The big ideas of mathematics

R. Ramanujam

Azim Premji University, Bengaluru
email: jam@imsc.res.in

Nature of Math Course, Week 4

February 8, 2022

What are big ideas?

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- Their meaning is independent of context and content, but is encapsulated in what they are and how they relate.
- They provide generic approaches to a wide range of ideas, encompassing viewpoints that cross boundaries.
- They apply across topic areas, with some generic capabilities that are not restricted to a particular domain.

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- Dichotomies: finite vs. infinite, discrete vs. continuous, stochastic vs. deterministic, existential vs constructive, ...

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- Dimensionality is not just a property of space but also a means of ordering knowledge.
- Repetition can be the source of accuracy, symmetry, or chaos.

Measurement

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- Random variation (spinners, coin tosses, covid tests)

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- Symmetry broken, as in the buckling of a cylindrical beam or the growth of a fertilised egg to a slightly asymmetrical adult animal.
- Fundamental as a model for the basic forces of nature, the structure of crystals, and the growth of organisms.

Algorithms

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- Iterative procedures leading to a variety of behaviours: explosion, decay, repetition, and chaos.
- Attributes: speed, efficiency, sensitivity, generality.