Intro to Maths 2 : Worksheet 2

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1. $f : X \to Y$ is a interjection
   $g : Y \to Z$ is a interjection

   Suppose f(g(x)) is not an interjection. That means two or more elements in X have the same pre-image in Z. For that either f(x) or g(x) has to be a interjection.
   Hence the composition of two 1-1 is always 1-1.

2. Say f(g(x)) is not a bijection. That means there is a element in Z that doesn't have a pre image in X. For that to happen either

   (a) an element $z \in Z$ doesn't have a pre-image in Y making $g(x)$ non-onto. (Contradiction)

   (b) an element $y \in Y$ doesn't have a pre-image in X making $f(x)$ non-onto. (Contradiction)
   Hence composition of two surjective func is always surjective.

3.
$$\begin{aligned} &(A \cap B) \subseteq A \\ \implies &f(A \cap B) \subseteq f(A) \end{aligned} \tag{1}$$

$$\begin{aligned} &(A \cap B) \subseteq B \\ \implies &f(A \cap B) \subseteq f(B) \end{aligned} \tag{2}$$

   From 1 and 2

$$f(A \cap B) \subseteq f(A) \cap f(B)$$

4. Let $f(A \cap B) \neq f(A) \cap f(B)$.
   i.e. $\exists\ x \in (A \cap B)$
   such that $f(x) \notin f(A) \cap f(B)$
   So either $f(x) \notin f(a)$ or $f(x) \notin f(b)$ which is impossible since $f(x) \in f(A \cup B)$.
   So by contradiction, $f(A \cap B) = f(A) \cup f(B)$

5. $X = \{1, 2, 3, 4, 5, 6\}$
   $Y = \{a, b, c, d\}$

   $f : X \to Y$
   $f(1) = a$

$f(2) = b$
$f(3) = c$
$f(4) = d$
$f(5) = d$

$A = \{1, 2, 3, 4\}$
$B = \{3, 4, 5, 6\}$
$A \cap B = \{3, 4\}$
$f(A \cap B) = \{b, c\}$
$f(A) \cap f(B) = \{b, c\}$
$f(A \cap B) = f(A \cap B)$ and $f(x)$ isn't one-one. So the converse isn't true.

6. When f is 1-1, the $cardinality(X) \geq cardinality(f(X))$.

   (a) $cardinality(X) = cardinality(f(X))$: When all the elements of set X are mapped to distinct elements in set Y

   (b) $cardinality(X) > cardinality(f(X))$: When any element in set Y has more than one pre-image in set X.

   When f in onto, the $cardinality(X) \geq cardinality(Y)$.

   (a) $cardinality(X) = cardinality(Y)$: one one function.

   (b) $cardinality(X) > cardinality(Y)$: Mutilple elements on set X can be mapped to a single element in set Y making it onto.

7. $f : X \to Y$ is a bijection
   $g : Z \to Y$ is a bijection
   $g^{-1} : Y \to Z$ is a bijection
   $f^{-1} : Y \to X$ is a bijection

   $g^{-1}f$ is $X \to Y \to Z$
   Let $x \in X$. X only has one image in Y i.e. $f(x)$
   f(x) only has one pre-image in Z i.e. $g^{-1}f(x)$
   Hence $g^{-1}f(x)$ is a one-one function.

   For $g^{-1}f$ to not be an onto function, there needs to be a $z \in Z$ such that it doesn't have a image in Y which is impossible since $g : Z \to Y$ is a bijection.
   So $g^{-1}f$ is a bijection.

8. X and Y are equivalent i.e. there is a bijection from X to Y.
   Let cardinality(X) $\neq cardinality(Y)$.

   (a) $cardinality(X)\ cardinality(Y)$: Its not a one-one function.

   (b) $cardinality(X) cardinality(Y)$: Its not a onto function.

$|x| = |y| = n$

every element $x \in$ has a unique $f(x) \in Y$.

9.

$$f(n) = \begin{cases} \frac{n}{2}, & n \text{ is even} \\ \frac{-(n+1)}{2}, & n \text{ is odd} \end{cases}$$

$f : \mathbb{N} \to \mathbb{Z}$

10.