

# Video Lectures On Artificial Intelligence

## Lecture 18

### Branch and Bound, Dijkstra's Algorithm

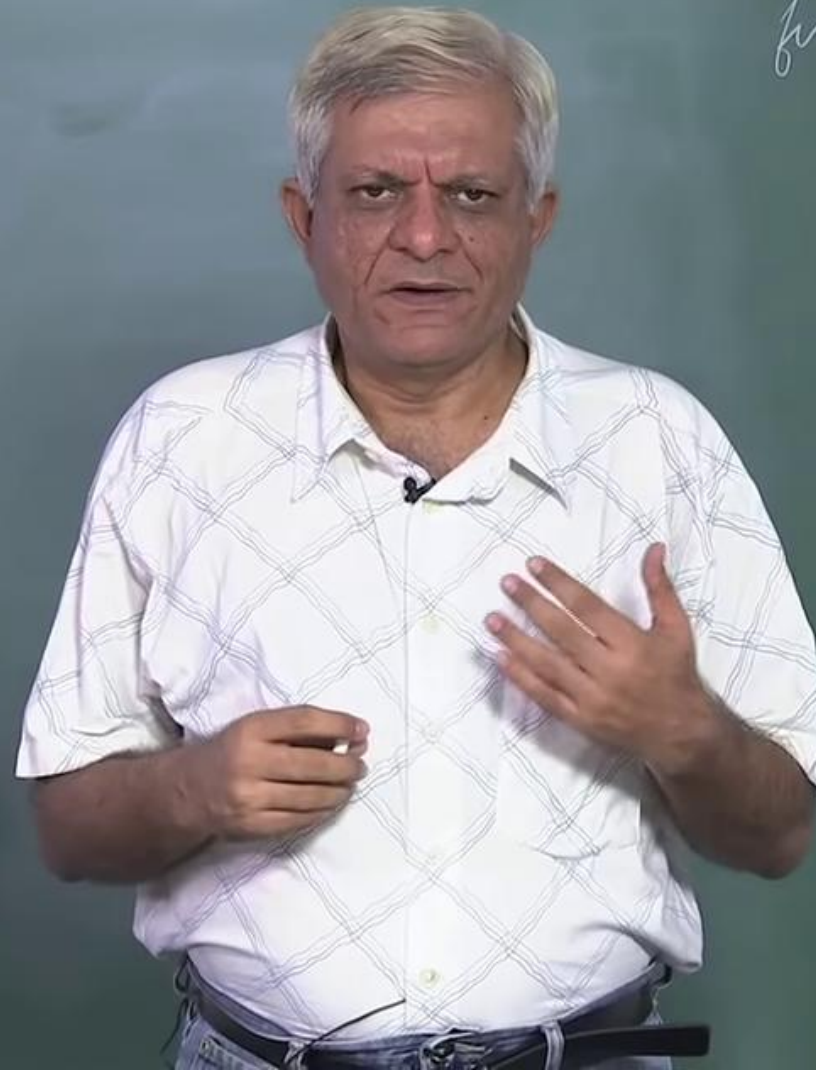
**Prof. Deepak Khemani**

Department of Computer Science  
IIT Madras



State Space Search  $\rightarrow$  Heuristic Search  $\rightarrow$  Optimization

State Space Search  $\rightarrow$  Heuristic Search  $\rightarrow$  Optimal  
 $\downarrow$   
find solutions faster



State Space Search  $\rightarrow$  Heuristic Search  $\rightarrow$  Optimization  
 $\downarrow$   
find solutions faster



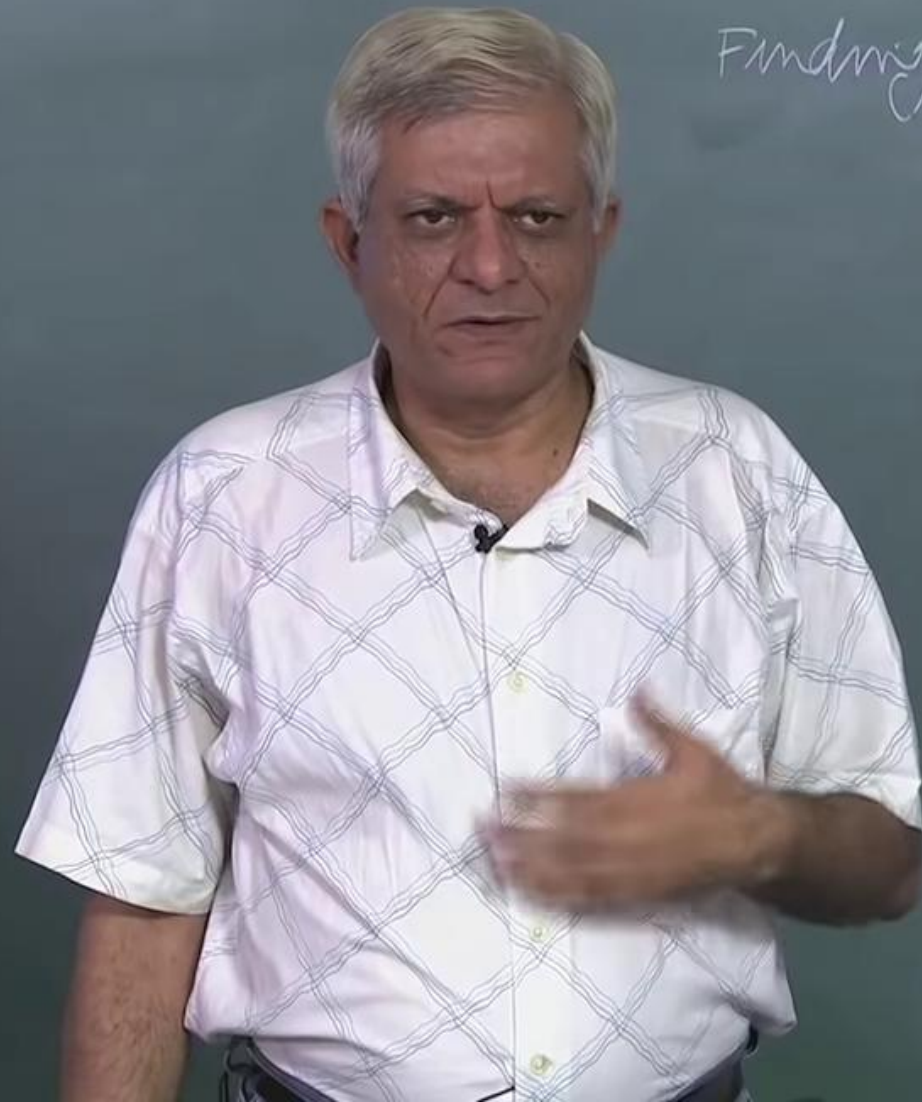
State Space Search  $\rightarrow$  Heuristic Search  $\rightarrow$  Optimal Search  
 $\downarrow$   $\downarrow$   
Finding optimal solutions find solutions faster



State Space Search  $\rightarrow$  Heuristic Search  $\rightarrow$  0  
 $\downarrow$   $\downarrow$   
Finding optimal solutions find solutions faster



State space search  $\rightarrow$  Heuristic  
 $\downarrow$   
Finding optimal solutions      find solution

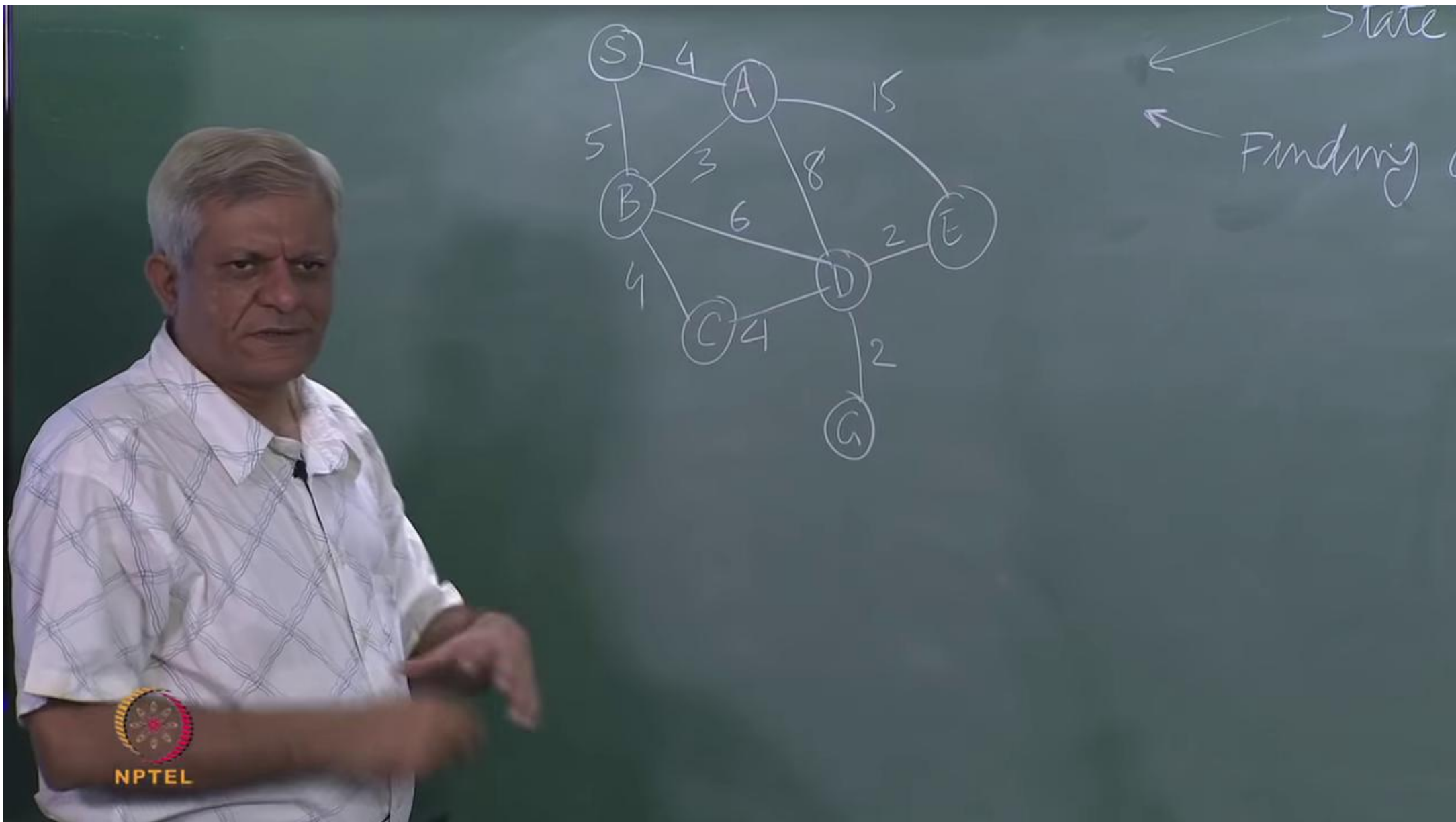


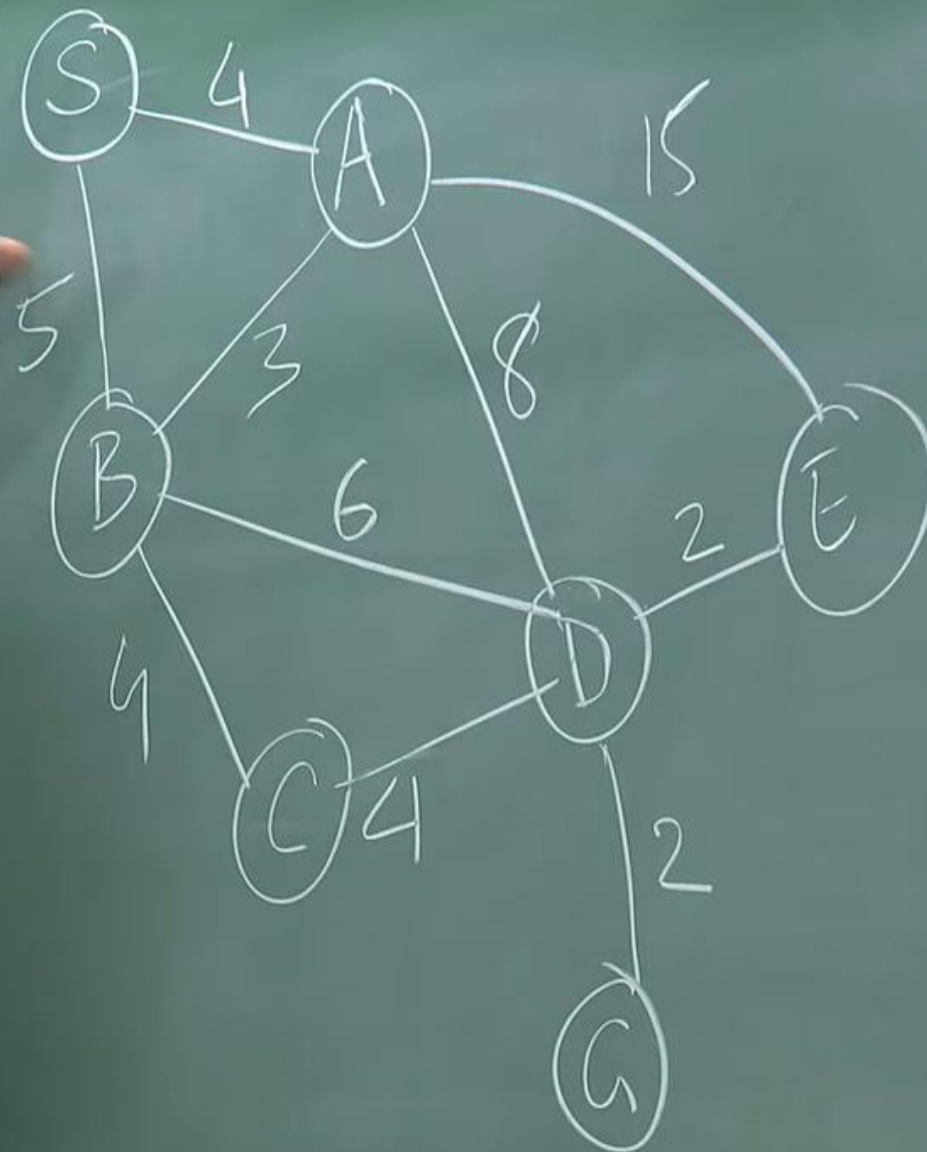
Finding opt

find solutions faster









Refine / Extend the least (estimated)  
cost partial solution



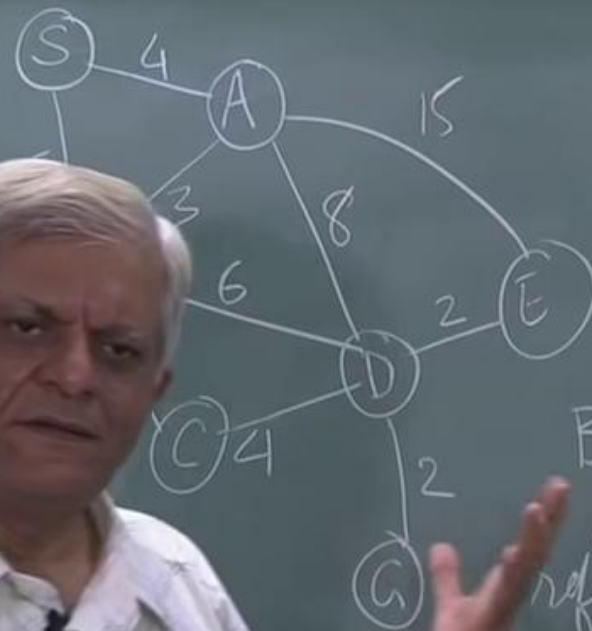
Refine / Extend the least (estimated)  
cost partial solution

Till such a solution is fully refined

Refine / Extend the least (estimated)  
cost partial solution

Till such a solution is fully refined





State Space Search  
Finding optimal solution

BRANCH & BOUND

refinement / extend

excluding some candidates

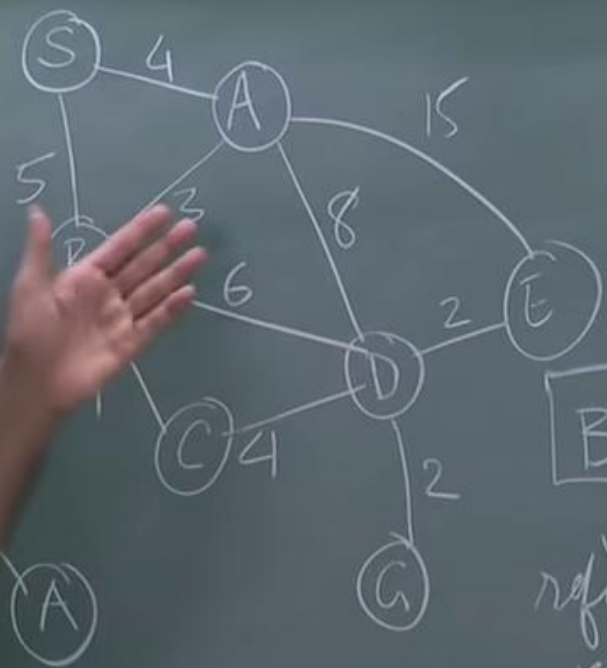
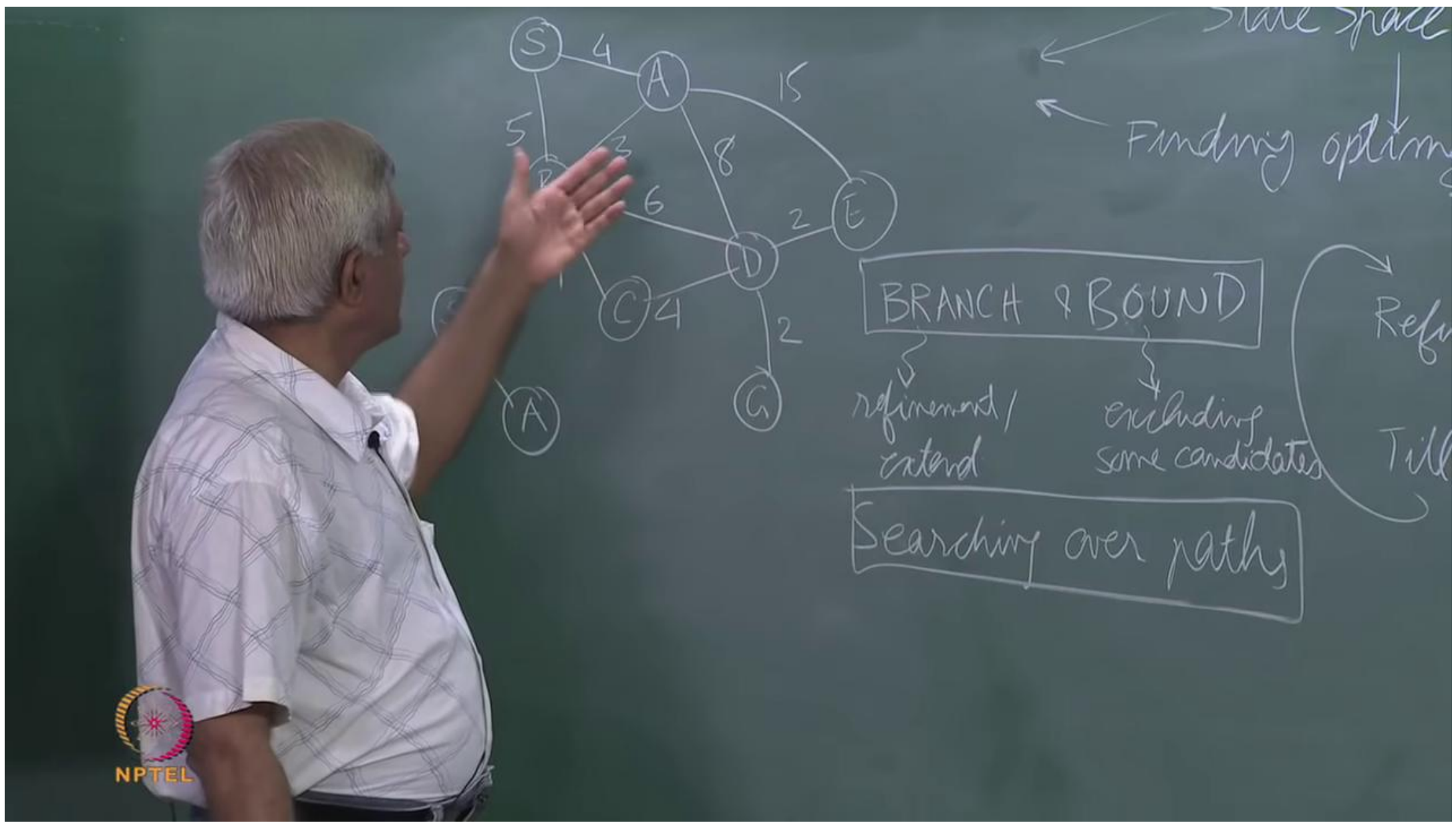
Refine / Extend

Till such

# BRANCH & BOUND

refinement/  
extend

excluding  
some candidates



State space  
Finding optimum

BRANCH & BOUND

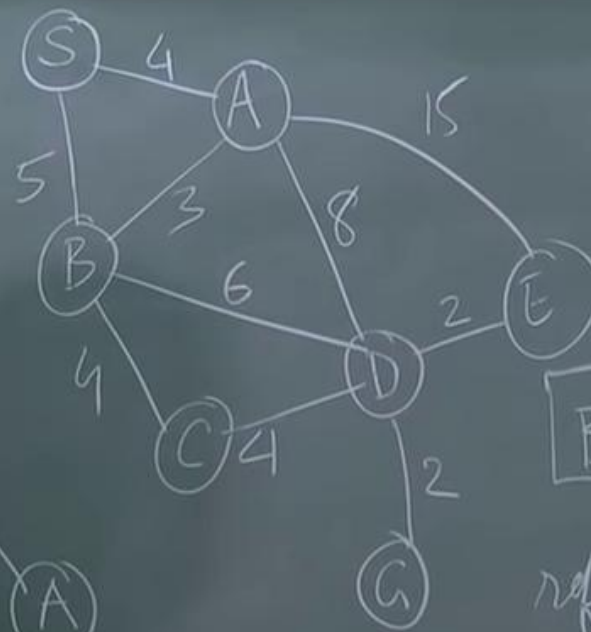
refinement/  
extend

excluding  
some candidates

Searching over paths

Refinement

Till



State Space

Finding optimum

BRANCH & BOUND

refinement/  
extend

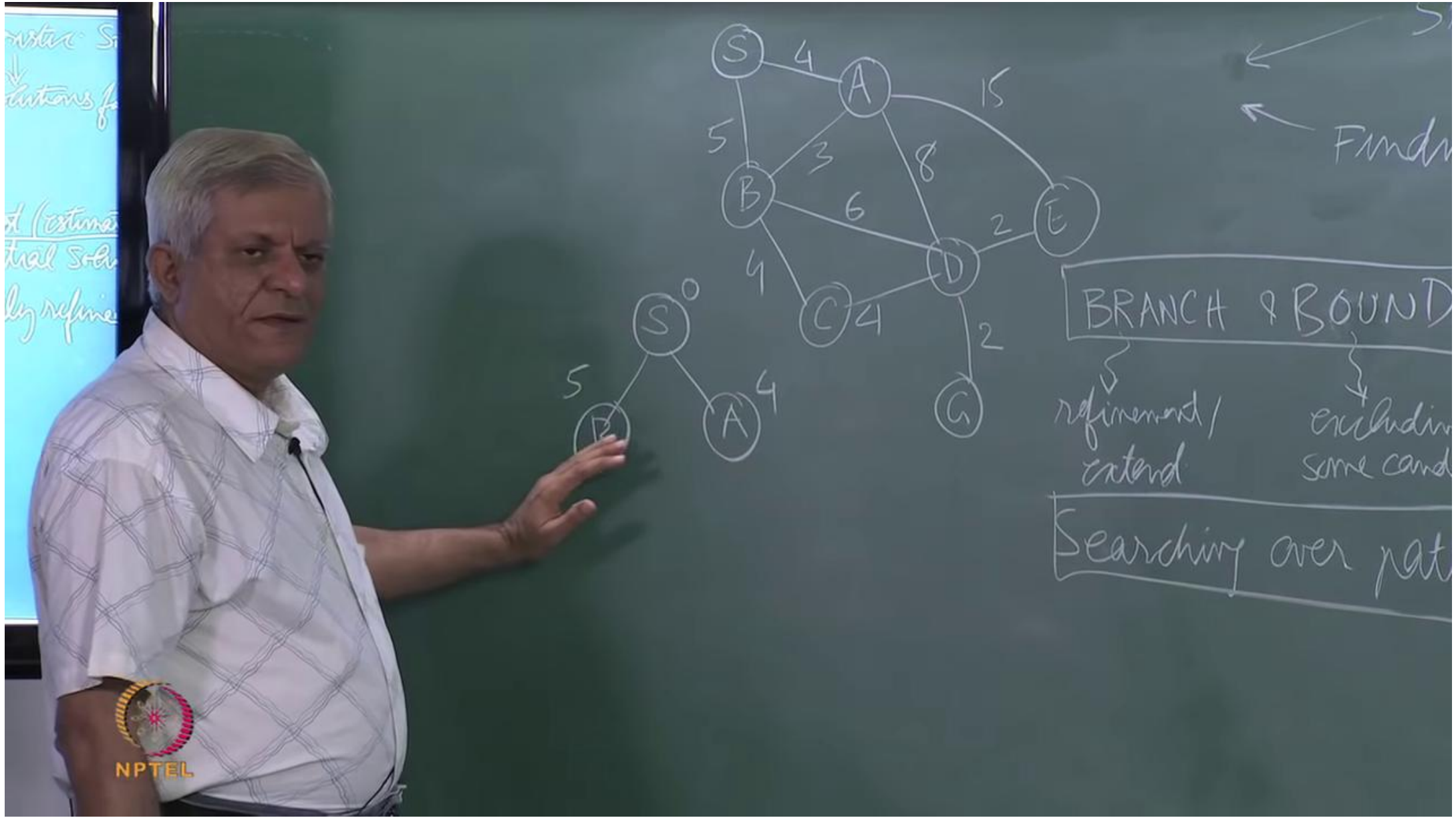
excluding  
some candidates

Refu

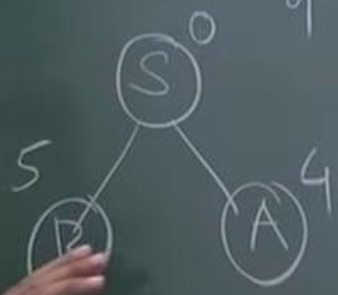
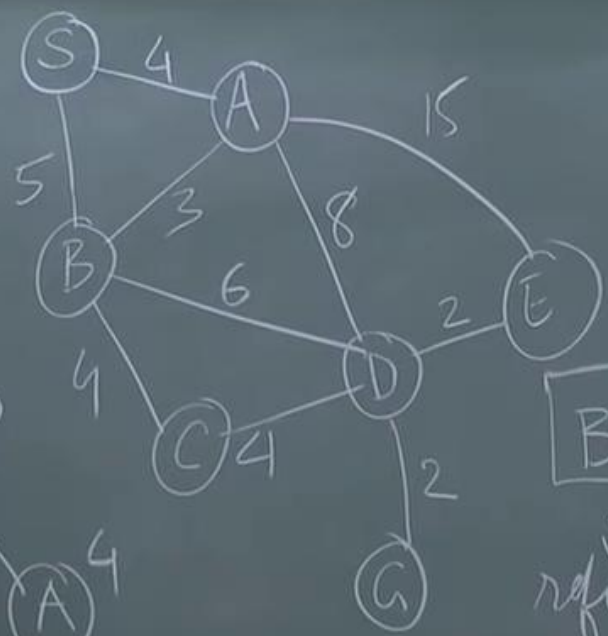
Till

Searching over paths





...ative S  
...utions f  
... (estimate  
...ial Solu  
...ly refine



BRANCH & BOUND

refinement/  
extend

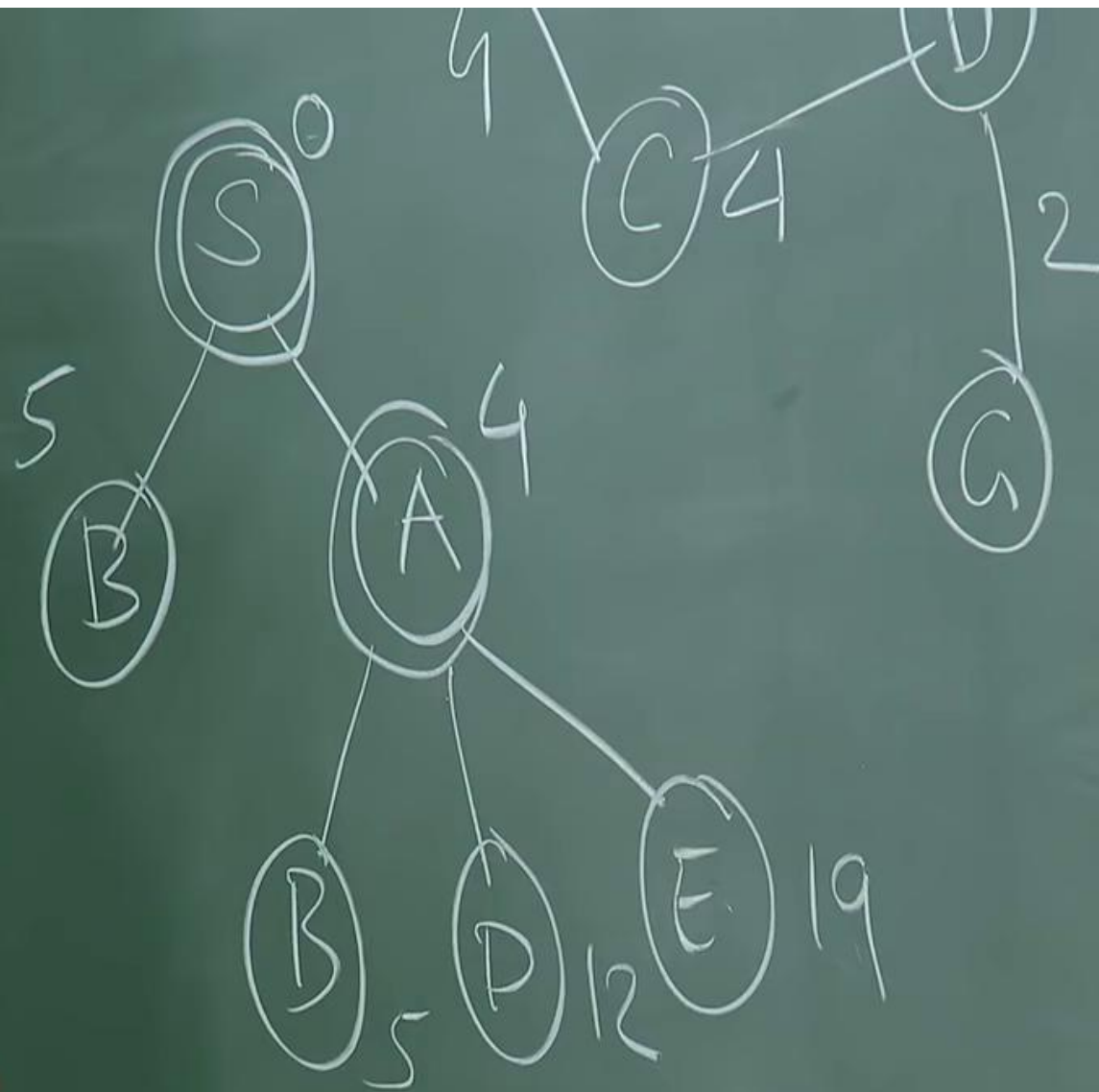
excluding  
some cand

Searching over pat

Find



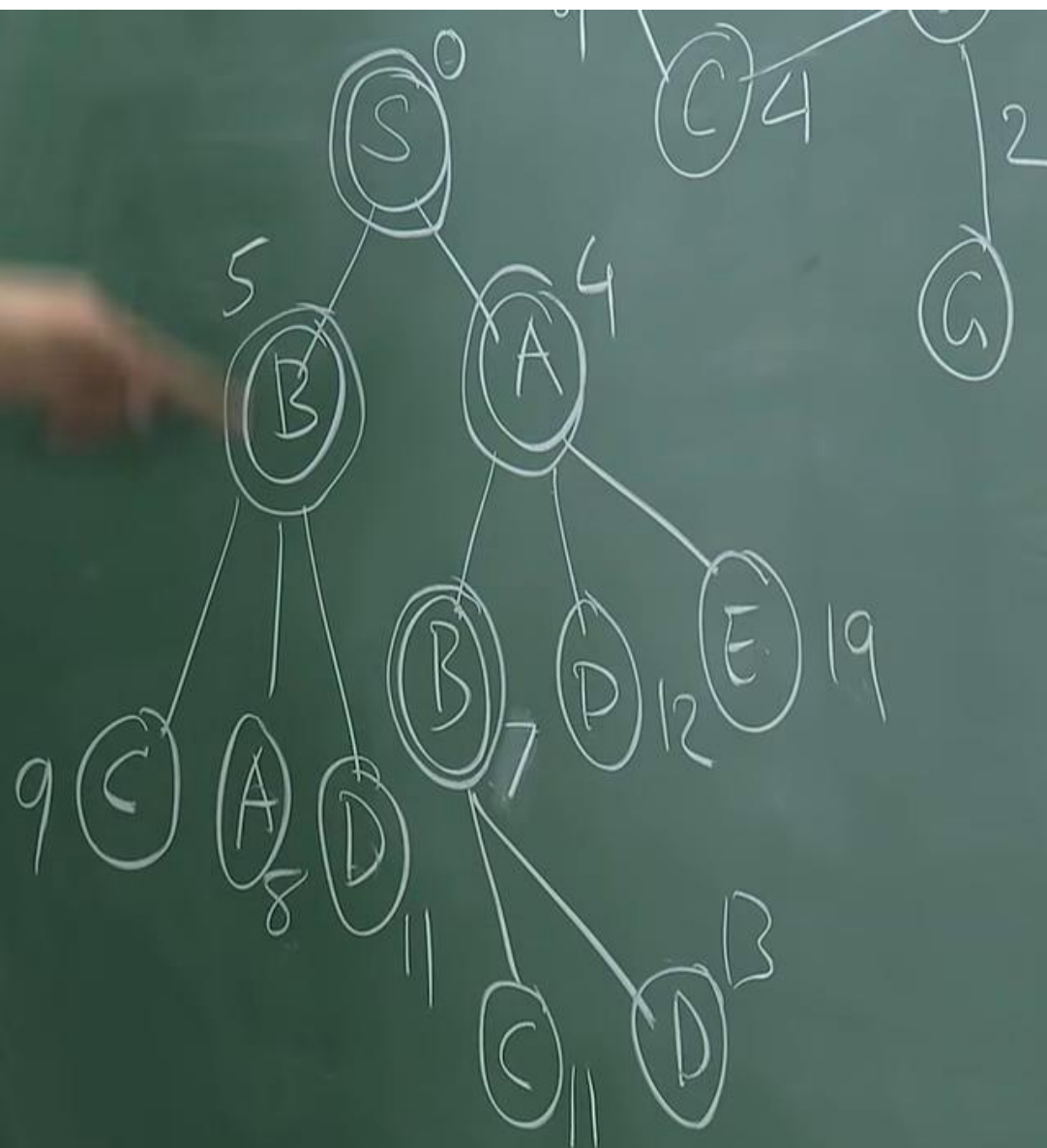




BRAN

refinement  
extension

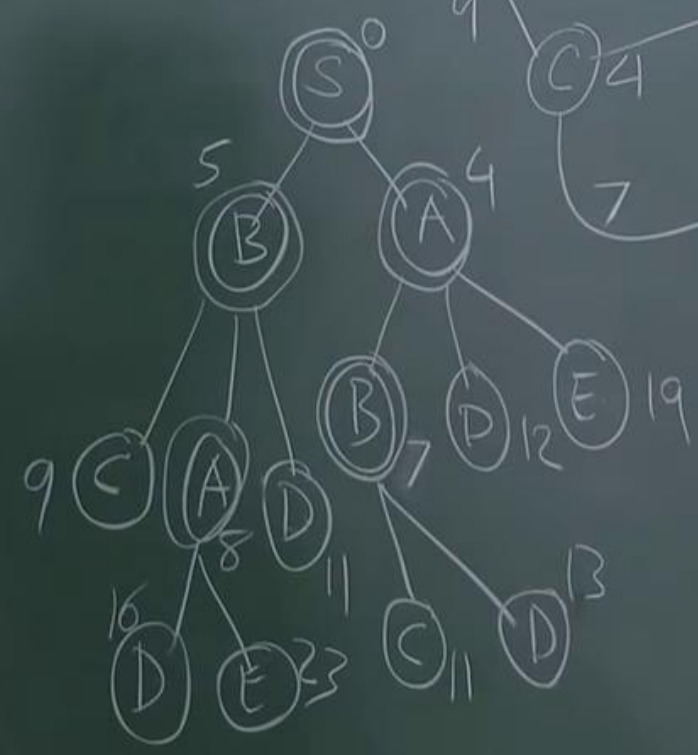
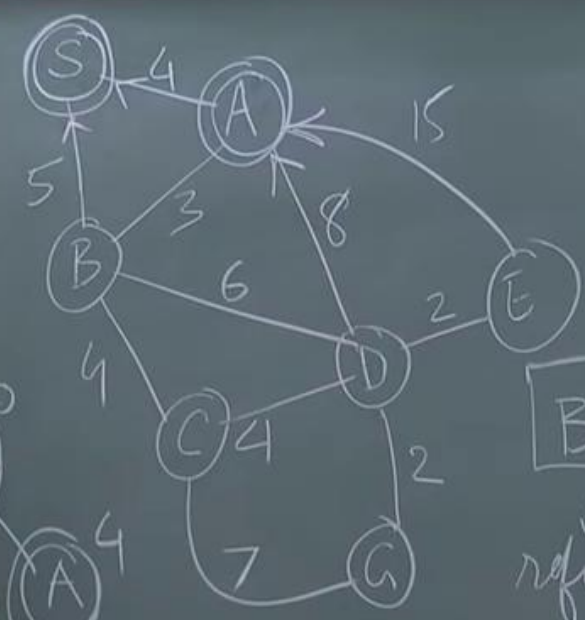
Search



BRANCH 4 IS  
↓  
refinement/  
extend

Searching over

Search → H  
find  
...  
// extend  
... a set



**BRANCH & BOUND**

refinement/  
extend

excluding  
some candidates

Searching over path

Start  
Finding



NPTEL

State

Finding

BRANCH & BOUND

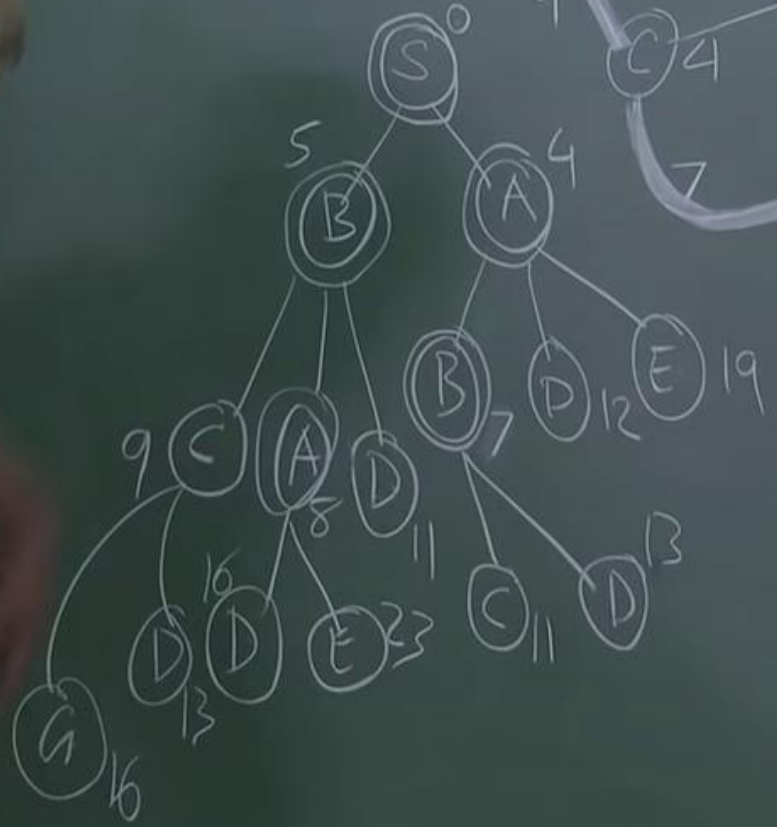
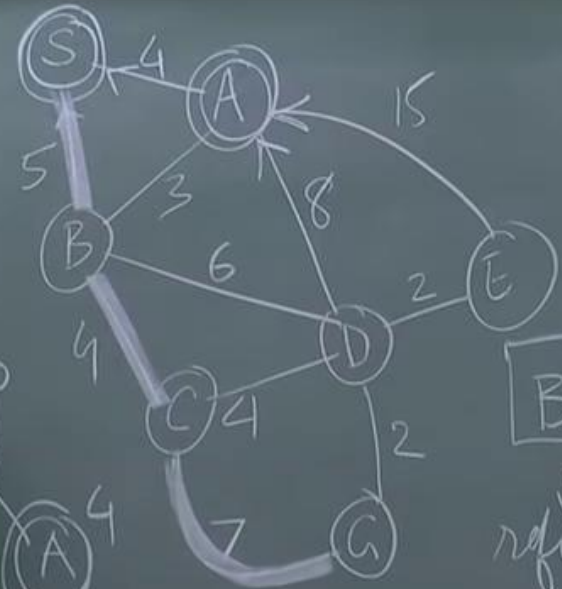
refinement / extend

excluding some candidates

Searching over paths



Since Search  $\rightarrow$  H  
 find  
 (goals)  
 extend the  
 Cost  
 half other solution



BRANCH & BOUND

refinement/  
extend

exclude  
some can

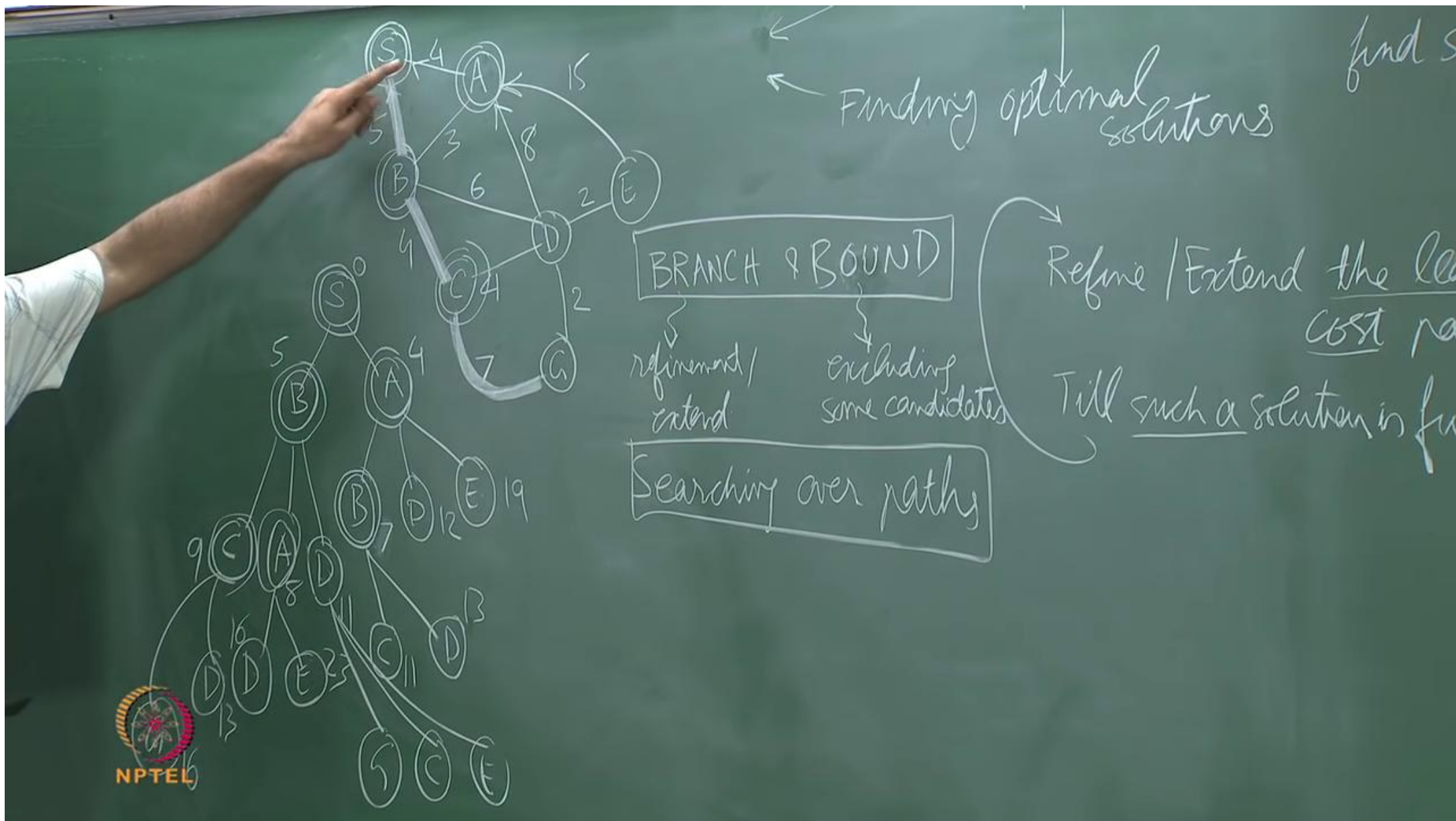
Searching over pa





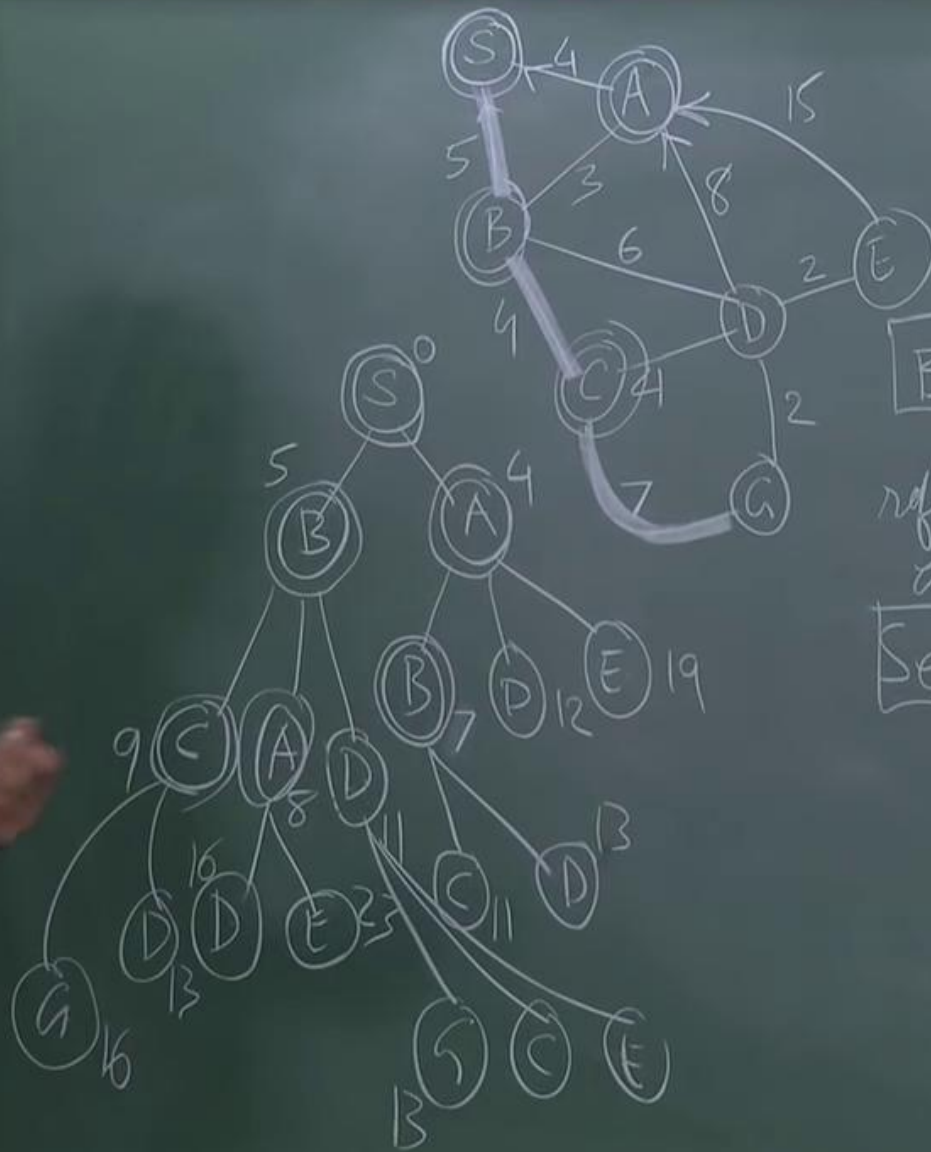
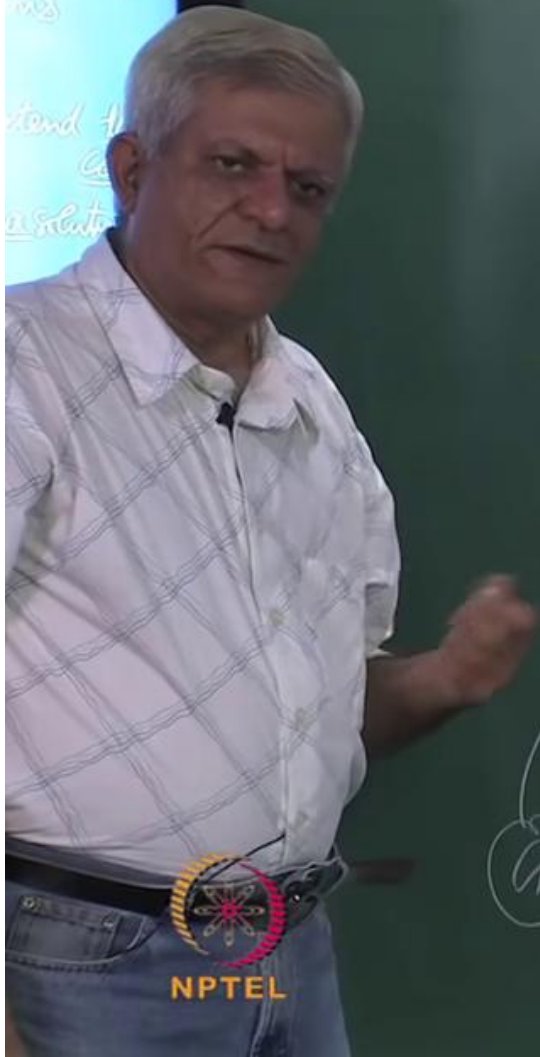
State Space Search → Heuristic Search → Optim  
↓  
Finding optimal solutions      find solutions faster

BOUND  
↓  
excluding some candidates  
Refine / Extend the least (estimated) cost partial solution  
Till such a solution is fully refined





$A \rightarrow H$   
 find  
 extend to  
 generate



State Space Search  
 Finding optimal solution

**BRANCH & BOUND**  
 { refinement / extend }  
 { excluding some candidates }

Searching over paths

Refine  
 Till solution

paths  $\rightarrow H$   
find  
extend the  
Cost  
a solution

State Space  $S$   
Finding optimal  $S$

Refine  
Till  $S$

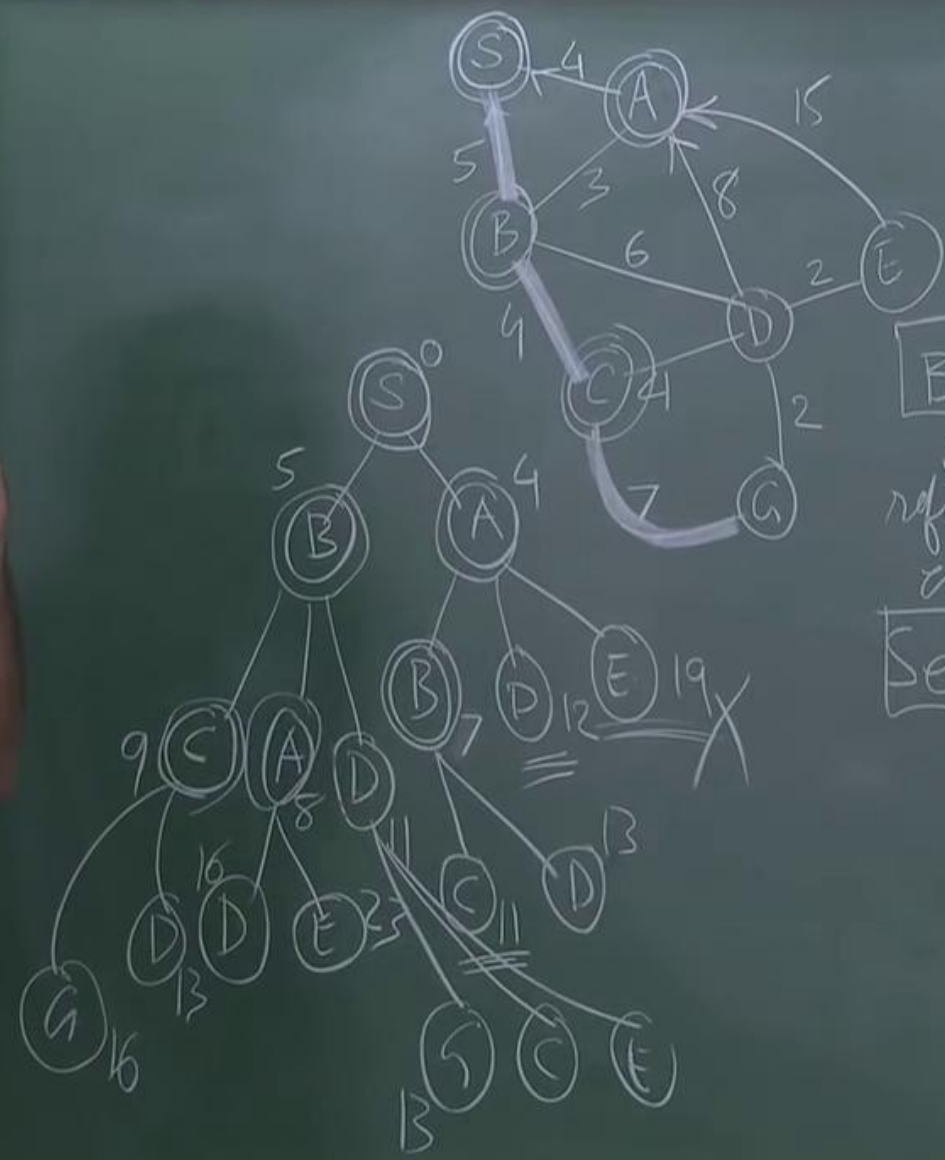
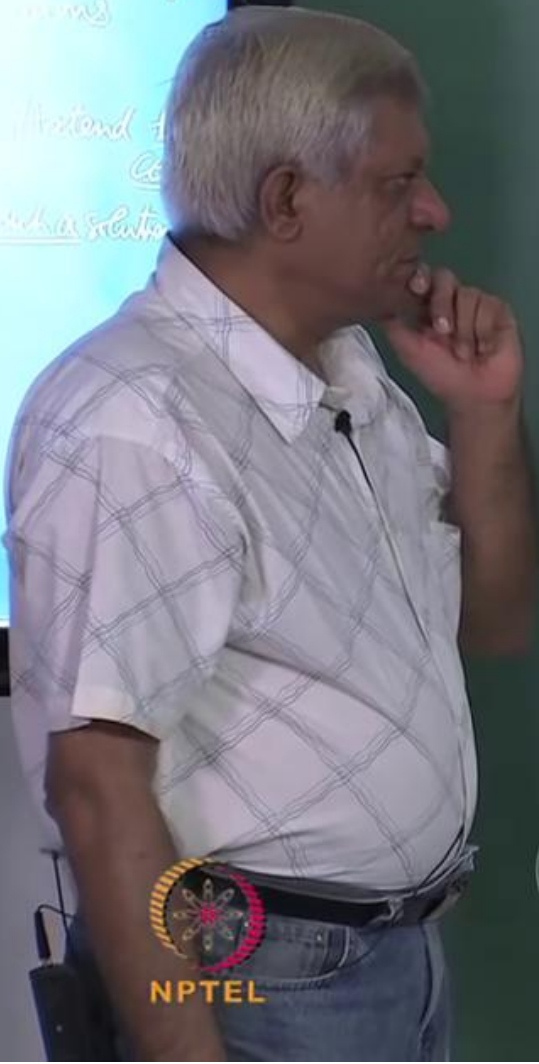
**BRANCH & BOUND**  
refinement/  
extend  
excluding  
some candidates

Searching over paths

NPTEL



search  $\rightarrow H$   
 find  
 nodes  
 extend to  
 a solution



State Space  
 Finding optimum  
**BRANCH & BOUND**  
 refinement / extend      excluding some candidates  
**Searching over paths**  
 Ref  
 Till

State Space Search → H

Finding optimal solutions

Refine / Extend the cost

Till such a solution is found



State Space Search → Heuristic Search → Optimization

Finding optimal solution      find solutions faster  
 $C = C^*$  (actual)

BRANCH & BOUND

pruning / extend  
excluding some candidates

searching over partial solutions

Refine and the least (estimated) cost partial solution  
continuously refined



State Space Search → Heuristic Search → Optimization

Finding optimal solutions

find solutions faster

$$C < C^*$$

actual

$C^*$

BRANCH & BOUND

pruning/extend

excluding some candidates

searching over paths

Refine  $f$

$f$  (estimated)

trial solution

refined

$C_i$





State Space Search → Heuristic Search → 0

↓  
Finding optimal solutions

↓  
find solutions faster

$C = C^*$  (actual)

$C < C^*$

$C > C^*$

Refine / Extend the least (estimated) cost partial solution

Till such a solution is fully refined

$C_1 < C^*$

$C_2 > C^*$

BR  
BOUND

finding candidates

th

The chalkboard features a graph on the left with nodes A, B, C, D, E. Node A is at the top, connected to B (weight 4), C (weight 3), and D (weight 8). Node B is connected to C (weight 6). Node C is connected to D (weight 4). Node D is connected to E (weight 2). Node E is connected to D (weight 2). There are also some other nodes and edges partially visible. The text on the board discusses State Space Search and Heuristic Search, with a focus on finding optimal solutions faster. It includes a comparison of cost C to optimal cost C\*, and a process of refining or extending the least cost partial solution until it is fully refined. There are also some mathematical expressions like C1 < C\* and C2 > C\*.

Search  $\rightarrow$  Heuristic Search  $\rightarrow$  Optimization

TSP  $\rightarrow$  Refinement Search

find solutions  $\rightarrow$  actual  
\*

Extend the least cost path

with a solution in

C B H M D

C	0	300	600	1000	2000
B	300	0			
H	600		0		
M	1000			0	
D	2000				0



NPTEL

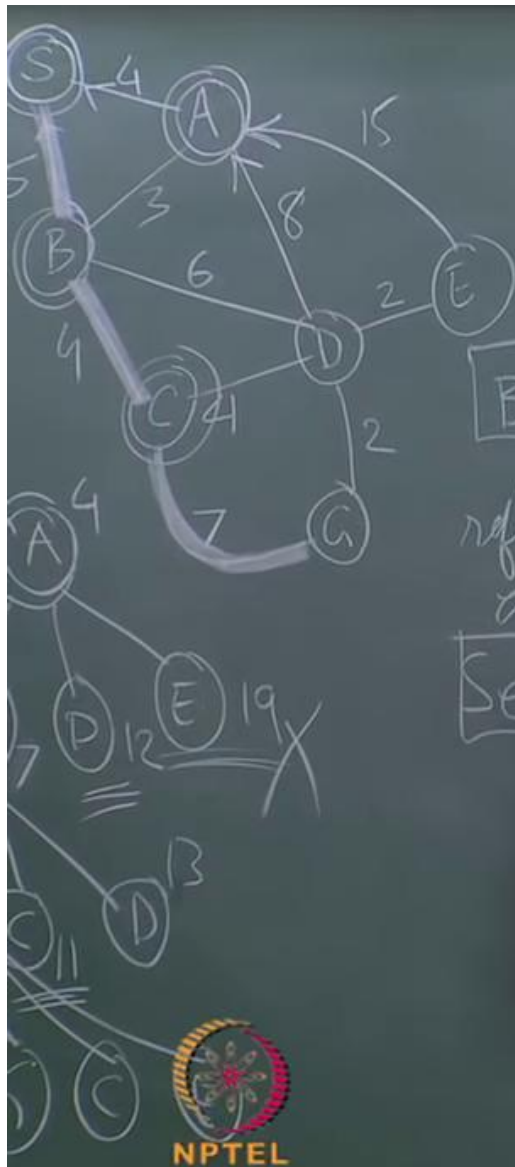
C B H M D

C	0	300	600	1000	2000
B	300	0	500	900	2100
H	600	500	0	700	1500
M	1000	900	700	0	1200
D	2000	2100	1500	1200	0



NPTEL





State Space Search  $\rightarrow$  Heuristic Search  $\rightarrow$  find solutions faster

Final optimal solutions  $\rightarrow$  Refine / Extend the least (estimated) cost partial solution

Branch & Bound  $\rightarrow$  refinement / extend Search

Until a solution is fully refined

$C < C^*$   
 $C > C^*$

$C_1 < C^*$   
 $C_2 > C^*$



State Space Search → Heuristic

Finding optimal solutions find so

**BRANCH & BOUND**

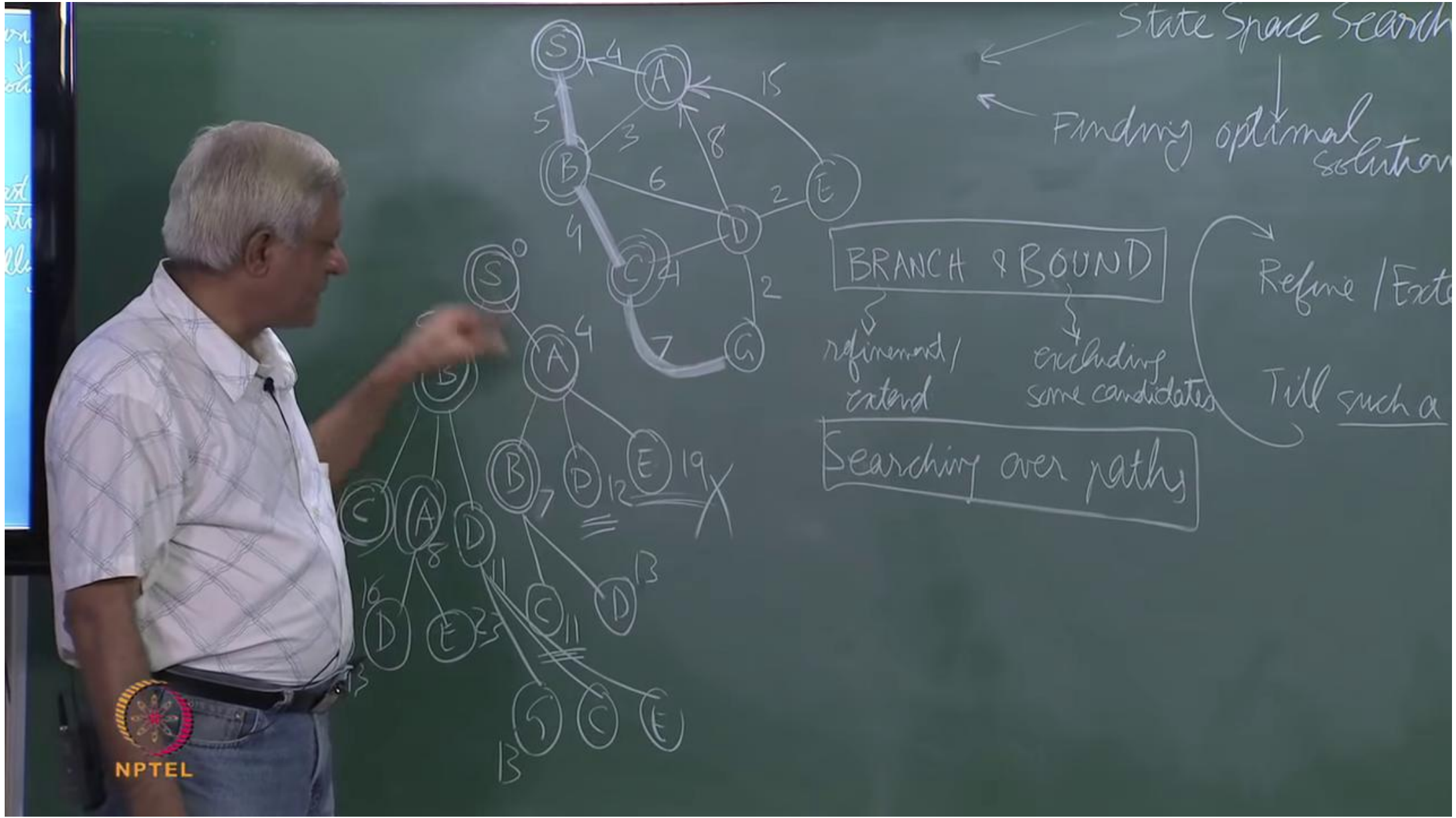
refinement/extend      excluding some candidates

Refine / Extend the least cost path

Till such a solution is found

Searching over paths

```
graph LR; S((S)) -- 4 --> A((A)); A -- 15 --> E((E)); E -- 2 --> D((D)); D -- 2 --> G((G)); S -- 5 --> C((C)); C -- 9 --> D; D -- 16 --> I((I)); I -- 10 --> G
```



State Space Search

Finding optimal solution

BRANCH & BOUND

refinement / extend

excluding some candidates

Refine / Extend

Till such a

Searching over paths

Brute Force Search → Heuristic Search → Optimization      TSP → Refinement

optimal solutions      faster solutions      actual

Refine / Extend      (estimated) Solution

Till solution is found       $C < C^*$        $C > C^*$

	C	B	H	M	D
C	0	300	600	1000	2000
B	300	0	500	900	2100
H	600	500	0	700	1500
M	1000	900	700	0	1200
D	2000	2100	1500	1200	0

C B H M D

C	0	300	600	1000	2000	900
B	300	0	500	900	2100	800
H	600	500	0	700	1500	1100
M	1000	900	700	0	1200	1600
D	2000	2100	1500	1200	0	2700

1/2



Search  $\rightarrow$  Optimization

TSP  $\rightarrow$  Refinement Search

(S) - set of all tours

actual  
 $C^*$

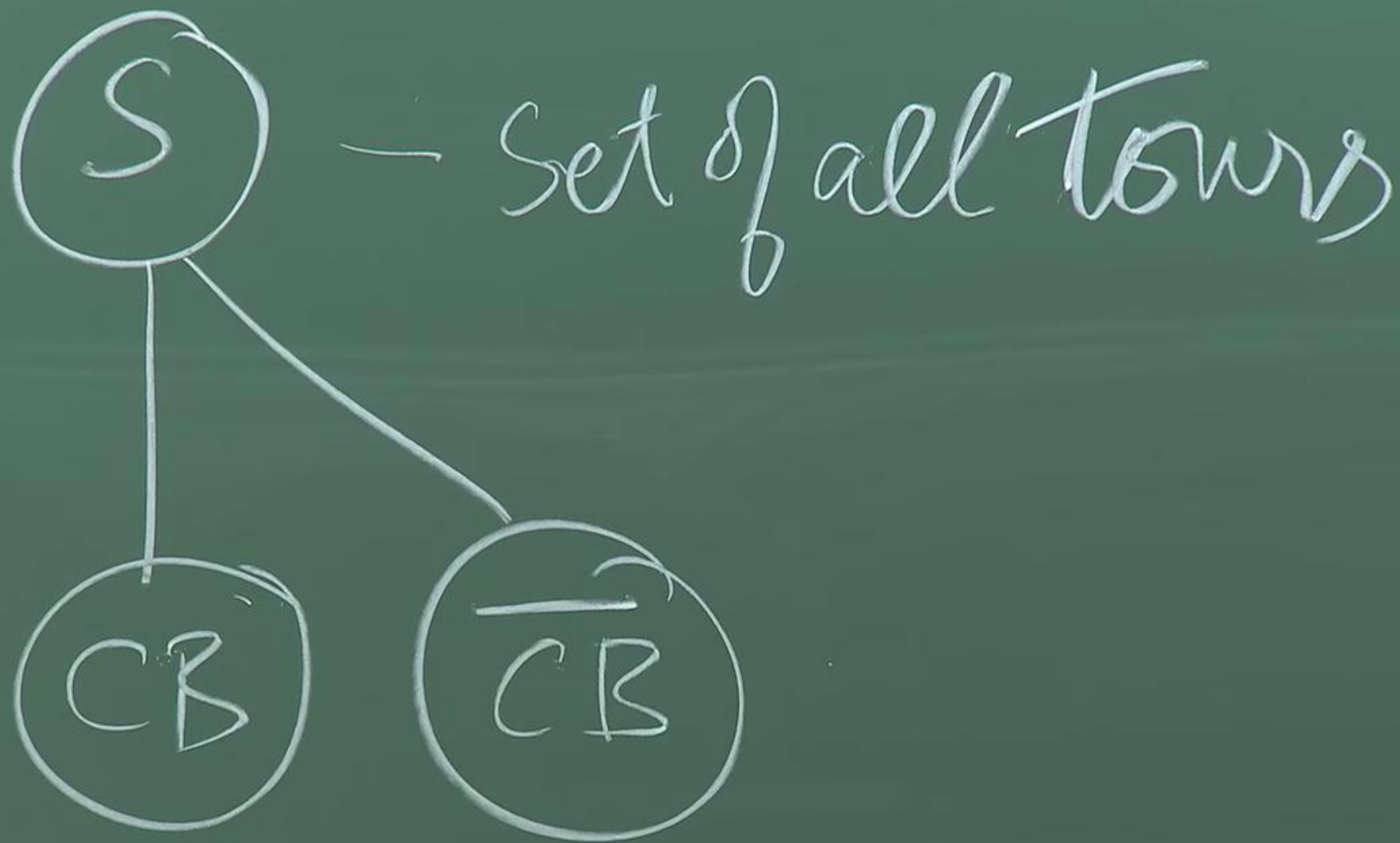
rel)  
than

$C^*$

$C^*$

	C	B	H	M	D	
C	0	300	600	1000	2000	900
B	300	0	500	900	2100	800
H	600	500	0	700	1500	1100
M	1000	900	700	0	1200	1600
D	2000	2100	1500	1200	0	2700

1/2



Heuristic Search → Optimization

TSP → Refinement Search

find solution after  
actual  
C\*

tend the least

Cost

solution is

	C	B	H	M	D	
C	0	300	600	1000	2000	900
B		0	500	900	2100	800
H			0	700	1500	1100
M				0	1200	1600
D					0	2700

CB CB



12

Stochastic Search  $\rightarrow$  Optimization

$\downarrow$   
solutions faster

$C = \{ \dots \}$

least (estimated)

partial

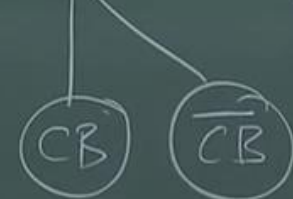
fully re

C B H M D

300	600	1000	2000
500	500	900	2100
500	700	500	
700		1200	
200			

TSP  $\rightarrow$  Refinement Search

$S$  - set of all



$\overline{CB}$

$CB$

900

900

800

800

1100

1100

1600

1600

2700

2700

$\frac{1}{2}$



→ Heuristic Search → Optimization

find solutions & iter

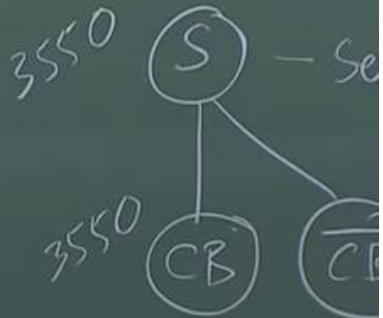
actual

$C^*$

the least  
cost part  
iteration is for

	C	B	H	M	D	
C	0	300	600	1000	2000	900
B	300	0	500	900	2100	800
H	600	500	0	700	1500	1100
M	1000	900	700	0	1200	1600
D	2000	2100	1500	1200	0	2700

TSP → Refinement Search



$\overline{CB}$	CB
900	900
800	800
1100	1100
1600	1600
1600	1600
2700	2700

1/2

= 3550

C B H M D

$\overline{CB}$

CB

C	0	300	600	1000	2000
B	300	0	500	900	2100
H	600	500	0	700	1500
M	1000	900	700	0	1200
D	2000	2100	1500	1200	0

900 + 700

900

800 + 600

800

1100

1100

1600

1600

2700

2700

State Space Search → Heuristic Search → Optimal Search

← Finding optimal solutions      find solutions faster  
 $C < C^*$        $C > C^*$        $C = C^*$  (actual)

H & BOUND

↓  
excluding some candidates

over paths

Refine / Extend the least (estimated) cost partial solution

Till such a solution is fully refined

(D)

(M)

(A)

(B) — (C)

$C_1 < C^*$

$C_2 > C^*$

C
B
H
M
D

3550  $S$  - set of all tours

3550

$CB$

$\overline{CB}$

$3550 + 650$

$HB$

$\overline{HB}$

$\overline{CB}$

$CB$

+700

900

+600

800



NPTEL



3550 (S) - set of all towns

3550 + 650

3550

CB

$\overline{CB}$

CB

$\overline{HB}$

3550

HB

$\overline{HB}$

900

search  $\rightarrow$  Heuristic Search  $\rightarrow$  Optimization

TSP  $\rightarrow$  Refinement  
3550

find solutions faster

actual

$C^*$

Extend the solution

such a solution

(D)

C B H M D

C	○	300	600	1000	2000
B	300	○	500	900	2100
H	600	500	○	700	1500
M	1000	900	700	○	1200
D	2000	2100	1500	1200	○

$\overline{CB}$

CB

$\overline{HB}$

900 + 700

900

(800) + 1600

800

+ 400

1100

1100

1600

1600

2700

2700

1/2

= 3550

Search  $\rightarrow$  Heuristic Search  $\rightarrow$  Optimization

find solutions faster  
(actual)

TSP  $\rightarrow$  Refinement Search

Extend the least cost

with a solution in

C B H M D

C	○	300	600	1000	2000
B	300	○	500	900	2100
H	600	500	○	700	1500
M	1000	900	700	○	1200
D	2000	2100	1500	1200	○

900 + 700

(800) + 1600

1100

1600

2700

CB

CB

HB

900

800

1100

1600

2700

+ 400

+ 200

1/2

= 3550

3550

3550

3550



Space Search  $\rightarrow$  Heuristic Search  $\rightarrow$  Optimization TSP  $\rightarrow$  Refinement

optimal solution  $\downarrow$  find solutions faster

$$C \leq C^*$$

Refine the least (estimated) partial solution fully refined

$$C_1 < C^*$$

$$C_2 > C^*$$

	C	B	H	M	D		$\overline{CB}$	$CB$
C	0	300	600	1000	2000	900	+700	900
B	300	0	500	900	2100	(800)	+1600	800
H	600	500	0	700	1500	1100		1100
M	1000	900	700	0	1200	1600		1600
D	2000	2100	1500	1200	0	2700		2700

1/2

=3550



increment search

$3550$   $(S)$  - set of all tours

$3550$

$(CB)$

$3550 + 650$   
 $(\overline{CB})$

~~$3550$~~

$(HB)$

$(\overline{HB})$



NPTEL

increment search

$3550$   $(S)$  - set of all tours

$3550$   $(CB)$

$3550 + 650$   $(\overline{CB})$

~~$3550$~~

$(HB)$

$(\overline{HB})$



NPTEL

Search  $\rightarrow$  Optimization

TSP  $\rightarrow$  Refinement Search

faster  
= actual

$$C < C^*$$

estimated)  
solution

ined

$$C_1 < C^*$$

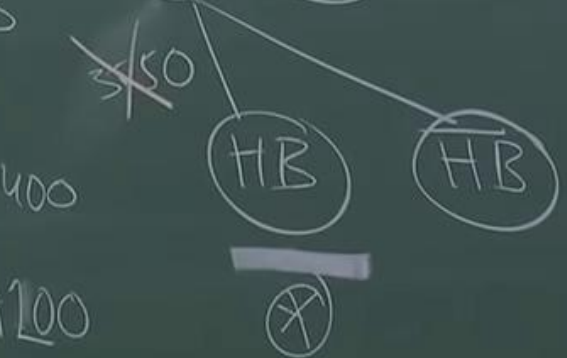
$$C_2 > C^*$$

	M	D
900	2000	900
1000	2100	1000
1100		1100
1200		1200
1300		1300
1400		1400
1500		1500
1600		1600
1700		1700
1800		1800
1900		1900
2000		2000
2100		2100
2200		2200
2300		2300
2400		2400
2500		2500
2600		2600
2700		2700

$$\frac{1}{2} = 3550$$

3550 S - set of all tours

3550 CB 3550 + 650 CB



State Space Search → Heuristic Search → Opt

← Finding optimal solutions

find solutions faster

$C \leq C^*$  (actual)

$C > C^*$

Refine / Extend the least (estimated) cost partial solution

Till such a solution is fully refined

(D)

BRANCH

Finding candidates

thy

(M)

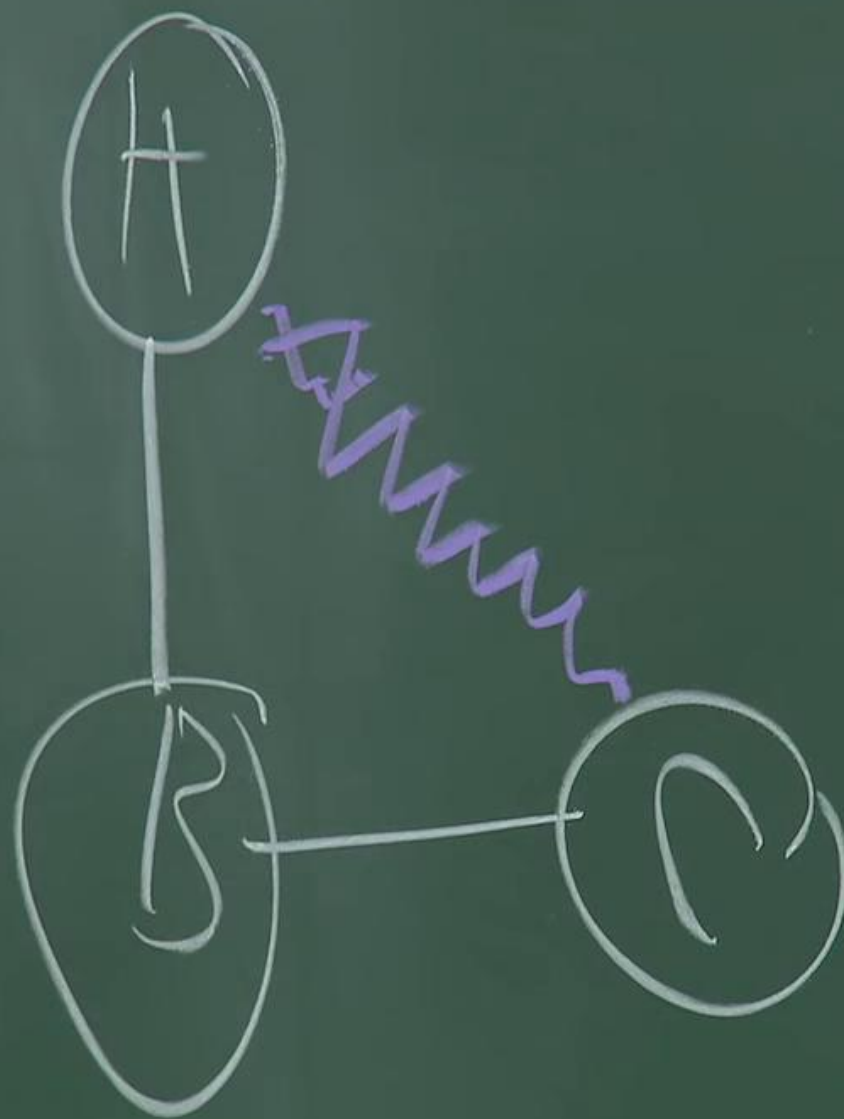
$C_1 < C^*$

$C_2 > C^*$

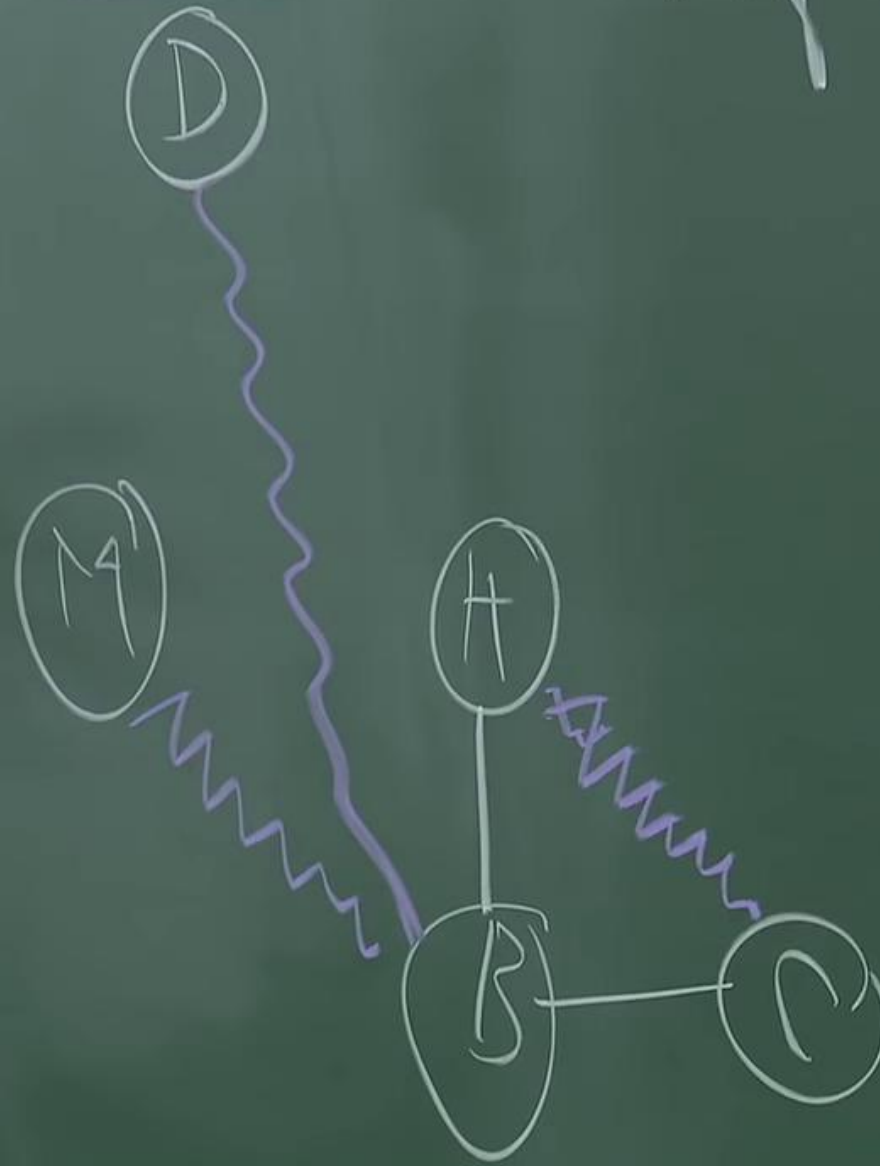
	C
A	0
B	300
H	600
M	1000
D	2000



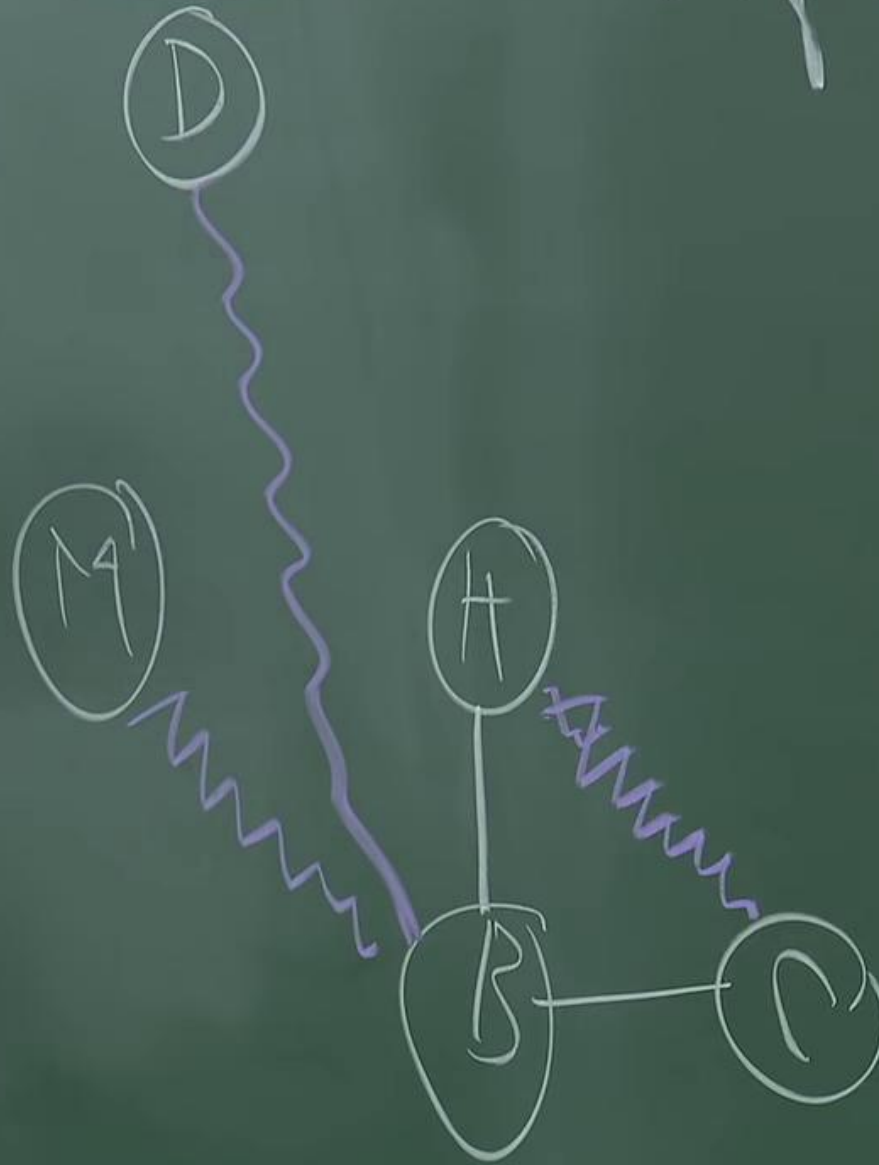
M




paths



paths





State Space Search → Heuristic Search → Optimizing

← Finding optimal solutions      find solutions faster

actual  
 $C < C^*$   
 $C > C^*$

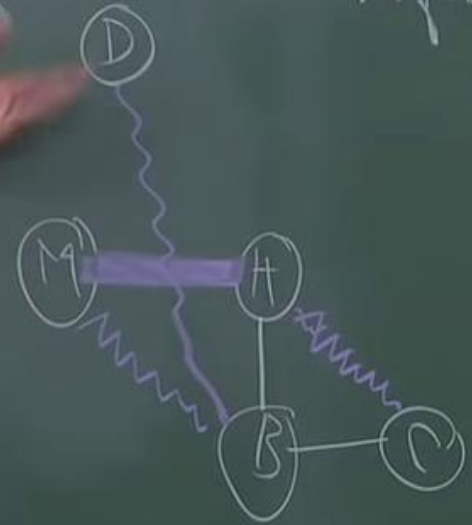
BOUND  
↓  
excluding some candidates

Refine / Extend the least (estimated) cost partial solution

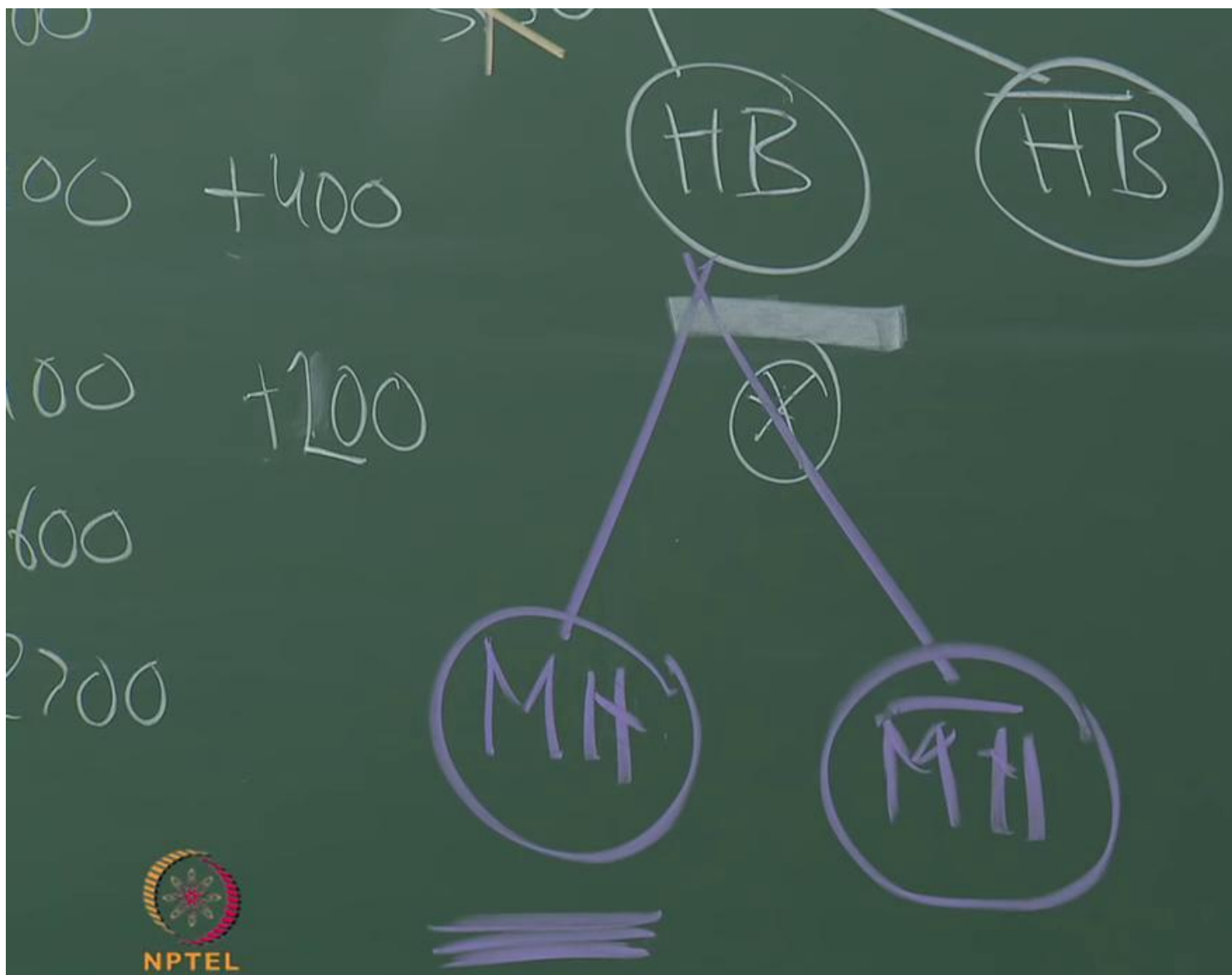
Till such a solution is fully refined

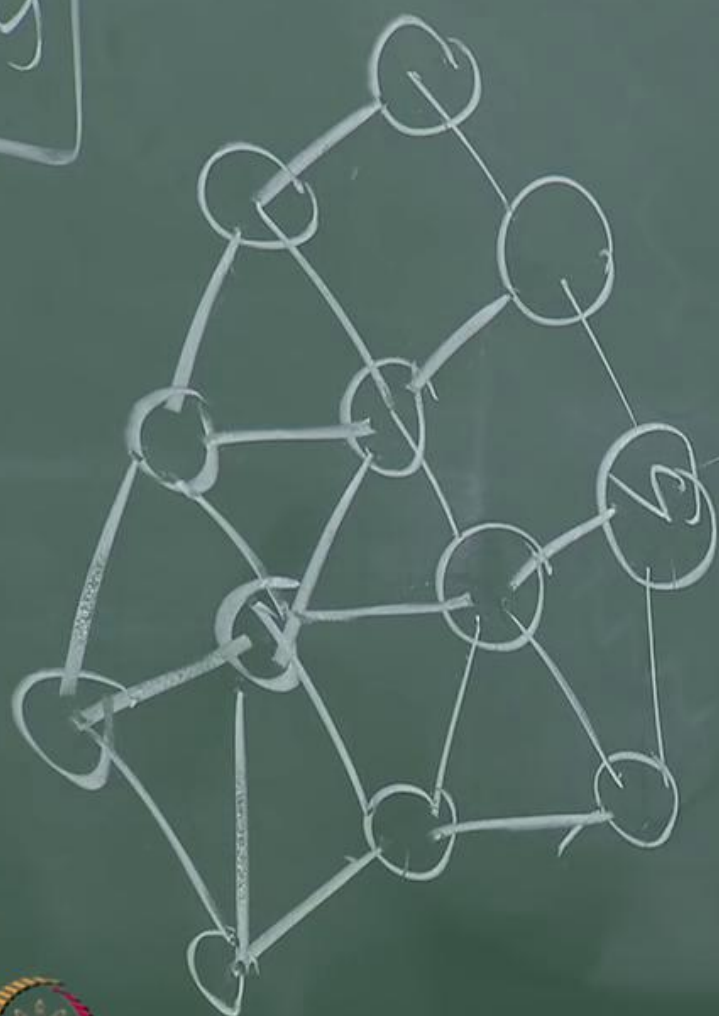
$C_1 < C^*$   
 $C_2 > C^*$

	C	B
C	0	300
B	300	0
H	600	<del>500</del>
M	1000	900
D	2000	2100









Goal

State Space Search  $\rightarrow$  Heuristic Search  $\rightarrow$  Optimal

Finding optimal solutions  $\rightarrow$  find solutions faster

$C < C^*$   
 $C > C^*$

Refine / Extend the least (estimated) cost partial solution  
 Till such a solution is fully refined

SOUND  
 excluding some candidates  
 path

$C_1 < C^*$   
 $C_2 > C^*$

Goal

NPTEL