Python Assignment 1

Aj Averett

Imports and Instruction

```
import numpy as np
import sympy as sp
from sympy import init_printing, Matrix
init_printing()
```

Defining a Matrix

```
A=np.array([[1,2,3],[4,5,6],[7,8,9],[2,4,6]])
A = Matrix(A)

print(f"{A[0]}, {A[:,0]}, {A[1,0]}") # Extracting elements from a matrix
```

```
1, Matrix([[1], [4], [7], [2]]), 4
```

Defining a Vector

```
x1, x2, x3 = [1, 2, 3], np.array([1,2,3]), np.array([[1],[2],[3]])
x1, x2, x3 = Matrix(x1), Matrix(x2), Matrix(x3)

#Showing these three methods produce equivalent vectors
print(f"{x1==x2}, {x1==x3}")
print(f"{x1[0]}, {x1[1]}, {x1[2]}")# Extracting elements from a vector
```

```
True, True
1, 2, 3
```

Question 1

Write the command that returns the entry of the matrix A that is in the 4th row and 2nd column.

```
print(A[3,1]) #Because of Zero-Indexing, arguments = n-1
```

```
4
```

Question 2

In what way did the matrix A change?

```
A=np.array([[1,2,3],[4,5,6],[7,8,9]]) #Matrix A is defined
A[2]=[7,8,10] #The third row is of A is assigned as [7,8,10]
A[1]=[1,1,1] #The second row is of A is assigned as [1,1,1]
Matrix(A)
```

$$\begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 1 \\ 7 & 8 & 10 \end{bmatrix}$$

Question 3

What do the following lines of code do? Explain.

```
A=np.array([[1,2,3],[4,5,6],[7,8,9]])
# The code below preforms Gaussian elimination on matrix A
A[1]=A[1]+A[0]*(-A[1,0]) # R_2 = R_2 + R_1 * negative of 1st element of R_2
A[2]=A[2]+A[0]*(-A[2,0]) # R_3 = R_3 + R_1 * negative of 1st element of R_3
A[2]=A[2]+A[1]*(-A[2,1]/A[1,1])
# R_3 = R_3 + R_1 * (negative of 2nd element of R_3 /2nd element of R_2)

Matrix(A)
```

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -6 \\ 0 & 0 & 0 \end{bmatrix}$$

Question 4

Find the solution to $A\vec{x} = \vec{b}$.

```
A = np.array([
    [1,2,3],
    [4,5,6],
    [7,8,10]])

b = np.array([-1,4,-10])
x = np.linalg.solve(A, b)

print(Matrix(A)*Matrix(x) == Matrix(b))
Matrix(x.round(3))
```

```
True
```

$$\begin{bmatrix} -14.667 \\ 35.333 \\ -19.0 \end{bmatrix}$$

Question 5

Verify your solution using an operator

```
print(Matrix(A @ x) == Matrix(b))
```

```
True
```

Question 6

Verify that $\vec{x} = A^{-1}\vec{b}$ using NumPy

```
A_inverse = np.linalg.inv(A)
Matrix((A_inverse @  b ).round(3))
```

$$\begin{bmatrix} -14.667 \\ 35.333 \\ -19.0 \end{bmatrix}$$

Question 7

Given k points in $\mathbb{R}^2$, we can find a polynomial of degree $k-1$ that passes through all k points. Find the equation of the 5th degree polynomial that passes through the points $(1,1)$, $(10,5)$, $(-10,6)$, $(2,100)$, $(-2,-30)$, and $(-1,60)$. Round your coefficients to three decimal places.

```
from sympy.printing.latex import print_latex
from sympy import linsolve

coord = {1:[1, 1],      2:[10, 5],
         3:[-10, 6],    4:[2, 100],
         5:[-2, -30],   6:[-1, 60],}


# Define A
a_i = np.array([])

for i in range(len(coord)):
    a_i = np.append(a_i, (coord[i+1][0]))

A = Matrix(np.array([a_i**5,    a_i**4,
                     a_i**3,    a_i**2,
                     a_i**1,    a_i**0]).T)


# Define b
b = np.array([])

for i in range(len(coord)):
    b = np.append(b, (coord[i+1][1]))

b = Matrix(b)


# Solve for x
c1, c2, c3, c4, c5, c6 = sp.symbols("c1 c2 c3 c4 c5 c6")
system = A, b
x = str(linsolve(system,c1, c2, c3, c4, c5, c6))\
    .replace('{','').replace('}','')\
    .replace('(','').replace(')','').split(',')

x_rounded = [round(float(num),3) for num in x]
```

The solution $\vec{x}$ rounded to three decimal places is:

```
Matrix(x_rounded)
```

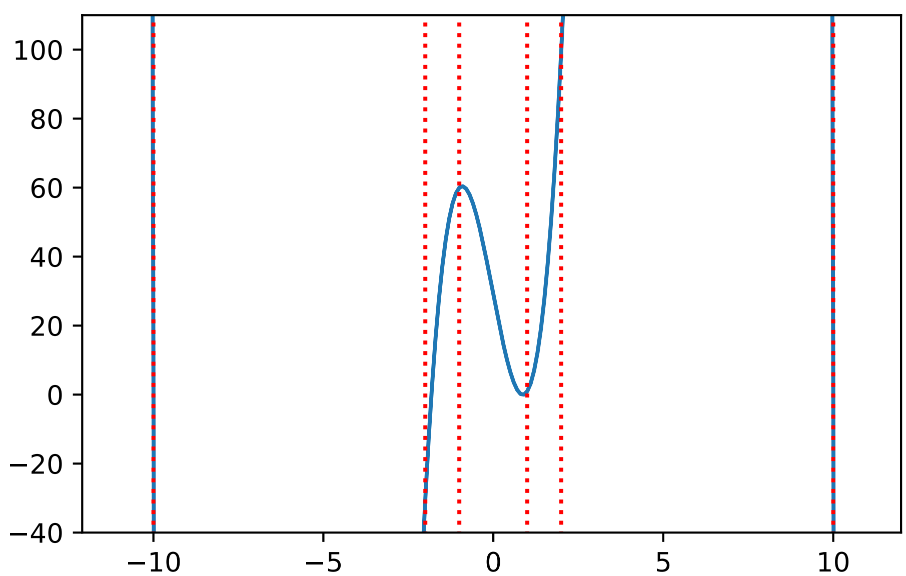
$$\begin{bmatrix} -0.212 \\ -0.018 \\ 21.728 \\ 1.591 \\ -51.015 \\ 28.927 \end{bmatrix}$$

Thus the $A\vec{x} = \vec{b}$ form of this would be as so:

$$\begin{bmatrix} 1.0 & 1.0 & 1.0 & 1.0 & 1.0 \\ 100000.0 & 10000.0 & 1000.0 & 100.0 & 10.0 \\ -100000.0 & 10000.0 & -1000.0 & 100.0 & -10.0 \\ 32.0 & 16.0 & 8.0 & 4.0 & 2.0 \\ -32.0 & 16.0 & -8.0 & 4.0 & -2.0 \\ -1.0 & 1.0 & -1.0 & 1.0 & -1.0 \end{bmatrix} \begin{bmatrix} -0.212 \\ -0.018 \\ 21.728 \\ 1.591 \\ -51.015 \\ 28.927 \end{bmatrix} = \begin{bmatrix} 1.0 \\ 5.0 \\ 6.0 \\ 100.0 \\ -30.0 \\ 60.0 \end{bmatrix}$$

This means that the 5th degree polynomial would look as so:

$$y = -0.212x^5 - 0.018x^4 + 21.728x^3 + 1.591x^2 - 51.015x + 28.927$$



Question 8

Find the reduced row echelon form of the following matrix. This matrix is created using list comprehension.

```
M=sp.Matrix([[x+5, x-4, (x-2)**2, x**2, x, 1] for x in [1,2,3,4,5,6]])
M.rref()[0]
```

$$\begin{bmatrix} 1 & 0 & 0 & \frac{4}{3} & \frac{4}{9} & \frac{1}{9} \\ 0 & 1 & 0 & \frac{8}{3} & \frac{5}{9} & -\frac{1}{9} \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Question 9

Find the command to evaluate the determinant of a matrix and use it to find the determinant of M.

```
M=sp.Matrix([[1, x, x**2, x**3, x**4, x**5] for x in [2,3,4,5,6,7]])
print(f"The determinant for the matrix defined above is: {M.det()}")
```

```
The determinant for the matrix defined above is: 34560
```