

Work Sheet Two

Introduction to Mathematical Thinking-2, 20 Jan 2020

Definition. Recall the definitions of domain, co-domain, range, 1-1 function, onto function, bijective function, finite set.

Now answer the following questions.

1. Show that composition of two injections (1-1 functions) is also an injection
2. Show that composition of two surjections (onto functions) is also a surjection
3. For any function f , show that $f(A \cap B) \subseteq f(A) \cap f(B)$.
4. Show that when f is a 1-1 function $f : X \rightarrow Y$, if A and B are subsets of X , then $f(A \cap B) = f(A) \cap f(B)$
5. Is the converse of above statement true, that is if $f(A \cap B) = f(A) \cap f(B)$ for every A, B subset of X , then f is 1-1
6. Let X and Y be finite sets. Let $f : X \rightarrow Y$, compare cardinalities of X , Y and $f(X)$. when f is 1-1 compare cardinality of X and $f(X)$. Compare cardinality of X and Y when the function is onto.
7. If f is a bijection from X to Y and g is a bijection from Z to Y , show that $g^{-1}f$ is a bijection from X to Z .
8. Show that, If X and Y are equivalent finite sets, then they have the same cardinality.
9. Show that, If X and Y are two finite sets with same cardinality, then they are equivalent (that is there is a bijection from X to Y .)
10. Give a bijection between the set of natural numbers and integers. Give a bijection between natural numbers and rational numbers.
11. Let f be a bijection, so that its inverse is defined. Show that $f^{-1}(A \cup B) = f^{-1}(A) \cup f^{-1}(B)$