Work Sheet Five

Introduction to Mathematical Thinking-2, 21 Feb 2020

Definition. Logical equivalence Two (molecular) statements P and Q are logically equivalent provided P is true precisely when Q is true. That is, P and Q have the same truth value under any assignment of truth values to their atomic parts. Recall that a statement is atomic if it cannot be divided into smaller statements, otherwise it is called molecular.

1. To verify that two statements are logically equivalent, you can make a truth table for each and check whether the columns for the two statements are identical. Check if the following statements are logically equivalent.
   1. $\neg P \vee Q$ and $P \implies Q$
   2. De Morgan's Laws. $\neg(P \wedge Q)$ is logically equivalent to $\neg P \vee \neg Q$.
   3. De Morgans laws . $\neg(P \vee Q)$ is logically equivalent to $\neg P \wedge \neg Q$

2. Use truth table to decide if the following is true or false. If you get more doubles than any other player then you will lose OR if you lose then you must have bought the most properties. (Hint: Write this as implication or implication. Assign all possible truth values to the parts of the implication).

3. Are the statements $(P \vee Q) \implies R$, $(P \implies R) \vee (Q \implies R)$ logically equivalent?

4. Make a truth table for the statement $(P \vee Q) \implies (P \wedge Q)$

5. Determine whether the following two statements are logically equivalent: $\neg(P \implies Q)$ and $P \wedge \neg Q$

6. Are the statements $P \implies (Q \vee R)$, $(P \implies Q) \vee (P \implies R)$ logically equivalent?

7. Simplify the following statements (so that negation only appears right before variables).
   a) $\neg(P \implies \neg Q)$
   b) $(\neg P \vee \neg Q) \implies \neg(\neg Q \wedge R)$.

8. Consider the statement, If a number is triangular(T) or square(S), then it is not prime (P) a) Make a truth table for the statement $(T \vee S) \implies \neg P$

b) If you believed the statement was false, what properties would a counterexample need to possess? Explain by referencing your truth table. c) If the statement were true, what could you conclude about the number 5657, which is definitely prime? Again, explain using the truth table.

9. Trump was telling you what he ate yesterday afternoon. He tells you, I had either popcorn or raisins. Also, if I had cucumber sandwiches, then I had soda. But I didn't drink soda or tea. Of course you know that Trump is the worlds worst liar, and everything he says is false. What did Trump u eat? Justify your answer by writing all of Trump' s statements using sentence variables (P,Q,R,S,T), taking their negations, and using these to deduce what Trump actually ate. (Hint: Write down three statements, and then take the negation of each (since he is a liar). You should find that Trump ate one item and drank one item. (Q is for cucumber sandwiches.)