```
N <- 10
y <- c(0, 1, 0, 0, 0, 0, 0, 0, 0, 1)
```

Write and fit a Stan program to model y as $\mathsf{Bernoulli}(\theta)$

```
bernoulli_code <- "
  data {
    int<lower = 0> N;
    int<lower = 0, upper = 1> y[N];
  }
  .
  .
  .
  .
  .
  .
"
N <- 10
y <- c(0, 1, 0, 0, 0, 0, 0, 0, 0, 1)
data <- list(N = N, y = y)
fit1 <- stan(model_code = bernoulli_code,
             data = data)
```

```
bernoulli_code <- "
  data {
    int<lower = 0> N;
    int<lower = 0, upper = 1> y[N];
  }
  parameters {
    real<lower = 0, upper = 1> theta;
  }
  .
  .
  .
"
N <- 10
y <- c(0, 1, 0, 0, 0, 0, 0, 0, 0, 1)
data <- list(N = N, y = y)
fit1 <- stan(model_code = bernoulli_code,
             data = data)
```

```
bernoulli_code <- "
  data {
    int<lower = 0> N;
    int<lower = 0, upper = 1> y[N];
  }
  parameters {
    real<lower = 0, upper = 1> theta;
  }
  model {
    y ~ bernoulli(theta);
  }
"
N <- 10
y <- c(0, 1, 0, 0, 0, 0, 0, 0, 0, 1)
data <- list(N = N, y = y)
fit1 <- stan(model_code = bernoulli_code,
             data = data)
```

Add a `generated quantities` block to make the predictions for:

$$t = \sum_{i=1}^{10} y_i$$

Use this to find the posterior predictive probability that the sum is equal to 5.

$$\Pr[\tilde{t} \mid y] = 5$$

In R:

```
theta <- as.matrix(fit1, pars = "theta")
y_rep <- sapply(theta,
  function(theta.i) rbinom(10, 1, theta))
t_rep <- apply(y_rep, 2, sum)
mean(t_rep == 5)
```

```
bernoulli_code <- "
  data { ... }
  parameters { ... }
  model { ... }
  generated quantities {
    .
    .
    .
    .
    .
  }
"
```

```
bernoulli_code <- "
  data { ... }
  parameters { ... }
  model { ... }
  generated quantities {
    int t_rep;
    {
      int y_rep[N];
      for (n in 1:N)
        y_rep[n] = bernoulli_rng(theta);
      t_rep = sum(y_rep);
    }
  }
"
fit2 <- stan(model_code = bernoulli_code,
             data = data)
mean(as.matrix(fit2, pars = "t_rep") == 5)
```

Add a beta$(5, 5)$ prior to θ and compare with the default

```
bernoulli_code <- "
  data { ... }
  parameters { ... }
  model {
    y ~ bernoulli(theta);
    .
  }
"
fit3 <- stan(model_code = bernoulli_code,
             data = data)
```

```
bernoulli_code <- "
  data { ... }
  parameters { ... }
  model {
    y ~ bernoulli(theta);
    theta ~ rbeta(5, 5);
  }
"
fit3 <- stan(model_code = bernoulli_code,
             data = data)
```

```
prior_comparison <- cbind(
  "Flat prior" =
    runif(4000, 0, 1),
  "Beta prior" =
    rbeta(4000, 5, 5)
)
mcmc_areas(prior_comparison)
```

```
posterior_comparison <- cbind(
  "Flat prior" =
    as.matrix(fit1, pars = "theta")[,1],
  "Beta prior" =
    as.matrix(fit3, pars = "theta")[,1]
)
mcmc_areas(posterior_comparison)
```