LTL :

$\varphi = p \mid \neg\varphi \mid \varphi \vee \varphi \mid X\varphi \mid \varphi\, \mathcal{U}\, \varphi$

$\rho = \rho_0\, \rho_1\, \rho_2 \ldots\ldots$ (Model for LTL formulas)

$\rho_i : P \longrightarrow \{\top, \bot\}$

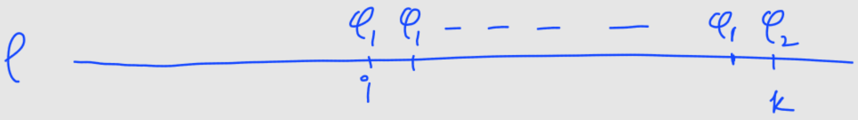
$\rho, i \models p$ if $\rho_i(p) = \top$

$\rho, i \models \neg\varphi$ if $\rho, i \not\models \varphi$

$\rho, i \models \varphi_1 \vee \varphi_2$ if $\rho, i \models \varphi_1$ or $\rho, i \models \varphi_2$

$\rho, i \models X\varphi$ if $\rho, i+1 \models \varphi$

$\rho, i \models \varphi_1\, \mathcal{U}\, \varphi_2$ if there is $k \geq i$ s.t. $\rho, k \models \varphi_2$ and for all $j \in \{i, i+1, \ldots, k-1\}$, $\varphi, j \models \varphi_1$



$F\varphi$ (sometime in the future, φ is true)
$:= \top\, \mathcal{U}\, \varphi$

$G\varphi$ (globally φ is true (at every point))

$:= \neg(F\neg\varphi)$

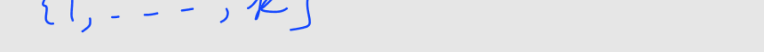
Eg: Printer shared by processors $\{1, \ldots, k\}$

g_i : processor i has been granted access to the printer

$F(g_i)$ $\quad\quad$ $X g_i$

r_i : processor i has requested access to the printer

$F(r_i\, \mathcal{U}\, g_i)$ $\quad\quad$ $GF r_i$:



(r_i appears infinitely often)

$G(r_i \Rightarrow r_i\, \mathcal{U}\, g_i)$

Satisfiability

Reduction to non-emptiness of the language of a Büchi automaton.

Vardi-Wolper construction : $|B_\varphi| = O(2^{|\varphi|})$

Closure of an LTL formula:

$CL'(\varphi)$:

$CL'(p) = \{p\}$

$CL'(\neg\varphi) = \{\neg\varphi\} \cup CL'(\varphi)$

$CL'(\varphi_1 \vee \varphi_2) = CL'(\varphi_1) \cup CL'(\varphi_2) \cup \{\varphi_1 \vee \varphi_2\}$

$CL'(X\varphi) = \{X\varphi\} \cup CL'(\varphi)$

$CL'(\varphi_1 \, U \, \varphi_2) = CL'(\varphi_1) \cup CL'(\varphi_2) \cup \{\varphi_1 \, U \, \varphi_2,\ X(\varphi_1 \, U \, \varphi_2)\}$

$CL(\varphi) = CL'(\varphi) \cup \{\neg\psi \mid \psi \in CL'(\varphi)\}$

identify $\neg\neg\psi$ with

Example: $\varphi = p \, U \, (q \vee \neg s)$

$CL(\varphi) = \{\varphi, p, q \vee \neg s, q, \neg s, s, X(p \, U (q \vee \neg s)), \neg p, \neg(q \vee \neg s),$
$\neg q, \neg X(p \, U (q \vee \neg s)), \neg\varphi\}$

Consider an LTL formula

$\varphi = (p \wedge \alpha) \vee ((q \wedge s) \, U \, \neg r)$

Ways to satisfy φ:

$p \wedge \alpha$
p, α

$(q \wedge s) \, U \, \neg r$
$\neg r$

$(q \wedge s) \, U \, \neg r$
$q \wedge s$
r
$X((q \wedge s) \, U \, \neg r)$

A
(3 groups)

P_1: For every $\varphi_1 \vee \varphi_2 \in CL(\varphi)$, $\varphi_1 \vee \varphi_2 \in A$ iff $\varphi_1 \in A$ or $\varphi_2 \in A$

P_2: For every $\neg\varphi_1 \in CL(\varphi)$, $\varphi_1 \in A$ iff $\neg\varphi_1 \notin A$

P_3: For every $\varphi_1 \, U \, \varphi_2 \in CL(\varphi)$, $\varphi_1 \, U \, \varphi_2 \in A$ iff $\varphi_2 \in A$
or ($\varphi_1 \in A$ and $X(\varphi_1 \, U \, \varphi_2) \in A$)

$A \subseteq CL(\varphi)$ is called an atom if A satisfies P_1, P_2, P_3.