Mathematical models of hysteresis

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Hysteresis may be defined as a *rate-independent memory effect,* and occurs in several phenomena in physics and other disciplines. Typical examples include plasticity, ferromagnetism, ferroelectricity, superconductivity, friction, flow in porous media, and so on.

Hysteresis was represented via continuous *hysteresis operators* in the 1970s by Krasnosel'skiĭ and co-workers [KrPo]:

$$w(t) = [\mathcal{F}(u(\cdot), w^0)](t) \qquad \forall t \geq 0. \tag{1}$$

Here by $u(\cdot)$ we denote the function $t \mapsto u(t)$, and by $\mathcal{F}$ a causal operator $C^0([0,T]) \to C^0([0,T])$.

Some models of hysteresis in elasto-plasticity may be represented via variational inequalities, and may thus be dealt within the framework of convex analysis. Others require a different approach.

Partial differential equations with hysteresis have been considered since the early 1980s. They include quasilinear parabolic and hyperbolic equations of the form

$$\frac{\partial}{\partial t}[u + \mathcal{F}(u)] + Au = f, \qquad \frac{\partial^2}{\partial t^2}[u + \mathcal{F}(u)] + Au = f. \tag{2}$$

Here $\mathcal{F}$ denotes a continuous hysteresis operator, and A is an elliptic operator. A suitable monotonicity-type condition is assumed for $\mathcal{F}$. The Cauchy problem for the first PDE is well-posed [Vi1,Vi3].

A different approach is in order in case of discontinuous hysteresis, e.g. for the simple *(delayed) relay operator.* Although this is single relation, it can be represented as a system of two variational inequalities. Existence and uniqueness of a weak solution for the first PDE was proved also in this case.

The Cauchy problem for the hyperbolic PDE above also has a weak solution [Vi2, Vi3]. This rests upon the dissipativity of hysteresis, and the related regularizing effect.

Details may be found e.g. in the monograph [Vi1] and the survey [Vi3]. A different approach is addressed in the recent monograph [MiRo].

References

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