# 5. Why we don't (usually) have to worry about multiple comparisons

- ▶ What is the multiple comparisons problem?
- Why don't we (usually) care about it?
- Some stories
- Statistical framework and multilevel modeling

## What is the multiple comparisons problem?

- Even if nothing is going on, you can find things
  - Data snooping
  - Overwhelmed by data and plausible "findings"
- ▶ "If not accounted for, false positive differences are very likely to be identified": 5% of our 95% intervals will be wrong

## Why don't I (usually) care about multiple comparisons?

- When looked at one way, multiple comparisons seem like a major worry
- ▶ But from another perspective, they don't matter at all:
  - ▶ I don't (usually) study phenomena with zero effects
  - ▶ I don't (usually) study comparisons with zero differences
  - ▶ I don't mind being wrong 5% of the time

#### Some stories

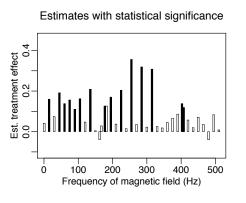
- "Beautiful parents have more daughters" (already discussed)
- ► SAT coaching in 8 schools
- Effects of electromagnetic fields at 38 frequencies
- Teacher and school effects in NYC schools
- Grades and classroom seating
- Comparing test scores across states

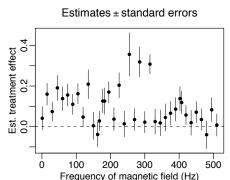
## SAT coaching in 8 schools

	Estimated treatment	Standard error of effect
School	effect, $y_j$	estimate, $\sigma_j$
A	28	15
В	8	10
C	-3	16
D	7	11
Е	-1	9
F	1	11
G	18	10
Н	12	18

- Separate experiment in each school
- Variation in treatment effects is indistinguishable from 0
- Multilevel Bayes analysis
  - Overlapping confidence intervals for the 8 school effects
  - Statements such as Pr (effect in A > effect in C)= 0.7

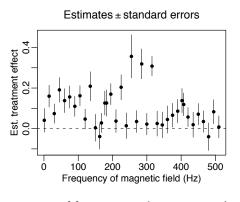
## Effects of electromagnetic fields at 38 frequencies

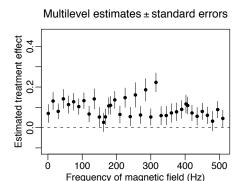




- Background: electromagnetic fields and cancer
- Original article summarized using p-values
- Confidence intervals show comparisons more clearly

## Separate estimates and hierarchical Bayes estimates





- Most comparisons are no longer statistically significant
- "Multiple comparisons" is less of a concern
- We moved the intervals together instead of widening them!

#### Teacher and school effects in NYC schools

- Goal is to estimate range of variation (How important are teachers? Schools?)
- ► Key statistic is year-to-year persistence (e.g., for teachers ranked in top 25% one year, how well do they do the next?)
- ▶ The "multiple comparisons" issue never arises!

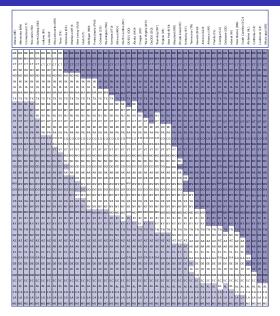
## Grades and classroom seating

- Classroom demonstration
- Assign students random numbers as "grades"
- ▶ Ask students with "grades" 0-25 to raise one finger, students with "grades" 75-100 to raise one hand
- ▶ Instructor scans the room to find a statistically significant comparison (e.g., "boys on the left side of the classroom have higher grades than girls in the back row")
- ▶ This is a pure multiple comparisons problem!

## Comparing test scores across states

- National Assessment of Educational Progress (NAEP)
- Comparing states: which comparisons are statistically significant?
- ▶  $50 \times 49/2$ : a classic multiple comparions problem!
- Our multilevel inferences

## Classical multiple comparisons inferences for NAEP



## Classical inferences for NAEP: close-up

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### Multilevel inferences for NAEP: close-up

#### Comparisons of Average Mathematics Scale Schores for Grade 4 Public Schools in Participating Jurisdictions

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#### NAEP: classical vs. multilevel

- Both procedures are algorithmic ("push a button")
- ▶ Both procedures treat 50 states exchangeably
- Multilevel inferences are sharper (more comparisons are "statistically significant")
- ► How can this be?

## Something for nothing? A free lunch?

- ► Classical multiple comparisons worries about  $\theta_1 = \theta_2 = \cdots = \theta_{50}$
- Not an issue with NAEP
- Multilevel model estimates the group-level variance, decides based on the data how much to adjust
- Classical procedure does not learn from the data

## Message from the examples

- Classical multiple comparisons corrections don't seem so important when we fit hierarchical models
- ▶ But they can be crucial for classical comparisons

#### Statistical framework

- ▶ Goal is to estimate  $\theta_j$ , for j = 1, ..., J (for example, effects of J schools)
- ▶ Comparisons have the form,  $\theta_j \theta_k$
- ▶ For simplicity, suppose data come from *J* separate experiments
- Type S errors
- Multilevel modeling as a solution to the multiple comparisons issue

## Type S (sign) errors

- ▶ I've never made a Type 1 error in my life
  - ▶ Type 1 error is  $\theta_i = \theta_k$ , but I claim they're different
  - ▶ I've never studied anything where  $\theta_j = \theta_k$
- ▶ I've never made a Type 2 error in my life
  - ▶ Type 2 error is  $\theta_j \neq \theta_k$ , but I claim they're the same
  - I've never claimed that  $\theta_j = \theta_k$
- But I make errors all the time!
- ▶ Type S error:  $\theta_1 > \theta_2$ , but I claim that  $\theta_2 > \theta_1$  (or vice versa)
- ► Type S errors can occur when we make *claims with confidence* (i.e., have confidence intervals for  $\theta_j \theta_k$  that exclude zero)
- ▶ We want to make fewer claims with confidence when we are less certain about the ranking of the  $\theta_j$ 's

## Multilevel (hierarchical) modeling

- ▶ Key parameter:  $\sigma_{\theta}$ , the sd of the true  $\theta_i$ 's
- Understand through special cases:
  - ho  $\sigma_{\theta} \approx 0$ : no variation
    - ▶ Multilevel model pools the estimated  $\theta_j$ 's toward each other
    - "Multiple comparisons" correction is done by shrinking comparisons
    - Very few claims with confidence (far fewer than 5%)
  - $\sigma_{\theta} \to \infty$ : large variation
    - ▶ Multilevel model is equivalent to estimating each  $\theta_j$  separately
    - "Multiple comparisons" corrections are not needed
- ▶ Bayesian multilevel modeling bounds the Type S error rate by automatically restricting the rate of claims with confidence

## Bayes shrinks the comparisons toward 0

► Partial pooling tends to reduce the number of statistically significant comparisons:

$$\begin{array}{rcl} \text{posterior E}(\theta_j - \theta_k) & = & \frac{\sigma_\theta^2}{\sigma_{\bar{y}}^2 + \sigma_\theta^2} (\bar{y}_j - \bar{y}_k) \\ \text{posterior sd}(\theta_j - \theta_k) & = & \sqrt{2}\sigma_{\bar{y}}\sigma_\theta \left/ \sqrt{\sigma_{\bar{y}}^2 + \sigma_\theta^2} \right. \\ \text{posterior z-score of } \theta_j - \theta_k : & \frac{(\bar{y}_j - \bar{y}_k)}{\sqrt{2}\sigma_{\bar{y}}} \cdot \frac{1}{\sqrt{1 + \sigma_{\bar{y}}^2/\sigma_\theta^2}} \end{array}$$

► Posterior mean of the difference is pulled toward 0, faster than the posterior sd decreases

## Summary on multiple comparisons

- Don't "fix" by altering p-values or (equivalently) by making confidence intervals wider
- Instead, multilevel modeling does partial pooling where necessary (especially when much of the variation in the data can be explained by noise), so that few claims can be made with confidence
- "Adjustments" are a dead end; "modeling" is forward-looking