

# Video Lectures On Artificial Intelligence

## Lecture 12 TSP Greedy Methods

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State Space Search  $\Rightarrow$  Optimization

State Space Search  $\rightarrow$  Optimization  
 $h(n)$  - Hill Climbing

State Space Search  $\Rightarrow$  Optimization  
h(n) - Hill Climbing  
OPTIMIZATION

Escaping local Optima  
 $\downarrow$   
Variable Neighbourhood Descent

State Space Search  $\rightarrow$  Optimization

$h(n)$  - Hill Climbing

$eval(n)$

$\downarrow$   
EXPLOITATION

Escaping local Optim

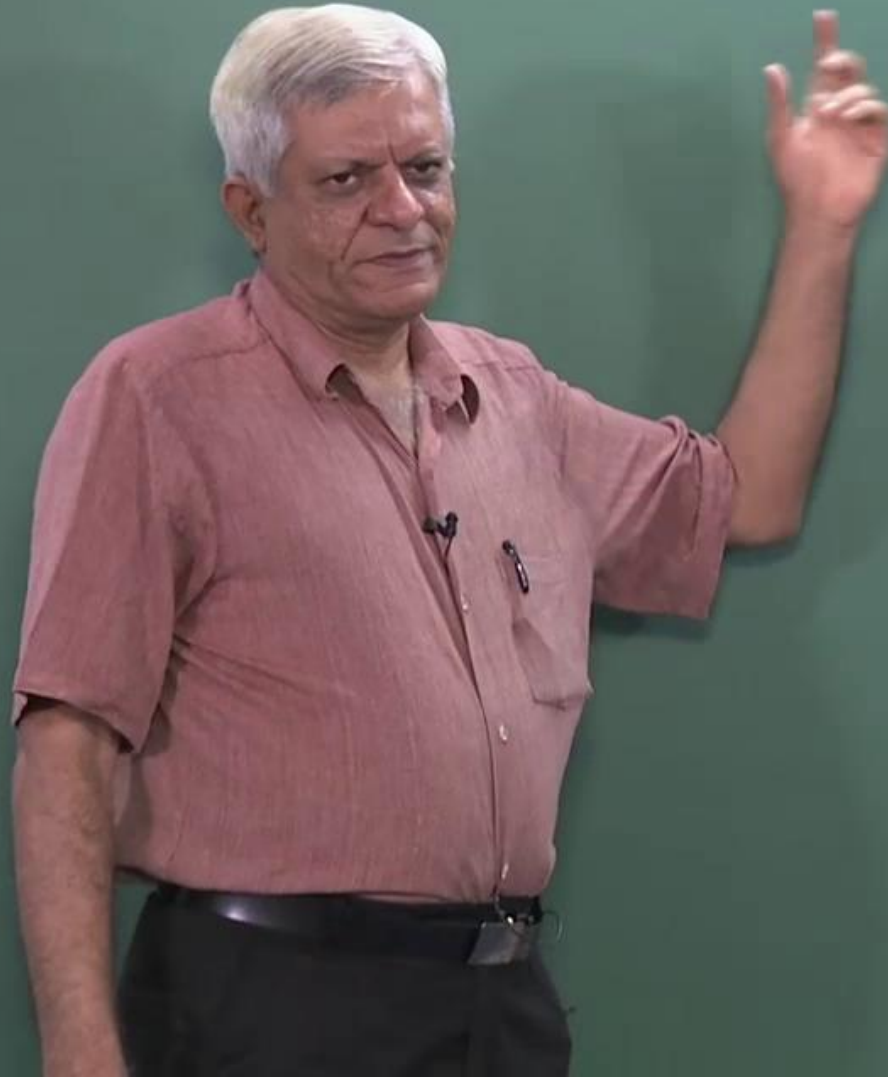
$\downarrow$   
Variable Neighborhood Descent

Local Optima

Steepest Descent

TSP

city<sub>1</sub> ... city<sub>n</sub>  
(1 ... n)





TSP

city<sub>1</sub> ... city<sub>n</sub>  
(1 ... n)

Complete

TSP City<sub>1</sub> ... City<sub>n</sub>

completely connected (1 ... n)



NPTEL



TSP  $city_1 \dots city_n$   
drilling circuit boards completely connected  $(1 \dots n)$   
thousands of "cities"

na  
t

TSP  $city_1 \dots city_n \rightarrow n!$   
drilling circuit boards completely connected  $(1 \dots n)$   
thousands of "cuts"

TSP  $city_1 \dots city_n \rightarrow n!$   
drilling circuit boards  $\swarrow$  connected  $(1 \dots n)$   
thousands of "cities"  $(271643859)$

TSP  $city_1 \dots city_n \rightarrow n!$

drilling circuit boards  
thousands of "cities"

connected  $(1 \dots n)$

(271643859)

TSP  $city_1 \dots city_n \rightarrow n! / n$

drilling circuit board  
thousands of "cities" fully connected  $(1 \dots n)$

(271643859)

$$g_n \rightarrow n! / 2^n \cdot \frac{(n-1)!}{2}$$

(271643859)



$$\#g_n \rightarrow n! / 2^n \cdot \frac{(n-1)!}{2}$$

(271643859)

100 var SAT  $\sim 10^{30}$



← circuit boards  
of "cities"  
TSP city<sub>1</sub> ... city<sub>n</sub> →  $n! / 2n = \frac{(n-1)!}{2}$   
completely connected (1 ... n) (13859)

100 var SAT ~  $10^{30}$   
100 var TSP ~  $10^{157}$

$$\#G_n \rightarrow n! / 2^n \cdot \frac{(n-1)!}{2}$$

(271643859)

100 var SAT  $\sim 10^{30}$

100 var TSP  $\sim 10^{157}$

$$n! \rightarrow n! / 2^n \cdot \frac{(n-1)!}{2}$$

(271643859)

100 var SAT  $\sim 10^{30}$

100 var TSP  $\sim 10^{157}$

drilling circuit boards  
thousands of

TSP city<sub>1</sub> ... city<sub>n</sub> →  $n! / 2^n$   $\frac{(n-1)!}{2}$   
 completely connected (1 ... n) (271643859)

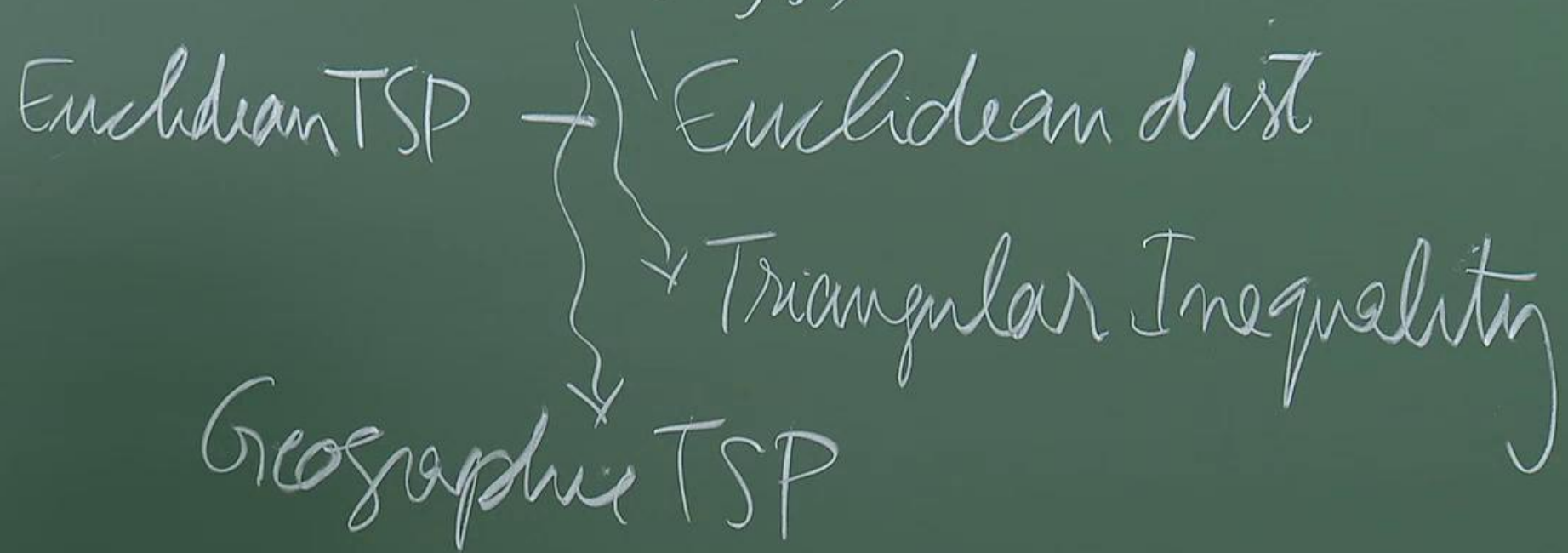
dist(i, j)  
 'Euclidean dist

100 var SAT ~ 10<sup>30</sup>  
 100 var TSP ~ 10<sup>157</sup>

$\text{dist}(i, j)$

Euclidean TSP — { Euclidean dist  
→ Triangular Inequality

$\text{dist}(i, j)$





completely connected  $(1 \dots n)$  (271643859)

$\text{dist}(i, j)$

Euclidean TSP  $\rightarrow$  Euclidean dist  
 $\rightarrow$  Triangular Inequality  
Geographic TSP

100 var SAT  $\sim 10^{30}$

100 var TSP  $\sim 10^{157}$

TSPLIB



<https://www.iwr.uni-heidelberg.de/groups/comopt/software/TSPLIB95/>



drilling circuit boards ← thousands of


TSP cities  $\dots$  city  $n \rightarrow n! / 2^n \cdot \frac{(n-1)!}{2}$   
completely connected  $(1 \dots n)$  (271643859)

dist  $(i, j)$

Euclidean TSP  $\rightarrow$   $\left\{ \begin{array}{l} \text{Euclidean dist} \\ \text{Triangular Inequality} \end{array} \right.$   
Geographic TSP

100 var SAT  $\sim 10^{30}$   
100 var TSP  $\sim 10^{15}$

TSPLIB  
optimal solution



TSP LIB

optimal solution

$\text{dist}(i, j)$

Euclidean TSP — { Euclidean dist  
    ↓  
    Triangular Inequality

TSP LIB

optimal solution



NPTEL

Perturbative

Constructive



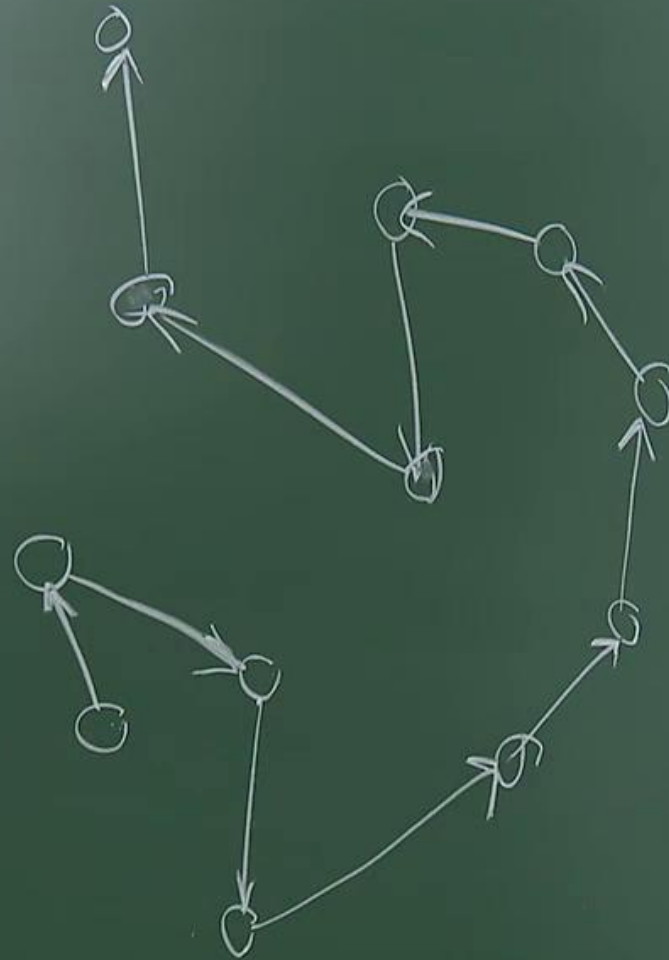
Alternative

## Constructive

Euclidean TSP  $\rightarrow$  Euclidean  
 $\rightarrow$  Triangulation  
 $\rightarrow$  Geographic TSP



Geographic TSP





$O(n)$

## Escaping local Optima

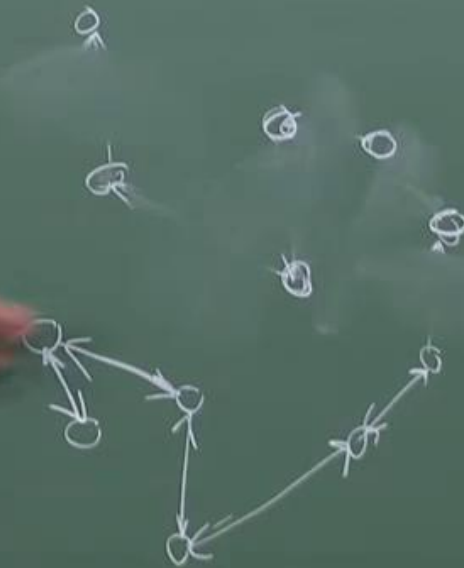
Variable Neighbourhood Descent

TSP  
drilling circuit boards  
thousands of "cities"  
completely connected

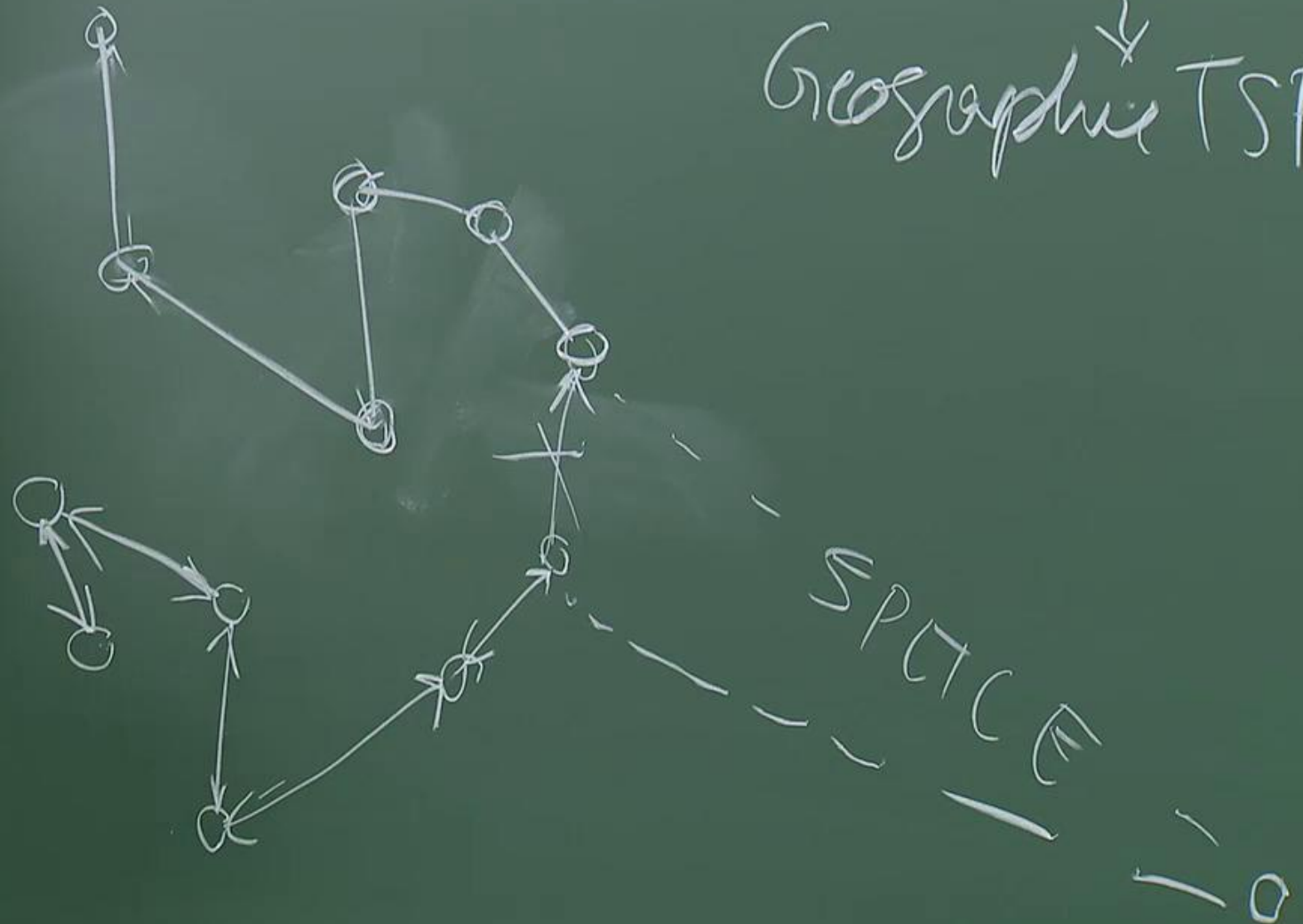
## Constructive

Euclidean TSP

Geographic



Triangulation  
Geographic TSP



Euclidean TSP

Euclidean dist

Triangular Inequality

Geographic TSP

Greedy Heuristic

Sort edges

TSP



SPCTC

# Escaping Local Optima

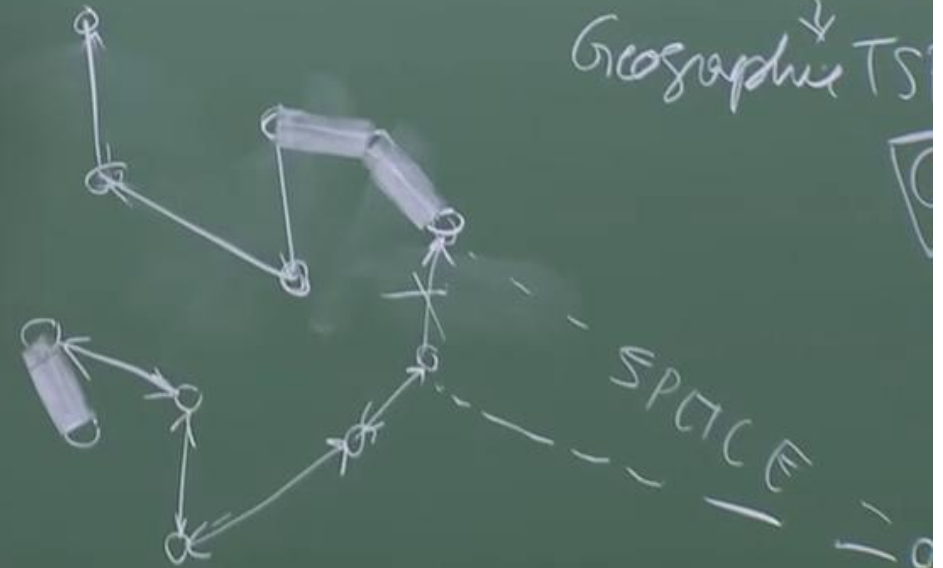
Variable Neighbourhood Descent

Permutation

TSP City  
drilling circuit boards  
thousands of "cities"  
completely connected

## Constructive

$dist(i, j)$   
Euclidean TSP  $\rightarrow$  Euclidean  
 $\rightarrow$  TSP  
Geographic TSP





Linear Inequality

TSP LIB

optimal solution

Savings Heuristic

Savings Heuristic

↓  
edges

Construct  $(n-1)$  tours of length 2



# Escaping Local Optima

Variable Neighbourhood Descent

drilling circuit boards  
thousands of "cities"

TSP cities  $1 \dots n$

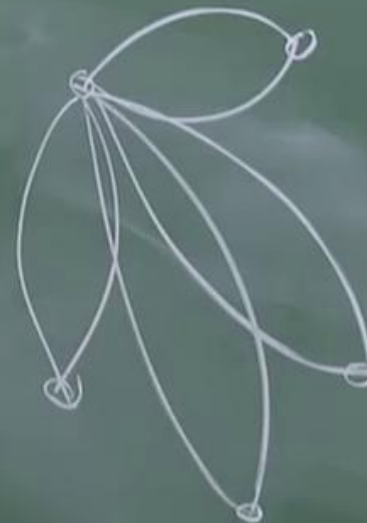
completely connected  $(1 \dots n)$

$\text{dist}(i, j)$

Constructive

Euclidean TSP  $\rightarrow$  Euclidean  
 $\rightarrow$  Triangular

Geographic TSP



Greedy

Sort

# Escaping Local Optima

Variable Neighborhood Descent

Perturbation

drilling circuit boards  
thousands of "cities"

TSP cities  $1 \dots n$

completely connected  $(1 \dots n)$

$dist(i, j)$

## Constructive

Euclidean TSP  $\rightarrow$  Euclidean  
 $\rightarrow$  Triangular

Geographic TSP

Greedy

Sort



NPTEL



Escaping Local Optima

Variable Neighbourhood Descent

Perturbative

drilling circuit boards  
thousands of "cities"

TSP Cities ...  $C_n$

completely connected  $(1 \dots n)$

$dist(i, j)$

Euclidean TSP  $\rightarrow$  Euclidean  
 $\rightarrow$  Triangular

Geographic TSP

Greedy Heuristic

$\downarrow$   
Sort edges

Instruction



ature

# Construction

Euclidean TSP  $\rightarrow$  Euclidean

$\rightarrow$  TSP

$\rightarrow$  Geographic TS



ature

# Construction

Euclidean TSP → Eu  
↓  
TSP  
↓  
Geographic TS



Escaping Local Optima

Multiple Neighborhood Descent

Portmanteau

holding constant bounds  
of "cities"

TSP Cities -  $n$

completely connected  $(1 \dots n)$

$dist(i, j)$

Euclidean TSP

Euclidean

Triangle

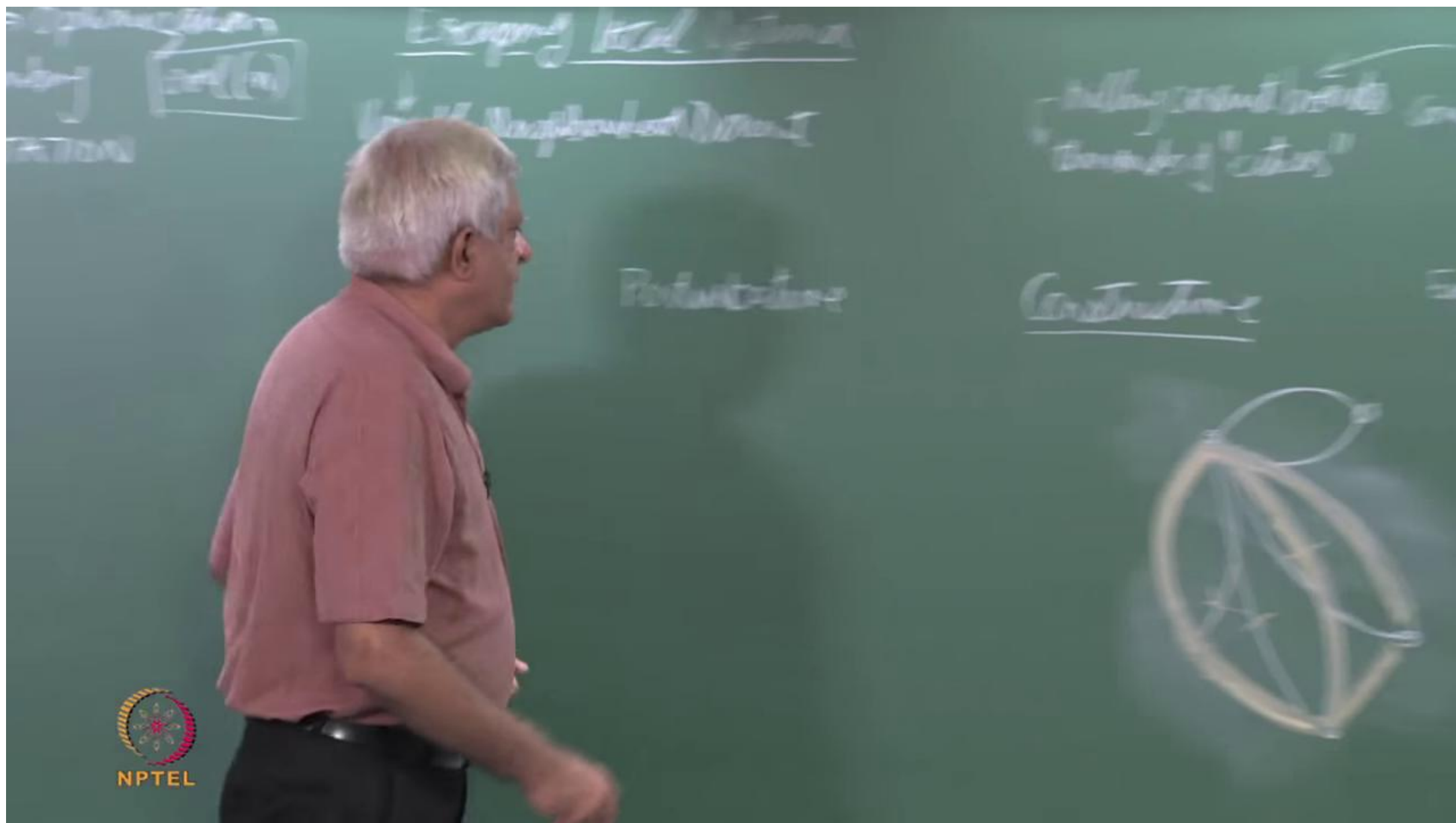
Geographic TSP

Greedy

Set of



NPTEL





de Space Search  $\rightarrow$  Optimization  
 $h(n)$  - Hill Climbing  $eval(n)$   
 $\downarrow$   
EXPLOIT

Escaping local Optima  
 $\downarrow$   
Variable Neighbourhood Descent

$\downarrow$  drill  
thru

Perturbative  
2-city exchange

Constr



Search  $\rightarrow$  Optimization  
 $h(n)$  - Hill Climbing  $eval(n)$   
 $\downarrow$   
EXPLOITATION

Escaping local Optima  
 $\downarrow$   
Variable Neighbourhood Descent

$\downarrow$  drilling circuit  
thousands of

Perturbation  
2-city exchange

Constructive

Perturbative

2-city exchange -  $n(2)$   
3-city exchange



NPTEL

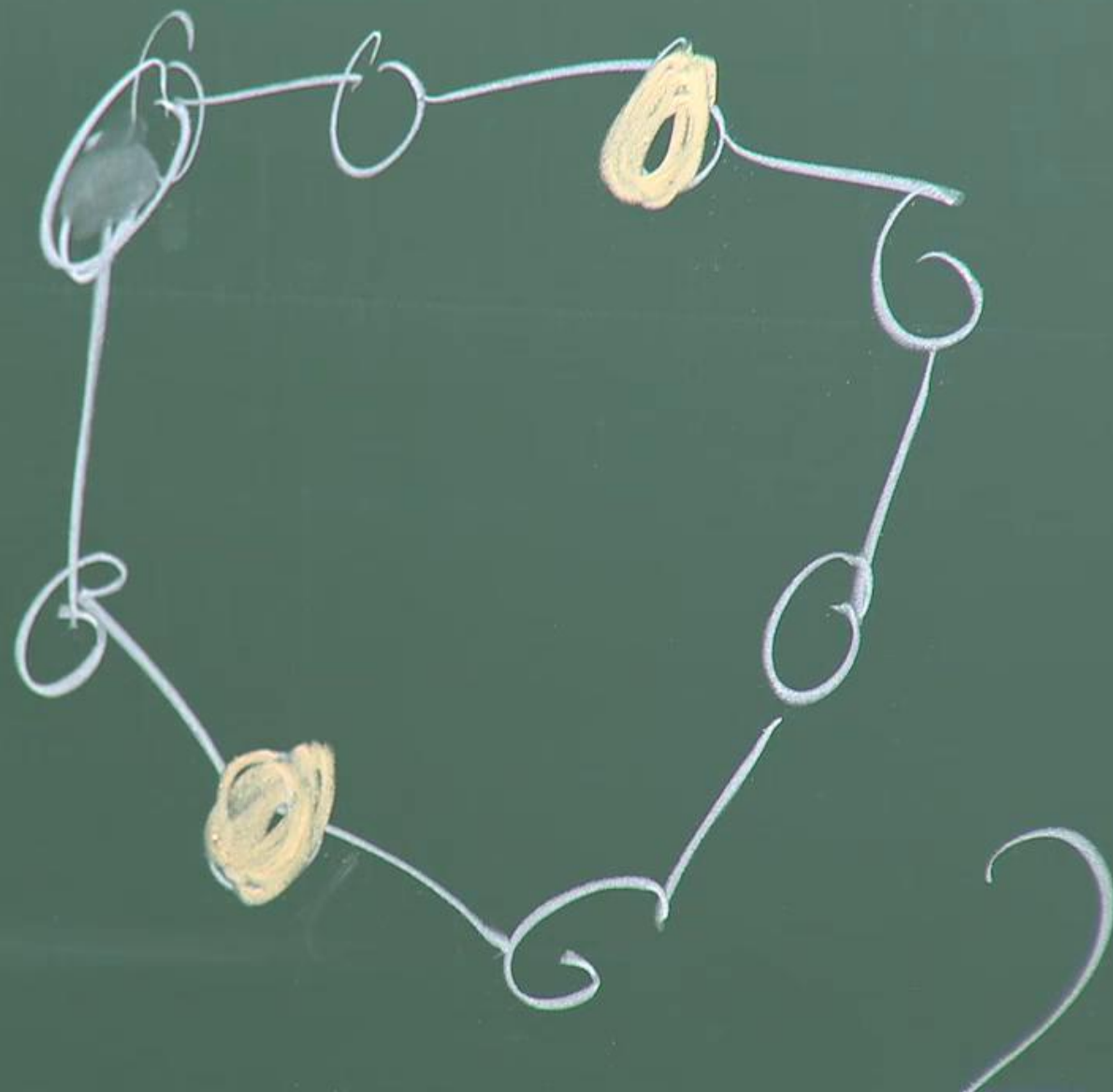
2-city exchange

3-city exchange

$$- n \binom{2}{2}$$

$$- n \binom{3}{3}$$

$$- n \binom{3! - 1}{0}$$



Perturbative

Const

2-city exchange -  $n \binom{n}{2}$

3-city exchange -  $n \binom{n}{3} (3! - 1)$

2-edge exchange



NPTEL



State Space Search  $\Rightarrow$  Optimization

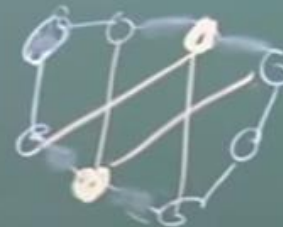
$h(n)$  - Hill Climbing

$eval(n)$

↓  
IMITATION

Escaping Local Optima

↓  
Variable Neighbourhood Descent

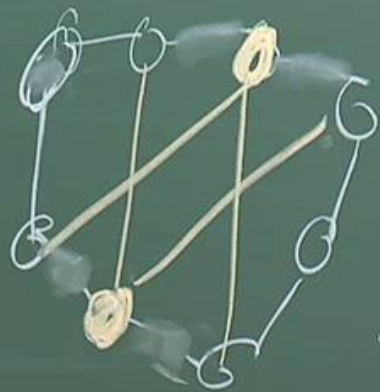


Perturbation

2-city exchange -  $n_2$   
3-city exchange -  $n_3$   
2-edge exchange -  $n_3$

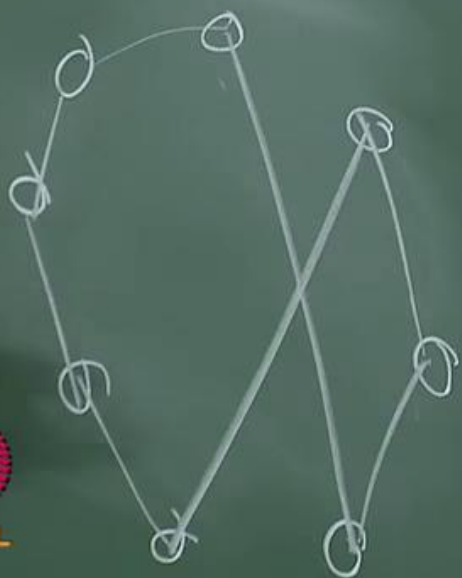






Perturbative

2-city exchange -  $n \binom{n-2}{2}$   
 3-city exchange -  $n \binom{n-3}{3} (3! - 1)$   
 2-edge exchange



State Space Search  $\rightarrow$  Optimization

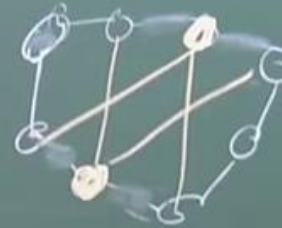
$h(n)$  - Hill Climbing

$eval(n)$

EXPLOITATION

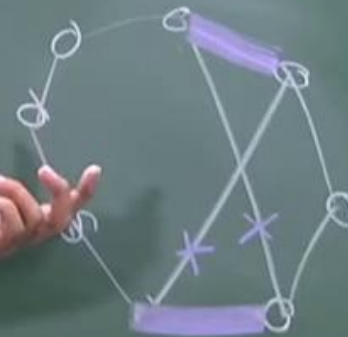
Escaping Local Optima

Variable Neighborhood Descent



Perturbation

2-city exchange -  $n_c$   
3-city exchange -  $n_c$   
2-edge exchange



State Space Search  $\rightarrow$  Optimization

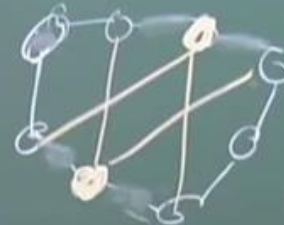
$h(n)$  - Hill Climbing

$eval(n)$

EVALUATION

Escaping Local Optima

Variable Neighborhood Descent



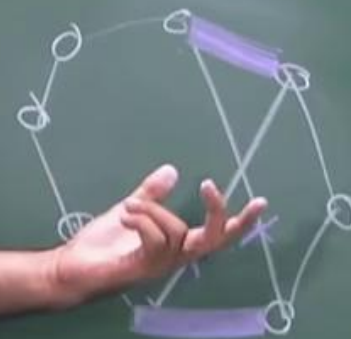
Perturbation

2-city exchange -  $n_c(2)$

3-city exchange -  $n_c(3)$

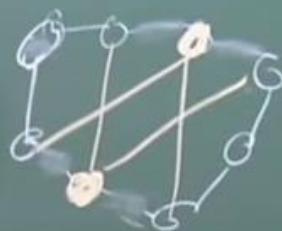
2 edge exchange

(2765 41398)



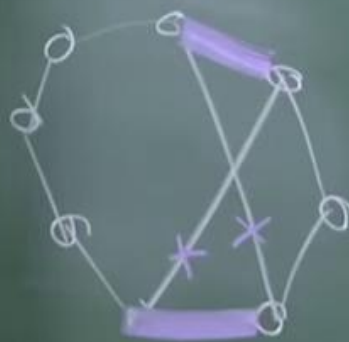
State Space Search  $\rightarrow$  Optimization  
 $h(n)$  - Hill Climbing  
 $eval(n)$   
 $\downarrow$   
 EXPLOITATION

Escaping Local Optima  
 $\downarrow$   
 Variable Neighbourhood Descent



Perturbative

2-city exchange -  $n_{(2)}$   
 3-city exchange -  $n_{(3)}(3! - 1)$   
 2-edge exchange



(27 654 | 398)

27 456 | 398



search  $\rightarrow$  Optimization

- Hill Climbing  $eval(n)$

$\downarrow$   
EXPLOITATION

Escaping local Optima

$\downarrow$   
Variable Neighborhood Descent

$\downarrow$  drilling circuit board  
thousands of "cities"

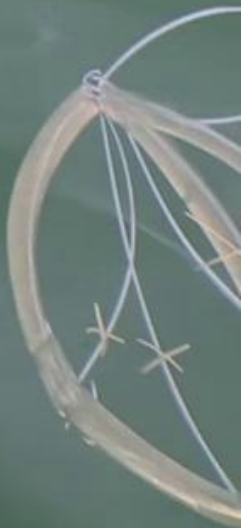
Perturbative

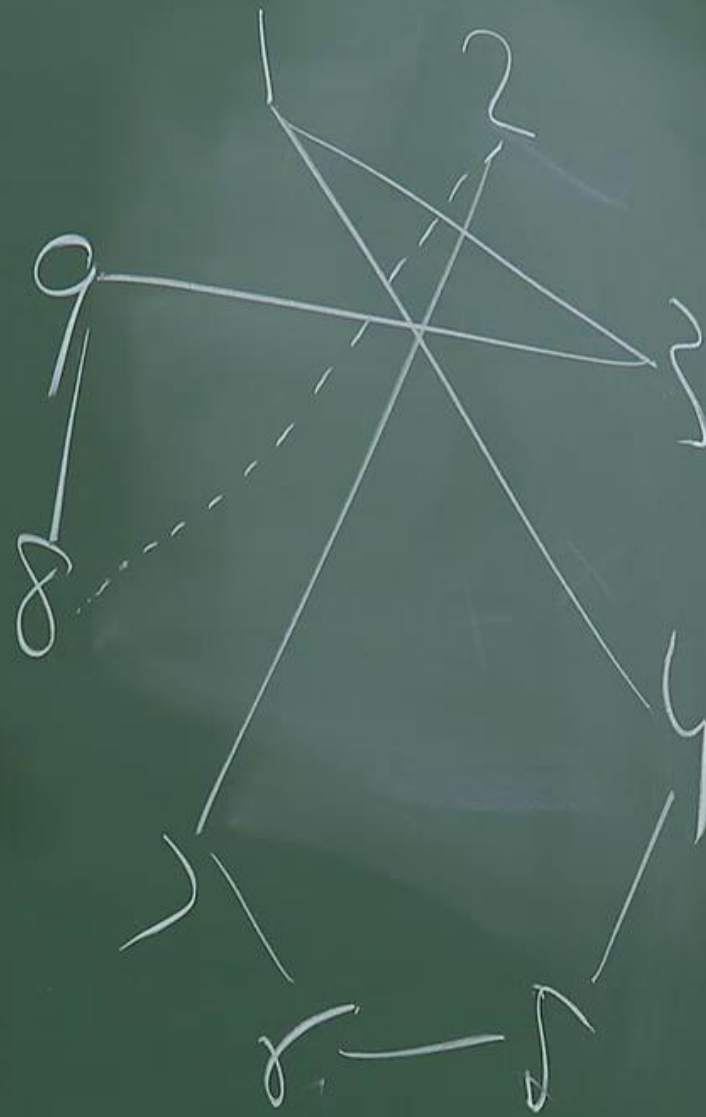
Constructive

2-city exchange -  $n(n-1)$   
3-city exchange -  $n(n-1)(n-2)$   
2-edge exchange

(27|654|398)

27|456|398





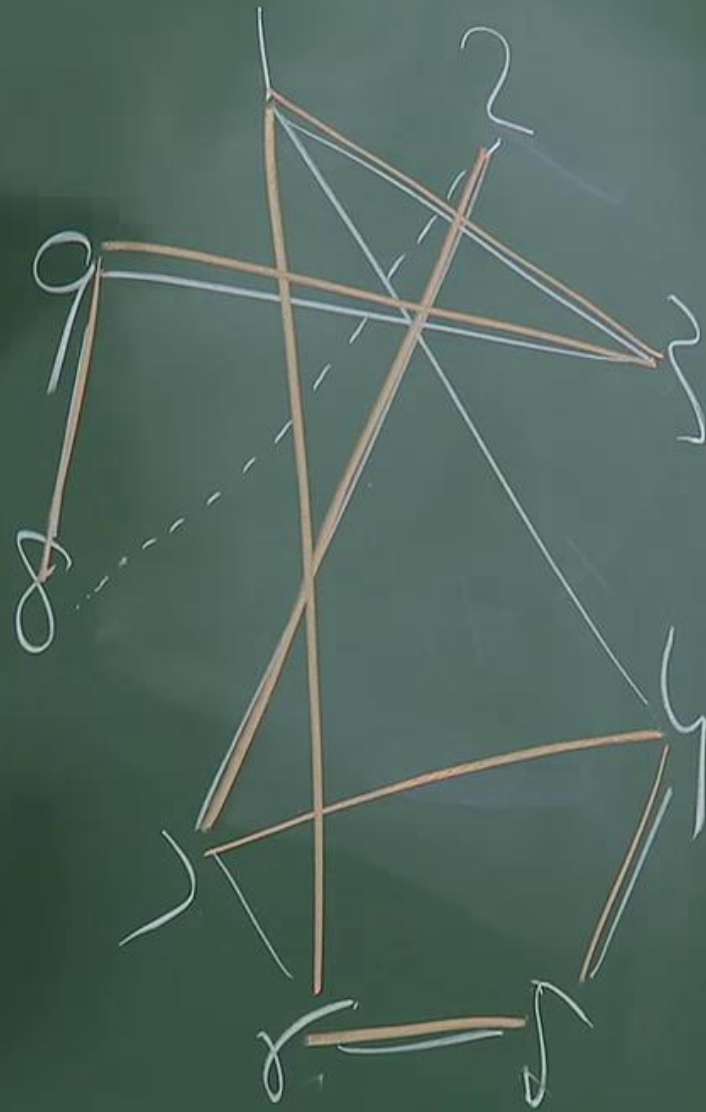
3-city tour

2 edge exc

(27|65

27 65



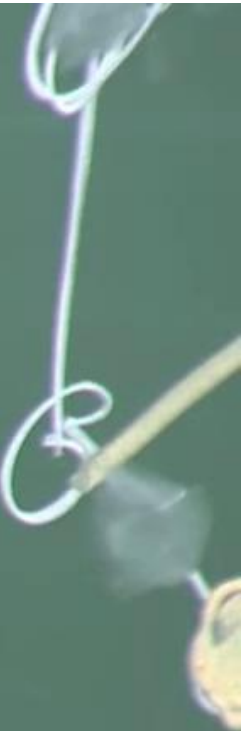
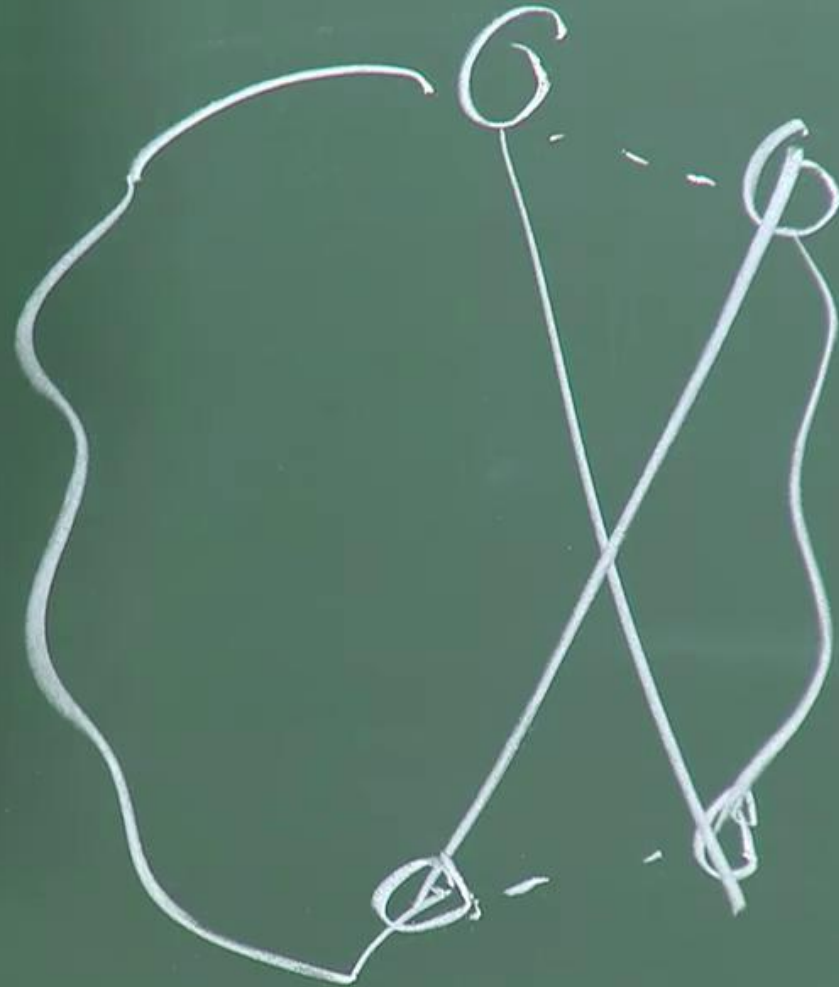


3-city tour

2 edge exc

(27|65

27 65



State Space Search  $\rightarrow$  Optimization

$h(n)$  - Hill Climbing

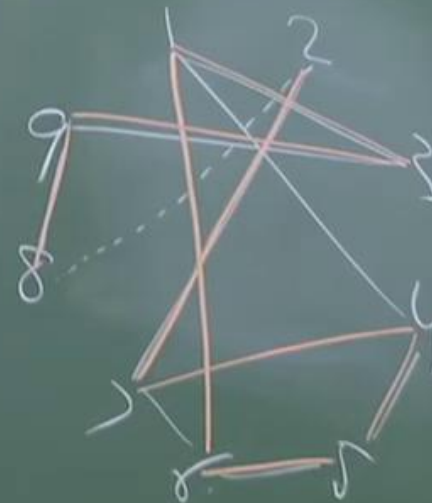
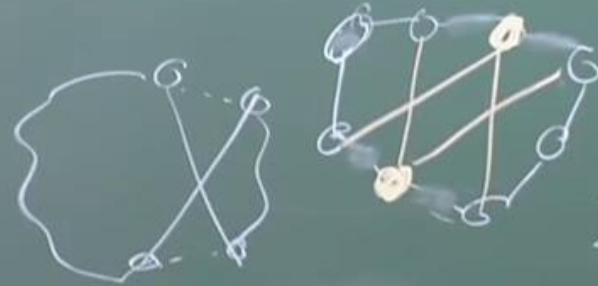
$eval(n)$

EXPLOITATION

Escaping local

Variable Neighbourhood

3-edge exchange



2-city

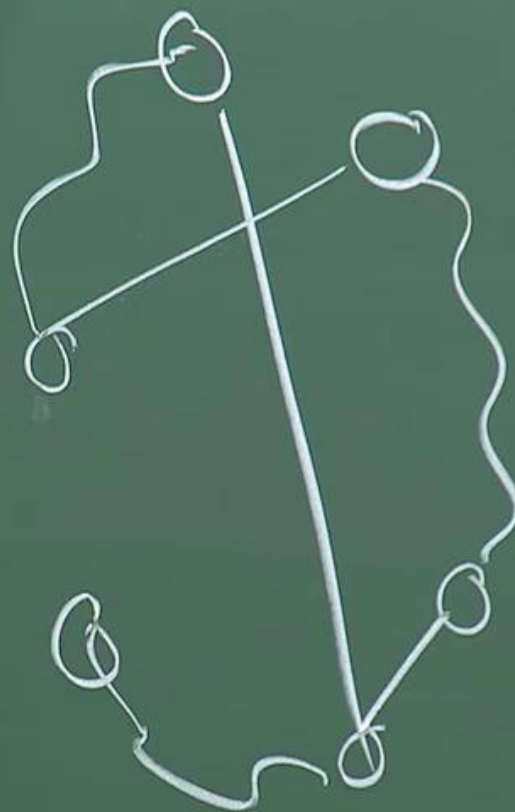
3-city

2-edge

(27)

27

# 3-edge exchange





State Space Search  $\Rightarrow$  Optimization

$h(n)$  - Hill Climbing

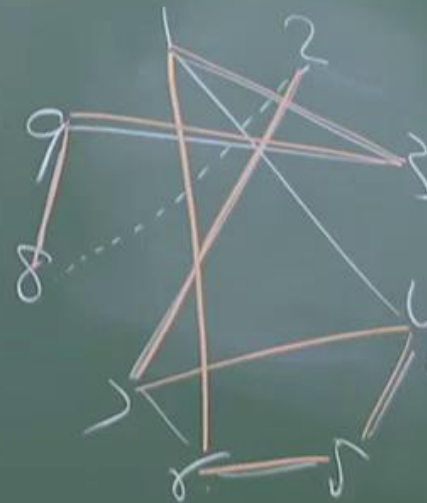
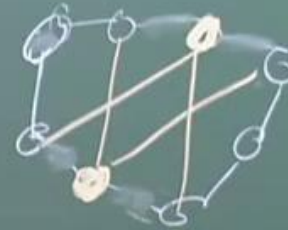
$eval(n)$

EXPLOITATION

Escaping Local

Variable Neighbourhood Descent

3-edge exchange



Perturbation

2-city exchange

3-city exchange

2-edge exchange

(27654)

27456

State Space Search  $\rightarrow$  Optimization

$h(n)$  - Hill Climbing

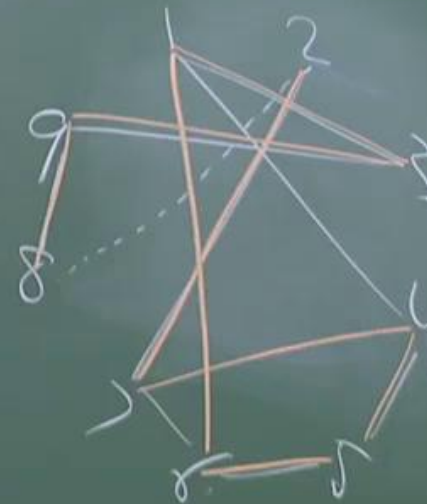
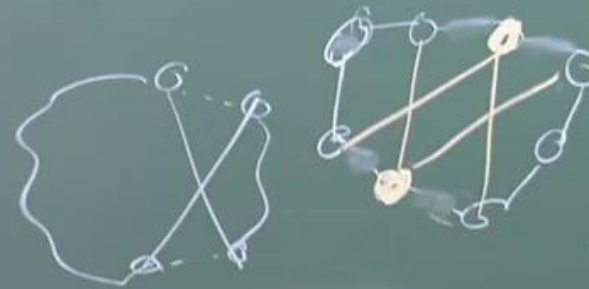
$eval(n)$

EXPLOITATION

Escaping Local

Variable Neighborhood

3-edge exchange



Portu

2-city exchange

3-city exchange

2-edge exchange

(27 | 65)

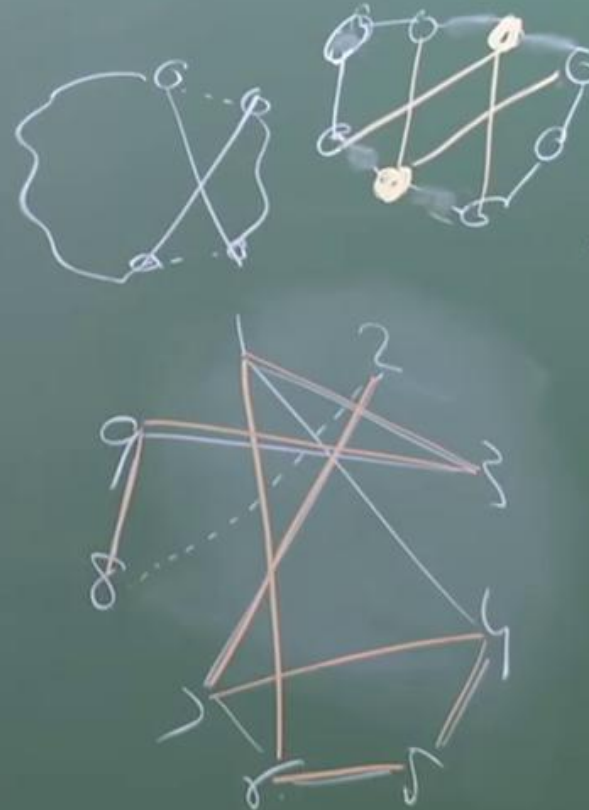
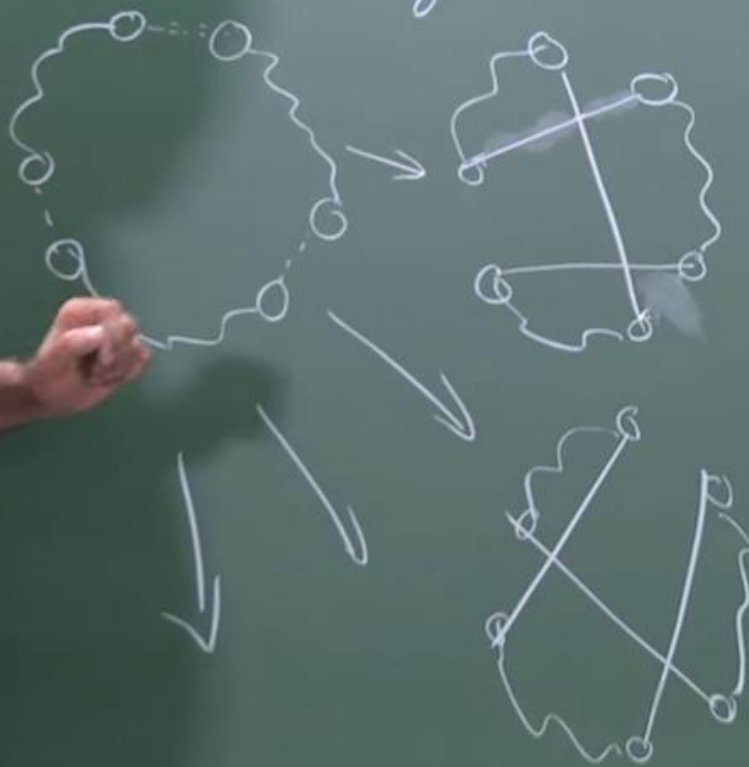
27 45



State Space Search  $\rightarrow$  Optimization  
 $h(n)$  - Hill Climbing  $eval(n)$   
 $\downarrow$   
 EXPLOITATION

Escaping Local  
 $\downarrow$   
 Variable Neighbourhood

3-edge exchange



Portals  
 2-city exch  
 3-city exch  
 2 edge exch

(27 | 65)  
 27 45

State Space Search  $\rightarrow$  Optimization

$h(n)$  - Hill Climbing

$eval(n)$

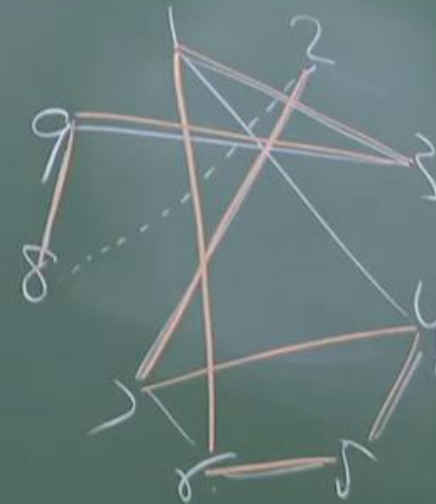
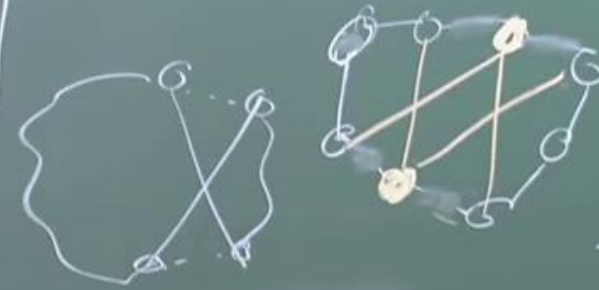
EXPLOITATION

Escaping Local

Variable Neighbourhood

3-edge exchange

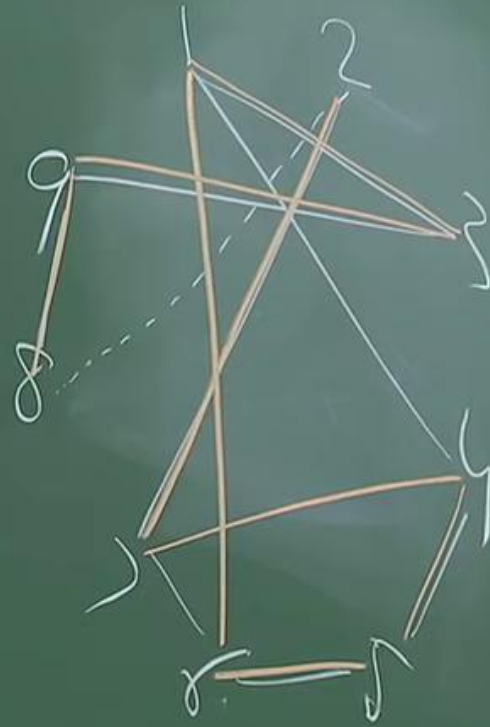
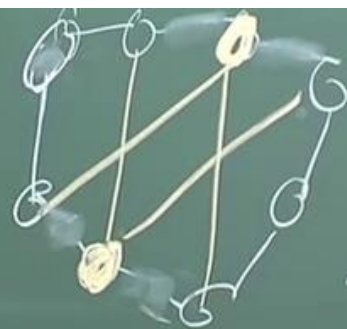
$m-1 \text{ } C_{3 \times 4}$



2-city ex  
3-city ex  
2-edge ex

(276)  
274

3-edge exchange  $|M-1| \binom{3 \times 4}{3}$



2-city  
3-city  
2-edge

(27)

27



Search  $\Rightarrow$  Optimization

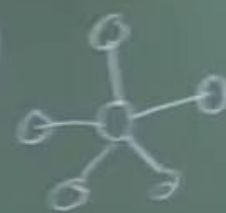
Full Search  $O(n!)$

EXPLOITATION

$n-1 C_{3 \times 4}$

## Escaping Local Optima

Neighbourhood Descent



drilling circuit board  
thousands of "cities"

Perturbation



Construction

city exchange

technique

$$n C_3 (3! - 1)$$

range

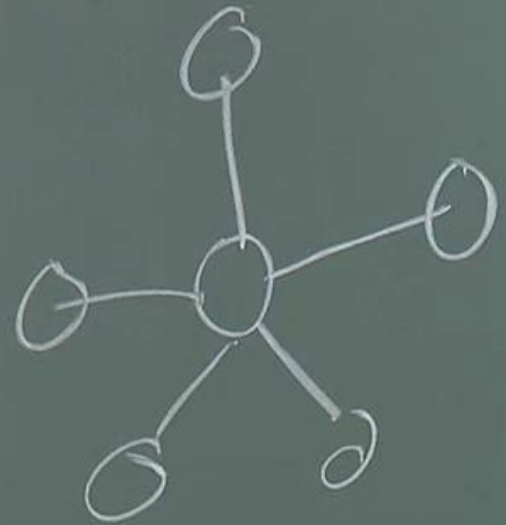
(27654 | 398)

27456 | 398



NPTEL

Descent



drilling circuit  
thousands of "cith"

turbatime



Construction



