Strategies in proving statements

Direct Proof
Proof by cases
Proof by contradiction
Proof by proving contrapositive
Computer Assisted Proof

Warm up exercises

1. If a is odd, then a^2 is also an odd number.
2. State the converse of the above statement. Prove or give counterexample.

Note that 1 can be proved using direct proof whereas the converse used contradiction.

Don't be mean

Geometric mean is less than or equal to the Arithmetic mean.
$x + y/2 \geq xy$

Don't be mean

$(x-y)^2 \geq 0$
But, $(x-y)^2 = (x+y)^2 - 4xy$
So, $(x+y)^2/4 \geq xy$.
Now take square roots on both sides.
QED (Quite Easily Done!)
Note that the idea for this proof comes by the observation that if we square both sides of the required inequality we get $4xy \leq (x^2 + 2xy + y^2)$. We observe that rearranging terms gives that we require $x^2 - 2xy + y^2$ to be greater than or equal to 0.

More Questions

Active engagement: What were the assumptions made in the theorem.
What does the theorem mean? Does it have geometric significance ? Can we generalise this?
How to we prove it?

Proof by Contrapositive

Using Truth tables we have shown that the statement "P implies Q" is logically equivalent to "Negation Q implies Negation P".
Example 1. Let x be an integer. If 7x+9 is odd, then x is odd.
Example 2. Let x be an integer. If $x^2 - 6x + 5$ is even then x is odd.
Example 3. Let x, y be reals. If $y^3 + yx^2 \leq x^3 + xy^2$, then $y \leq x$.
Example 4. Let x, y be integers. If $5 \nmid xy$, then $5 \nmid x$ and $5 \nmid y$.
Example 5. Let a, b be integers and n a natural number. If $12a12b \pmod{n}$, then$n \nmid 12$

Computer Assisted Proof

The four-color theorem states that any map in a plane can be colored using four-colors in such a way that regions sharing a common boundary (other than a single point) do not share the same color. This problem is sometimes also called Guthrie's problem after F. Guthrie, who first conjectured the theorem in 1852. The conjecture was then communicated to de Morgan and thence into the general community. In 1878, Cayley wrote the first paper on the conjecture.

4 colour problem

This result was finally obtained by Appel and Haken (1977), who constructed a computer-assisted proof that four colors were sufficient. However, because part of the proof consisted of an exhaustive analysis of many discrete cases by a computer, some mathematicians do not accept it. However, no flaws have yet been found, so the proof appears valid. A shorter, independent proof was constructed by Robertson et al. (1996; Thomas 1998).
(source Wolfram mathematics)