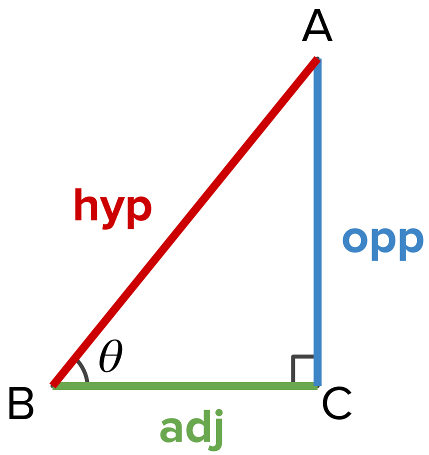
Trigonometry

Trigonometric Ratio:

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} \qquad \operatorname{cosec}\theta = \frac{\text{hyp}}{\text{opp}}$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} \qquad \sec\theta = \frac{\text{hyp}}{\text{adj}}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}} \qquad \cot\theta = \frac{\text{adj}}{\text{opp}}$$



$$\sin\theta . \operatorname{cosec}\theta = 1$$

$$\cos\theta . \sec\theta = 1$$

$$\tan\theta . \cot\theta = 1$$

$$\frac{\sin\theta}{\cos\theta} = \tan\theta$$

$$\frac{\cos\theta}{\sin\theta} = \cot\theta$$

Trigonometric ratios of angles

$\text{Sin}(90-A) = \cos(A)$

$\text{Cos}(90-A) = \sin A$

$\text{Tan}(90-A) = \cot A$

$\text{Sec}(90-A) = \text{cosec } A$

$\text{Cosec}(90-A) = \sec A$

$\text{Cot}(90-A) = \tan A$

Trigonometric identities

$\text{Sin}^2 A + \cos^2 A = 1$

$1 + \tan^2 A = \sec^2 A$

$1 + \cot^2 A = \text{cosec}^2 A$

Trigonometric Ratios of Common angles

Angles (A)	Sin A	Cos A	Tan A	Cosec A	Sec A	Cot A
0^0	0	1	0	Not defined	1	Not defined
30^0	1/2	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$	2	$\frac{2}{\sqrt{3}}$	$\sqrt{3}$
45^0	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1	$\sqrt{2}$	$\sqrt{2}$	1
60^0	$\frac{\sqrt{3}}{2}$	1/2	$\sqrt{3}$	$\frac{2}{\sqrt{3}}$	2	$\frac{1}{\sqrt{3}}$
90^0	1	0	Not defined	1	Not defined	0

Total Marks: 1

Q. In a rectangle **ABCD**, **AB = 20 cm**, **∠BAC = 60°**. The length of the side **BC** is ______ .

A) $10\sqrt{3}$ cm

B) $20\sqrt{3}$ cm

C) $30\sqrt{3}$ cm

D) $40\sqrt{3}$ cm

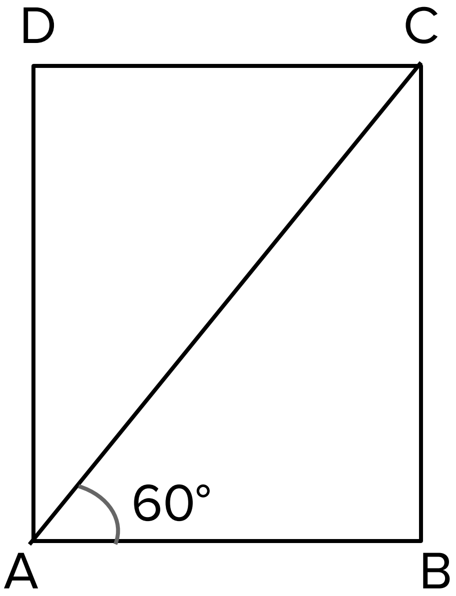
SOLUTION

In Δ **ABC,**

$$\tan 60^\circ = \frac{BC}{AB}$$

$$\Rightarrow \quad \sqrt{3} = \frac{BC}{20}$$

$$\Rightarrow \quad BC = 20\sqrt{3} \text{ cm}$$



Total Marks: 1

Q. In a rectangle **ABCD**, **AB = 20 cm**, **∠ BAC = 60°**. The length of the side **BC** is ______ .

A) $10\sqrt{3}$ cm

B) $20\sqrt{3}$ cm

C) $30\sqrt{3}$ cm

D) $40\sqrt{3}$ cm

Total Marks: 2

Q. Find the value of $8\tan^2\theta - 8\sec^2\theta$.

SOLUTION

$$8\tan^2\theta - 8\sec^2\theta = 8(\tan^2\theta - \sec^2\theta)$$

$$= -8(\sec^2\theta - \tan^2\theta)$$

$$= -8(1) \qquad \because \sec^2\theta - \tan^2\theta = 1$$

$$= -8$$

Total Marks: 3

Q. Evaluate without using trigonometric tables:

$$\frac{3\cos 55^0}{7\sin 35^0}-\frac{4\left(\cos 70^0 . \cos ec20^0\right)}{7\left(\tan 5^0 . \tan 25^0 . \tan 45^0 . \tan 65^0 . \tan 85^0\right)}$$

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SOLUTION

$$\cos 55^0 = \cos\left(90^0 - 35^0\right) = \sin 35^0$$

$$\cos 70^0 = \cos\left(90 - 20\right) = \sin 20^0$$

$$\tan 5 = \cot 85^0 \tan 25^0 = \cot 20^0$$

$$\Rightarrow \frac{3 \sin 35^0}{7isn35^0} - \frac{4\left(\sin 20^0 \cos ec\, 20^0\right)}{7\left(\cot 85^0 \tan 85^0 \cot 65^0 \tan 65^0 \tan 45^h\right)}$$

$$= \frac{3}{7} - \frac{4}{7} = -\frac{1}{7}$$

Total Marks: 3

Q. If $\cot\theta = \frac{7}{8}$, evaluate:

$$(i)\ \frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$$

$$(ii)\ \cot^2\theta$$

SOLUTION

In right ΔABC, $\angle B = 90^\circ$ and $\angle A = \theta$

$\cot\theta = \frac{7}{8}$ .... (1) [Given]

But in right ΔABC, $\cot\theta = \frac{AB}{BC}$ ..... (2)

From (1) and (2), $\frac{AB}{BC} = \frac{7}{8} = \frac{7k}{8k}$

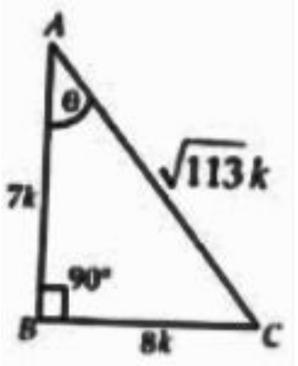
$\Rightarrow AB = 7k$ and $BC = 8k$

$\therefore AC^2 = AB^2 + BC^2 = (7k)^2 + (8k)^2$ (Pythagoras theorem)

$AC = \sqrt{49k^2 + 64k^2} = \sqrt{113k^2} = \sqrt{113}k$

$\therefore \sin\theta = \frac{BC}{AC} = \frac{8k}{\sqrt{113}k} = \frac{8}{\sqrt{113}}$

$\cos\theta = \frac{AB}{AC} = \frac{7k}{\sqrt{113}k} = \frac{7}{\sqrt{113}}$



SOLUTION

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Now,

$$(i)\ \frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)} = \frac{\left(1+\frac{8}{\sqrt{113}}\right)\left(1-\frac{8}{\sqrt{113}}\right)}{\left(1+\frac{7}{\sqrt{113}}\right)\left(1-\frac{7}{\sqrt{113}}\right)}$$

$$\frac{1^2-\left(\frac{8}{\sqrt{133}}\right)^2}{1^2-\left(\frac{7}{\sqrt{113}}\right)^2} = \frac{1-\frac{64}{113}}{1-\frac{49}{113}} = \frac{\frac{113-64}{113}}{\frac{113-49}{113}} = \frac{\frac{49}{113}}{\frac{64}{113}} = \frac{49}{64}$$

$$(ii)\ \cot^2\theta = \left(\frac{7}{8}\right)^2 = \frac{49}{64}.$$

Total Marks: 4

Q. Prove that:

$$\left(\frac{1}{\sec^2\theta-\cos^2\theta}+\frac{1}{\cos ec^2\theta-\sin^2\theta}\right)\times \sin^2\theta.\cos^2\theta$$

$$=\frac{1-\sin^2\theta\cos^2\theta}{2+\sin^2\theta\cos^2\theta}$$

SOLUTION

$L.H.S.$

$$= \left(\frac{1}{\sec^2\theta - \cos^2\theta} + \frac{1}{\cos ec^2\theta - \sin^2\theta}\right)\sin^2\theta\cos^2\theta$$

$$= \left[\frac{1}{\frac{1}{\cos^2\theta} - \cos^2\theta} + \frac{1}{\frac{1}{\sin^2\theta} - \sin^2\theta}\right]\sin^2\theta\cos^2\theta$$

$$= \left[\frac{\cos^2\theta}{1-\cos^4\theta} + \frac{\sin^2\theta}{1-\sin^4\theta}\right]\sin^2\theta\cos^2\theta$$

$$= \left[\frac{\cos^2\theta}{(1+\cos^2\theta)(1-\cos^2\theta)} + \frac{\sin^2\theta}{(1-\sin^2\theta)(1+\sin^2\theta)}\right]\sin^2\theta\cos^2\theta$$

$$= \left[\frac{\cos^2\theta}{(1+\cos^2\theta)\sin^2\theta} + \frac{\sin^2\theta}{\cos^2\theta(1+\sin^2\theta)}\right]\sin^2\theta\cos^2\theta$$

$$= \frac{\cos^4\theta}{1+\cos^2\theta} + \frac{\sin^4\theta}{1+\sin^2\theta}$$

SOLUTION

$$= \frac{\left[\cos^4\theta\left(1+\sin^2\theta\right)+\sin^4\theta\left(1+\cos^2\theta\right)\right]}{\left(1+\cos^2\theta\right)\left(1+\sin^2\theta\right)}$$

$$= \frac{\cos^4\theta+\sin^2\theta\cos^4\theta+\sin^4\theta+\cos^2\theta\sin^4\theta}{\left(1+\cos^2\theta\right)\left(1+\sin^2\theta\right)}$$

$$= \frac{\sin^4\theta+\cos^4\theta+\sin^2\theta\cos^2\theta\left(\cos^2\theta+\sin^2\theta\right)}{\left(1+\cos^2\theta\right)\left(1+\sin^2\theta\right)}$$

$$= \frac{\left(\sin^2\theta\right)^2+\left(\cos^2\theta\right)^2+2\sin^2\theta\cos^2\theta-\sin^2\theta\cos^2\theta}{\left(1+\cos^2\theta\right)\left(1+\sin^2\theta\right)}$$

$$= \frac{\left(\sin^2\theta+\cos^2\theta\right)^2-\sin^2\theta+\cos^2\theta}{1+\sin^2\theta+\cos^2\theta+\sin^2\theta\cos^2\theta}$$

$$= \frac{1-\sin^2\theta\cos^2\theta}{1+1+\sin^2\theta\cos^2\theta} = \frac{1-\sin^2\theta\cos^2\theta}{2+\sin^2\theta\cos^2\theta} = R.H.S.$$

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Reach out to me @

gopal.paliwal@vedantu.com