Analogical Models and Homogenization Disagree

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We shall show that the outcome of the theory of homogenization is at variance with a classical engineering technique for constructing rheological models of mechanical materials.

Analogical models. The mechanical constitutive behavior of several materials is often represented via discrete models, that are built up by arranging few elements. These are representative of the basic behavior of the material, viz., elasticity, viscosity and plasticity in continuum mechanics. Similar constructions are also used elsewhere, e.g., in electro-magnetism. In the case of univariate models, one may also use the pictorial image of parallel and serial arrangements.

Although apparently these models are not derived from any representation of the mesoscopic structure, here we address the following question:

may these models be retrieved by homogenizing an underlying lower-scale structure?

We shall answer in the negative: the main reason stays in an (unjustified) *mean-field hypothesis,* that is at the basis of these constructions.

Homogenization via Scale Transformations. An alternative approach is illustrated, that rests on the following steps:

(i) formulation of a (space-distributed) model for an inhomogeneous material, with constitutive data that depend periodically on a x/η;

(ii) derivation of a *two-scale model* as $\eta \to 0$;

(iii) *upscaling,* i.e. derivation of a coarse-scale model by averaging the mesoscopic fields over a reference set $\mathcal{Y}$;

(iv) *downscaling,* i.e., retrieval of a two-scale solution from a coarse-scale solution.

This shows that the single-scale and two-scale models are equivalent. This entitles us to regard the coarse-scale model as the genuine *homogenized model.*

Variational Approach. This program is here developed for an initial- and boundary-value problem for a class of elasto-visco-plastic materials, see [4].

This process is given a variational formulation, which allows one to restate the homogenization result in terms of De Giorgi's Γ-*convergence.*

This is just an example of a more general set-up: analogous results have been derived for scale transformation for elasto-visco-plastic material of Prandt-Reuss-type, for electro-magnetic materials, for phase-transitions, see references.

References

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