Computing a One-Way ANOVA

Step 1: Understanding our Data and Research Question

Amount (in ounces) Coffee Consumed Across				
	Italian Coffee Drinker	French Coffee Drinker	American Coffer Drinker	Total
	9	7	18	
	8	9	16	
	6	6	16	
	8	6	11	
	10	6	15	
	4	11	23	
	6	6	19	
	12	12	10	
	7	10	19	
	7	7	11	
Mean				
Std. Dev.				
Variance				

Data Background:
Here we have daily coffee consumption (in ounces) data for 30 individuals across three groups, Italian, French, and American; we have three groups (k=3) each with 10 individuals (n=30).

Research Question:
Our research, or alternative, hypothesis (Ha) is that there is a difference among the amount of coffee consumed on a daily basis among the three countries, and we can represent it as:

Ha: mean group 1 ≠ mean group 2 ≠ mean group 3

Within this, the null hypothesis (Ho) is that there is difference among our three groups, and we can represent it as:

Ho: mean group 1 = mean group 2 = mean group 3

To determine if there a difference among groups, we will conduct an ANOVA, the steps below outline the process of conducting an ANOVA manually, so to speak.

Step 2: Calculating Group and Grand Mean, Variance and Standard Deviation

Mean Calculation

Amount (in ounces) Coffee Consumed Across				
	Italian Coffee Drinker	French Coffee Drinker	American Coffer Drinker	Total
	9	7	18	
	8	9	16	
	6	6	16	
	8	6	11	
	10	6	15	
	4	11	23	
	6	6	19	
	12	12	10	
	7	10	19	
	7	7	11	
Mean	7.70	8.00	15.80	10.50
Std. Dev.				
Variance				

Group Means:
Within each of our groups, we calculate our mean scores, a walkthrough for each is below:

Italian Group:
9 + 8 + 6 + 8 + 10 + 4 + 6 + 12 + 7 + 7 = 77/10 = 7.7

French Group:
7 + 9 + 6 + 6 + 6 + 11 + 6 + 12 + 10 + 7 = 80/10 = 8

American Group:
18 + 16 + 16 + 11 + 15 + 23 + 19 + 10 + 19 + 11 = 158/10 = 15.8

All Groups: (note we know the totals for each group, we need to use the total number or responses, 30)
77 + 80 + 158/30 = 10.5

Variance Calculation

	Amount (in ounces) Coffee Consumed Across			
	Italian Coffee Drinker	**French Coffee Drinker**	**American Coffer Drinker**	**Total**
	9	7	18	
	8	9	16	
	6	6	16	
	8	6	11	
	10	6	15	
	4	11	23	
	6	6	19	
	12	12	10	
	7	10	19	
	7	7	11	
Mean	7.70	8.00	15.80	10.50
Std. Dev.				
Variance	5.12	5.33	17.51	23.22

Variance:

Variance is a measurement of the spread between numbers in a data set. The variance measures how far each number in the set is from the mean. Variance is calculated by taking the differences between each number in the set and the mean, squaring the differences (to make them positive) and dividing the sum of the squares by the number of values in the set. Note that since we have a sample, our denominator is n-1

Italian Group:
(9-7.70)^2 + (8-7.70)^2 + (6-7.70)^2 + (8-7.70)^2 + (10-7.70)^2 + (4-7.70)^2 + (6-7.70)^2 + (12-7.70)^2 + (7-7.70)^2 + (7-7.70)^2 = 46.1/(10-1) = 5.12

French Group:
(7-8)^2 + (9-8)^2 + (6-8)^2 + (6-8)^2 + (6-8)^2 + (11-8)^2 + (6-8)^2 + (12-8)^2 + (10-8)^2 + (7-8)^2 = 48/(10-1) = 5.33

American Group:
(18-15.8)^2 + (16-15.8)^2 + (16-15.8)^2 + (11-15.8)^2 + (15-15.8)^2 + (23-15.8)^2 + (19-15.8)^2 + (10-15.8)^2 + (19-15.8)^2 + (11-15.8)^2 = 157.6/(10-1) = 17.51

All Groups: (9-10.50)^2 + (8-10.50)^2 + (6-10.50)^2 + (8-10.50)^2 + (10-10.50)^2 + (4-10.50)^2 + (6-10.50)^2 + (12-10.50)^2 + (7-10.50)^2 + (7-10.50)^2 + (7-10.50)^2 + (9-10.50)^2 + (6-10.50)^2 + (6-10.50)^2 + (6-10.50)^2 + (11-10.50)^2 + (6-10.50)^2 + (12-10.50)^2 + (10-10.50)^2 + (7-10.50)^2 + (18-10.50)^2 + (16-10.50)^2 + (16-10.50)^2 + (11-10.50)^2 + (15-10.50)^2 + (23-10.50)^2 + (19-10.50)^2 + (10-10.50)^2 + (19-10.50)^2 + (11-10.50)^2 = 673.5/(30-1) = 23.22

Standard Deviation Calculation

	Amount (in ounces) Coffee Consumed Across			
	Italian Coffee Drinker	**French Coffee Drinker**	**American Coffer Drinker**	**Total**
	9	7	18	
	8	9	16	
	6	6	16	
	8	6	11	
	10	6	15	
	4	11	23	
	6	6	19	
	12	12	10	
	7	10	19	
	7	7	11	
Mean	7.70	8.00	15.80	10.50
Std. Dev.	2.26	2.31	4.18	4.82
Variance	5.12	5.33	17.51	23.22

Standard Deviation:

Standard deviation is a measure of the dispersion of a set of data from its mean. Standard deviation is calculated as the square root of variance by determining the variation between each data point relative to the mean

Italian Group: √5.12 = 2.26

French Group: √5.33 = 2.31

American Group: √17.51 = 4.18

All Groups: √23.22 = 4.82

Step 3: Calculating ANOVA Components:

Determine Degrees of Freedom and Critical F Value, Calculate SStotal, SSbetween, SSwithin, MSbetween

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	Italian Coffee Drinker	French Coffee Drinker	American Coffer Drinker	Total
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The ANOVA uses two degrees of freedom, (1) Degrees of Freedom Between and (2) Degrees of Freedom Within

Degrees of Freedom Between is calculated by subtracting 1 from the number of groups we have:
We have three groups in our sample, so df between = 3 – 1 = 2

Degrees of Freedom within is calculated by determining df for each group and then adding them together:
df Italian Coffee Drinker = 10 – 1 = 9
df French Coffee Drinker = 10 – 1 = 9
df American Coffee Drinker = 10 – 1 = 9
df within = 9 + 9 + 9 = 27

Looking at an F-table, we don't have a match for our df values and instead use the closest match which is df within = 26 and df between = 2 which gives us a critical value of 3.37 when p = .05, if we get an F value equal to or greater than 3.37, we reject Ho, if we get an F value less than 3.37 we fail to reject Ho.

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Sum of Squares
SStotal = the sum of squares of all the observations, regardless of which treatment produced them from the grand mean, we actually already calculated this earlier as part of our variance calculation for All Groups above, our SStotal is the numerator in the calculation, SStotal = 673.5

SSbetween (also known as SStreat) = the sum of squares (deviations) of the group means from the grand mean
SSbetween = in our example SSbetween is calculated as (number of group observations *(group mean - grand mean)^2), looking at our first group--Italian Coffee Drinker--we have 10 observations, with a group mean of 7.7 and we know the grand mean (the mean in our total column that we calculated earlier) is 10.5, therefore for this group, SSbetween is input as: 10*(7.7-10.50)^2. The entire SSbetween calculation is below:
SSbetween = (10*(7.7-10.50)^2) + (10*(8-10.50)^2) + (10*(15.8-10.50)^2) = 421.8

SSwithin = (also known as SSerror) = the sum over the sums of squared deviations of scores around each group's mean. We calculated sums of squared deviations earlier as part of our variance calculations for each of our three groups. Similar to SStotal, each of the values come from the numerators of our variance calculations, therefore SSwithin = 46.1 + 48 + 157.6 = 251.7

Also, you may have noticed SStotal = SSbetween + SSwithin which means that we could have reworked the SStotal equation to determine SSwithin, SSwithin = SStotal – SSbetween.

Step 4: Conducting Our ANOVA Analysis and Determining Significance

Mean Square (Variance Estimate) Calculation

The ANOVA's outcome is an F ratio which is calculated as the variance between groups divided by the variance within each of our groups. Within this in mind, we will need to compute one two more pieces of information so that we can conduct our ANOVA analysis

MSbetween is the variation due to the interaction between the samples and is calculated as SSbetween/df between, so for our example, MSbetween = 421.8/2 = 210.9 (recall we determined each of these values above)
If sample means are close to each other, the Grand Mean will be a similar value and MSbetween this will be small

MSwithin is the variation due to differences within individual samples, and is calculated as SSwithin/df within, so for our example, MSwithin = 251.7/27 = 9.32 (similarly here, recall we determined each of these values above)
MSwithin considers each sample independently so there is no interaction between samples involved.

ANOVA Analysis

Between and Within Values

Source	**Sum of Squares**	**Degrees of Freedom**	**Mean Square (Variance Est.)**	**F Ratio**
Between	SSbetween	K - 1	MSbetween = SSbetween/ K - 1	MSbetween/ MSwithin
Within	SSwithin	N - K	MSwithin = SSwithin/ N - K	
Total	SStotal = SSbetween + SSwithin	N - 1		

F Ratio

Source	**Sum of Squares**	**Degrees of Freedom**	**Mean Square (Variance Est.)**	**F Ratio**
Between	421.8	2	210.9	22.63
Within	251.7	27	9.32	
Total	673.5			

Effect Size - Eta Squared

Effect Size

	SSbetween/ SStotal	421.8/ 673.5	0.63
			Large Effect

Recall earlier, we determined that the critical value for our F ratio (the point at which we determine to reject or fail to reject the Ho) to be 3.37. Looking at the F ratio for our ANOVA analysis which is highlighted in yellow, we see that it is 22.63, which is greater than our 3.37 critical value. Therefore, we reject our Ho which suggests that there is no difference among groups when it comes to amount of coffee consumed on a daily basis.

Calculating & Interpretating Eta Sqaured Effect Size

SPSS does not provide Eta Squared effect size for ANOVA, instead it reports Partial Eta Squared only, therefore, we will need to compute the effect size by "hand". The equation is SSbetween/SStotal.

Partial eta sqaured should be interpreted as the proportion of variance of the test variable that is a function of the grouping variable. A value of zero indicates that the differnece in the mean scores is equal to 0 whereas a value of 1 indicates that the sample means differ and the scores within each group do not differ (perfect replication).

What is a small versus large eta squared is dependent on the investigation. However, eta sqaured values of .01, .06, and .14 are, by convention, interpreted as small, medium, and large effect sizes, respectively.

Next Steps: (To be done in SPSS)

Testing Equal Variances Assumption using Levene's Analysis

Recall that the ANOVA assumes that variances of the populations from which our samples are drawn are equal. Therefore to assess the assumption, we would use Levene's analysis to test homogeneity of variances. Specifically, Levene's tests the null hypothesis that the population variances are equal. If the resulting p-value of Levene's test is less than 0.05, we would assume the obtained differences in our sample variances are unlikely to have occurred based on random sampling from a population with equal variances. Therefore, we would reject the null hypothesis of equal variances and conclude that there is a difference between the variances in the population.

Conducting Post Hoc Analyses

Recall from the notes above, the ANOVA test tells us whether we have an overall difference between our groups, but it does not tell us which specific groups differed – post hoc tests do. Because post hoc tests are run to confirm where the differences occurred between groups, they should only be run when we have a shown an overall statistically significant difference in group means (i.e., a statistically significant one-way ANOVA result). Post hoc tests attempt to control the experimentwise error rate (usually alpha = 0.05) in the same manner that the one-way ANOVA is used instead of multiple t-tests.

If our data meet the assumption of homogeneity of variances (i.e. our Levene's is NOT significant, its p-value is GREATER than 0.05), use Tukey's honestly significant difference (HSD) post hoc test. Note when we use SPSS Statistics, Tukey's HSD test is simply referred to as "Tukey" in the post hoc multiple comparisons dialogue box.

If our data did not meet the homogeneity of variances assumption, (i.e. our Levene's IS significant, its p-value is LESS than 0.05) we we would likely conduct the Games Howell post hoc test.