

The language of mathematics

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A quote

Mathematics as a language

Mathematics is the language in which God has written the universe.

[The universe] cannot be read until we have learnt the language and become familiar with the characters in which it is written. It is written in mathematical language, and the letters are triangles, circles and other geometrical figures, without which means it is humanly impossible to comprehend a single word.

Opere Il Saggiatore

Galileo Galilei

Why a language?

Why should we even think of mathematics as a language?

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- ▶ Mathematics meets this definition of a language. Linguists who do not consider math a language cite its use as a written rather than spoken form of communication.
- ▶ Math is a universal language. The symbols and organisation to form equations are the same in every country of the world.

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- ▶ A syntax and a grammar
- ▶ A semantics that attaches meaning to syntax
- ▶ A narrative or discourse

Mathematics meets all of these requirements.

Characteristics

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- ▶ Precision?
- ▶ "Context free" ness?

Language as difficulty

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- ▶ The concept of **variable**
- ▶ The **equality** symbol
- ▶ The use of **quantifiers**
- ▶ Use of **local** definitions

The mess that variables make

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- ▶ *Consider the line $x = k$ where k is a constant. A **constant** that can be any number ?!?*
- ▶ The use of quantifiers and careful use of **free** variables **by us**, not necessarily by children, can clarify things easily.

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- ▶ We need good translations between the two, and a training in logic can help the educator with this.

Making meaning

Consider the following situations where a teacher makes use of a phrase in context. What do you think goes on inside the student's head ?

- ▶ is linear in P .
- ▶ is a cubic polynomial with leading coefficient $1/3$.
- ▶ vanishes when $v = c$.
- ▶ halves when n is multiplied by 4.

The burden of English

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- ▶ Often, practical considerations replace mother-tongue by the language of the region.
- ▶ There is also considerable debate on when English should be introduced as a subject.

Mothertongue?

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Skipped!

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- ▶ No mother speaks in this tongue!
- ▶ The language of **literate discourse** is alien to children in general, and acts hegimonically on first generation learners.

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- ▶ The word **naarkaram** is as alien as **quadrilateral**.

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and **only then** reason about things. W. W. Sawyer*

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- ▶ Education located entirely in the world of abstractions strengthens the agenda of conflating history and myth.
- ▶ A language devoid of local, physical reality erects further barriers of caste, class and gender.

Language and exclusion

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- ▶ A student negotiates multiple language layers: home language, school language, textbook language and the language of each academic discipline.
- ▶ All these languages can be sites of oppression and hegemony.

What is to be done?

We need to fight the tyranny of the written word as the sole marker of achievement.

- ▶ The education system should enable the students' owned language to be heard in the classroom.
- ▶ Language is a powerful means of exclusion, but it is also the instrument of liberation.
- ▶ The language of mathematics is a critical component of classroom transformation.