Intro to Maths 2 : Assesment

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Part A

1. In S_3, find 4 elements with order 2.

   Elements in S_3:
   () $\rightarrow$ identity element
   (1,2)
   (2,3)
   (1,3)
   (1,2,3)
   (1,3,2)

   Elements with order 2:
   $(1,2)$ since $(1,2) * (1,2) = ()$
   $(1,3)$ since $(1,3) * (1,3) = ()$
   $(3,2)$ since $(1,2) * (3,2) = ()$
   () since its identity and can take any order.

2. Show that following sets are groups:

   (a) $X = \{a + \sqrt{2}b\,; a, b \in Z\}$
   It is a group under addition.
   Closure : $(a + \sqrt{2}b) + (a' + \sqrt{2}b') = (a + a') + \sqrt{2}(b + b')$
   Since $(a + a')$ and $(b + b') \in Z$, It follows closure.

   Associative: Since $a, b, \sqrt{2} \in R$ and Real numbers are always associative under addition, this set follows associative.

   Identity: The identity element is 0 i.e. $a, b = 0$

   Inverse: $a + \sqrt{2}b + x = 0$
   $x = -a - \sqrt{2}b$
   Inverse is $-a - \sqrt{2}b$

(b) $y = \{a + \iota b\,; a, b \in z, i^2 = 1\}$
It is a group under addition.
Closure : $(a + \iota b) + (a' + \iota b') = (a + a') + \iota(b + b')$
Since $(a + a')$ and $(b + b') \in Z$, It follows closure.

Associative:
$(a + \iota b) + [(c + \iota d) + (e + \iota f)] = [(a + \iota b) + (c + \iota d)] + (e + \iota f) = (a + c + e) + \iota(b + d + f)$
So it is associative.

Identity: The identity element is 0 i.e. $a, b = 0$

Inverse: $(a + \iota b) + x = 0$
$x = (-a - \iota b)$
Inverse is $(-a - \iota b)$

Part B

mod $G = 24$ List subgroups of order :

(a) Order 2: $\{(), (12)\}, \{()(23)\}, \{()(34)\}, \{()(24)\}, \{()(13)\}, \{()(14)\}\{(), (13)(24)\}, \{(), (23)(14)\}, \{(), (12)(34)\}$
(b) Order 3: $\{(), (132), (123)\}, \{(), (124), (142)\}, \{(), (134), (143)\}, \{(), (234), (243)\}$
(c) Order 6:$\{(), (123), (12), (23), (13), (321)\}, \{(), (124), (12), (142), (24), (14)\}, \{(), (134), (13), (14), (34), (143)\},$
(d) Order 12: Even permutation group.

Part C

Let $G = Z$ and $H = 4Z$:

(a) Show that H is a subgroup
It is a subgroup under addition:
$n \in Z, G$
Identity: Inverse is 0. $4n + 0 = 4n$
Inverse: -4n. $4n + (-4n) = 0$ i.e. identity

(b) Find left cosets of H
G={...-2,-1,0,1,2,3...}
H={...-8,-4,0,4,8...}
For finding cosets we'll multiply elements of Left cosets(L) i.e. gH: {...-8+n, -4+n, 0+n, 4+n, 8+n...}

(c) Find right cosets of H
Right cosets(R) are i.e. Hg: {...n+(-8), n+(-4), n+0, n+4, n+8...}

(d) Can you find a bijection between set of left cosets and right cosets.
Since addition is commutative, both right cosets and left cosets are same i.e. their cardinality is also same. So a bijection from the set of left cosets to a set of right cosets does exist.
$L \to R : k + n \to n + k$ k $\in H$