SOLVING EQNS IN STRINGS ON MAKANINS ALGORITHM

– (Claudio Gutiérrez)

→ Word eqns

$$xxabxby = ababbabab\,abbbaabbaa$$

- This is a simple pattern matching problem. However it becomes a word eqn when variables are on both sides.

$$xaxby = bybyx$$

- First solution: Lentin, Plotkin, Sickmann(70's) gave algorithm which gives solution if eqn has one otherwise it runs forever.
- In 1977, Makanin solved the decidablity problem for wood eqns & proved it decidable. ~~The~~
- Later Jaffar, extended Makanin's algo to give all solution.
- Plandowski, Rytter, Jez used recompression to solve it in PSPACE.

Definitions:

- $C = \{a_1 \ldots a_r\}$ constant
- $\mathcal{V} = \{v_1, x_2 \ldots\}$ variables.
- $w \in C \cup \mathcal{V}$ word
- $|w| =$ length of word.

exponent of periodicity $(w) = p$ if $w = u v^p z$ $u, v, z \in C \cup$
$v \neq \phi$

$w = a\, abab\, b$

$w = a\,(ab)^2\, b$

$\boxed{Eop(w) = 2}$

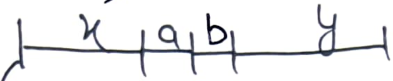
- $\mathcal{E}(w_1, w_2)$: word eqn $w_1 = w_2$
- Unifier $(\mathcal{E})$: $U = (U_1 \ldots U_n)$ such that $w_1 = w_2$ when variables $(x_1, x_2, \ldots x_n)$ are replaced by $(U_1 \ldots U_N)$.

Basically a solution.

- $EoP(U)$ is the maximal exponent of periodicity of the words U_i ??

Graphical representation

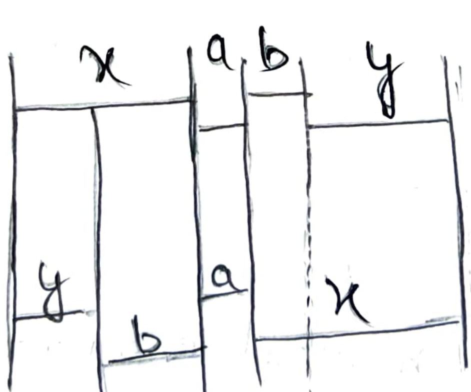
$$xaby = ybax$$



- boundaries.
- unit length is given to constants.
- length of variable can change.

Now we overlap and try to find such boundaries for the system in which the words b/w the boundaries are same.

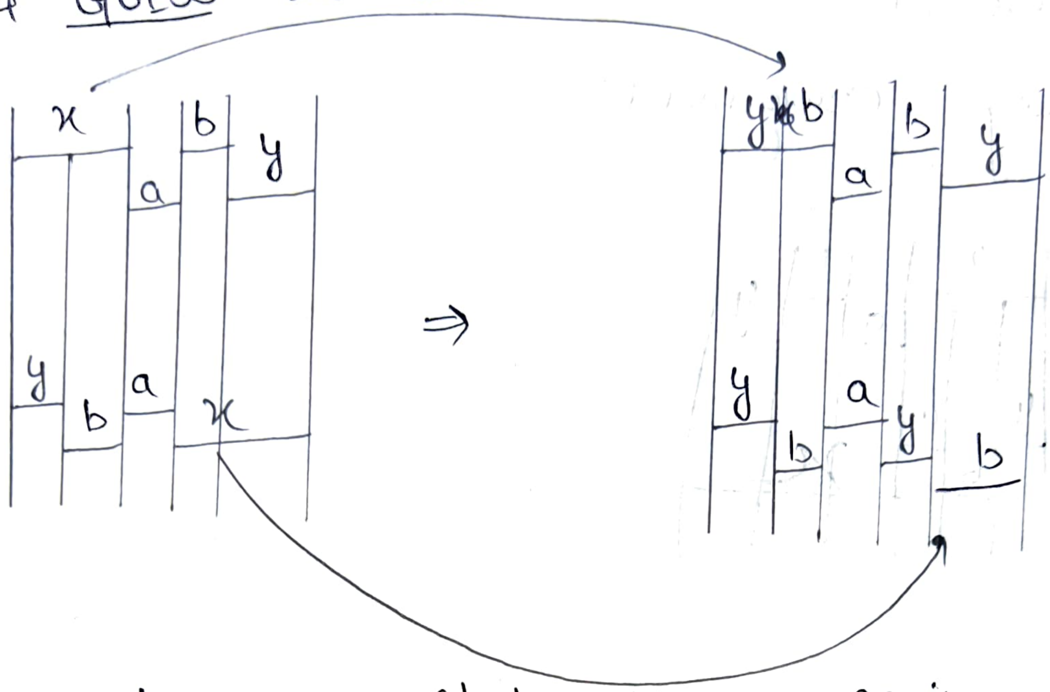
One such arrangement of boundaries.



Now we replace variable from left to right.

$$\boxed{x = yb}$$

- Now we replace all occurances of x with 'yb' & <u>GUESS</u> the new boundaries.



Now we start replacing again.

$y = y$ ✓

$b = b$ ✓

$a = a$ ✓

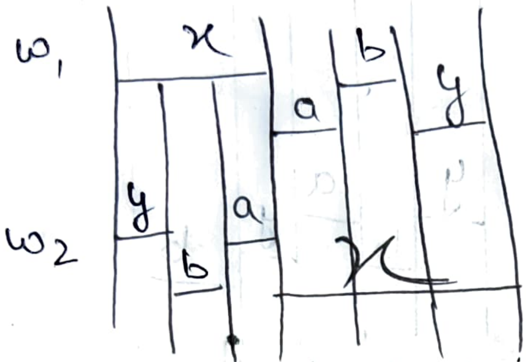
$\boxed{b = y}$

Now replace ~~to~~ all y with 'b'.

So we get : $\boxed{\begin{array}{l} y = b \\ x = yb = bb \end{array}}$ <u>Solution</u>

Another example

boundary arrangement:



(b)

$y \; b \; a$? ?

$x = y b a$?

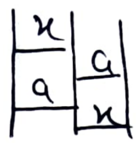
$a = y$

$y = a$

Problems:

→ number of occurance of some variable starts growing after replacement.

→ endless loop. ($xa = ax$) ??



$\underline{x = a}$??

→ what to do if there is no evident replacement. (b) ??

Solution: ~~One~~ ~~vo~~

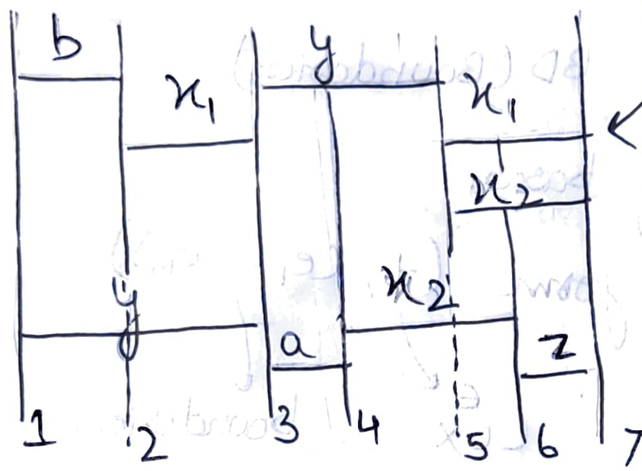
- Each variable can only occur twice in a word eqn.
- Any word eqn w/ more than two occurances of a variable can be converted into system of eqns w/ atmost two occurances

$$b\underline{x}y\underline{x} = ya\underline{x}z$$

$$\hookrightarrow \left\{ \begin{array}{l} bx_1yx_1 = yax_2z \\ x_1 = x_2 \end{array} \right\}$$

Generalised eqns: (Representation of graphical form in eqns)

$$\left\{\begin{array}{l} b x_1 y x_1 = y a x_2 z \\ x_1 = x_2 \end{array}\right\}$$



GEN(bxyx= yaxz):

$C = \{a, b\}$

$X = \{x_1, x_2, y, z\}$

$BD = \{1, 2 \ldots 7\}$

$BS = \{(b, (1,2)), (a, (3,4)), (x_1, (2,3)), (x_1, (5,7)),$
$(y, (3,5)), (y, (1,3)), (x_2, (4,6)), (x_2, (5,7))$
$(z, (6,7))\}$

GE consists of:

(1) C : constants

X : variables

(2) liner ordered set BD (Boundaries)

(3) finite set BS of bases.

bs is of form $(t, (e_1 \cdots e_n))$

$\in C \cup X$ boundaries.

Condition 1: for each variable in X, there are only two bases called duals denoted by x & $\bar{x}$. thier boundary sequence E_x & $E_{\bar{x}}$ must be same. length.

This ensures that atmost two occurance of a variable happens.

Condition 2: For each base of a constant; the boundary sequence has exactly two consecutive elements.

~~Notations~~

~~En~~ • Pair (i,j) of boundaries is called <u>indecomposable</u> if $\underline{j = i+1}$

• Column (b_s) = $(\text{left}(bs), \text{right}(bs))$

• base is empty if column is empty

• eqn is solved if ~~all~~ variable bases are empty.

<u>Definition</u> : Solution (Unifier) for a GE ~~is~~ is a function 'f'. that ~~for some~~. assigns each in-de-composable column to a word.

• For each constant (c) $\underset{\text{Unifier } L}{U}(\text{col}(bs)) = c$
~~w/ colu~~

• $\underbrace{U(\text{col}(x)) = U(\text{col}(\bar{x}))}$

basically both occurances of the variable get same word.

• U is strict if $U(i, i+1) \neq \phi$ for all $i \in BD$.

• The exponent of periodicity of U is maximal exponent of periodicity of the words $U(\text{col}(x))$??

Def 4:

Tranformation algorithm

$GE = (C, X, BD, BS)$

Definition 7: Carrier of GE 'x_c'

x_c : Smallest left boundary i.e. leftmost variable.

- if there are two leftmost variables choose the one w/ biggest rightmost boundary i.e. the bigger of the two.

$l_c = Left(x_c)$

$r_c = right(r_c)$

critical boundary ~~cr~~ $cr = \min\{LEFT(y) : r_c \in col(y)\}$

if cr is empty ; $\underline{cr = r_c}$

this basically means that choose the leftmost boundary in all the coulumns that r_c is a part of ??

Definition 10: Let b_s be a non carrier base of GE.

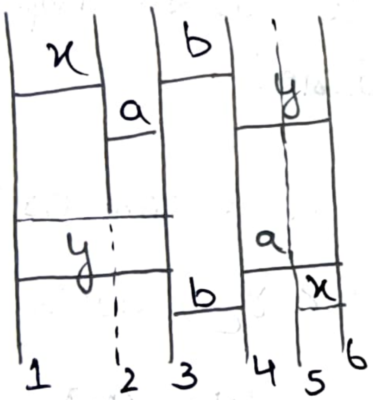
→ b_s is superflous if $col(b_s) = (i, j) < l_c$

→ b_s is transport if $l_c \leq left(b_s) < c_r$

or

$col(bs) = (cr, cr)$

→ b_s is fixed if it is not superflous or transport.

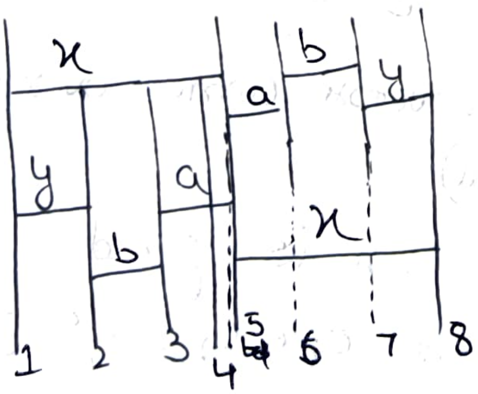


Carrier : y (leftmost, biggest)

$l_c = 1$

$r_c = 3$

$cr = 3$.

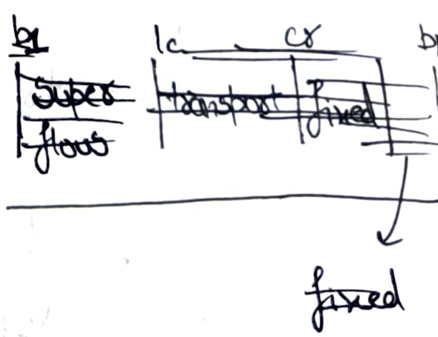


Carrier : x

l_c : 1

r_c : 5

cr : 4



$\Big|^{b_1}$ super flows $\Big|^{l_c}$ transport $\Big|^{c_r}$ fixed $\Big|^{r_c}$ fixed $\Big|^{b_M}$

these are the bases we are moving.

<u>Notation</u>: For each i such that $l_c \leq i \leq r_c$;
we will introduce a new symbol i^{tr}
and denote $tr(E_x) = tr(e_1, e_2 \dots e_n) = (e_1^{tr}, \dots e_n^{tr})$

After transport of 'x' what happens to the boundaries.

<u>Definition 9</u>: print of GE is a linear order ($\leq$) on $l_c < i < r_c$ such that.
~~on~~ ~~l_c i^{tr}~~

if $j<k$ before transport then $j^{tr} < k^{tr}$??

- $\leq$ extends the order of BD and $j^{tr} < k^{tr}$ for $l_c \leq j \leq k \leq r_c$
- $tr(E_c) = \bar{E_c}$ (the structure carries over to dual)
- if x is tranport, $\bar{x}$ is fixed then if for some $e_i \in E_x$, $e_i^{tr} = \bar{e_i}$ then $tr(E_x) = E_{\bar{x}}$.
- Constants remain unchanged.