

# Video Lectures On Artificial Intelligence

## Lecture 17 Population Based Methods II

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2-4-5-6-9-3-1-7-8

2-4-5-6-9-3-1-7-8  
3-7-1-9-4-2-5-6-8

Path expansion 2-4-5-6-9-3-1-7-  
3-7-1-9-4-2-5-6-  
↑

8	1	6
3	5	7
4	9	2

Path representation 2-4-5-6-9-3-1-7-8 GAs  
-7-1-9-4-2-5-6-8

Partially mapped  
(PMX) crossover

6



Path representation

2-4-5-6-9-3-1-7  
3-7-1-9-4-2-5-6

illegally mapped  
crossover

6 9 3 1  
9 4 2 5

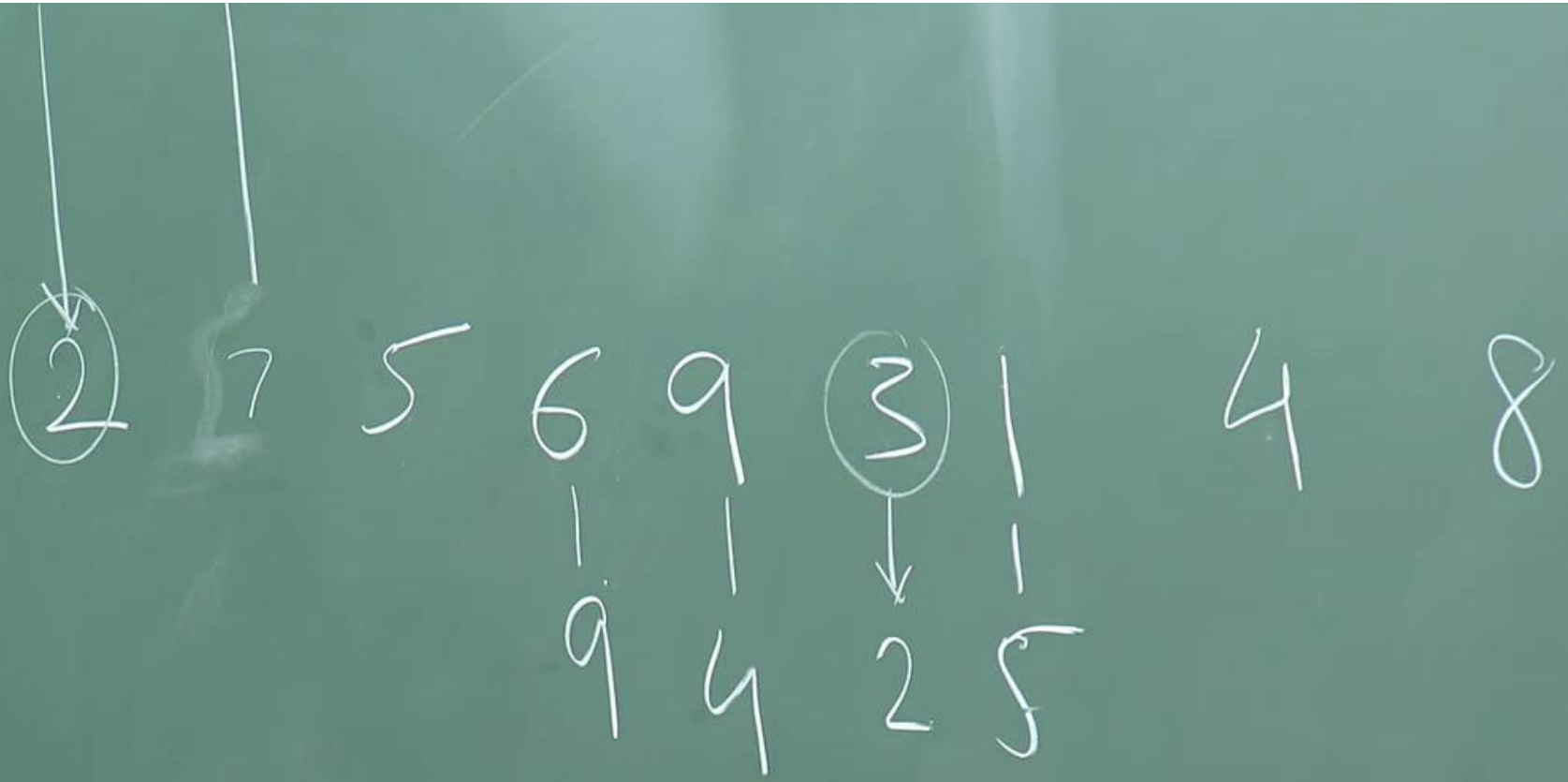
6 9 3 1  
1 1 1 1  
9 4 2 5



red  
saver

(C<sub>1</sub>)

(C<sub>2</sub>)



Path representation  $\textcircled{1} 2-4-5-6-9-3-1-7-8$

$\textcircled{P2} \textcircled{3}-7-1-9-4-2-5-6-8$

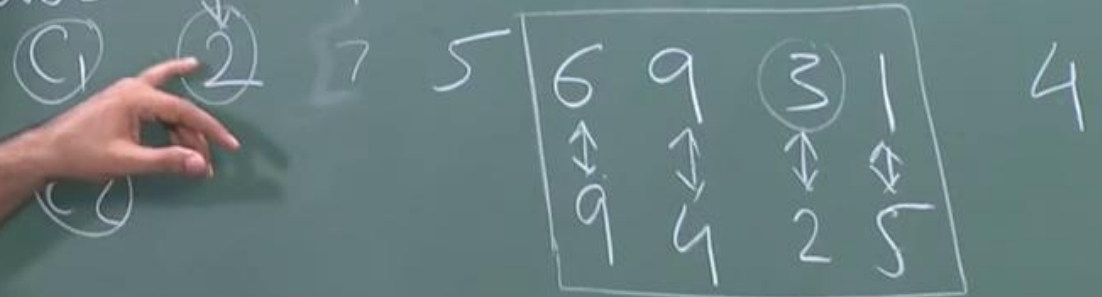
mapped  
crossover

$\textcircled{C1}$



Path representation  $\textcircled{1} 2-4-5-6-9-3-1-7-$   
 $\textcircled{P2} \textcircled{3}-7-1-9-4-2-5-6-$

fully mapped  
crossover



ged  
ssaver

(C1)

2

7

5

6

9

3

1

4

8

(C2)

3

6

1

9

4

2

5

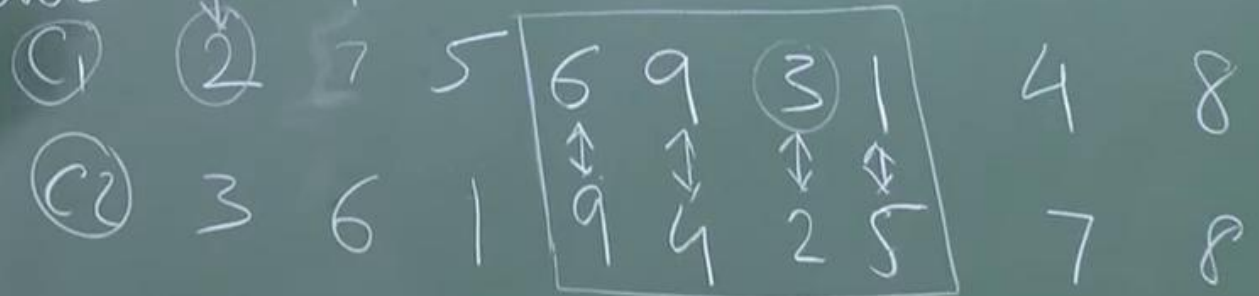
7

8



Path representation  $\textcircled{1} 2-4-5-6-9-3-1-7-8$   
 $\textcircled{P2} \textcircled{3}-7-1-9-4-2-5-6-8$

Partially mapped  
(PMX) crossover



Order Crossover

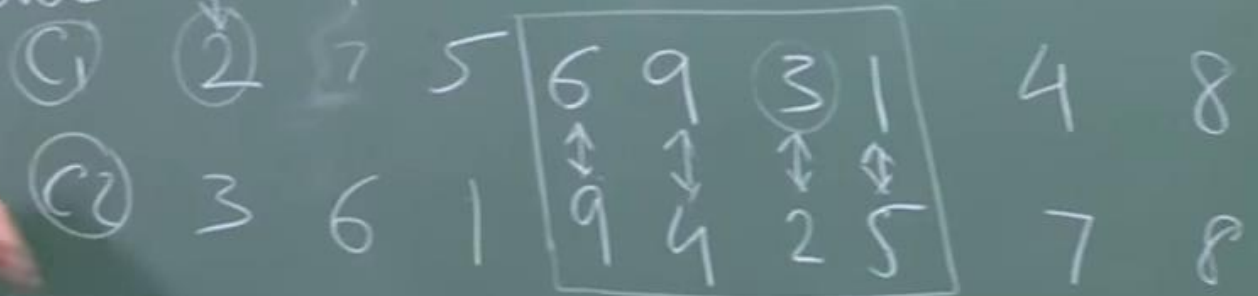
6 9 3 1  
 9 4 2 5



Path representation 2-4-5-6-9-3-1-7-8

(P2) 3-7-1-9-4-2-5-6-8

centrally mapped  
(MX) crossover



crossover

6 9 3 1  
9 4 2 5

6 9 3 1  
9 4 2 5

7 4 2 5 8

---



As for 1st

# ORDINAL REPRESENTATION

# ADJACENCY REPRESENTATION

# ADJACENCY REPRESENTATION

---

1 2 3 4 5 6 7 8 9

# ADJACENCY REPRESENTATION

---

1 2 3 4 5 6 7 8 9



NPTEL

# ADJACENCY REPRESENTATION

9	5	7	2	6	8	1	3	4
1	2	3	4	5	6	7	8	9

Path representation 2-4-5-6-9-3-1-7-8

7-1-9-4-2-5-6-8

ADJ

9

Partially mapped  
(PMX) crossover

Order

6	9	3	1
↕	↕	↕	↕
9	4	2	5

6 9 3 1  
9 4 2 5

7 4 2 5 8



2-4-5-6-9-3-1-7-8 GHS for 158

3-7-1-9-4-2-5-8

ADJACENCY REPRESENTATION

9	5	7	2	6	8	1	3	4
1	2	3	4	5	6	7	8	9

2 7 5 6 9 3

3 6 1 9 4

6 9

9 4

2

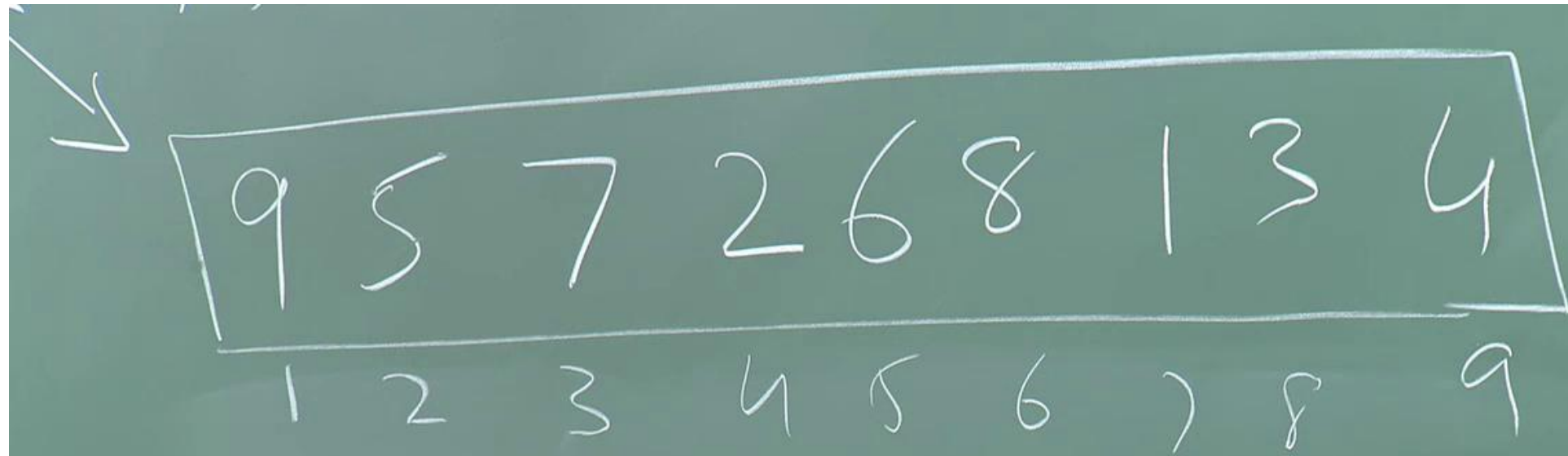
NPTEL



9 5 7 2 6 8 1 3 4

1 2 3 4 5 6 7 8 9

2 1 . . . . .



9	5	7	2	6	8	1	3	4
1	2	3	4	5	6	7	8	9



P2

9	5	7	2	6	8	1	3	4
---	---	---	---	---	---	---	---	---

1 2 3 4 5 6 7 8 9

P1

7	4	1	5	6	9	8	2	3
---	---	---	---	---	---	---	---	---

Alternating Crossover

- Choose from alternating  
parents

6-9-5-1-7-8

9-4-2-5-8

ADJACENCY REPRESENTATION

P2

9	5	7	2	6	8	1	3	4
1	2	3	4	5	6	7	8	9

P1

7	1	5	6	9	8	2	3
---	---	---	---	---	---	---	---

Alternating Crossover  
- choose from alternating parents

(3) 7

(1) X

NPTEL



P2 

9	5	7	2	6	8	1	3	4
---	---	---	---	---	---	---	---	---

1 2 3 4 5 6 7 8 9

8 P1 

7	4	1	5	6	9	8	2	3
---	---	---	---	---	---	---	---	---

Alternating Crossover

- Choose from alternating parents

4 2 5 8

(C3) 7

(1) X



NPTEL

P2 

9	5	7	2	6	8	1	3	4
---	---	---	---	---	---	---	---	---

1 2 3 4 5 6 7 8 9

P1 

7	4	1	5	6	9	8	2	3
---	---	---	---	---	---	---	---	---

Alternating Crossover  
 - Choose from alternating parents

2 5 8  
 (C3) 7

(P1) 3



P2 9 5 7 2 6 8 1 3 4  
 1 2 3 4 5 6 7 8 9

8  
 8  
 P1 7 4 1 5 6 9 8 2 3

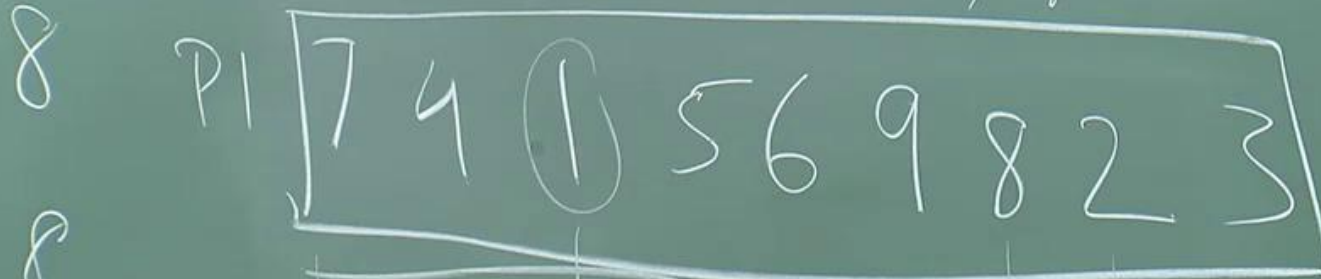
Alternating Crossover  
 - Choose from alternating parents

4 2 5 8

3 7 2

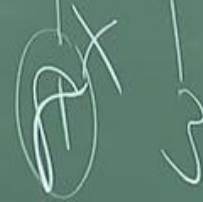
8 3

1 2 3 4 5 6 7 8 9



Alternating Crossover  
- Choose from alternating parents

4 2 5 8



Subtour Crossover

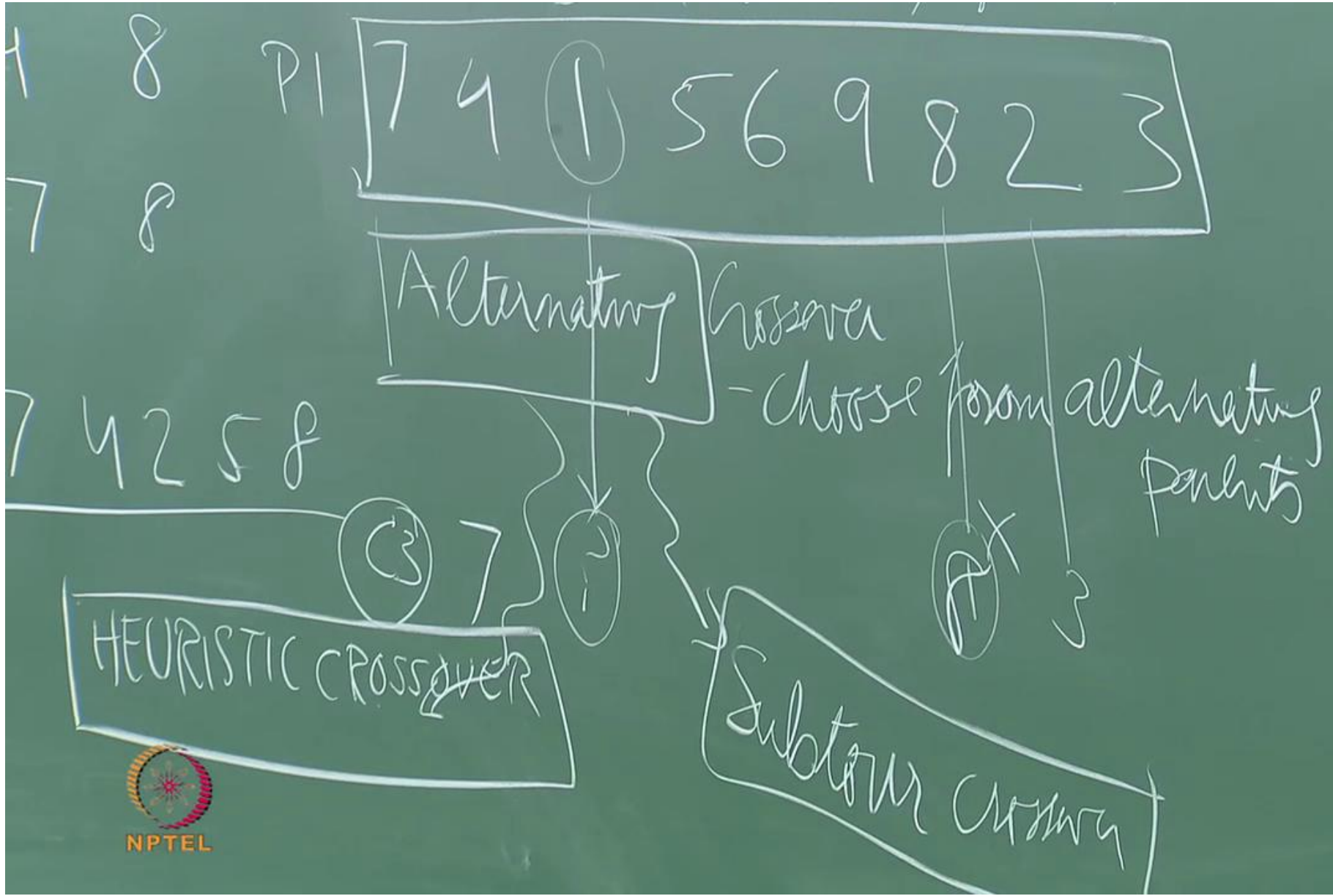


Partially mapped  
(PMX) crossover

Order Crossover

6	9	3	1
↕	↕	↕	↕
9	4	2	5

6	9	3	1
<hr/>			
9	4	2	5





$\begin{array}{c|c} 1 & 6 \\ \hline 4 & 2 \\ 2 & 5 \\ 5 & 8 \end{array}$

# ADJACENCY REPRESENTATION

$\begin{array}{ccc} 6 & 9 & 3 \\ \updownarrow & \updownarrow & \updownarrow \\ 9 & 4 & 2 \end{array}$

$\begin{array}{cc} 6 & 9 \\ \hline 9 & 4 \end{array}$



P1  $\begin{array}{|c|c|c|c|c|c|c|c|c|} \hline 9 & 5 & 7 & 2 & 6 & 8 & 1 & 3 & 4 \\ \hline 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ \hline \end{array}$   
 $\begin{array}{|c|c|c|c|c|c|c|c|c|} \hline 7 & 4 & 1 & 5 & 6 & 9 & 8 & 2 & 3 \\ \hline \end{array}$

Alt

Crossover

- choose from alternating parents

$\begin{array}{c} (3) 7 \\ (1) 5 \end{array}$

CROSSOVER

$\begin{array}{c} (4) 3 \\ (5) 6 \end{array}$

Subtour crossover

P2 9 5 7 2 6 8 1 3 4  
 1 2 3 4 5 6 7 8 9

P1 7 4 1 5 6 9 8 2 3

Alternating Crossover

- Choose from alternating parents

for TSP

ADJACENCY REPRESENTATION

1	5	7	2	6	8	1	3	4
1	2	3	4	5	6	7	8	9
7	4	1	5	6	9	8	2	3

Alternating

Crossover

- choose from alternating parents



Crossover

Subtour

ORDINAL REPRESENTATION

2



NPTEL



# ORDINAL REPRESENTATION

2-4-5-6-9-3-1-7-8

CENCY REPRESENTATION

2-4-5-6-9-3-1-7-8



alternating  
parents



# Reference Index

1 2 3 4 5 6 7 8 9

# ORDINAL REPRESENTATION

2-4-5-6-9-3-1-7-8

Reference Index

1 ~~2~~ 3 4 5 6 7 8 9

2

# ORDINAL REPRESENTATION

2-4-5-6-9-3-1-7-8

Reference Index

~~1~~/~~2~~/~~3~~/~~4~~/~~5~~/~~6~~/~~7~~/~~8~~/~~9~~

2 3 3 3 5 2 1 1 1

# ORDINAL REPRESENTATION

2-4-5-6-9-3-1-7-8

Reference Index

~~1~~/~~2~~/~~3~~/~~4~~/~~5~~/~~6~~/~~7~~/~~8~~/~~9~~

2 3 3 3 5 2 1 1 1



Path representation 2-4-5-6-9-3-1-7-8

(P2) 3-9-4-2-5-6-8

Partially mapped crossover (PMX)

ADJAC

P2 

9	5
1	2

  
P1 

7	4
---	---

  
Alter

6	9	3	1
	↕	↕	↕
4		2	5

4 8  
7 8

Order Cross

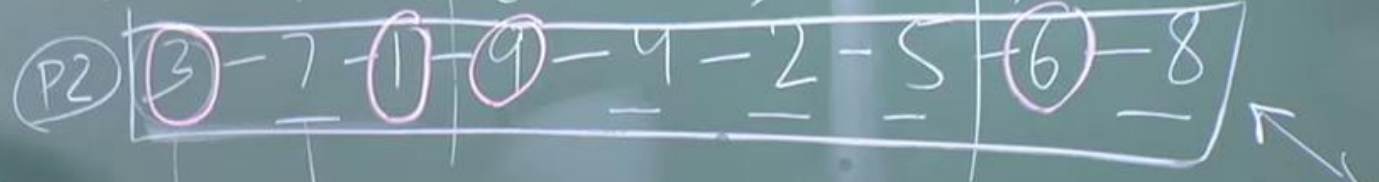
9 3 1  
9 4 2 5

7 4 2 5 8

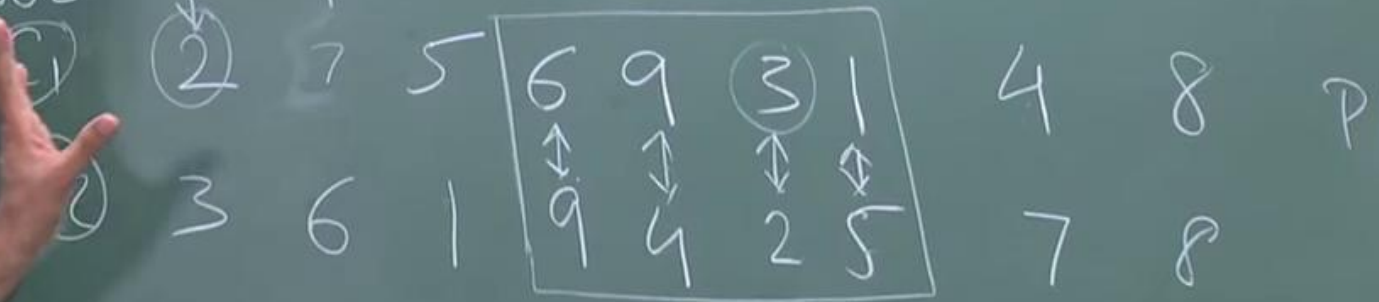
(C3) 7

HEURISTIC CROSSOVER

Path Representation 2-4-5-6-9-3-1-7-8

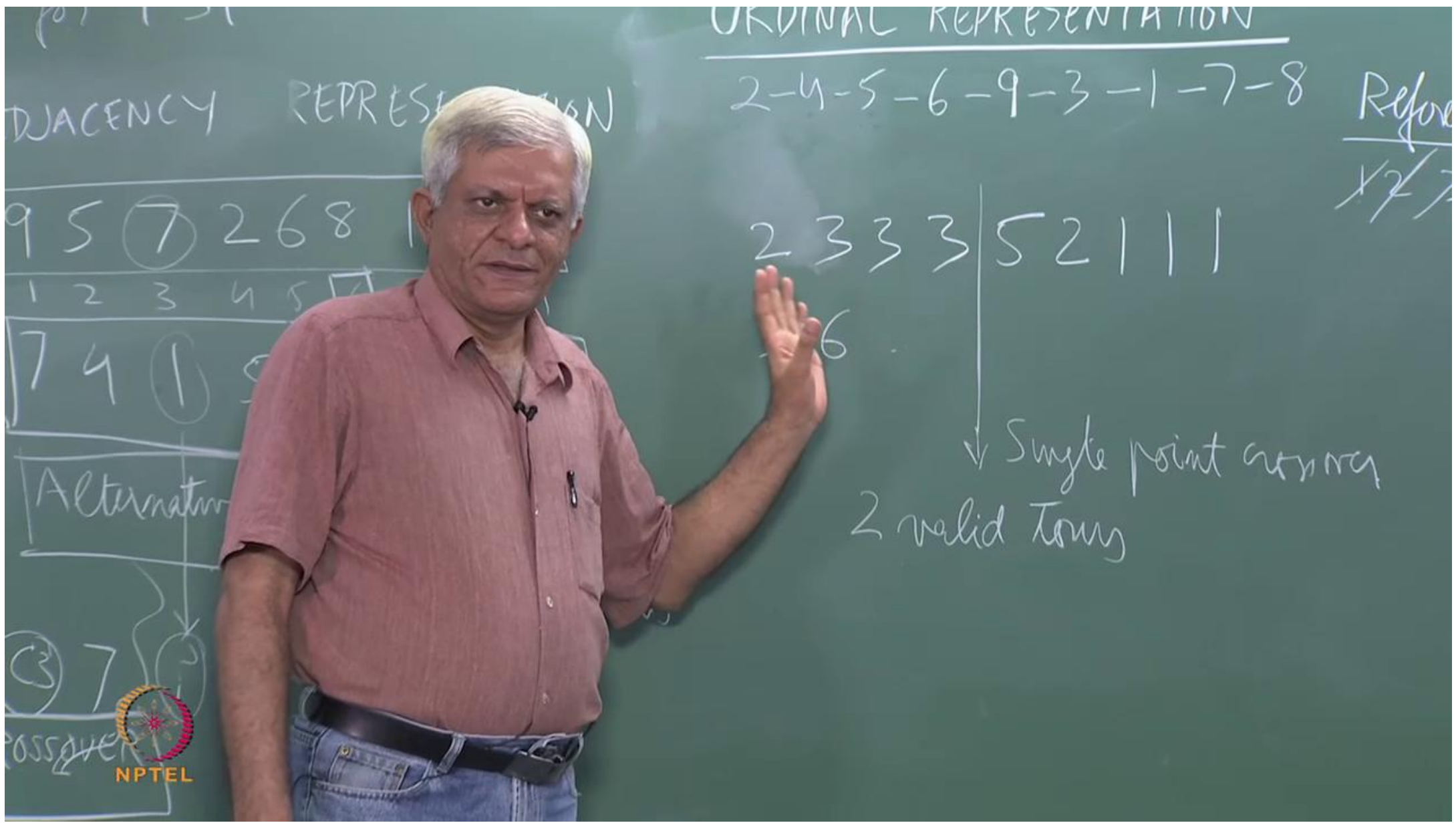


mapped  
crossover



6 9 3 1	7 4 2 5 8
9 4 2 5	

HEURISTIC



ADJACENCY REPRESENTATION

9 5 7 2 6 8 1  
1 2 3 4 5 6

7 4 1 5

Alternation

3 7 1  
crossover  
NPTEL

ORDINAL REPRESENTATION

2-4-5-6-9-3-1-7-8

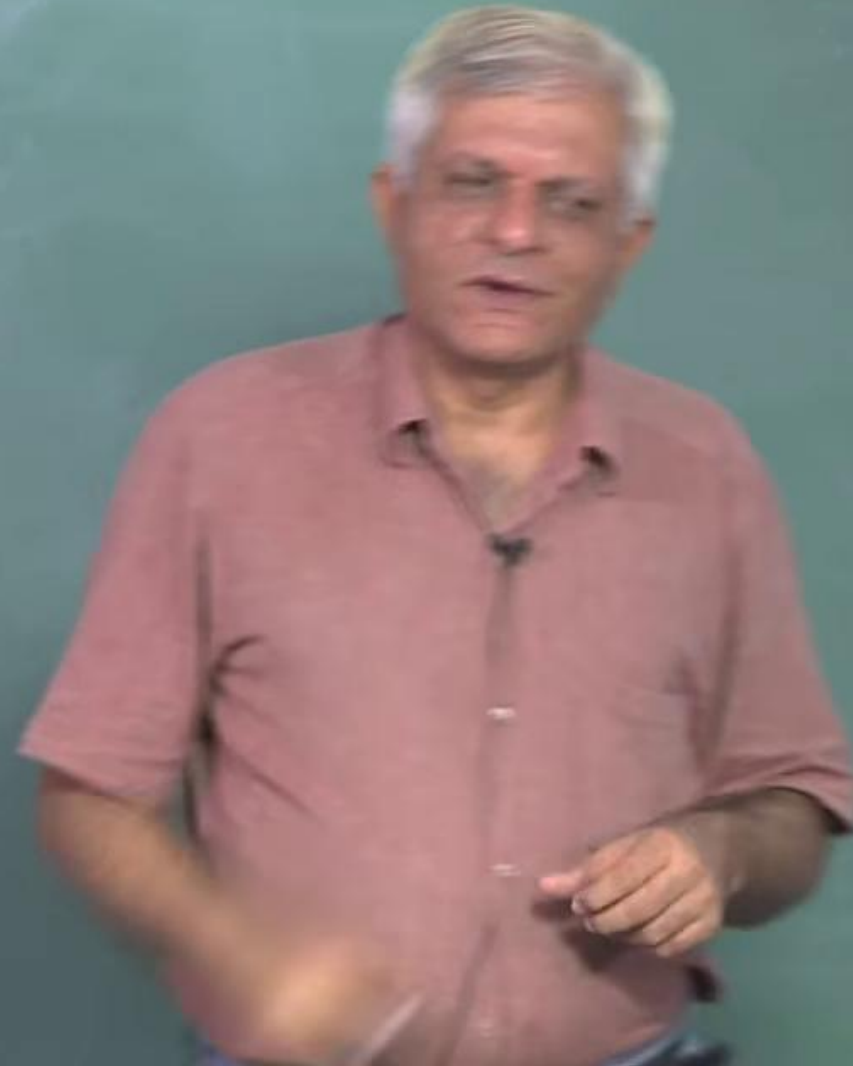
Reform  
1/2/7

2 3 3 3 | 5 2 1 1 1

6

↓ Single point corner  
2 valid Tours

# Population based methods





Population based methods — Culture

↓  
GAs

recombination at  
genotype level

Selection

phenotype level

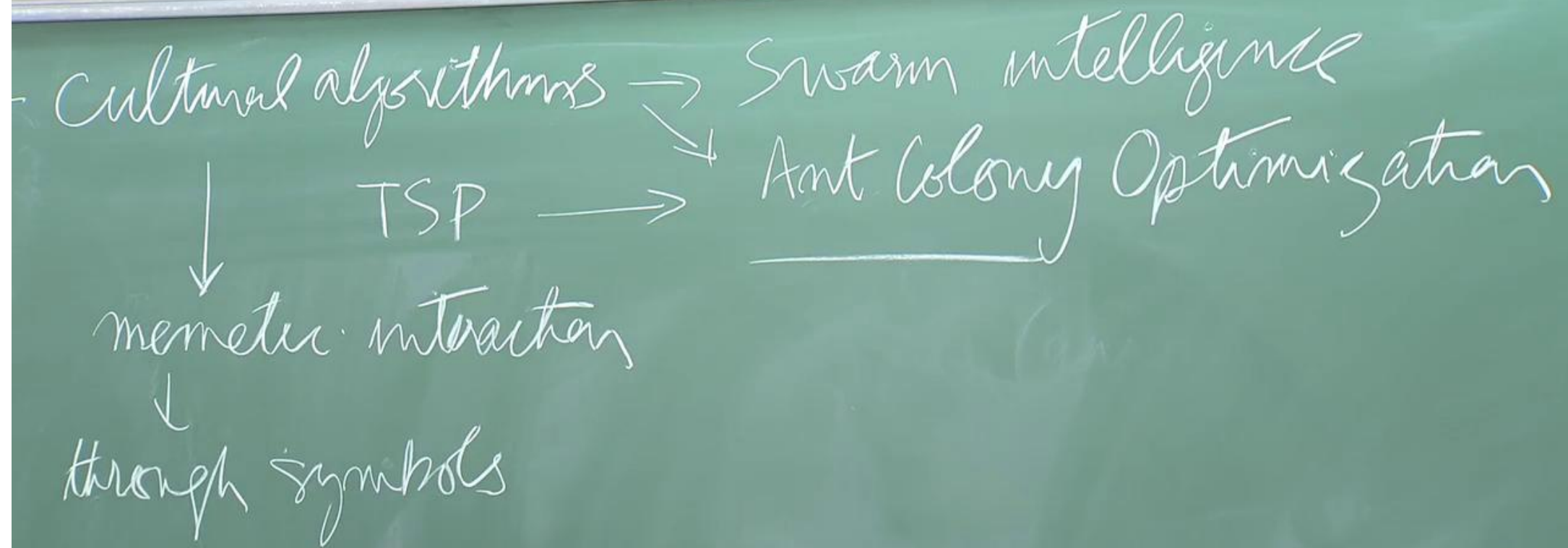
Population based methods — Cultural algorithms

TSP —

recombination at  
genotype level

then { phenotype } level





Population based methods

↓  
GAs

[phenotype] at  
the level

Selection [genotype] level

— Cultural algorithms → SW

TSP → Am

↓  
memetic interaction

↓  
through symbols → P

Population based methods



recombination at  
genotype level

then

phenotype level

Cultural algorithms



S



Ad

TSP



memetic interaction



through symbols

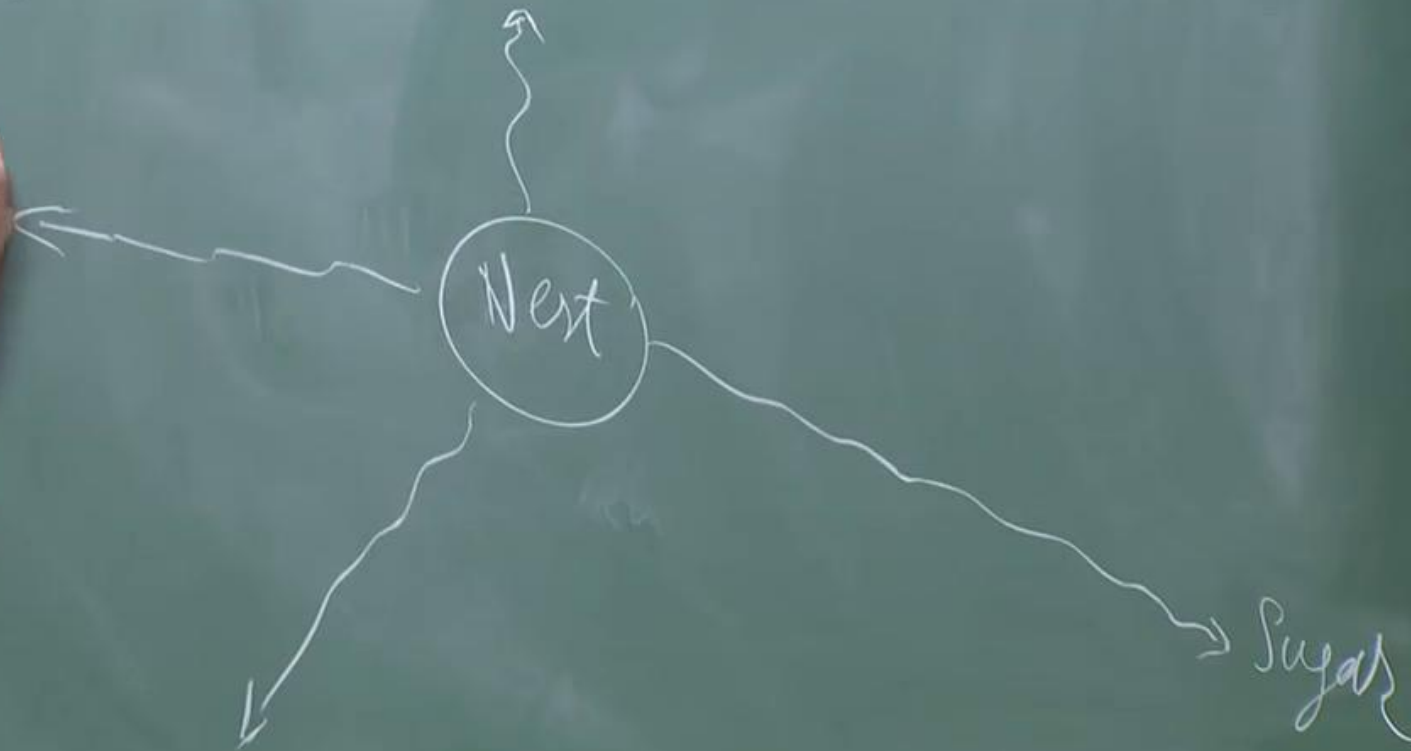


NPTEL





Cultural algorithms → Swarm intelligence  
↓ TSP → Ant Colony Optimization  
↓ memetic algorithm  
↓ through ants → PHEROMONE TRAILS







Columnar map minimization → Swarm Intelligence  
↓ TSP → Ant Colony Optimization

↓  
memetic inter-  
↓  
through symbiosis

→ PHEROMONE TRAILS

Nest

Sugar

Colony algorithms → Swarm Intelligence  
↓ TSP → Ant Colony Optimization

memetic interaction  
↓  
through symbols

PROMONE TRAILS

Nest



DORIGO et al

Ant Colony Optimization  
(for TSP)

Population

↓  
GAs

recon  
gen

Selection  
ph



DORIGO et al Ant Colony Optimization  
(for TSP)

Popula  
↓  
GAs

Let there be  $N$  cities  
 $M$  ants

Sele



NPTEL



DORIGO et al Ant Colony Optimization  
(for TSP) Popular  
Let there be  $N$  cities  
 $M$  ants  
↓  
GAs

Each ant constructs a tour in a greedy fashion  
Sele



DORIGO et al Ant Colony Optimization  
(for TSP)

Let there be  $N$  cities  
 $M$  ants

Popul  
↓  
GAs

Each ant constructs a tour in a greedy fashion  
deposits pheromone on the tour it constructs

Sel

DORIGO et al Ant Colony Optimization  
(for TSP)

Let there be  $N$  cities  
 $M$  ants

Popul  
↓  
GAs

Each ant constructs a tour in a greedy fashion  
↳ deposits pheromone on the tour it constructs

$T_{ij}$

DORIGO et al Ant Colony Optimization  
(for TSP)

Let there be  $N$  cities  
 $M$  ants

Popul  
↓  
GA's

Each ant constructs a tour in a greedy fashion

Self

deposits pheromone on the tour it constructs

$T_i(t+n)$

DORIGO et al Ant Colony Optimization  
(for TSP)

Let there be  $N$  cities  
 $M$  ants

Popul  
↓  
GAs

each ant constructs a tour in a greedy fashion

Sele

deposits pheromone on the tour it constructs

$$\tau_{ij}(t+n) \leftarrow$$



DORIGO et al Ant Colony Optimization  
(for TSP)

Let there be  $N$  cities  
 $M$  ants

Population  
↓  
GAs

Each ant constructs a tour in a greedy fashion

Select

↳ deposits pheromone on the tour it constructs

$$T_{ij}(t+n) \leftarrow (1-\lambda) T_{ij}(t)$$


coefficient of evaporation



Each ant constructs a tour in a greedy fashion

Select

deposits pheromone on the tour it constructs


$$\tau_{ij}(t+n) \leftarrow (1-\lambda)\tau_{ij}(t) + \tau_{ij}(n)$$

coefficient of evaporation

Let there be  $N$  cities  
 $M$  ants

Each ant constructs  $\tau_{ij}(t)$  in a greedy fashion  
 on the tour it constructs

Selection  $\left\{ \begin{array}{l} \text{recombination at} \\ \text{genotype level} \\ \text{phenotype level} \end{array} \right.$

$\tau_{ij}(t) = \lambda \tau_{ij}(t) + \Delta \tau_{ij}(t+n)$

$\sum_{k=1}^n \Delta \tau_{ij}^k$

NPTEL

GA's

recom  
gen  
Selection  
level

TSP → Ant Colon  
↓  
memory interaction  
↓  
through symbols → PHEROM

constructs

$$+ \Delta \tau_{ij}(t)$$

$$\Delta \tau_{ij}^k = 0 \text{ if ant does not traverse edge } ij$$





SP)  
↓  
GAs

recombination  
genetic level

Selection

by  
ashion

at constructs

+  $\Delta T_{ij}(t)$

ration

↓  
TSP → Ant Colon  
↓  
memetic interaction  
↓  
through symbols → PHERO

$$\Delta T_{ij}^k = 0 \text{ if ant does not traverse edge } ij$$
$$= Q / L_k$$

$$\Delta \tau_{ij}^k = 0 \text{ if ant does not traverse edge } ij$$

$$= Q / L_k \text{ — cost of tour found by ant } k$$





Optimization (for TSP)      Population based methods — cultural algorithms

↓  
GAs

$$P \propto [T_{ij}]^{\alpha} [n_{ij}]^{\beta}$$

tour in a greedy fashion

more on the tour it constructs

$$(1-\lambda)T_{ij}(t) + \Delta T_{ij}$$

coefficient of evaporation

$$\Delta T_{ij}^k = 0 \text{ if ant does not trace } ij$$

$$= Q / L_k \text{ — cost of } ij \text{ by } k$$



methods — Cultural algorithms →


$P \propto [T_{ij}]^{\alpha} [N_{ij}]^{\beta} \rightarrow \text{VISIBILITY}$

$\propto \frac{1}{\text{cost}(i, j)}$

Ant C

PHE

— 0 if ant does not traverse edge



The image shows a chalkboard with handwritten mathematical formulas and text. A hand is pointing at the equation  $P \propto [T_{ij}]^{\alpha} [N_{ij}]^{\beta}$ . To the right of this equation, an arrow points to the word 'VISIBILITY', which is then followed by another arrow pointing to the formula  $\propto \frac{1}{\text{cost}(i, j)}$ . At the top left, the word 'methods' is followed by a dash and 'Cultural algorithms', with an arrow pointing to the right. On the top right, 'Ant C' is written. Below 'VISIBILITY', the letters 'PHE' are visible. At the bottom, a line of text starts with '— 0 if ant does not traverse edge'.

Population based methods — Cultural algorithms → Swarm

↓  
GAs

Ant C

TSP)

greedy  
fashion

from it constructs

$(t) + \Delta T_{ij}(t+1)$

evaporation

Ant od at  
moves to c  
with

$$P \propto [T_{ij}]^{\alpha} [T_{ij}]^{\beta} \rightarrow \text{VISIBILITY} \propto \frac{1}{\text{cost}(i,j)}$$

PHE

$$= 0 \text{ if ant does not traverse edge } i,j$$

$$= Q / L_k \text{ — cost of tour found by ant } k$$





methods — Cultural algorithms → visibility

$$P \propto \frac{[T_{ij}]^\alpha [n_{ij}]^\beta}{\sum_k [T_{(k)}]^\alpha [n_{(k)}]^\beta} \rightarrow \text{VISIBILITY}$$

$$\propto \frac{1}{\text{cost}(i,j)}$$

set of allowed cities



NPTEL

$\tau^k$

— 0 if ant does not traverse edge

# DORIGO et al Ant Colony Optimization (for TSP)

Let there be  $N$  cities  
 $M$  ants

Each ant constructs a tour (in a greedy fashion)

deposits pheromone on the tour it can

$$T_{ij}(t+n) \leftarrow (1-\lambda)T_{ij}(t) + \tau$$

Coefficient of evaporation

