Constructing the mathematical language

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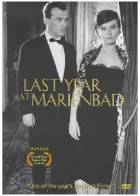
Rules of the game



Last year at Marienbad by Alan Resnais

- Character M is always trying to persuade another to play a card game with him.
- Cards arranged in rows of 1, 3, 5, 7.
- They take turns to pick any number of cards from one row. The one who takes the last card loses.
- M is always polite, lets the other start. The other always loses.

Nim

Nim, from the German word nehmen, analysed in 1904 by C. L. Bouton of Harvard University.'

- Two players I and II, move alternately.
- The game is played with m piles of counters.
- When a player moves, she picks a pile and removes some non-zero many counters from that pile.
- When a player cannot move, he loses (and the other wins).

Ingredients

Every game has three main ingredients:

- The set of players, often $\{I, II\}$. In general, $[n] = \{1, 2, \ldots, n\}$.
- The rules of the game, that specify, at any game position, whose turn it is to move, what moves are applicable, and the resulting new game position after any move.
- Outcomes or winning conditions, that specify at which positions the game is over, and perhaps depending on the course of play, the outcome at those positions.

Backward induction

Zermelo 1913: In every finite extensive form game of perfect information, we can compute whether player i can win (or not).

- Theorem: Backward induction shows who wins, gives a winning strategy in the case of win / lose games, and an NE for general games.
- Note that the game arena for any Nim heap is acyclic and hence the unfolding is a finite tree, so BI applies.

End of theory?

Backward induction completely solves finite extensive form games of perfect information, so we might as well go home.

- So the Nim game is solved, isn't it ?

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- So the Nim game is solved, isn't it ?
- If we are only interested in existence of winning strategies, this suffices. If we also wish to look at the structure of strategies, this leaves us quite unsatisfied.
- Indeed, in the case of Nim, combinatorial analysis offers more.