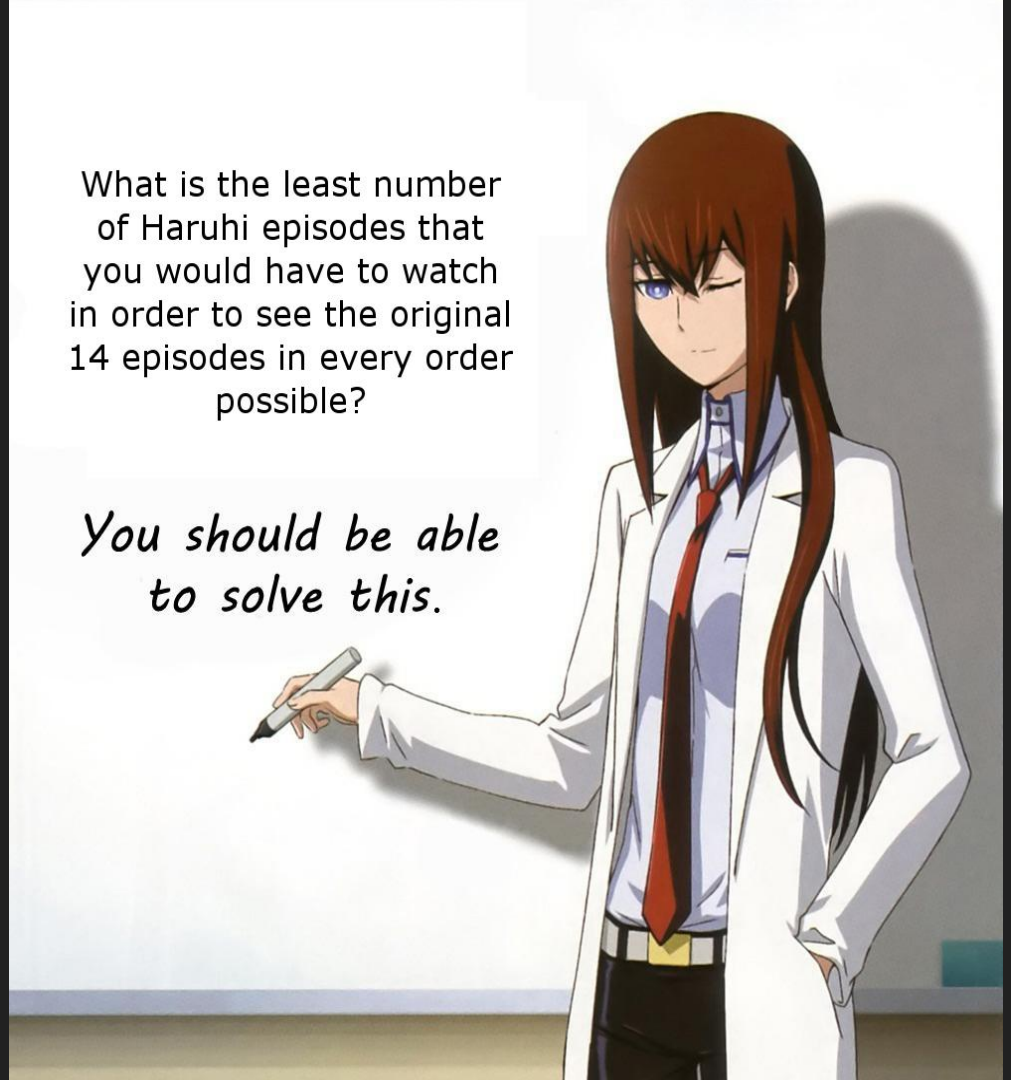

What is the least number of Haruhi episodes that you would have to watch in order to see the original 14 episodes in every order possible?
You should be able to solve this.

SUPERPERMUTATIONS

PERMUTATIONS

Superpermutation : A string which contains all the possible permutations of given characters. (overlapping allowed)

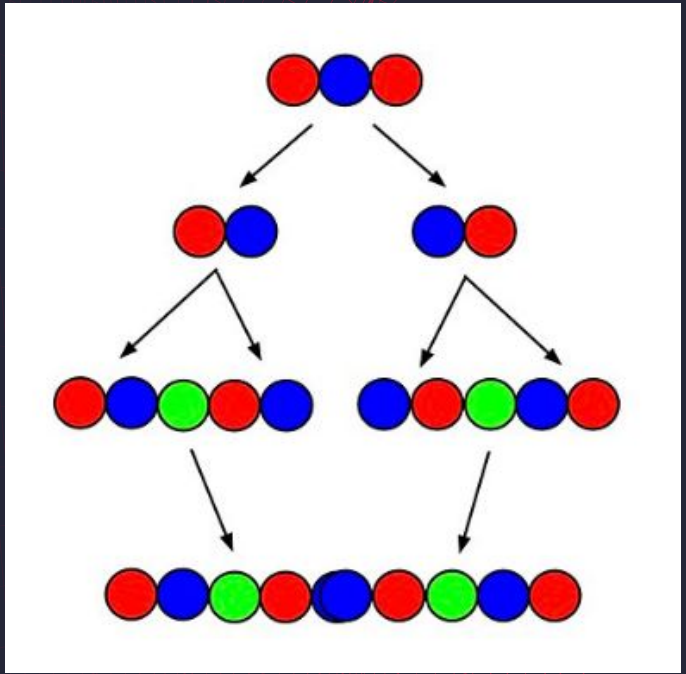
Minimal Superpermutation : Length of the shortest super permutation.

Quest for finding the minimal superpermutation

A trivial lower bound

$L(n) >= n+n!-1$

The Recursive algorithm to build superpermutations.



Given a superpermutation on $n-1$ symbols, to obtain one with n symbols you perform the following steps:

1. Write out the permutations in the original superpermutation, in the order in which they appear.
2. Duplicate each of them, placing the new symbol n between the two copies.
3. Squeeze the result back together again, making use of all available overlaps.

> Gives a superperm of length n!+(n-1)!+(n-2)!.....1!

> By exhaustive computer search we know that this algorithm gives the minimal superperm upto n=5.

> How do we know the resultant string has all the permutations

a(1) = 1

a(2) = 3

a(3) = 9

a(4) = 33

a(5) = 153

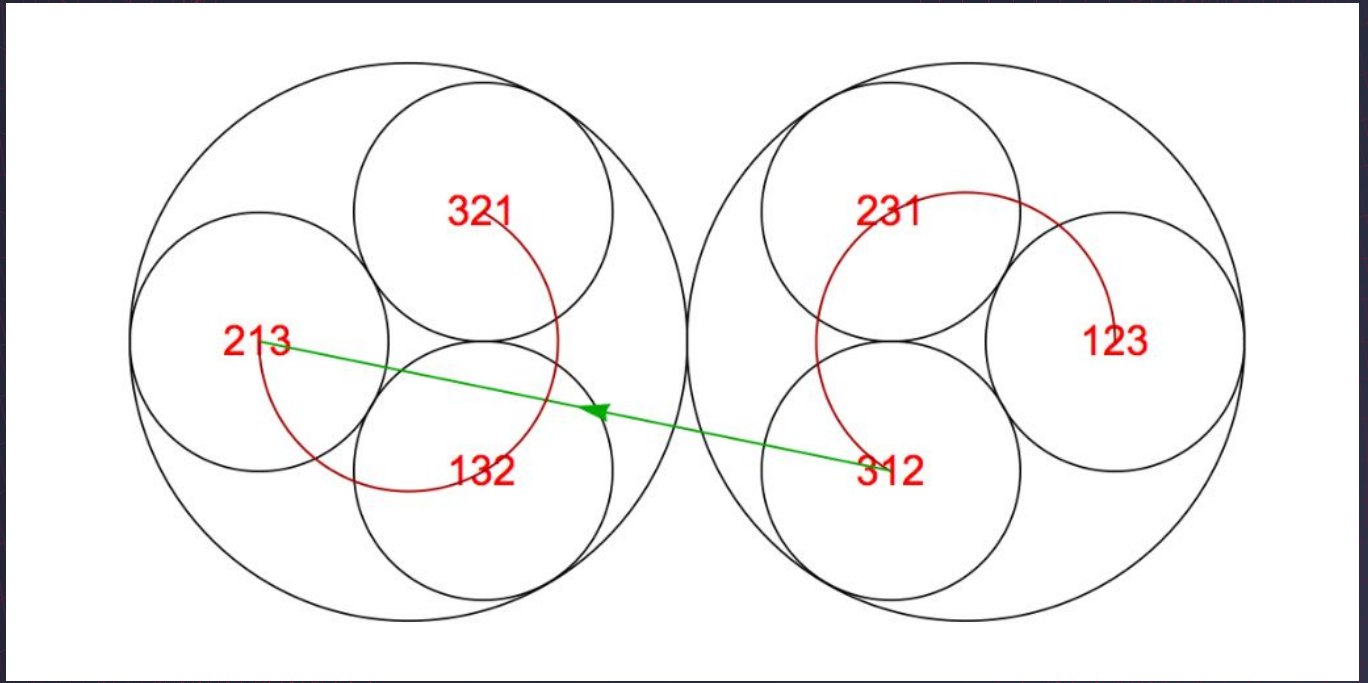
a(6) = 873 ????????

For n=6, it gives 873 which now we know is not the minimal superperm.

SuperPermutation as graphs

Robin Houston's paper :
https://www.researchgate.net/publication/264979617_Tackling_the_Minimal_Superpermutation_Problem

Weighted graph path for n = 3



Credit : Greg Egan

123456123451623451263451236451326451362451364251364521364512346512341562341526341523641523461523416523412563412536412534612534162534126534123564123546123541623541263541236541326543126453162435162431562431652431625431624531642531462531426531425631425361425316452314652314562314526314236154231654231564213564215362415362145362154362153462135462134562134652136425134621536421563421653421635421634521634251634215643251643256143256413256431265432165432615342613542613452613425613426513426153246513246531246351246315246312546321546325146325416325461325463124563214563241563124563124653214653241653246153264153261453261543265143625143652143562143526143521643521463521436512436152436125436124536124365142356142351642351462351426351423651432654136254136524135624135261352416352413654213654123

(8) (PDF) Tackling the Minimal Superpermutation Problem. Available from: https://www.researchgate.net/publication/264979617_Tackling_the_Minimal_Superpermutation_Problem [accessed Feb 18 2020].

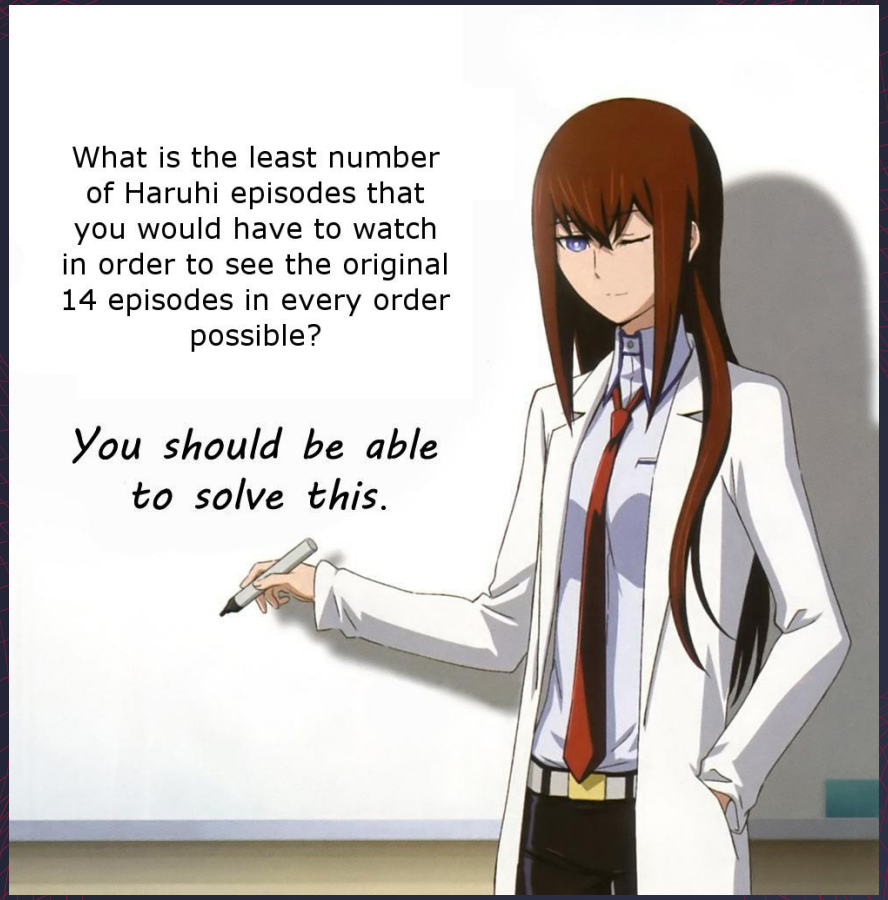
Now that he found a smaller super-perm for n=6, for any n>6, we can we can get a super perm that is one smaller than what we would originally get from 1! + 2! + 3!.....n! by applying the same recursion method on that super-perm.

Now we know

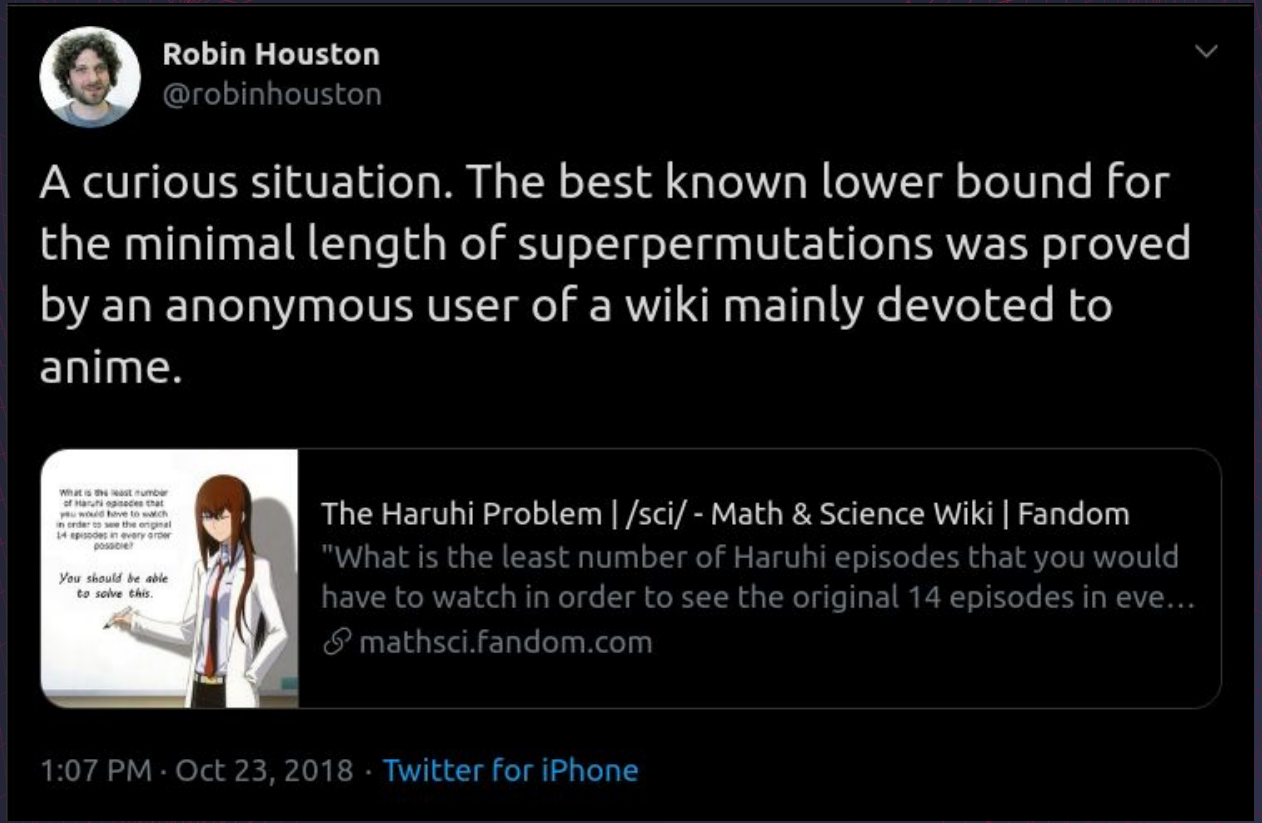
$$n! + (n-1) \leq L(n) \leq \Sigma n! \quad ; n<6$$

$$n! + (n-1) \leq L(n) \leq \Sigma n!-1 \quad ; n\geq 6$$

Haruhi Suzumiya Problem



Original archived thread : https://warosu.org/sci/thread/S3751105



A lower bound on the length of the shortest superpattern

Anonymous 4chan Poster, Robin Houston, Jay Pantone, and Vince Vatter

October 25, 2018

This proof is inspired by that posted anonymously at

http://mathsci.wikia.com/wiki/The_Haruhi_Problem

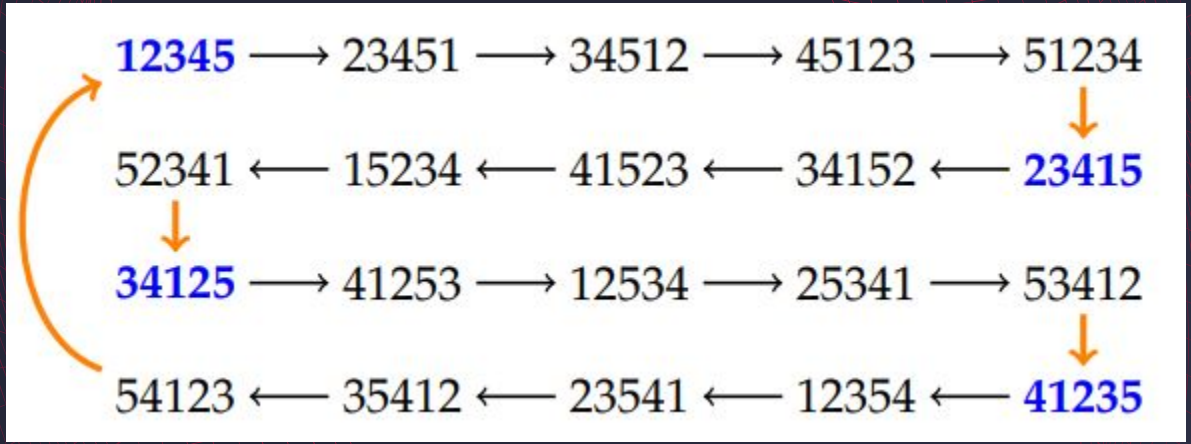
which itself was taken from a 4chan discussion archived at

https://warosu.org/sci/thread/S3751105#p3751197

Lower bound proposed by anon :

$L(n) \geq n! + (n-1)! + (n-2)! + (n-3)$

Example of 2 loop for n=5



Proposition : If a 2-loop is generated by π, then it is generated by all $n - 1$ permutations obtained by fixing the last entry of π and cyclically permuting the other entries, i.e., by π and the permutations

Now we know

$$n! + (n-1)! + (n-2)! + n-3 \leq L(n) \leq \Sigma n! \quad ; n<6$$

$$n! + (n-1)! + (n-2)! + n-3 \leq L(n) \leq \Sigma n!-1 \quad ; n\geq 6$$

References

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