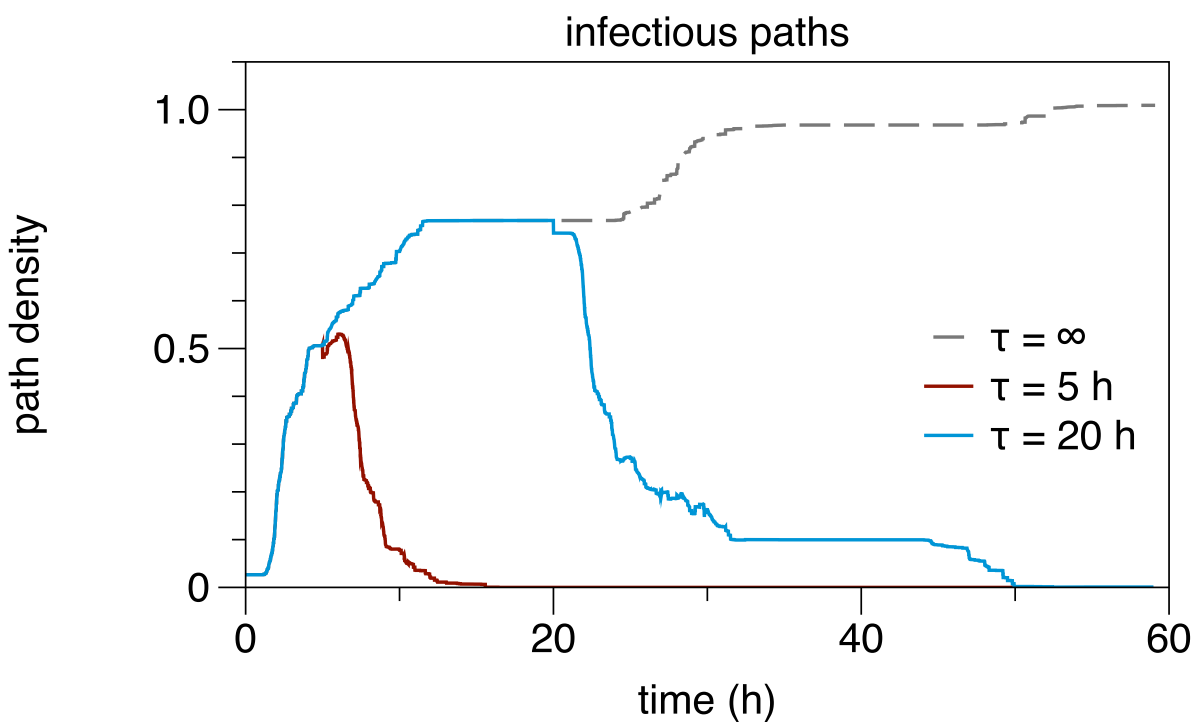


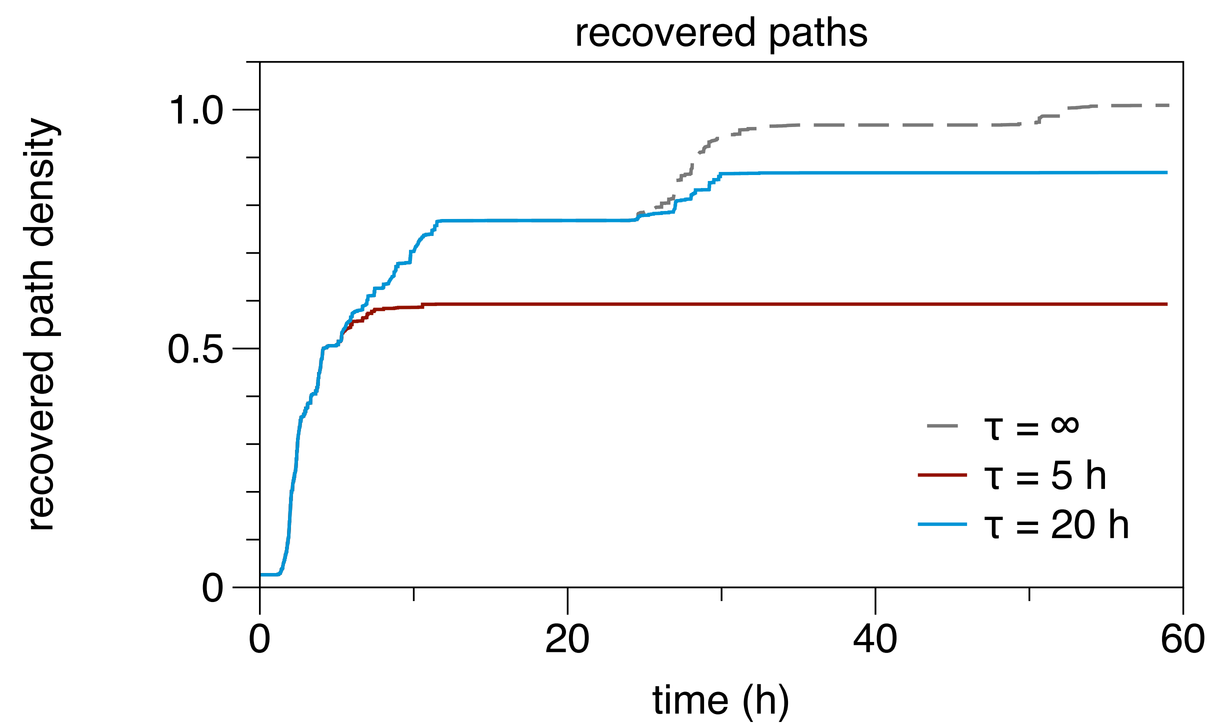
critical infectious period

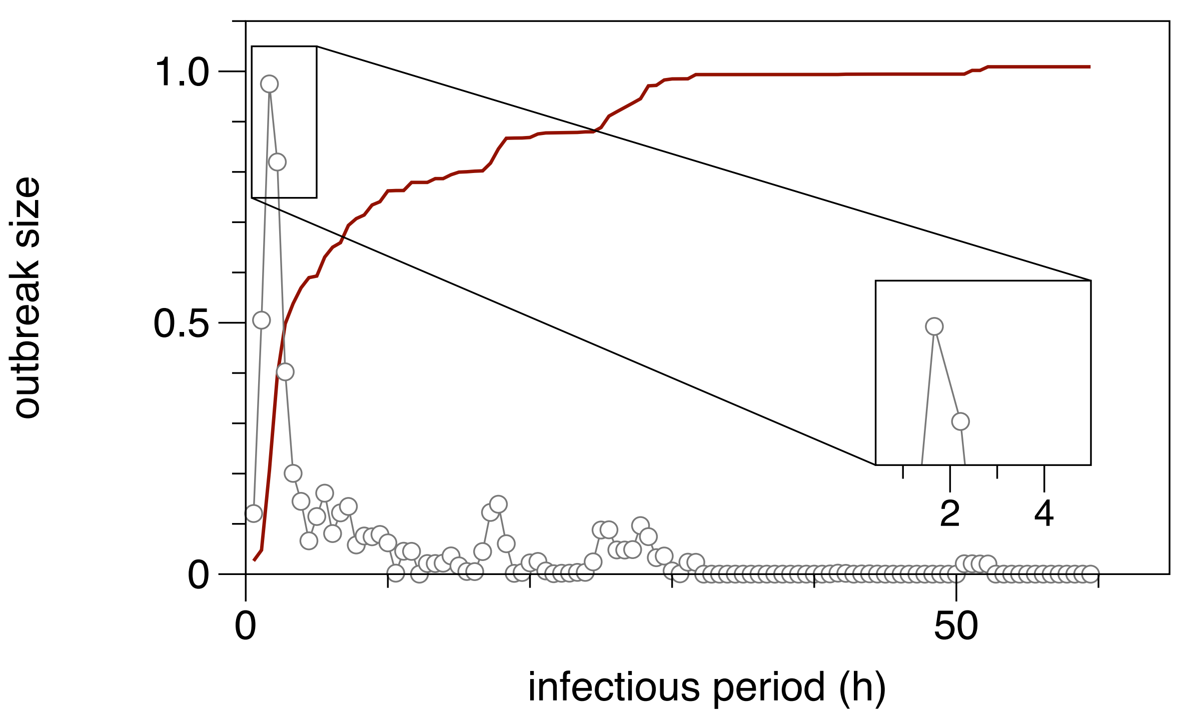
$$\tau \sim 3\ y$$

conference contacts

results





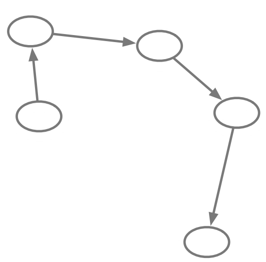


critical infectious period

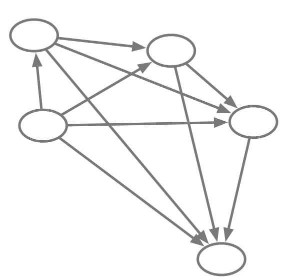
$\tau \sim 2\ h$

recap.

network

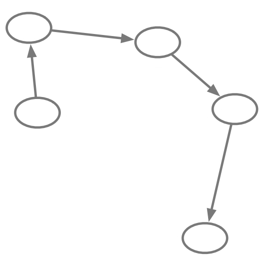


perspective:
accessibility

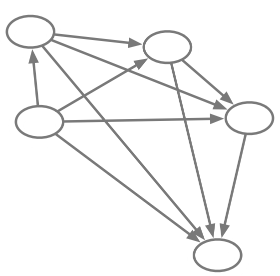


recap.

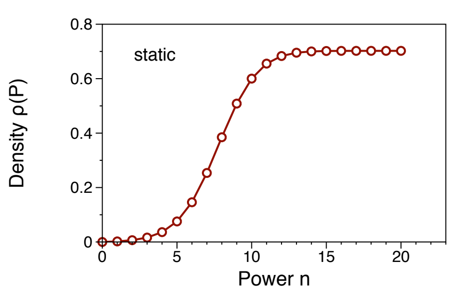
network

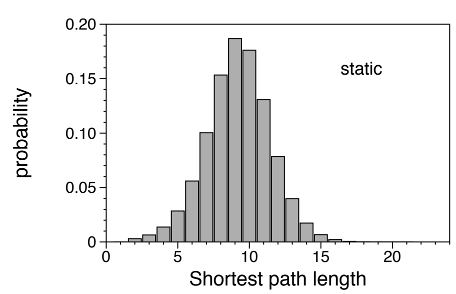


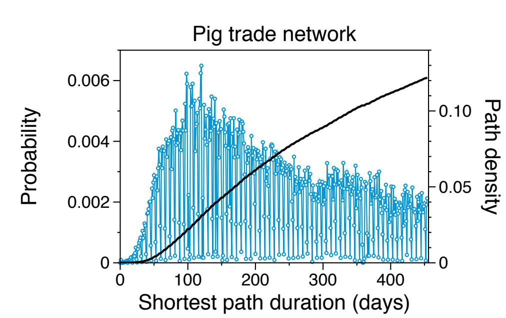
perspective:
accessibility



path lengths/durations

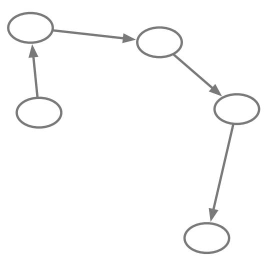




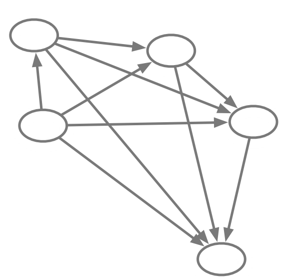


recap.

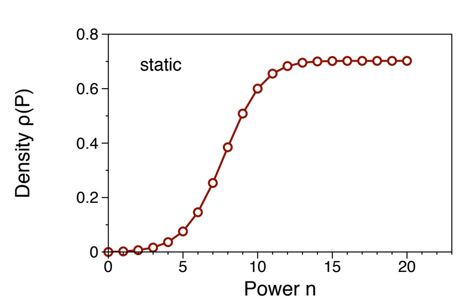
network

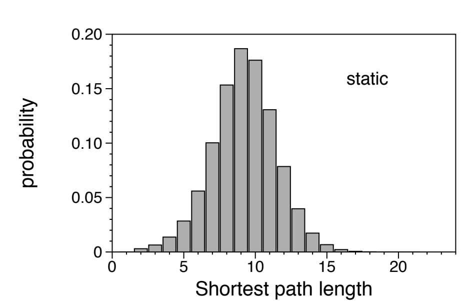


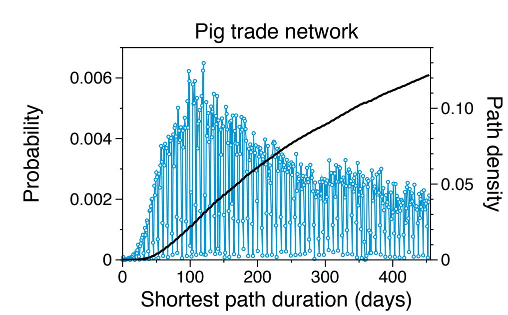
perspective:
accessibility



path lengths/durations







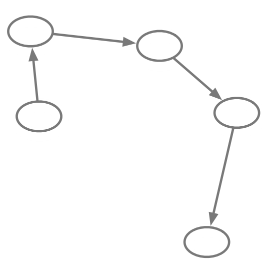
causal fidelity

temporal accessibility

$$\mathcal{P}_n = \prod_{i=1}^{n} (\mathbf{I} + \mathbf{A}_i)$$

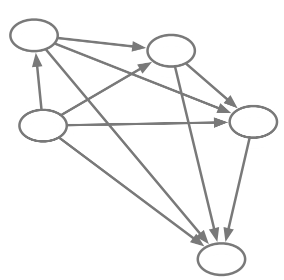
recap.

network

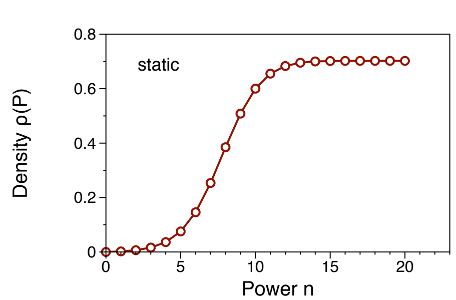


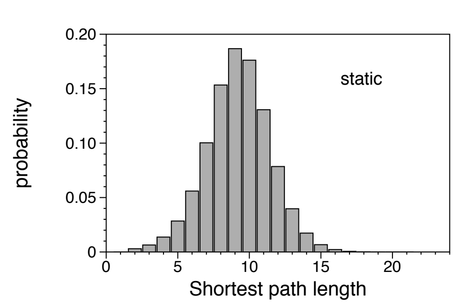
incidence formulation

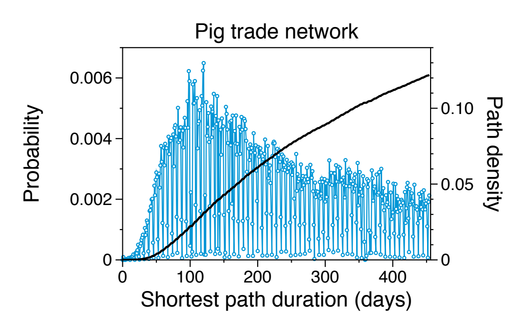
perspective: accessibility



path lengths/durations







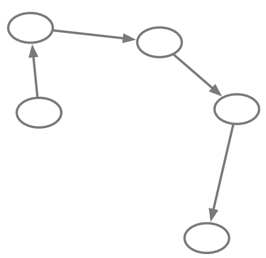
causal fidelity

temporal accessibility

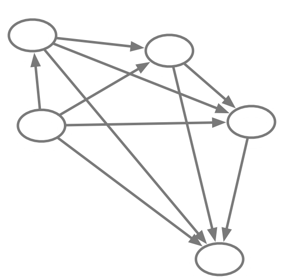
$$\mathcal{P}_n = \prod_{i=1}^{n} (\mathbf{I} + \mathbf{A}_i)$$

recap.

network

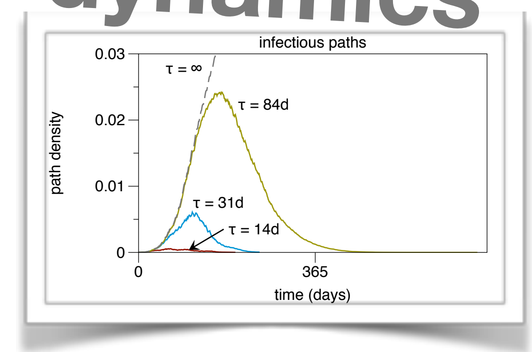


perspective:
accessibility

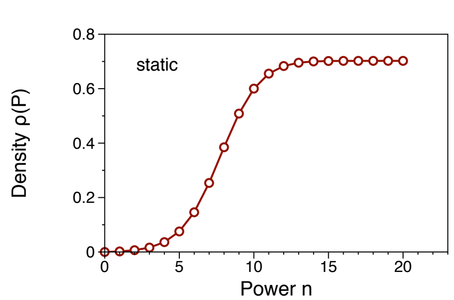


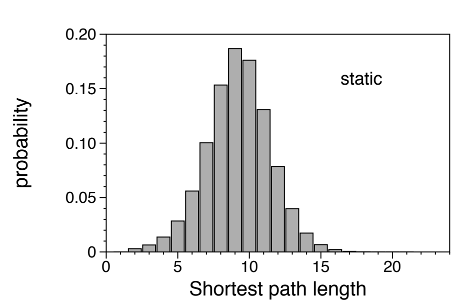
incidence
formulation

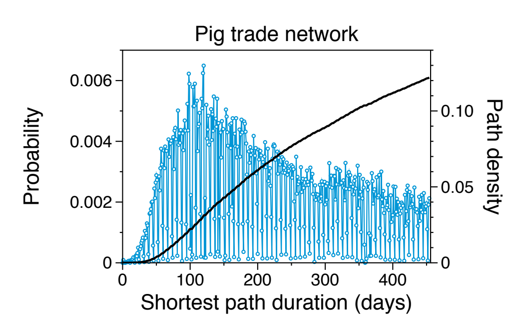
SIR
dynamics



path lengths/durations







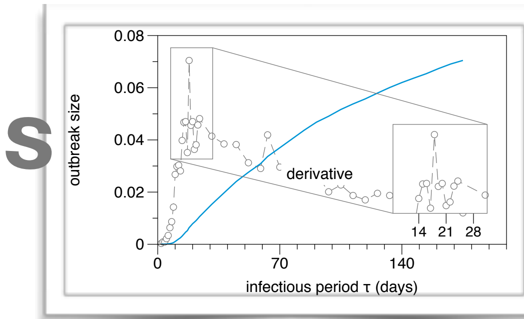
causal
fidelity

temporal
accessibility

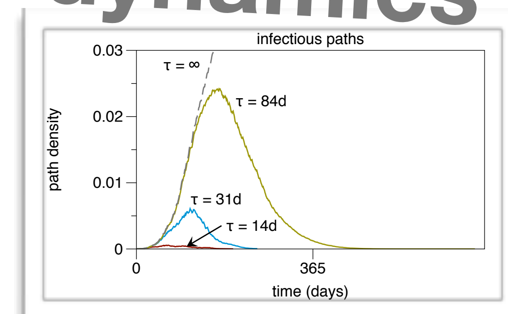
$$\mathcal{P}_n = \prod_{i=1}^{n} (\mathbf{I} + \mathbf{A}_i)$$

recap.

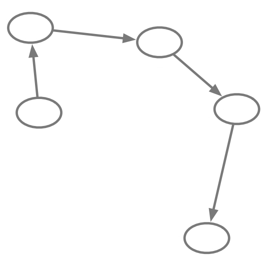
critical infectious period



SIR dynamics

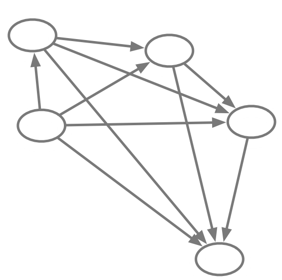


network



incidence formulation

perspective: accessibility

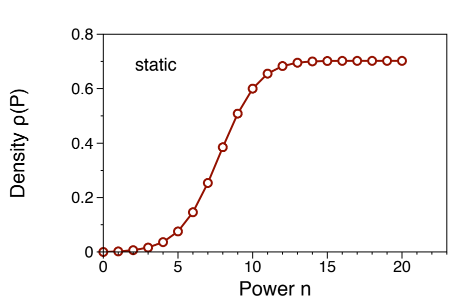


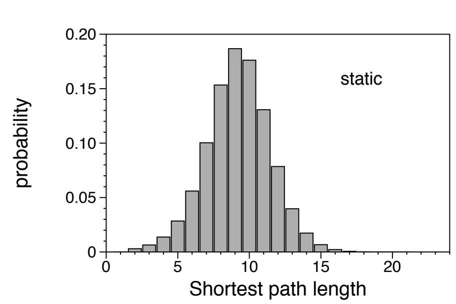
causal fidelity

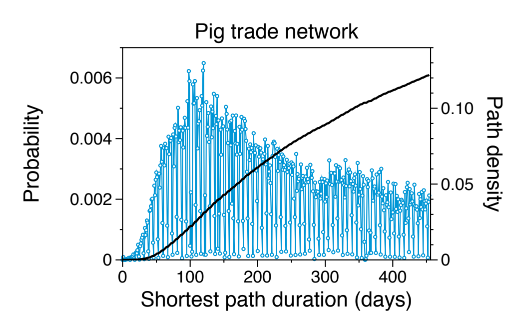
path lengths/durations

temporal accessibility

$$\mathcal{P}_n = \prod_{i=1}^{n} (\mathbf{I} + \mathbf{A}_i)$$







papers.

Unfolding Accessibility provides a Macroscopic Approach to Temporal Networks

Hartmut H K Lentz et al.
Phys. Rev. Lett, 2013

Infections on Temporal Networks— A Matrix-Based Approach

Andreas Koher et al.
PLOS ONE, 2016

Disease Spread through Animal Movements

Hartmut H K Lentz et al.
PLOS ONE, 2016