Video Lectures On

Artificial Intelligence

Lecture 20

Admissibility of A*

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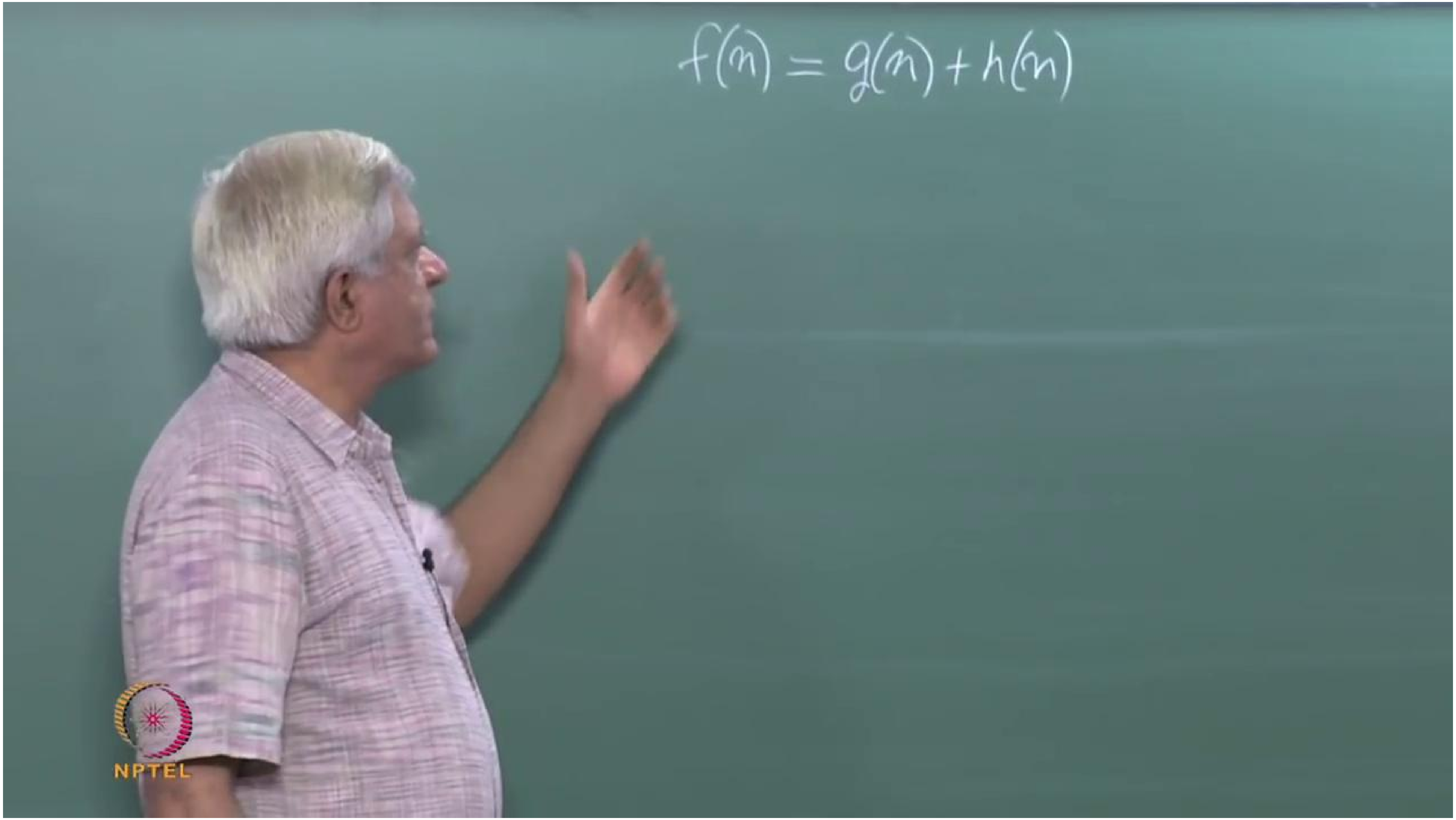
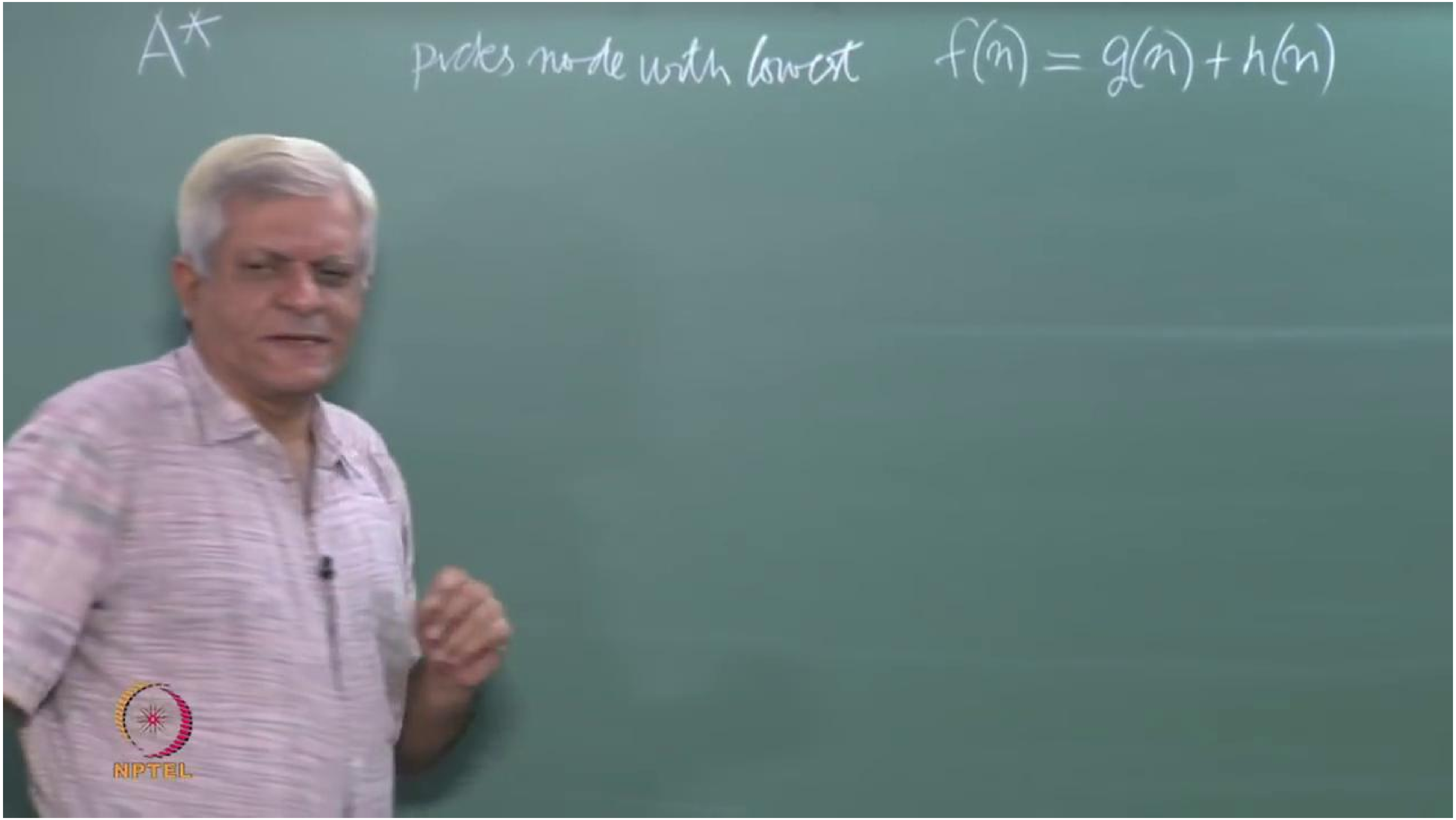
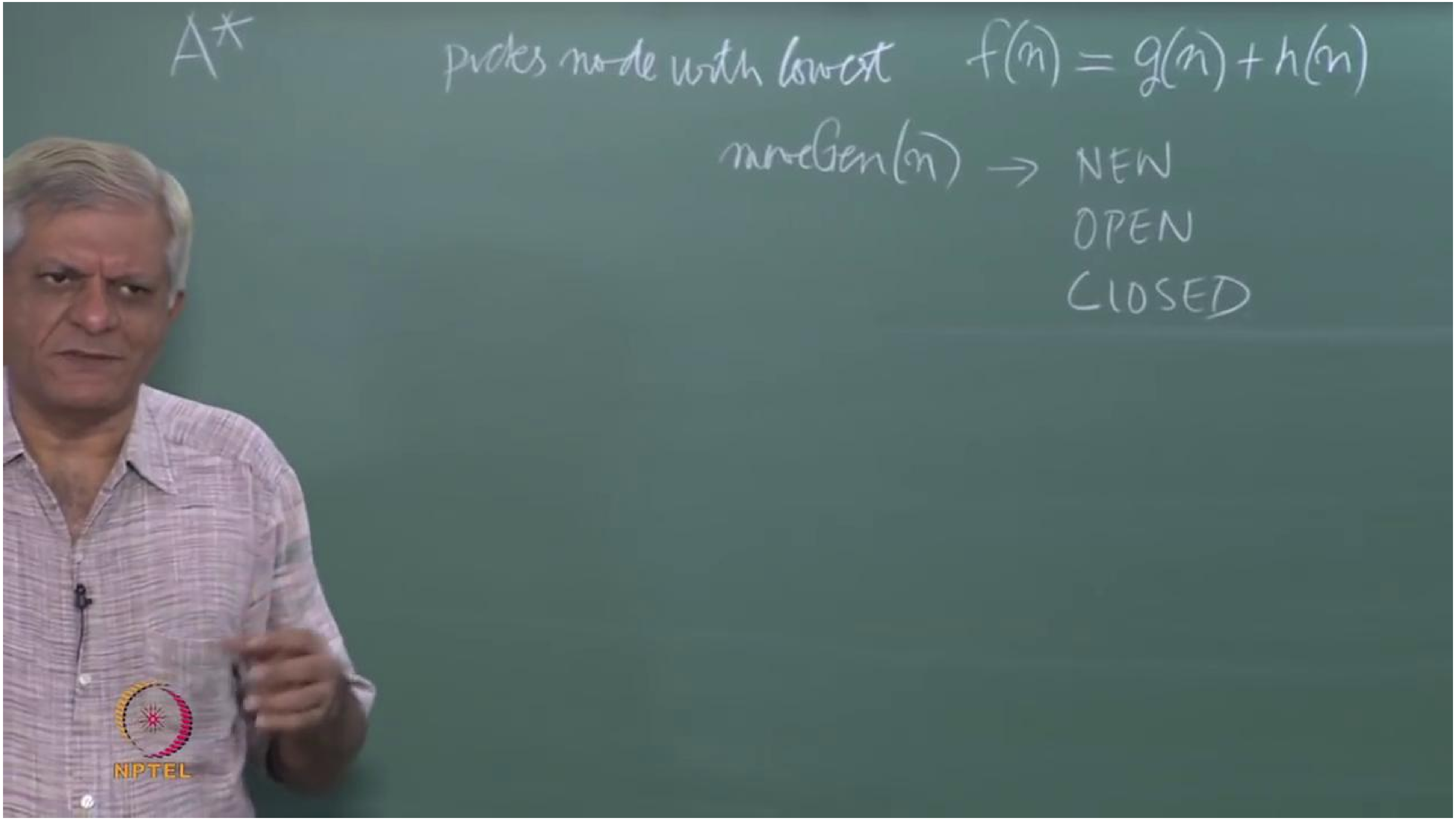
Department of Computer Science
IIT Madras

NPTEL


$f(n) = g(n) + h(n)$

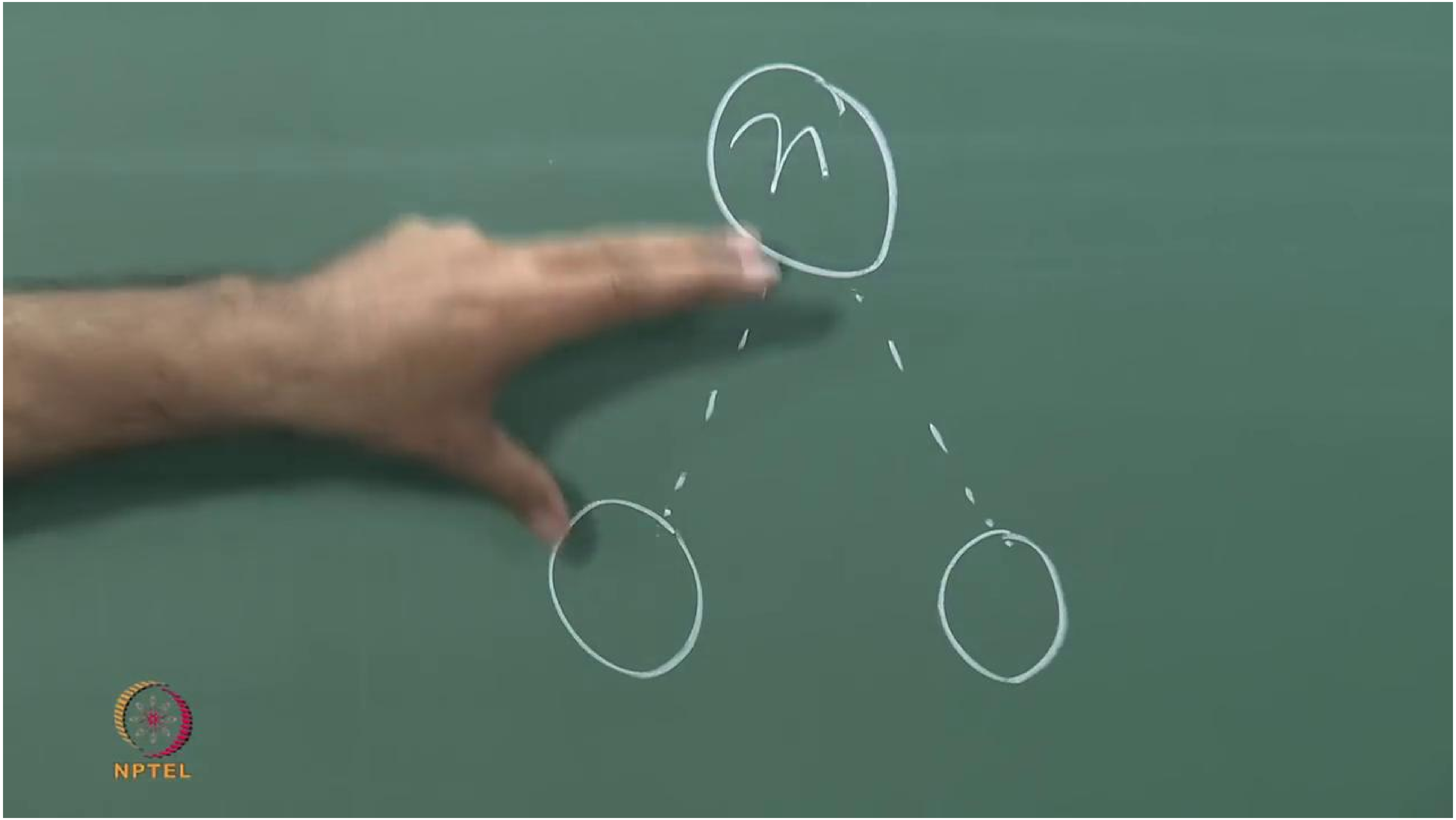

A*
picks node with lowest
f(n) = g(n) + h(n)
NPTEL

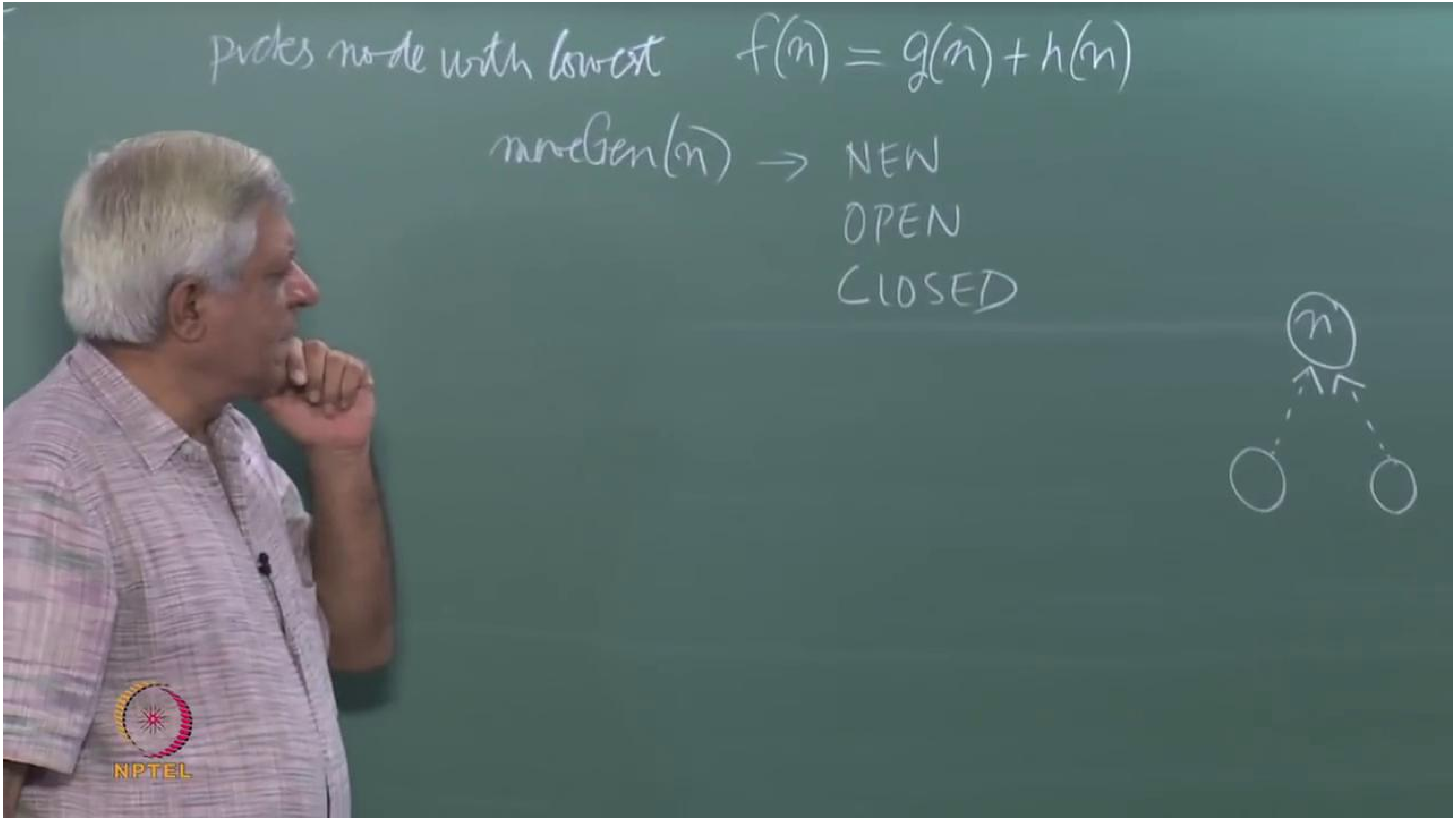

A*
picks node with lowest $f(n) = g(n) + h(n)$
moveGen(n) → NEW
OPEN
CLOSED

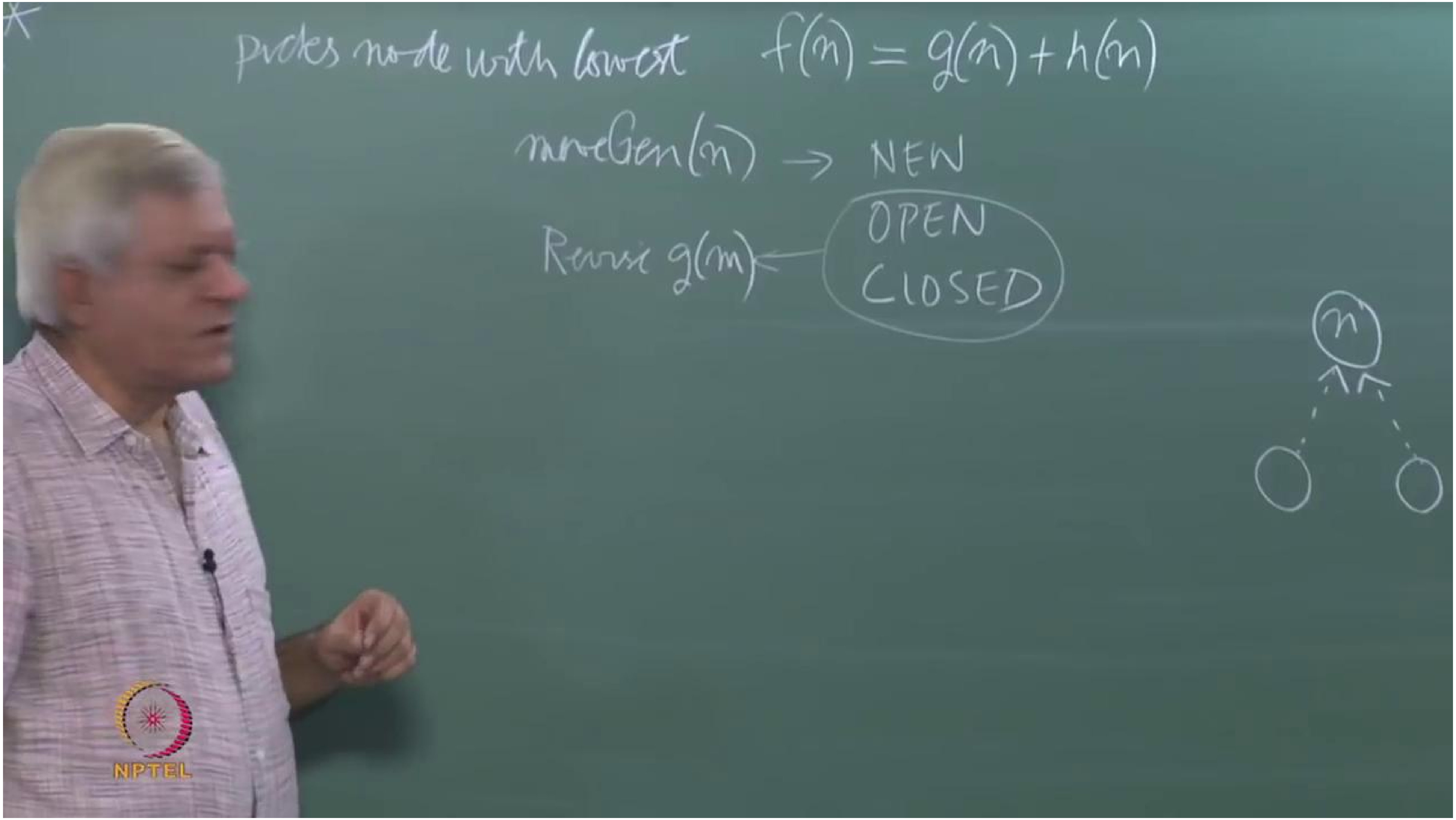
moveGen(n) → NEW

OPEN

CLOSED

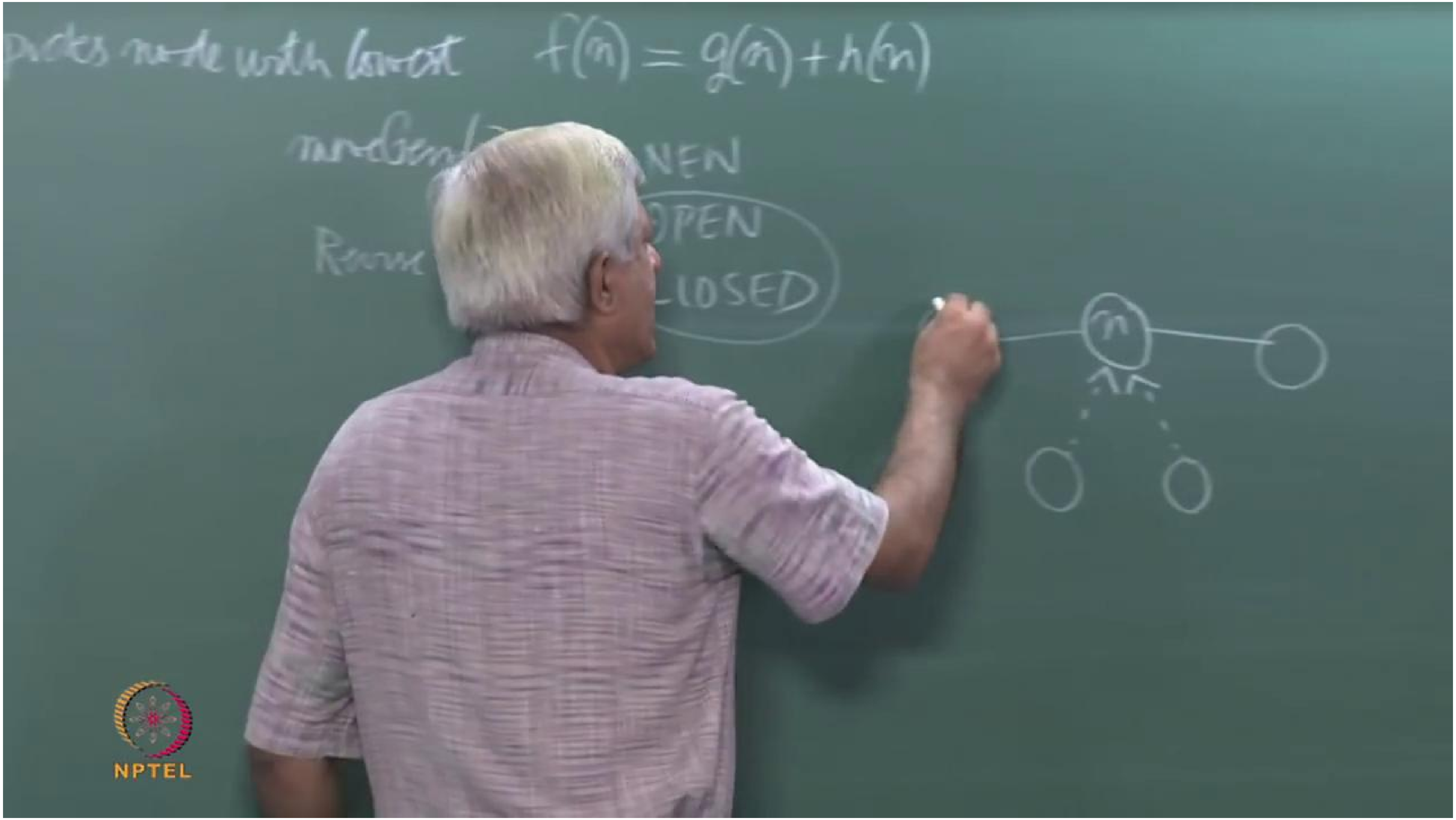

n

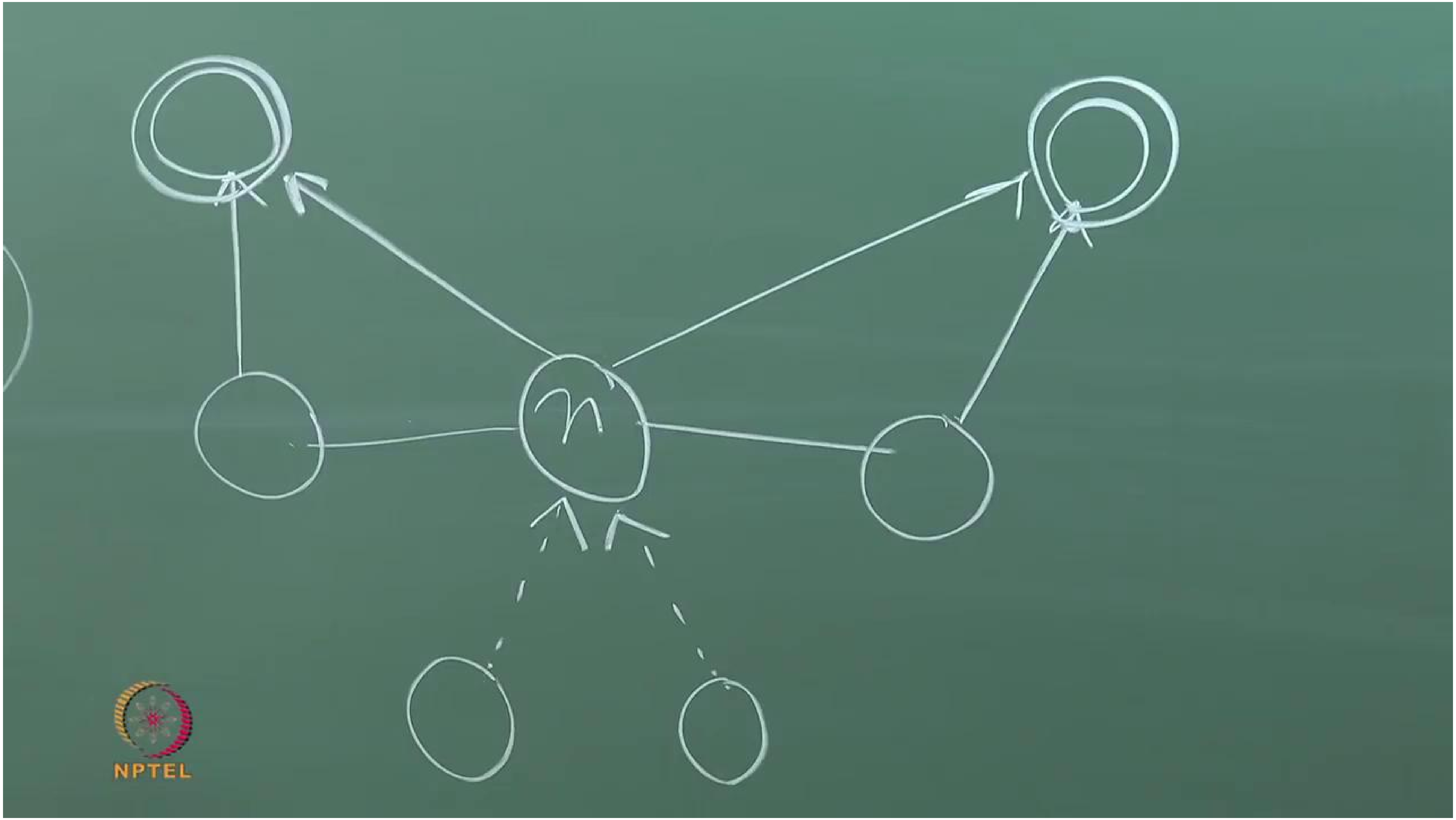

picks node with lowest $f(n) = g(n) + h(n)$
moveGen(n) → NEW
OPEN
CLOSED
n
NPTEL

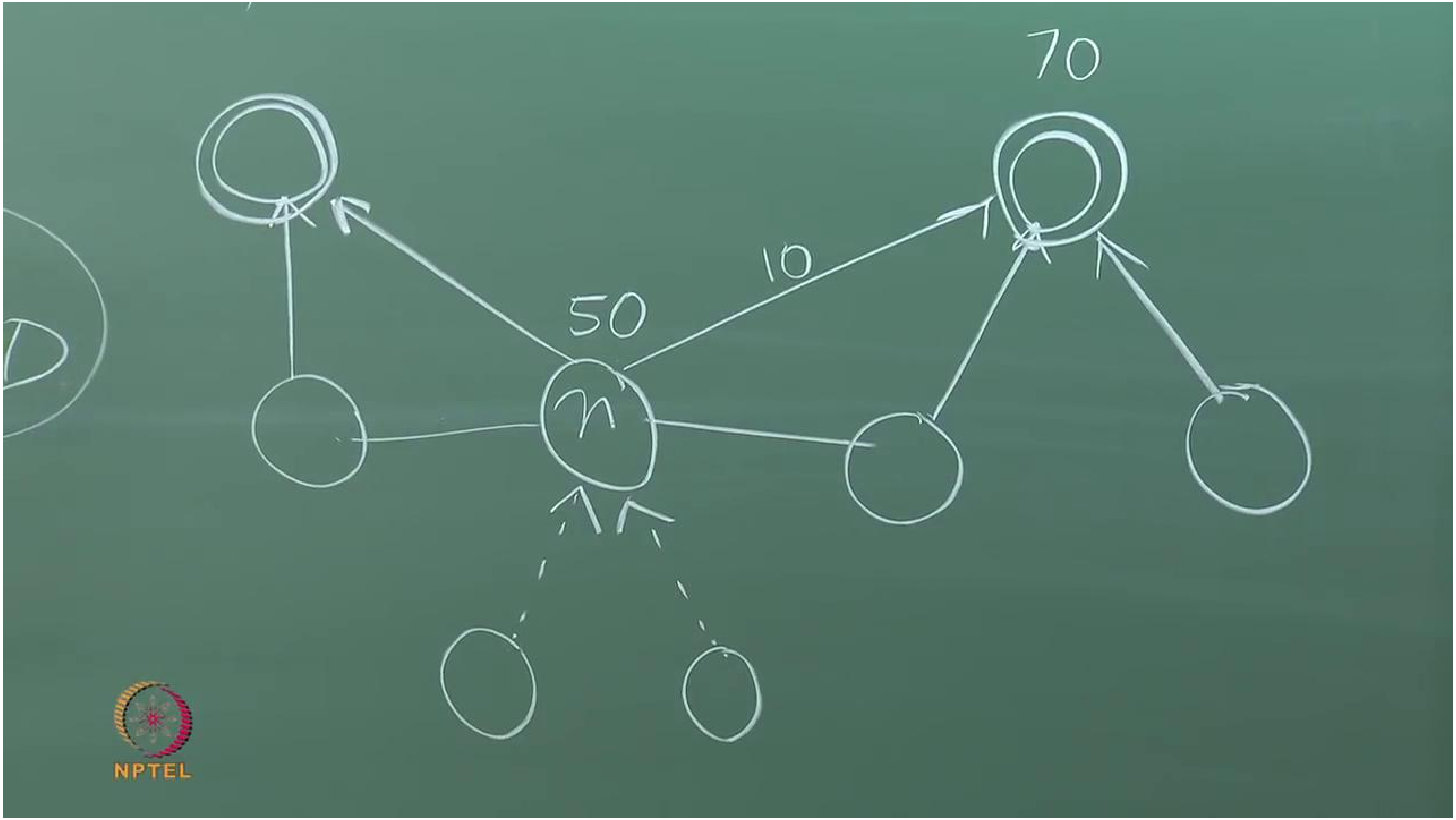

picks node with lowest $f(n) = g(n) + h(n)$
moveGen(n) → NEW
Revise g(m) ← OPEN CLOSED
n
NPTEL

moveGen(n) $\rightarrow$ NEW

Revise g(m) $\leftarrow$ OPEN CLOSED


picks node with lowest
f(n) = g(n) + h(n)
OPEN
n


n

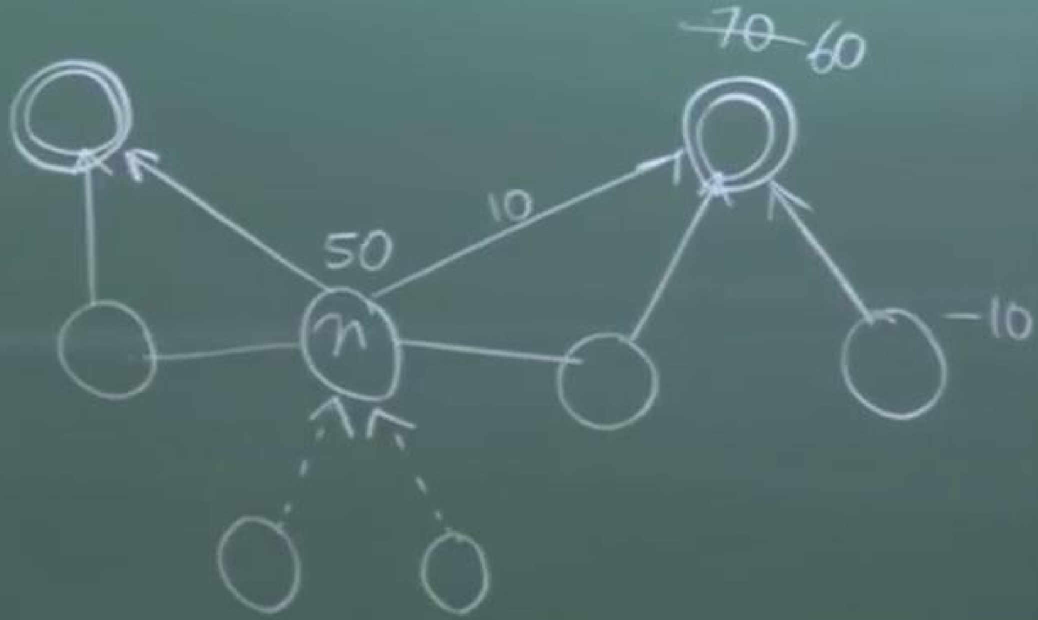

70
10
50
n

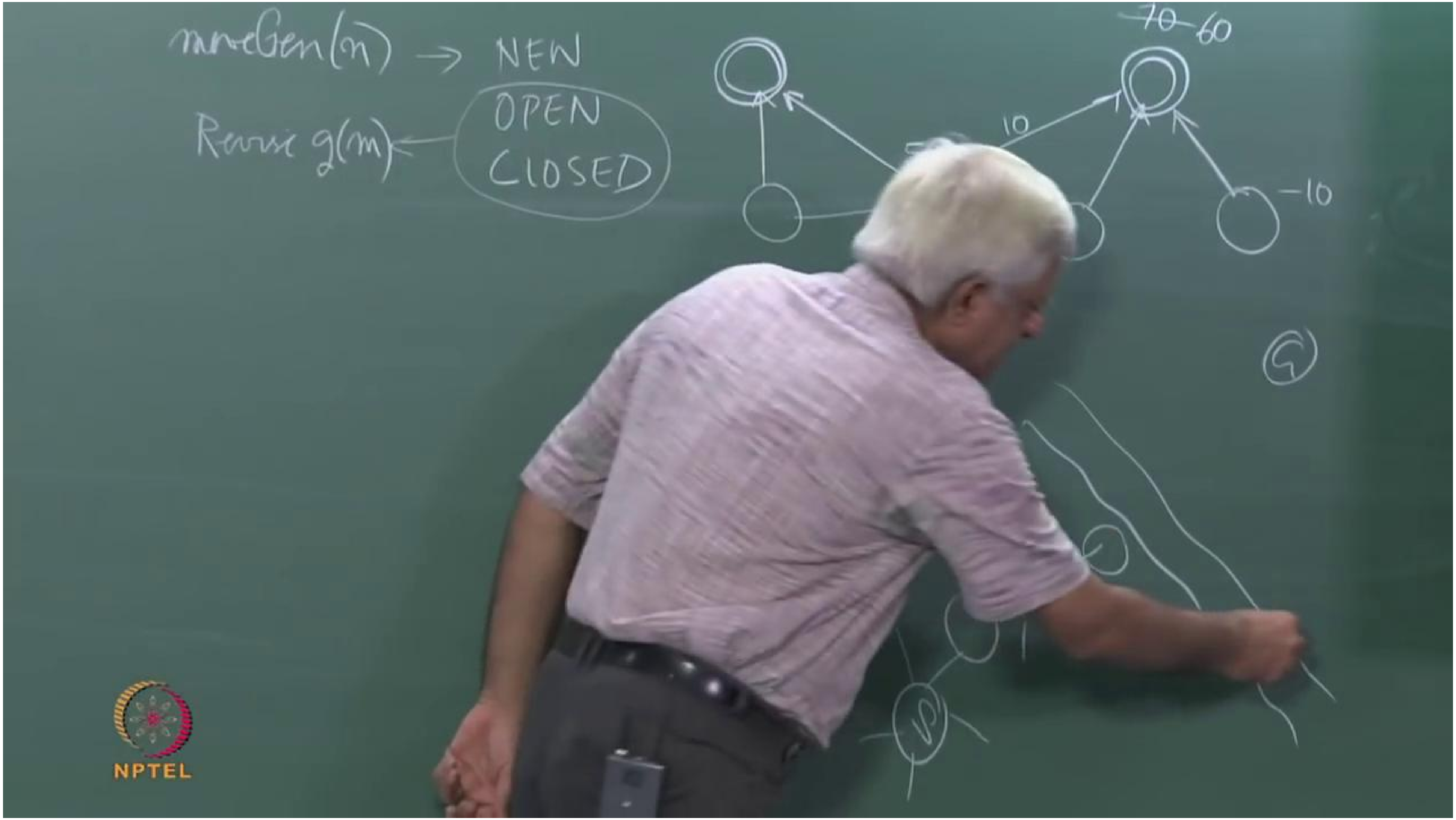
with lowest $f(n) = g(n) + h(n)$

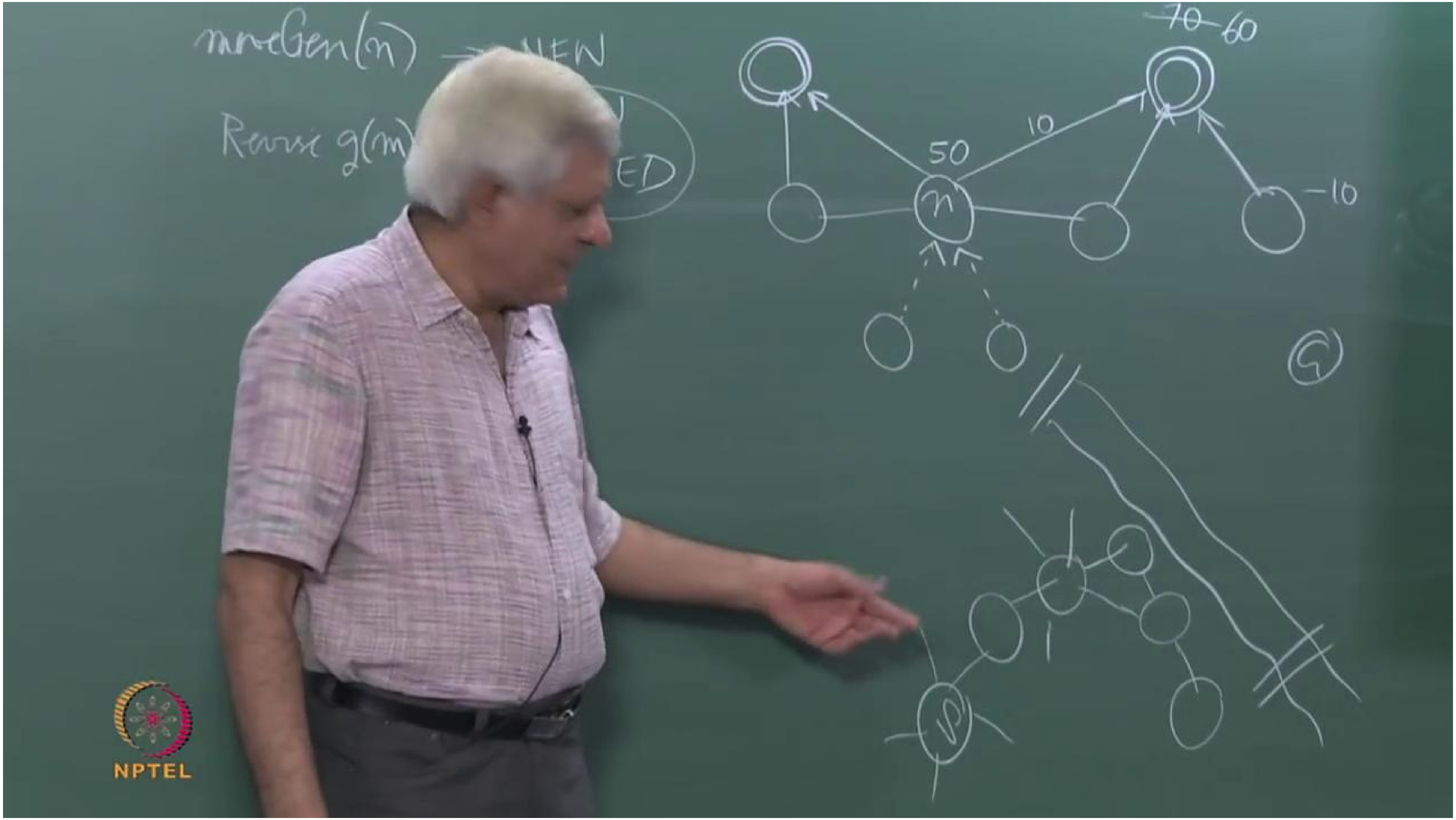
$(n) \rightarrow$ NEW

OPEN

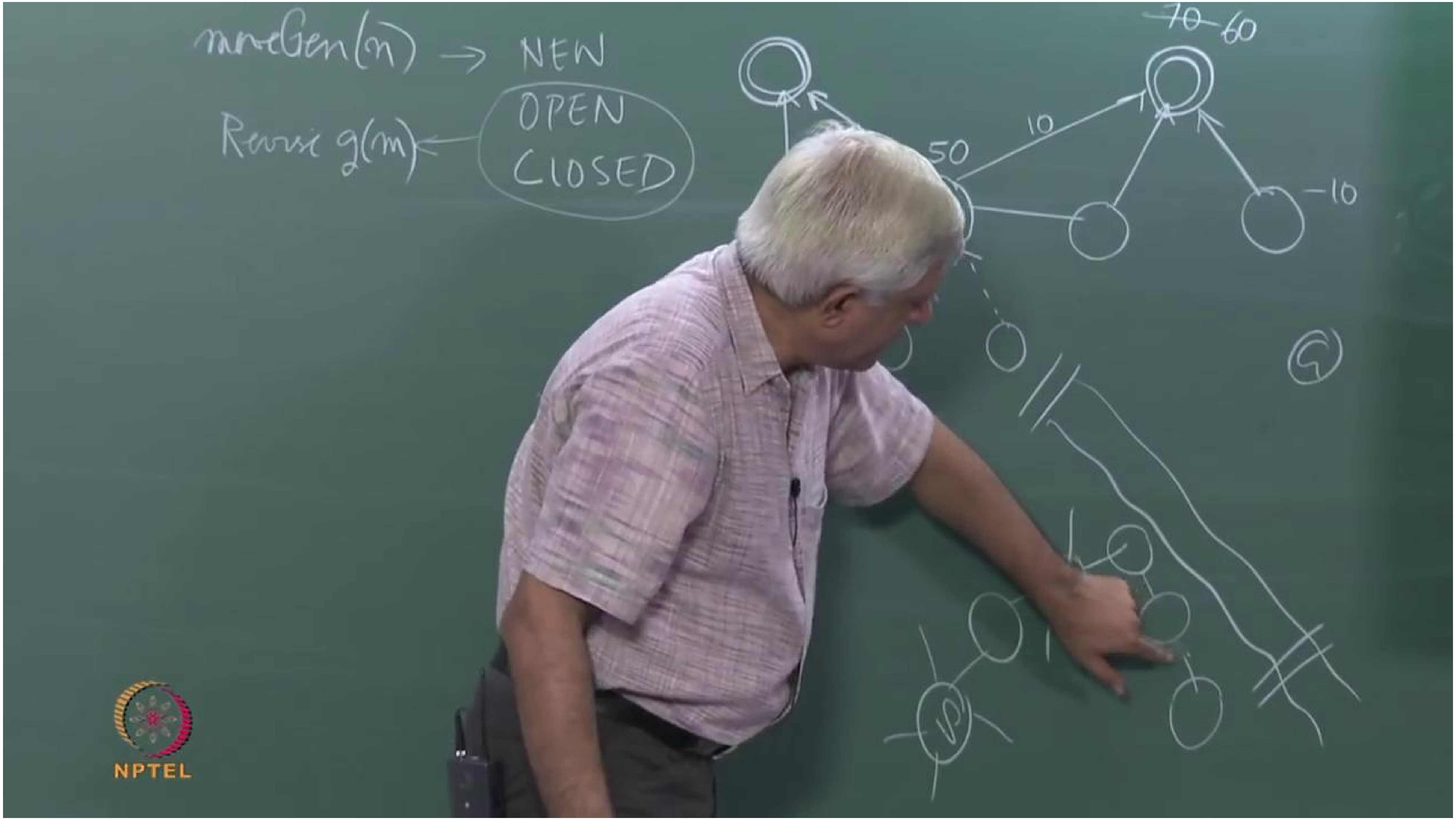
CLOSED

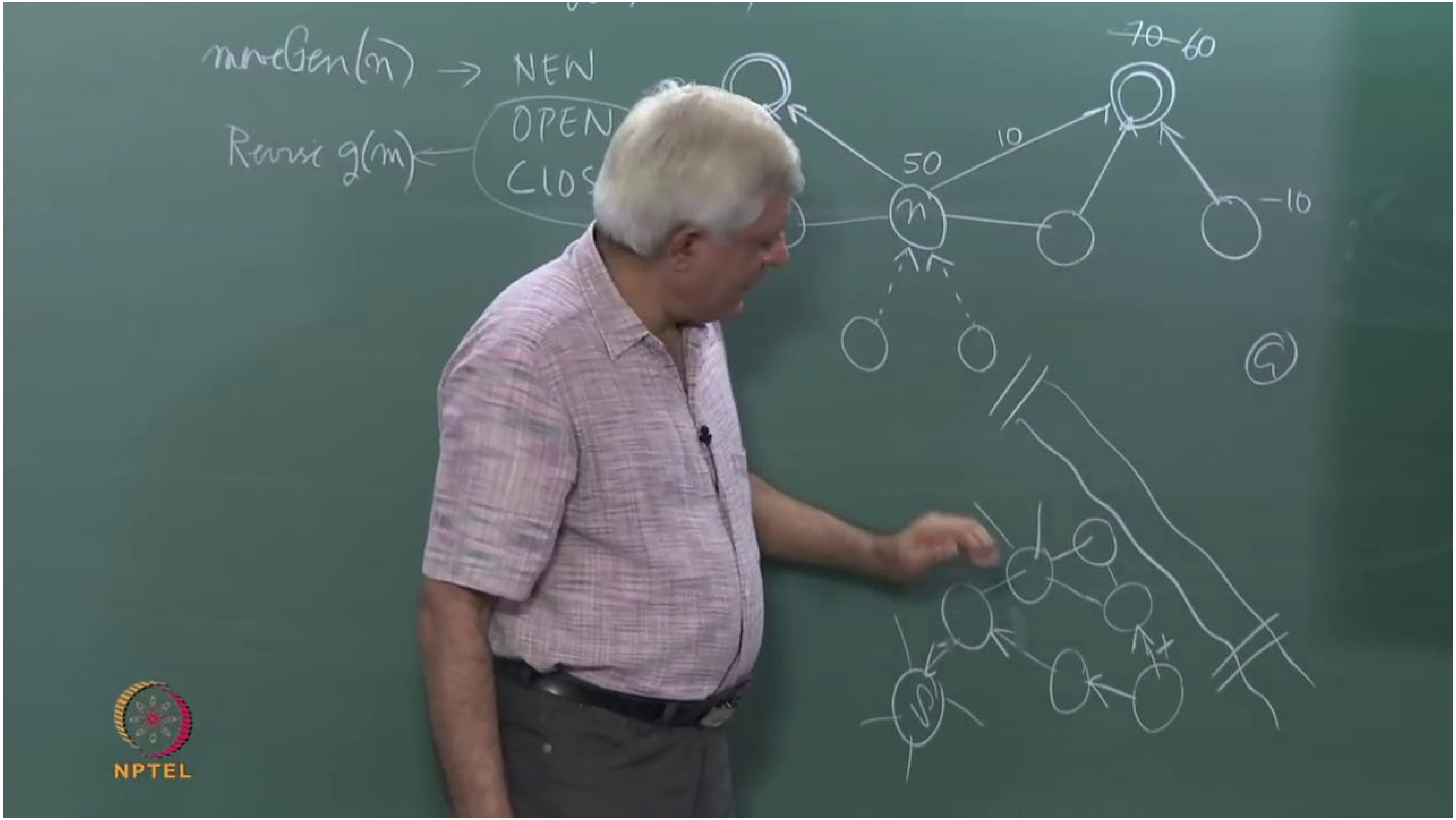


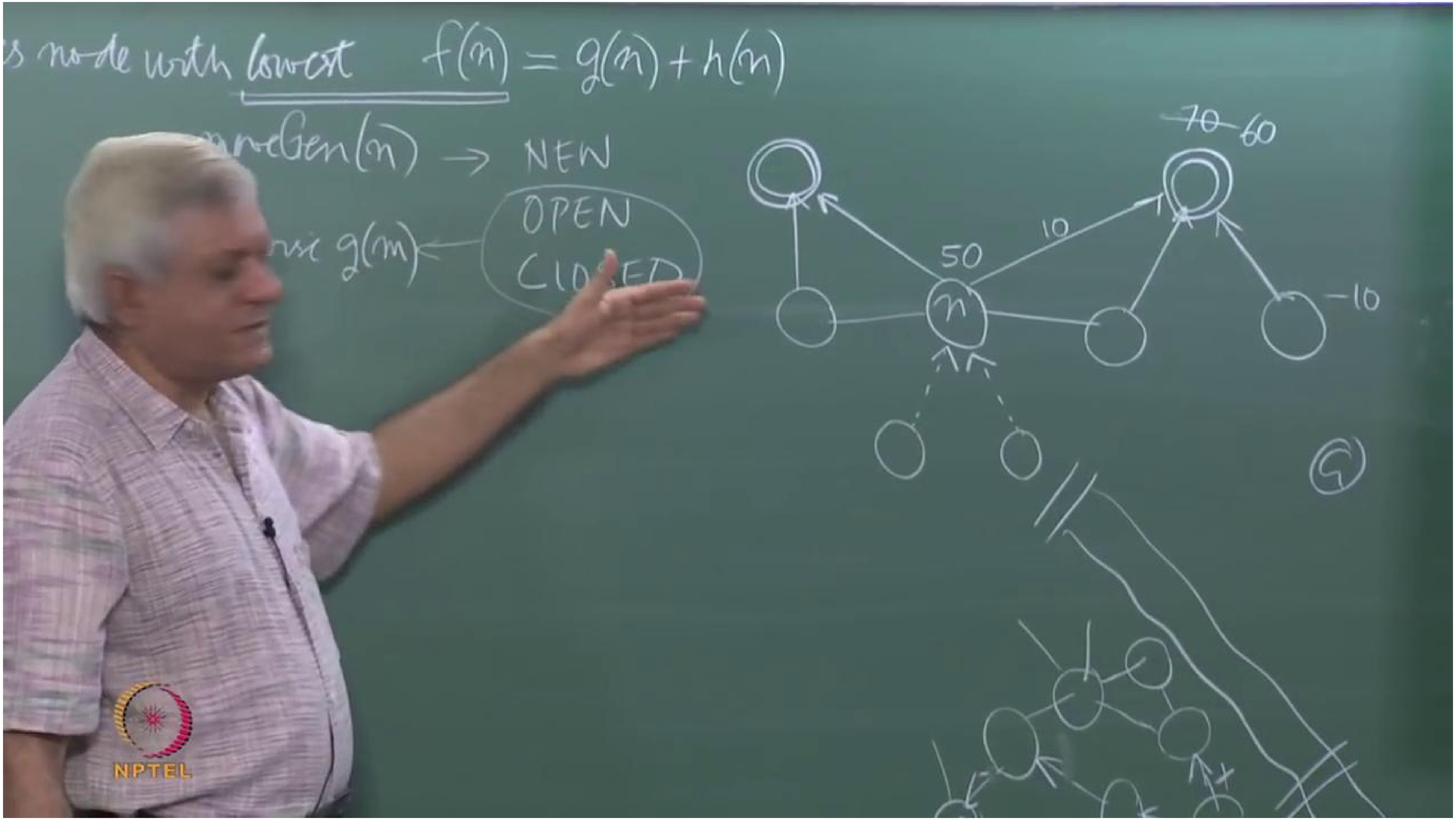

moveGen(n) → NEW
OPEN
CLOSED
Revise g(m)
70 60
10
-10
G
NPTEL



moveGen(n) → NEW
Revise g(m)
ED
~~70~~ 60
10
50
n
−10
G
p
NPTEL


moveGen(n) → NEW
OPEN
CLOSED
Revise g(m)
50
10
70
60
-10
G
NPTEL


moveGen(n) → NEW
OPEN
CLOS
Revise g(m)
~~70~~ 60
10
50
n
-10
G
S
NPTEL

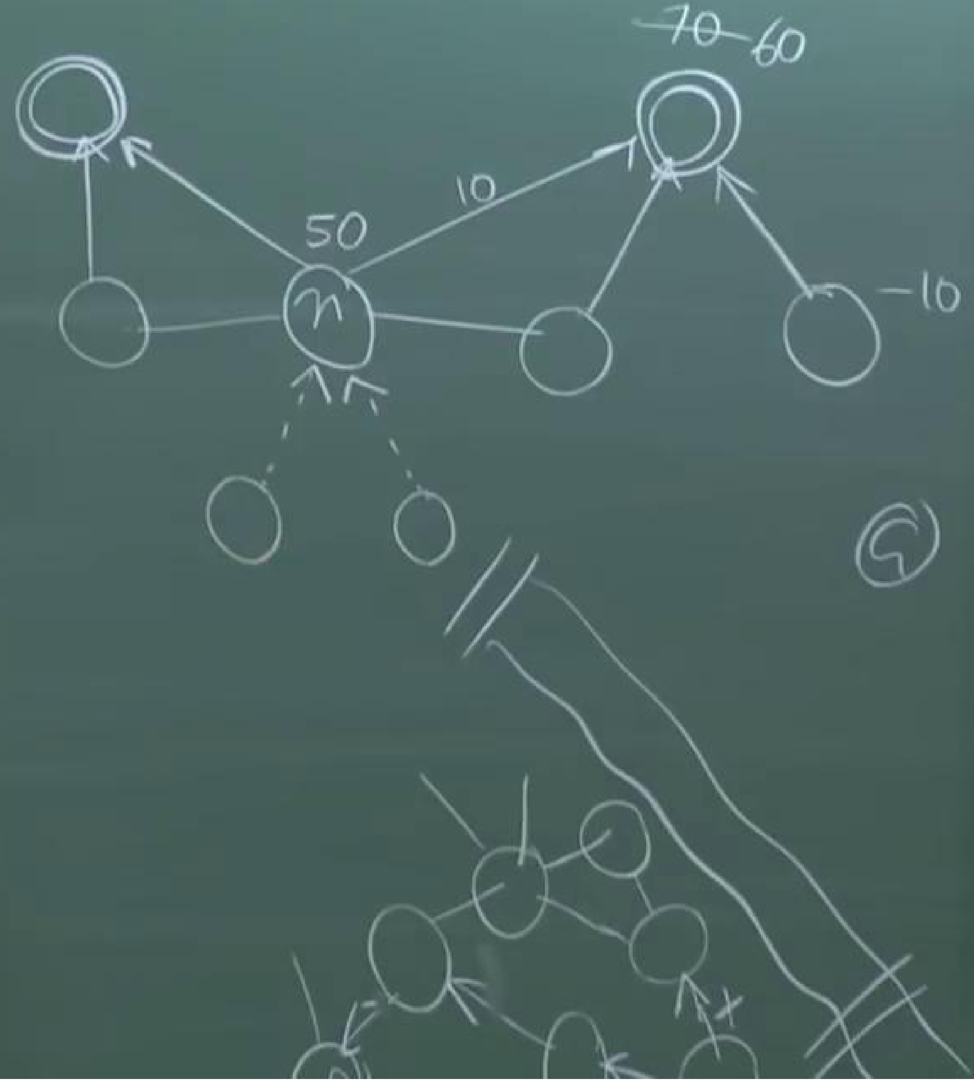


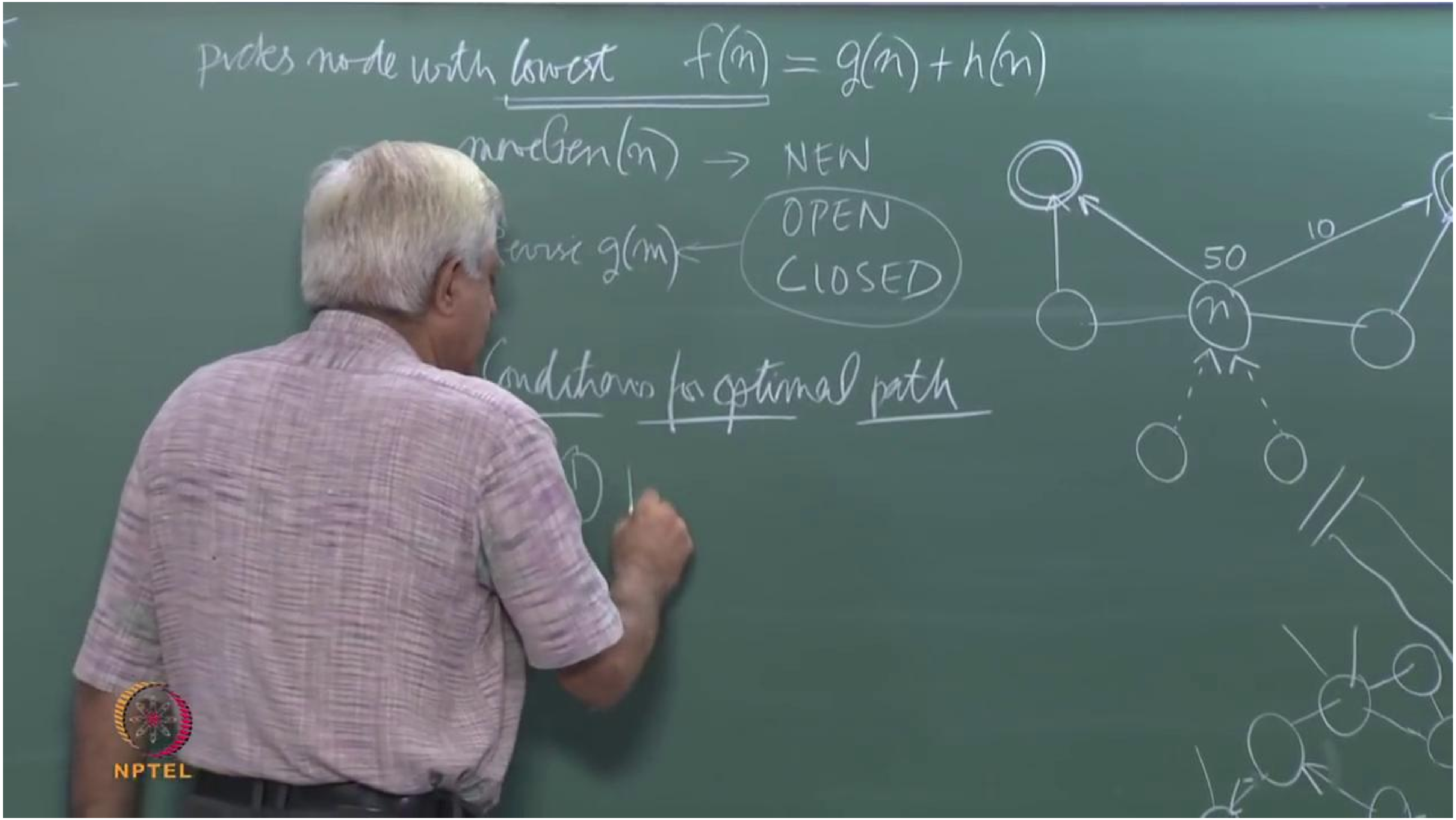
s node with lowest f(n) = g(n) + h(n)
moveGen(n) → NEW
OPEN
CLOSED
g(m)
70 60
10
50
n
-10
G

es node with lowest $f(n) = g(n) + h(n)$

moveGen(n) → NEW

Re... OPEN CLOSED





picks node with lowest f(n) = g(n) + h(n)
moveGen(n) → NEW
OPEN
CLOSED
revise g(m)
Conditions for optimal path
50
10
n
NPTEL

Conditions for optimal path

1. Finite branching factor

NPTEL

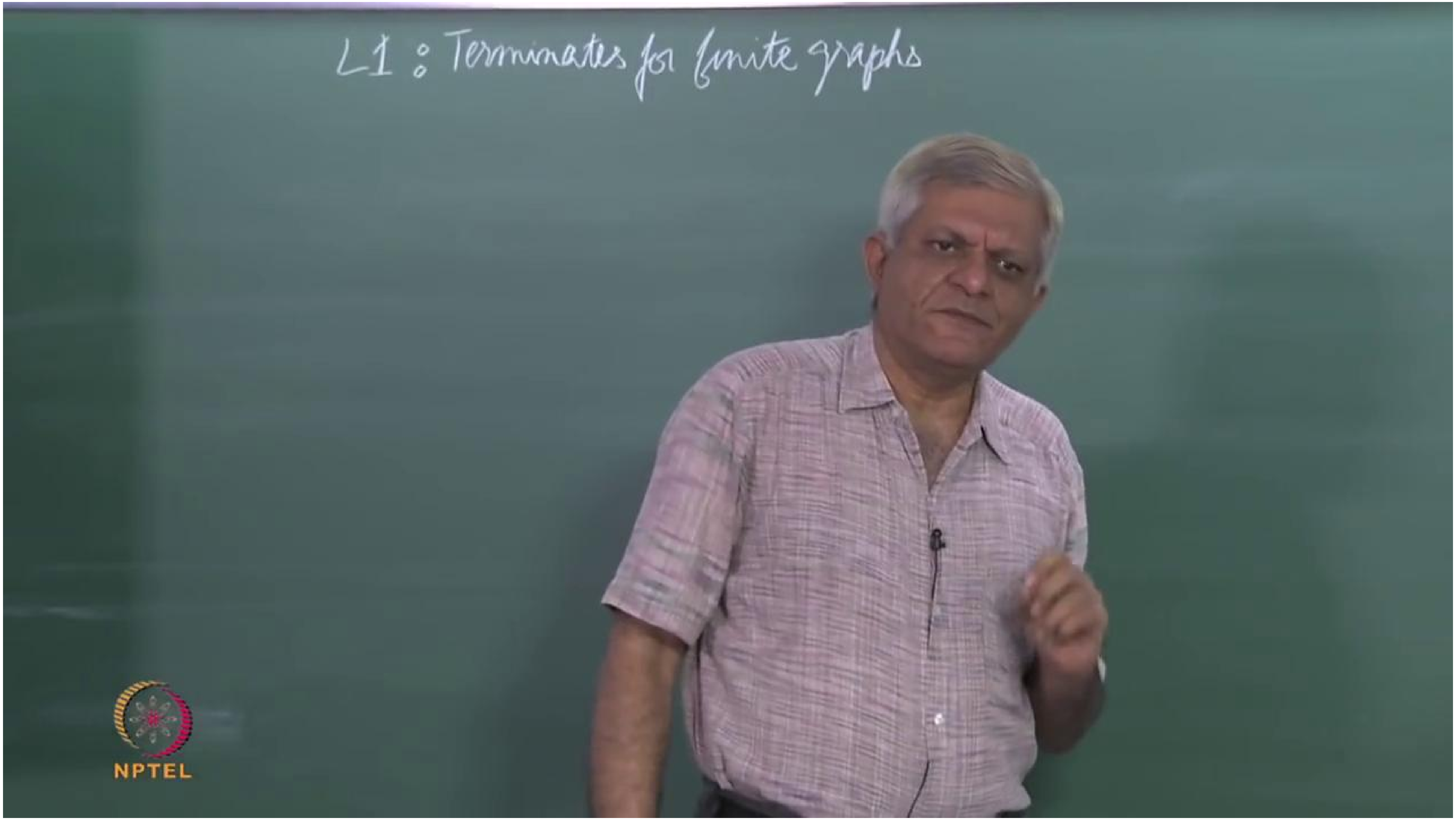
Conditions for optimal path

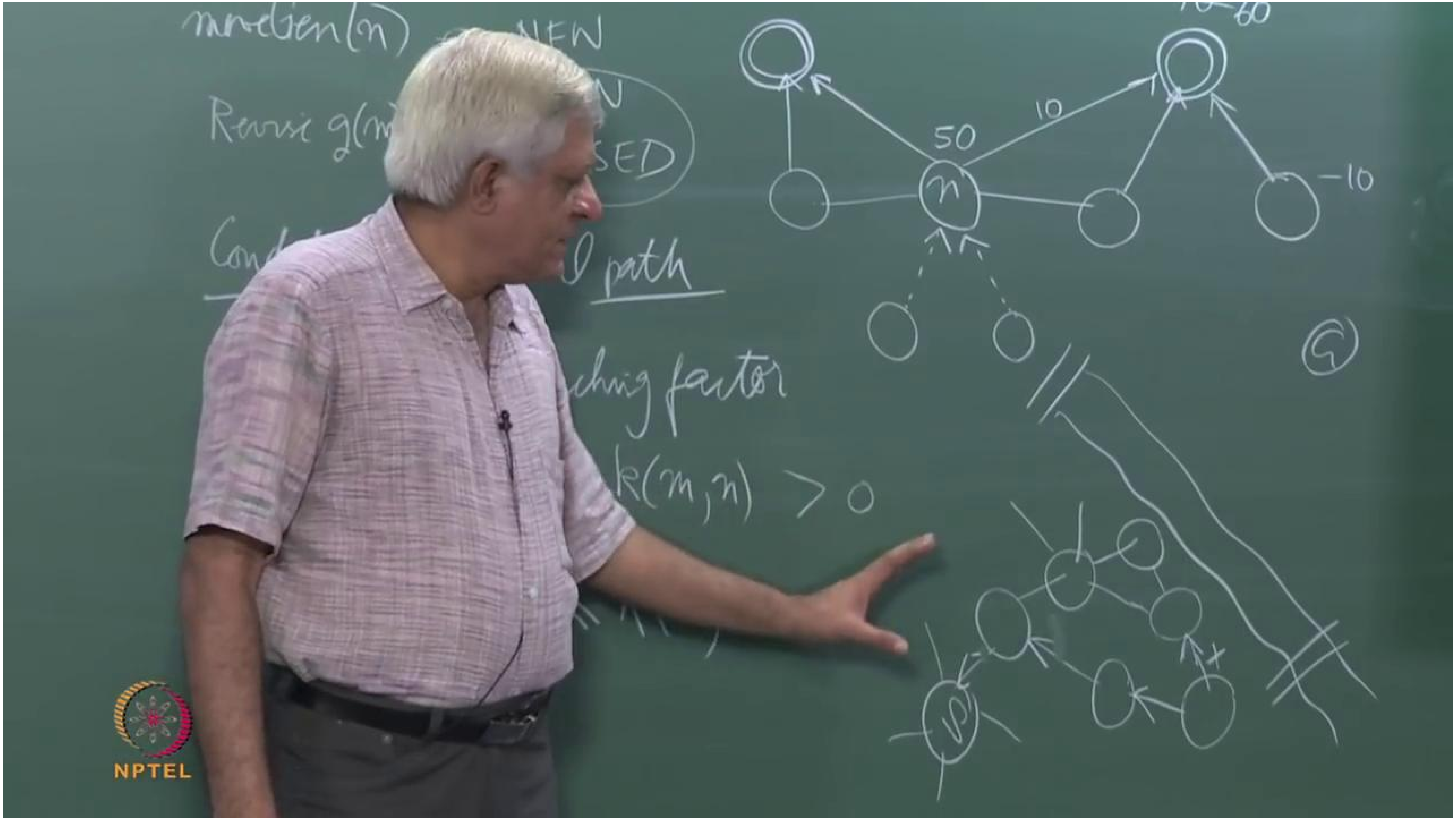
(1) Finite branching factor

(2) for each edge $k(m, n)$

Conditions for optimal path

1. Finite branching factor
2. for each edge $k(m, n) > 0$
3. $h(n) \leq h^*(n)$

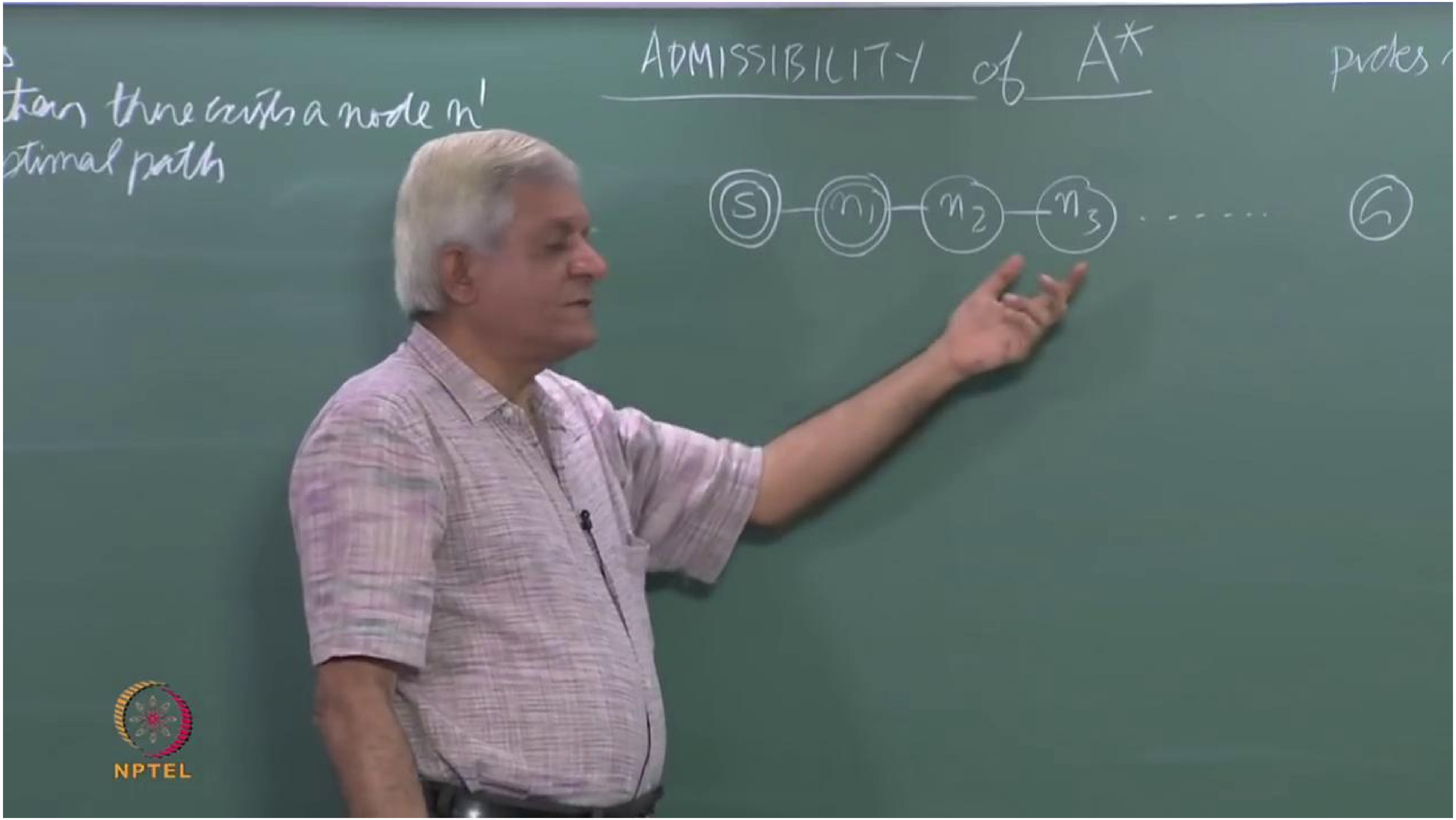

L1 : Terminates for finite graphs
NPTEL


NEW
50
10
−10
path
k(m,n) > 0
NPTEL

L1 : Terminates for finite graphs

L2 : At all times before termination there exists a node n' on OPEN which is on an optimal path

$$S - n_1 - n_2 - n_3 \cdots\cdots G$$


ADMISSIBILITY of A*
there exists a node n'
path
S
n1
n2
n3
G
NPTEL

L1 : Terminates for finite graphs

L2 : At all times before termination there exists a node n' on OPEN which is on an optimal path.

Further $f(n') \leq f^*(S)$

ADMISSIBILITY of A

ates for finite graphs
times before termination there exists a node n'
EN which is on an optimal pa
then $f(n') \leqslant f^*(S)$

$S - n_1 - n_2 - n$

$$f(n') = g(n') + h(n')$$

...ates for finite graphs
... times before termination there exists a node n'
...EN wh... an optimal path.
...then ... $f^*(S)$

ADMISSIBILITY of A...

S — n_1 — n_2 — n...

$$f(n') = g(n') + h(n')$$
$$= g^*(n') + h(n')$$

$$f(n') = g(n') + h(n')$$

$$= g^*(n') + h(n')$$

$$\leq g^*(n') + h^*(n')$$

$$
\begin{aligned}
f(x') &= g(x') + h(x') \\
&= g^*(x') + h(x') \\
&\leq g^*(x') + h^*(x') \\
&\leq f^*(x')
\end{aligned}
$$

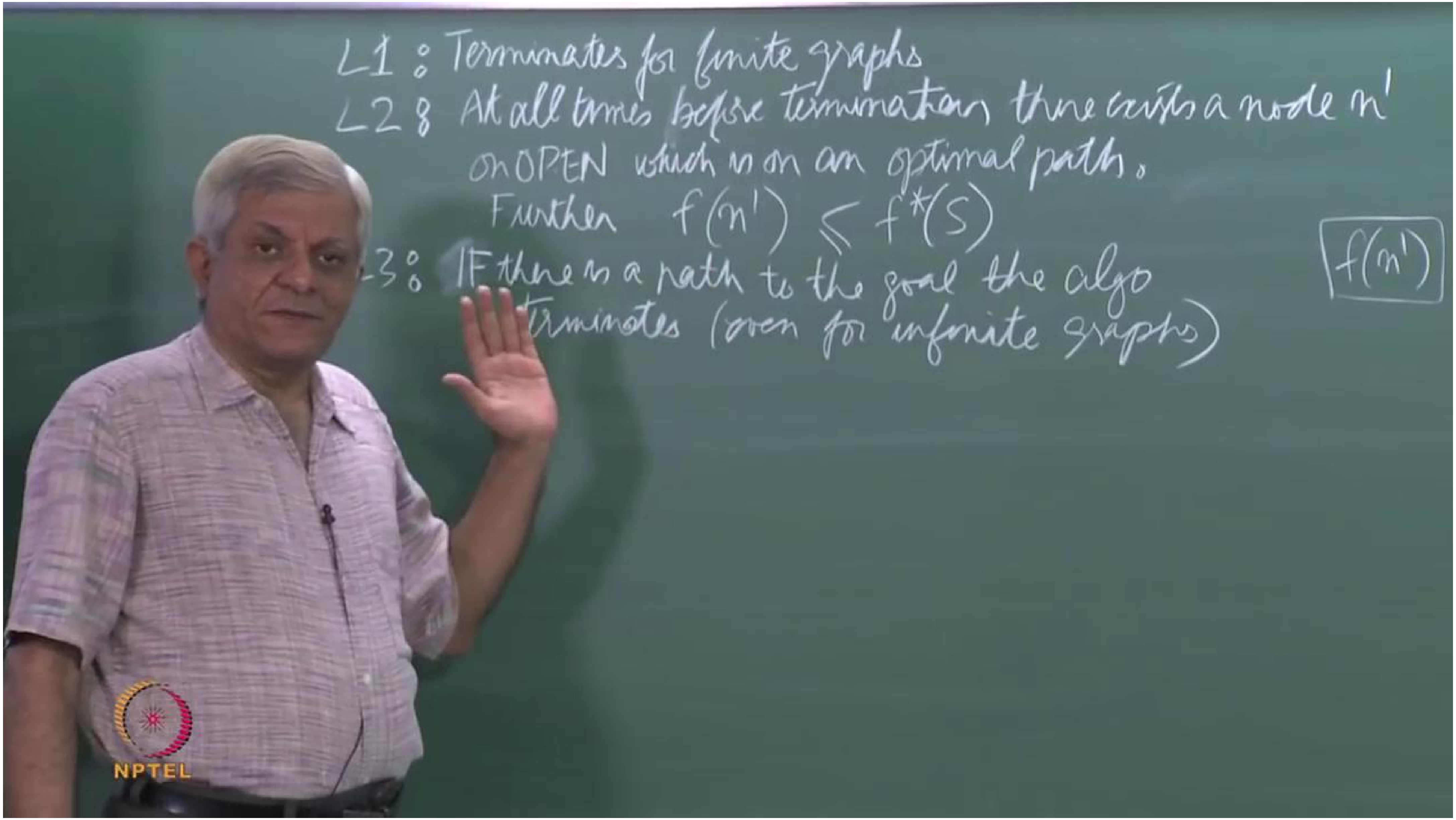
$$
\begin{aligned}
f(n') &= g(n') + h(n') \\
&= g^*(n') + h(n') \\
&\leq g^*(n') + h^*(n') \\
&\leq f^*(n') = f^*(S)
\end{aligned}
$$

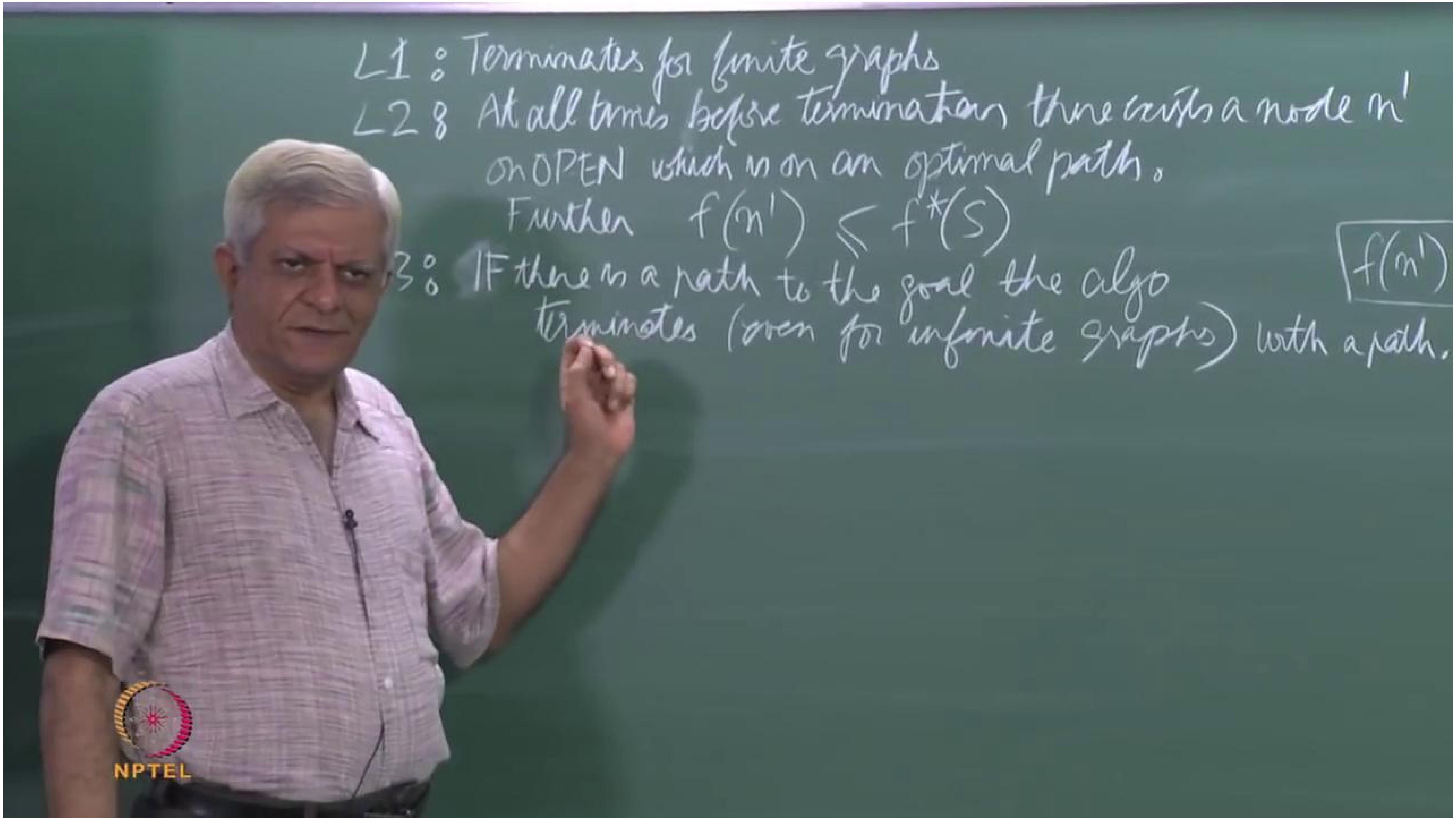
L1 : Terminates for finite graphs

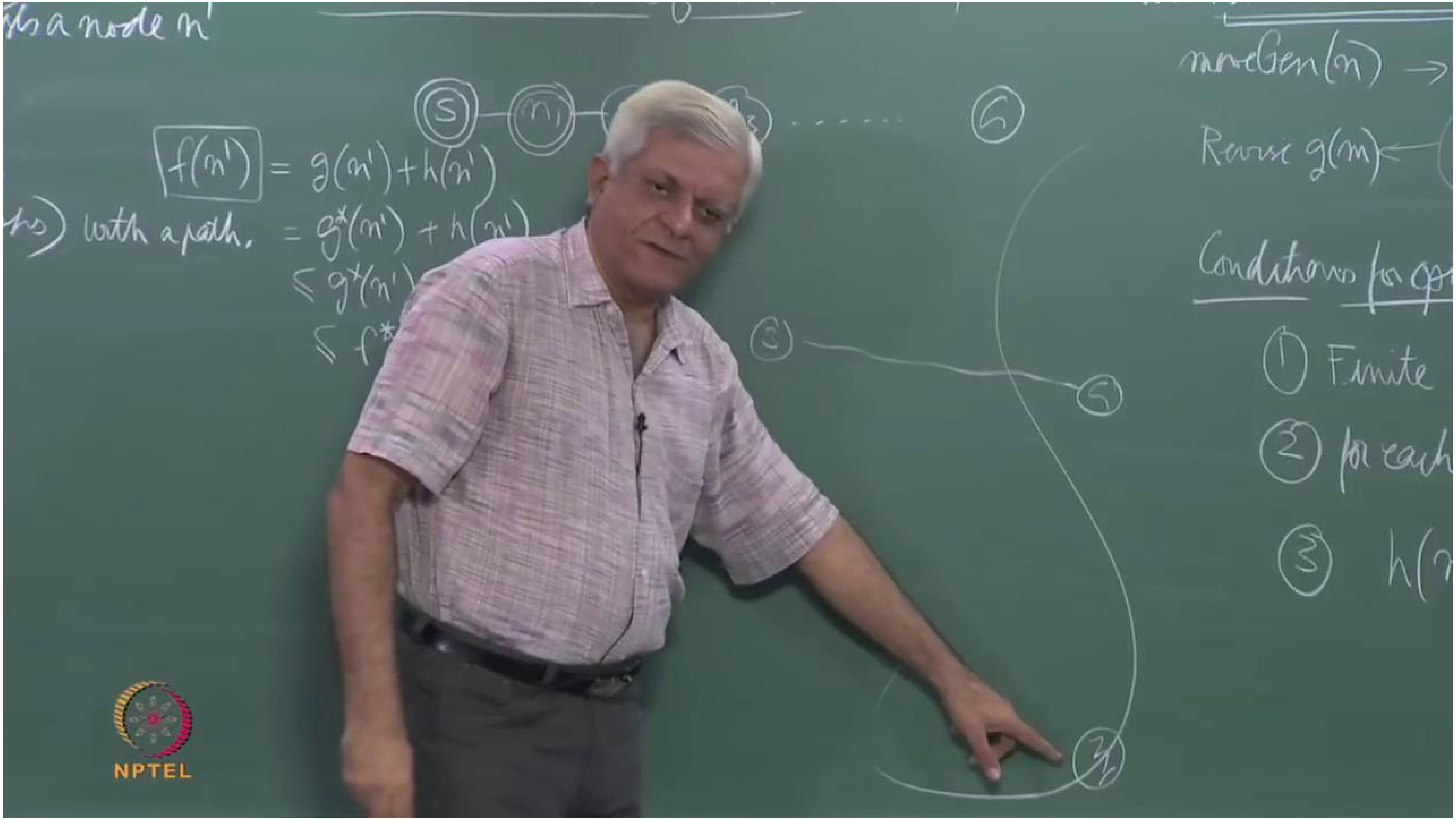
L2 : At all times before termination there exists a node n' on OPEN which is on an optimal path.

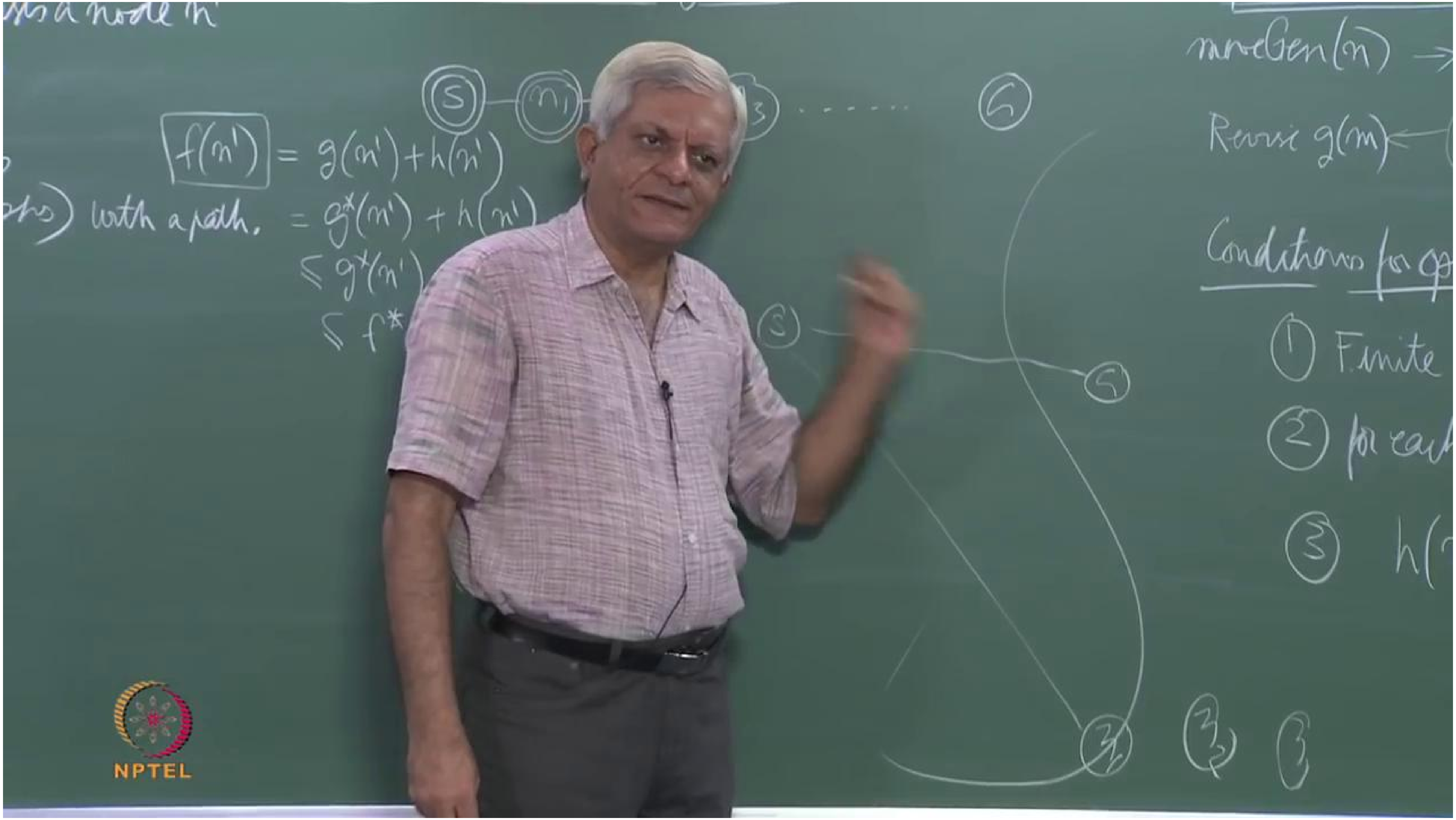
Further $f(n') \leq f^*(S)$

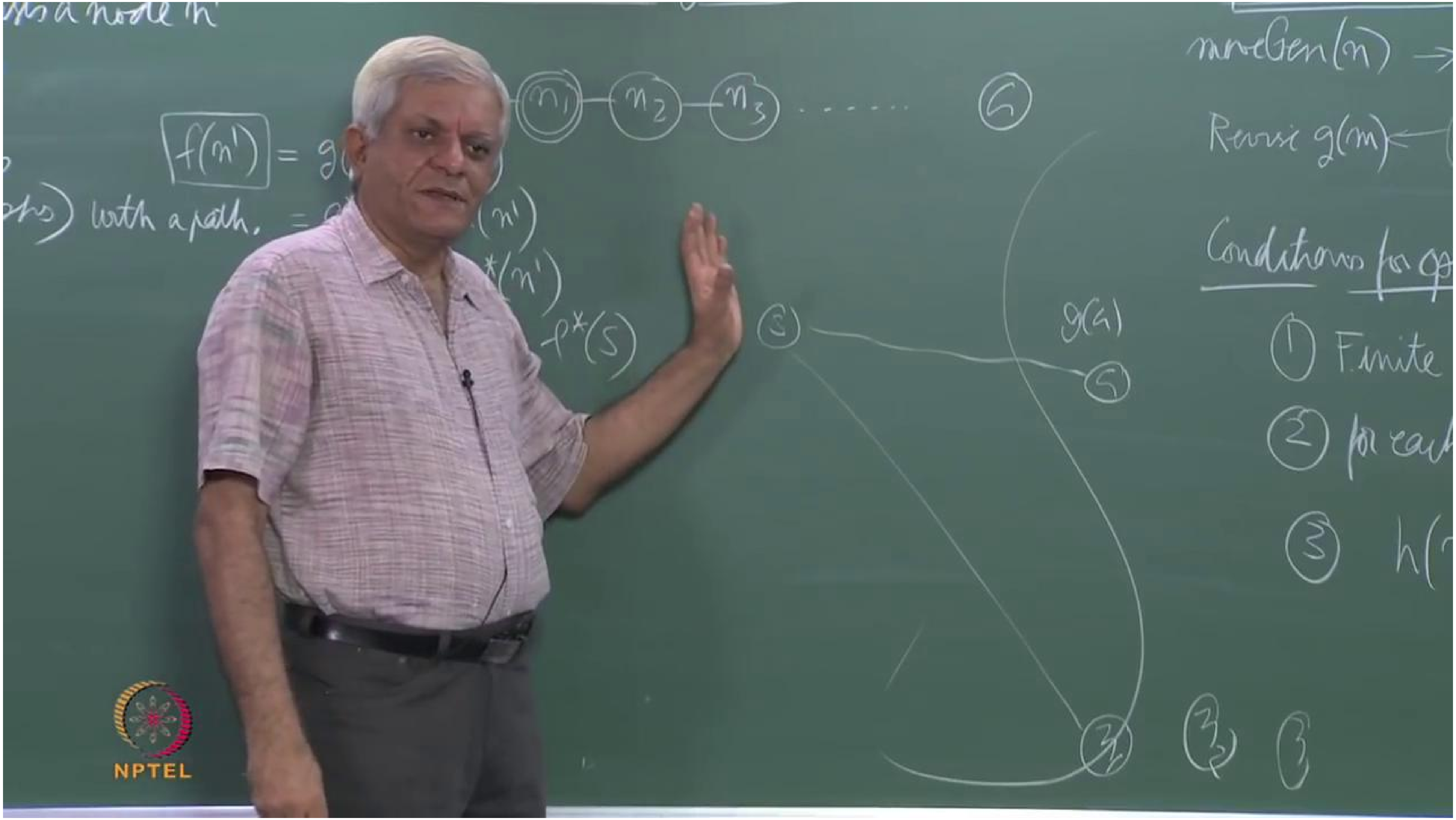
$\boxed{f(n')}$


L1 : Terminates for finite graphs
L2 : At all times before termination there exists a node n'
on OPEN which is on an optimal path.
Further $f(n') \leq f^*(S)$
L3 : IF there is a path to the goal the algo
terminates (even for infinite graphs)
$\boxed{f(n')}$
NPTEL


L1 : Terminates for finite graphs
L2 : At all times before termination there exists a node n' on OPEN which is on an optimal path.
Further $f(n') \leq f^*(S)$
L3 : IF there is a path to the goal the algo terminates (even for infinite graphs) with a path.
$f(n')$


a node n'
f(n') = g(n') + h(n')
with a path. = g*(n') + h(n')
≤ g*(n')
moveGen(n) →
Reverse g(m)
Conditions for
1 Finite
2 for each
3 h(n
NPTEL


moveGen(n)
Revise g(m)
with a path.
Finite
NPTEL



moveGen(n) →
Revise g(m)
Conditions for op
1 Finite
2 for each
3 h(
f(n') = g
with a path.
g(G)
NPTEL

L1: Terminates for finite graphs

L2: At all times before termination there exists a node n' on OPEN which is on an optimal path.

Further $f(n') \leq f^*(S)$

$\boxed{f(n')}$

L3: IF there is a path to the goal the algo terminates (even for infinite graphs) with a path.

L4: A^* is admissible

NPTEL

L2: At all times before termination there exists a node n'
on OPEN which ... optimal path.
Further f ... C*(S)

L3: IF there is a pa... ...al the algo
terminates ... infinite graphs) with a path.

L4: A* is

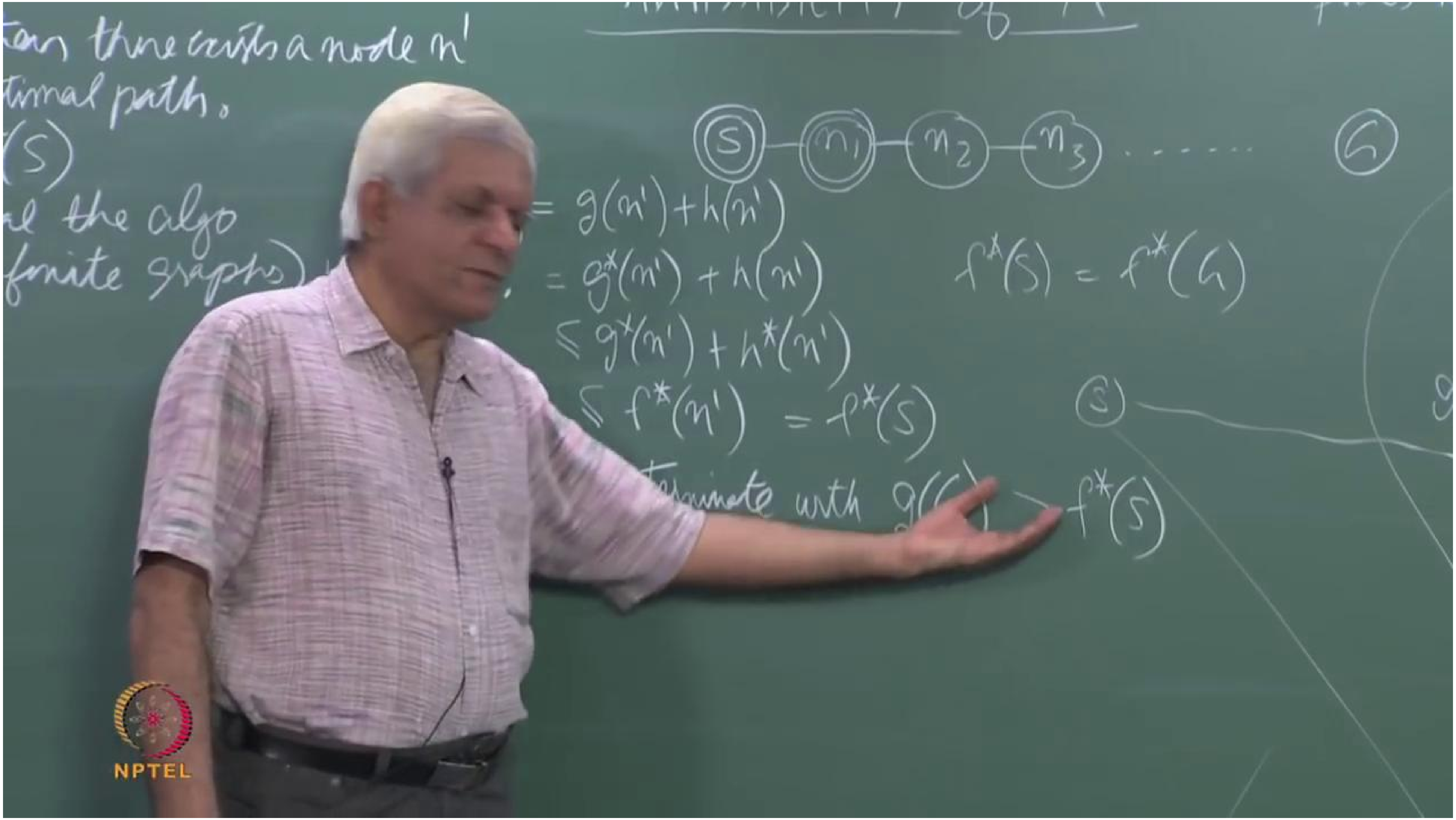
$\boxed{f(n')} = g(n'$
$= g^*(n$
$\leq g^*(n$
$\leq f$

Let A* t

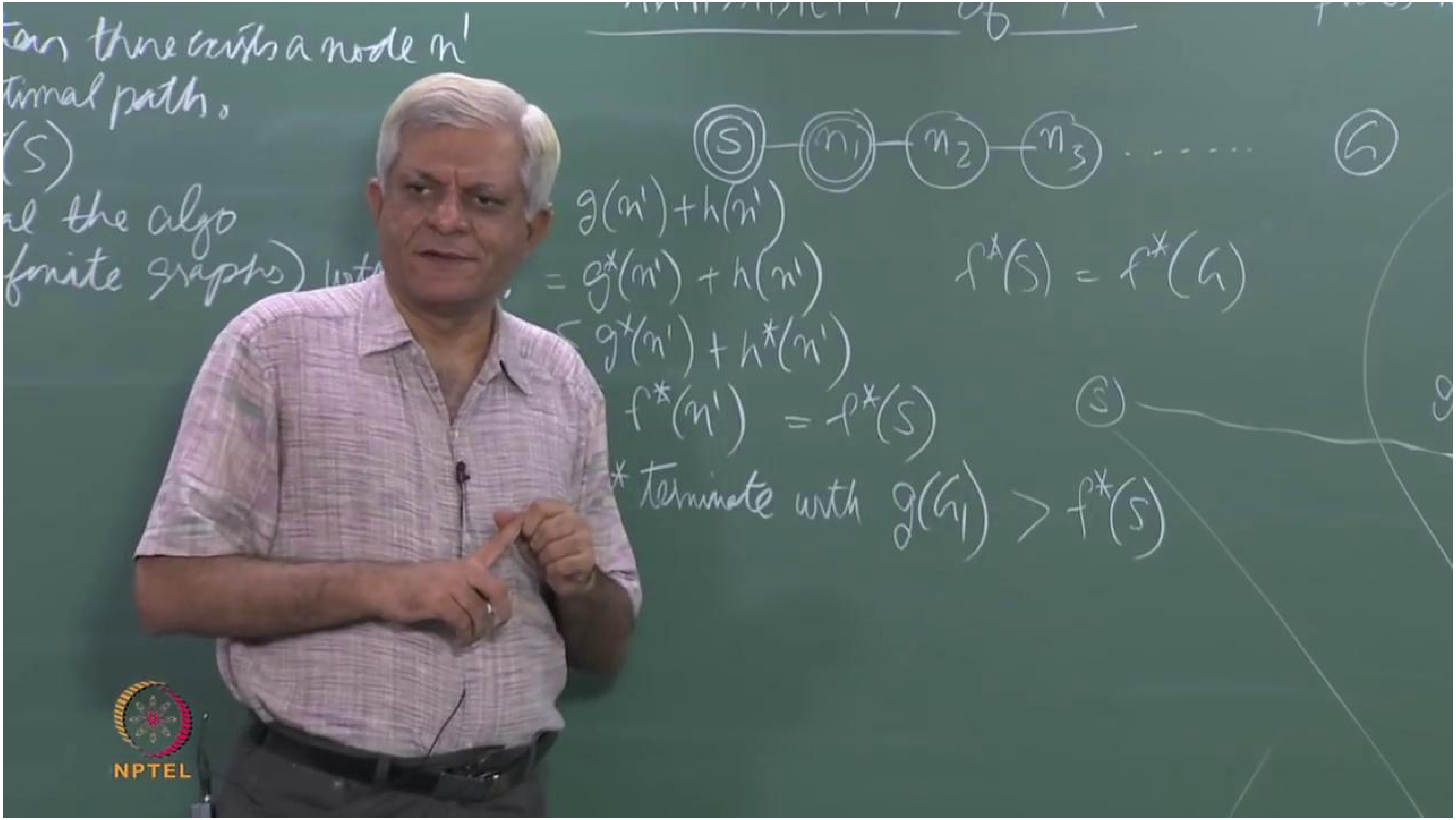
$$\leq g^*(n') + h^*(n')$$

$$\leq f^*(n') = f^*(S)$$

Let A^* terminate with $g(G_1) > f^*(S)$



there exists a node n'
timal path.
(S)
the algo
finite graphs)
S
n1
n2
n3
G
= g(n') + h(n')
= g*(n') + h(n')
≤ g*(n') + h*(n')
≤ f*(n') = f*(S)
f*(S) = f*(G)
S
with
f*(S)
NPTEL



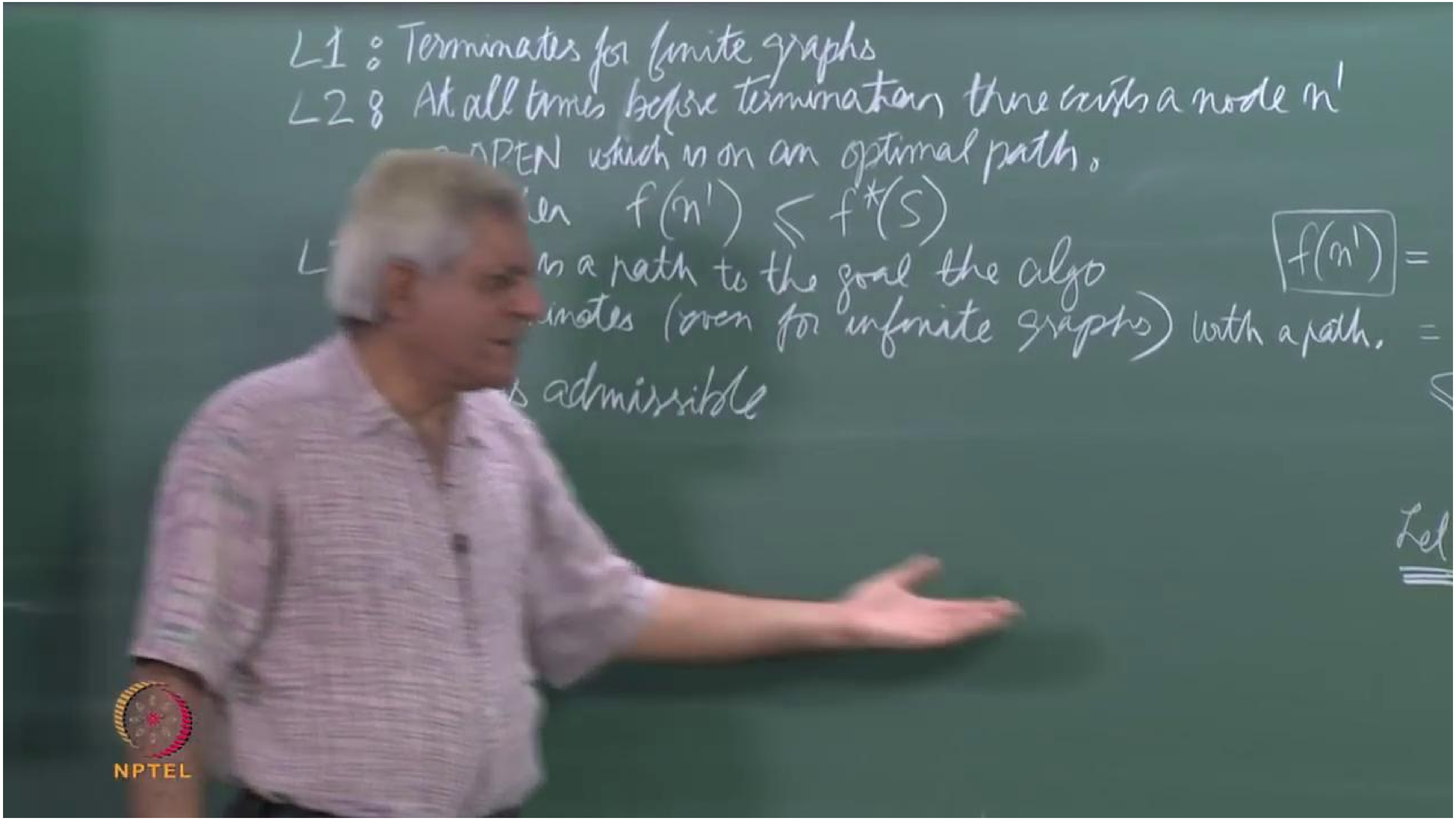
ean there exists a node n'
timal path.
(S)
l the algo
finite graphs)
S
n1
n2
n3
G
g(n') + h(n')
= g*(n') + h(n')
g*(n') + h*(n')
f*(n') = f*(S)
f*(S) = f*(G)
S
* terminate with g(G) > f*(S)
NPTEL

$\leq f^*(n') = f^*(S)$

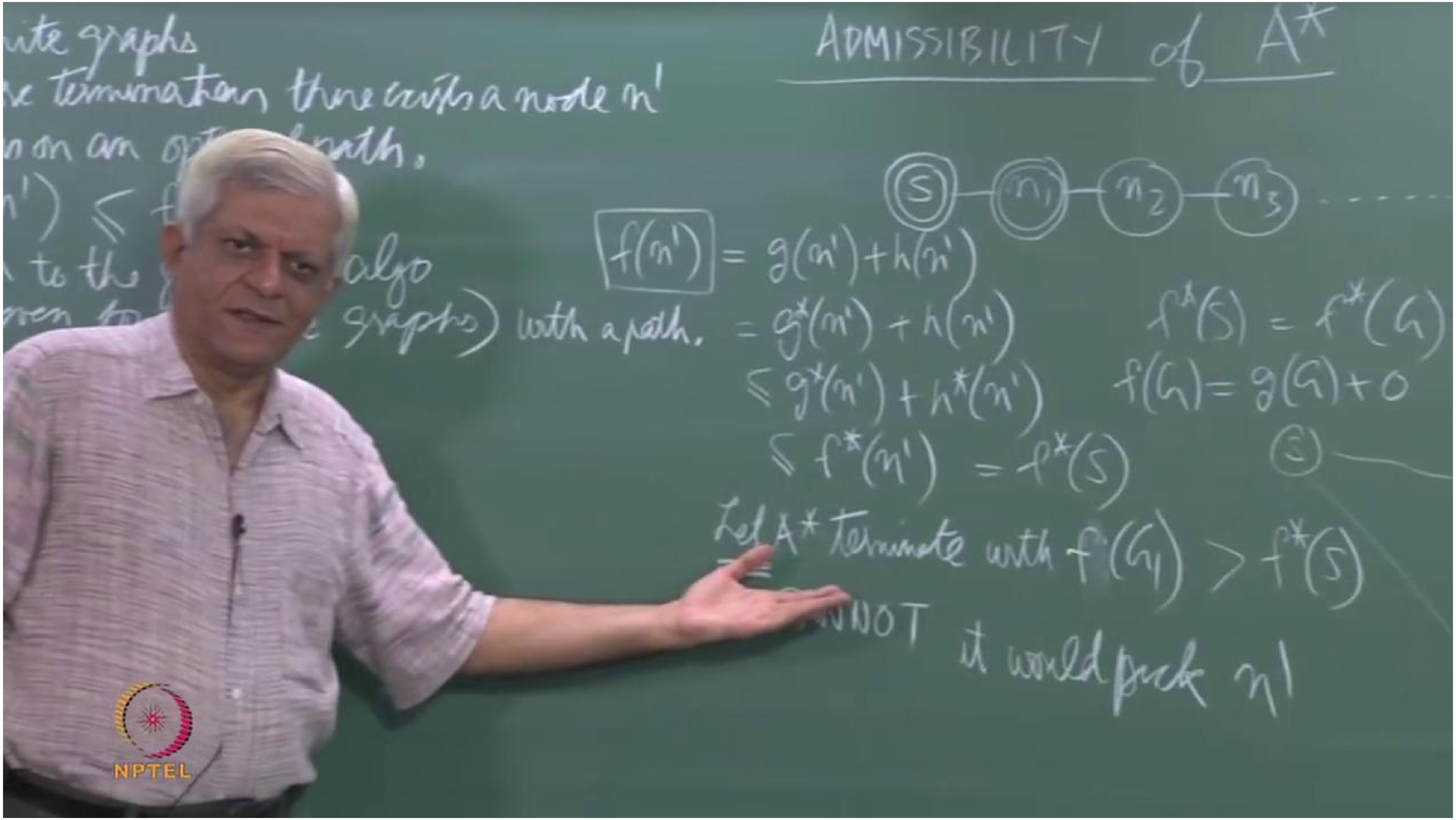
S

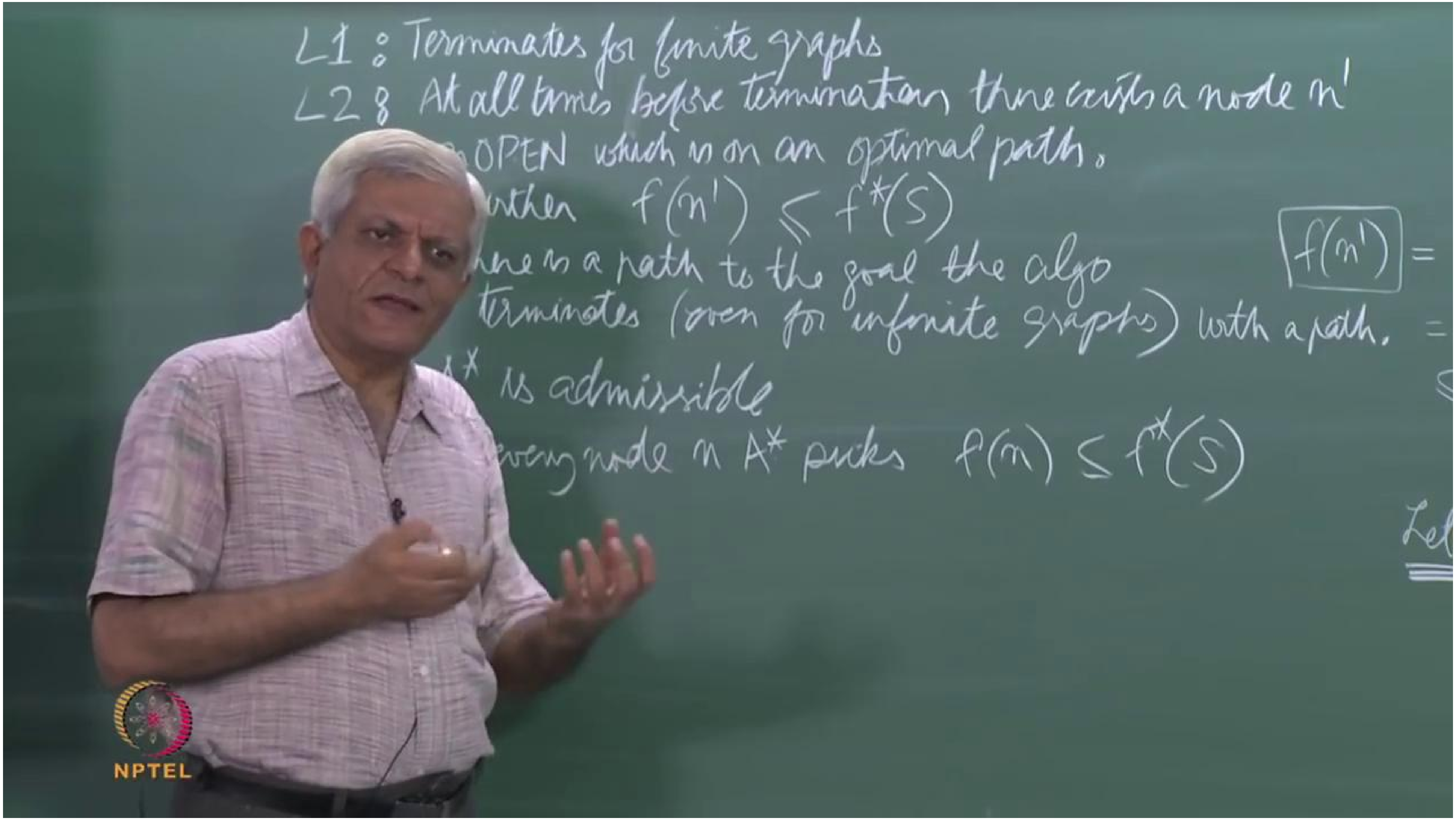
Let A^* terminate with $g(G_1) > f^*(S)$

CANNOT it would pick n'



L1 : Terminates for finite graphs
L2 : At all times before termination there exists a node n'
OPEN which is on an optimal path.
en f(n') ≤ f*(S)
is a path to the goal the algo
minates (even for infinite graphs) with a path.
admissible
f(n') =
=
NPTEL


ADMISSIBILITY of A^*
$\boxed{f(n')} = g(n') + h(n')$
$= g^*(n') + h(n')$
$\leq g^*(n') + h^*(n')$
$\leq f^*(n') = f^*(S)$
$f^A(S) = f^*(G)$
$f(G) = g(G) + 0$
Let A^* terminate with $f(G_1) > f^*(S)$
it would pick n'
with a path.
algo



L1 : Terminates for finite graphs
L2 : At all times before termination there exists a node n'
OPEN which is on an optimal path.
f(n') ≤ f*(S)
is a path to the goal the algo
terminates (even for infinite graphs) with a path.
is admissible
node n A* picks f(n) ≤ f*(S)
f(n') =
NPTEL

L1: Terminates for finite graphs

L2: At all times before termination there exists a node n' on OPEN which is on an optimal path.

Further $f(n') \leq f^*(S)$

L3: If there is a path to the goal the algo terminates (even for infinite graphs) with a path.

L4: A^* is admissible

For every node (n) A^* picks $f(n) \leq f^*(S)$

$\boxed{f(n')} =$

$=$

$\leq$

$\leq$

Let

L1 : Terminates for finite graphs

L2 : At all times before termination there exists a node n'
on OPEN which is on an optimal path.
Further $f(n') \leq f^*(S)$

IF there is a path to the goal the algo
terminates (even for infinite graphs) with a path.

1 : A^* is admissible

5 : For every node n A^* picks $f(n) \leq f^*(S)$

:

:

$\boxed{f(n')}$ =

=

$\leq$

Let

L1 : Terminates for finite graphs

L2 : At all times before termination there exists a node n' on OPEN which is on an optimal path.

Further $f(n') \leq f^*(S)$

: IF there is a path to the goal the algo terminates (even for infinite graphs) with a path.

4 : A^* is admissible

: For every node n A^* picks $f(n) \leq f^*(S)$

h_2 is MORE INFORMED than h_1 , $h_2(n) > h_1(n)$

$\boxed{f(n')} = g$

$= g$

$\leq$

$\leq$

Let A

L1 : Terminates for finite graphs

L2 : At all times before termination there exists a node n' on OPEN which is on an optimal path.

Further $f(n') \leqslant f^*(S)$

$\boxed{f(n')} = g$

L3 : IF there is a path to the goal the algo terminates (even for infinite graphs) with a path.

L4 : A^* is admissible

L5 : For every node n A^* picks $f(n) \leq f^*(S)$

L6 : h_2 is MORE INFORMED than h_1 , $h_2(n) > h_1(n)$

Let A

L2: At all times before termination there exists a node n' on OPEN which is on an optimal path.

Further $f(n') \leq f^*(S)$

L3: IF there is a ... the goal the algo termin... for infinite graphs) with a path.

L4: A^* ...

L5 ... A^* picks $f(n) \leq f^*(S)$

L6 ... MED than h_1 , $h_2(n) > h_1(n)$

... n by A^*_2

$\boxed{f(n')} =$

=

Let

L2: At all times before termination there exists a node n' on OPEN which is on an optimal path.

Further $f(n') \leq f^*(S)$

$\boxed{f(n')} =$

L3: IF there is a path to the goal the algo terminates (even for infinite graphs) with a path.

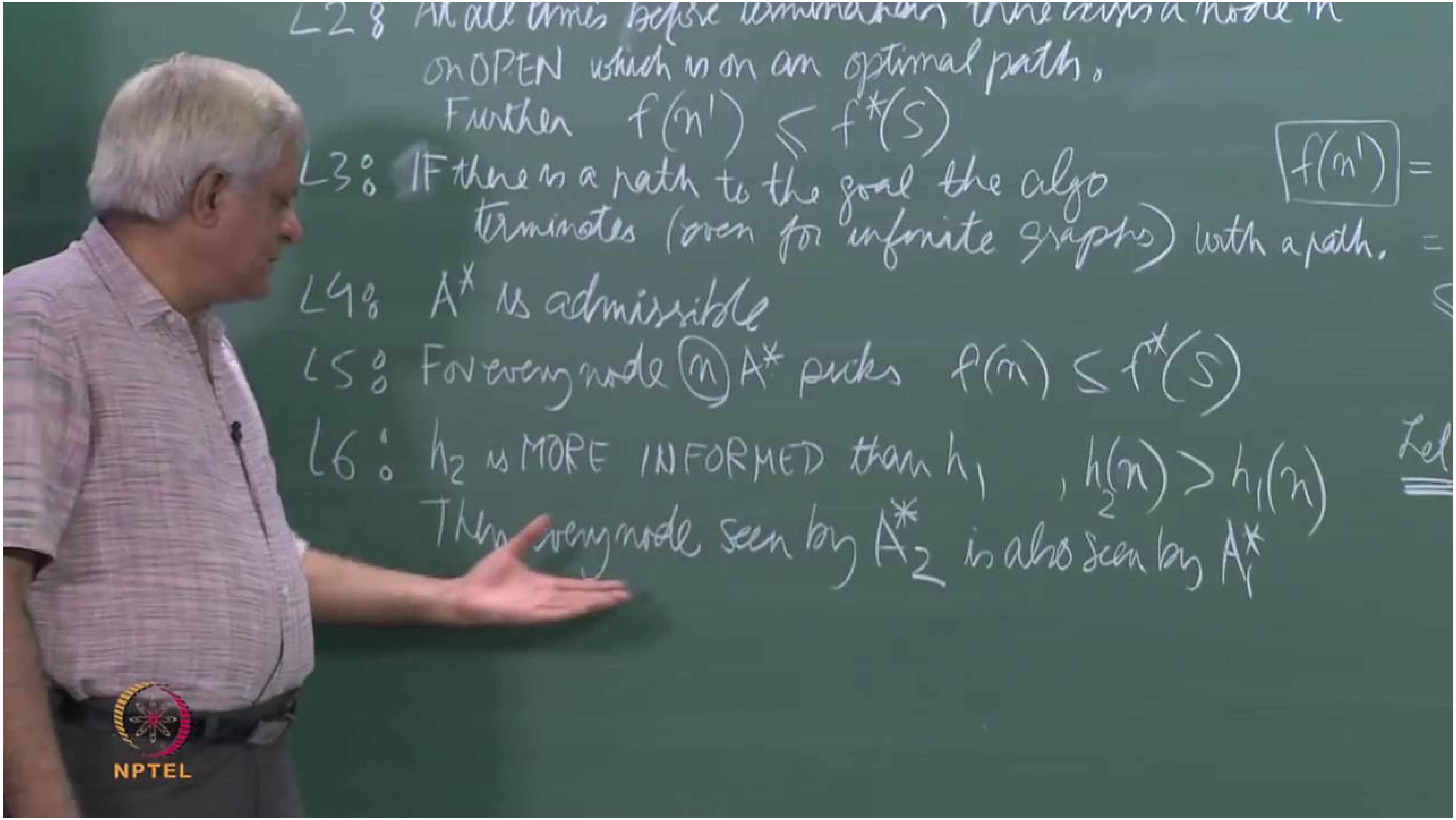
L4: A^* is admissible

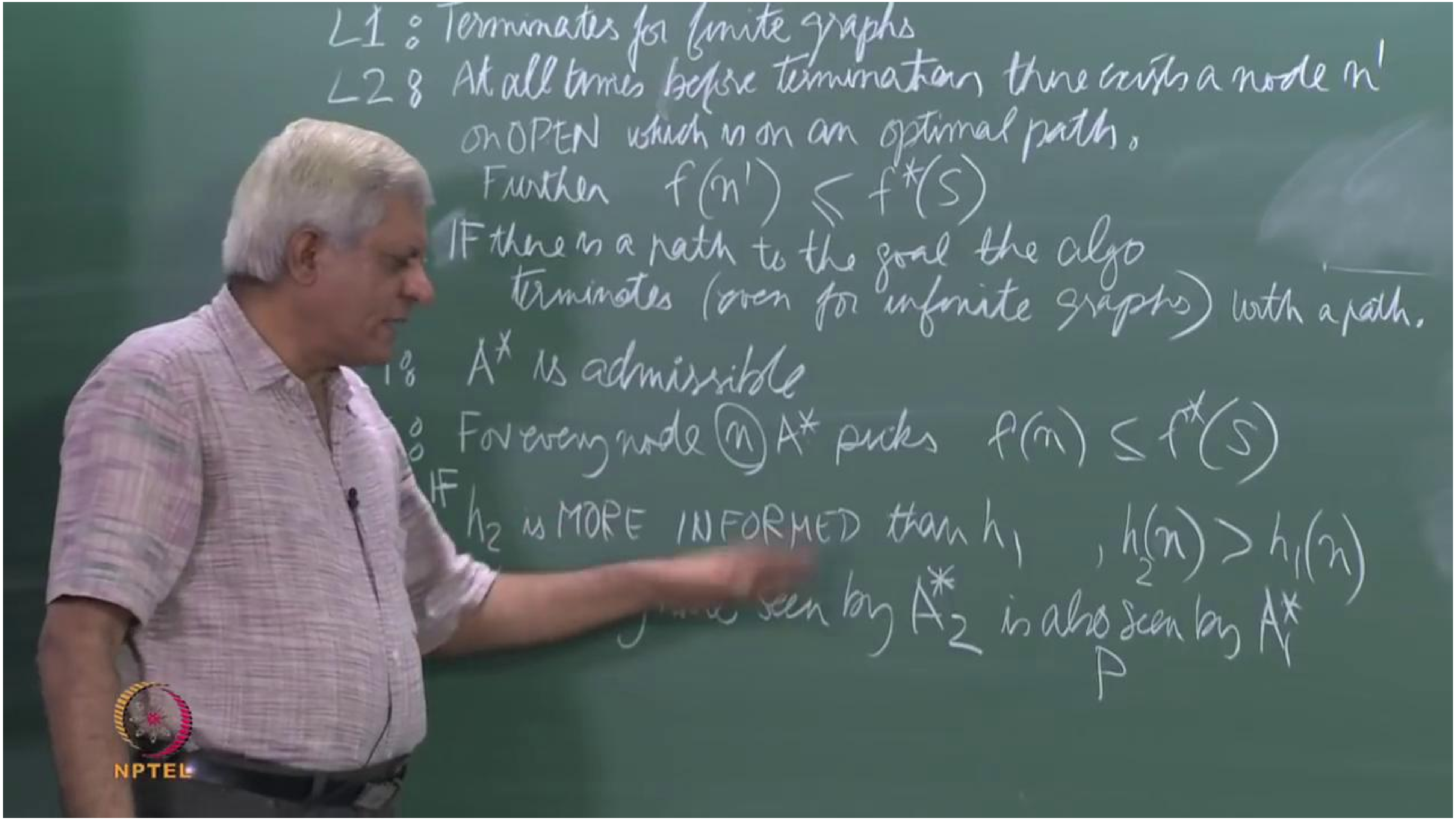
L5: For every node n A^* picks $f(n) \leq f^*(S)$

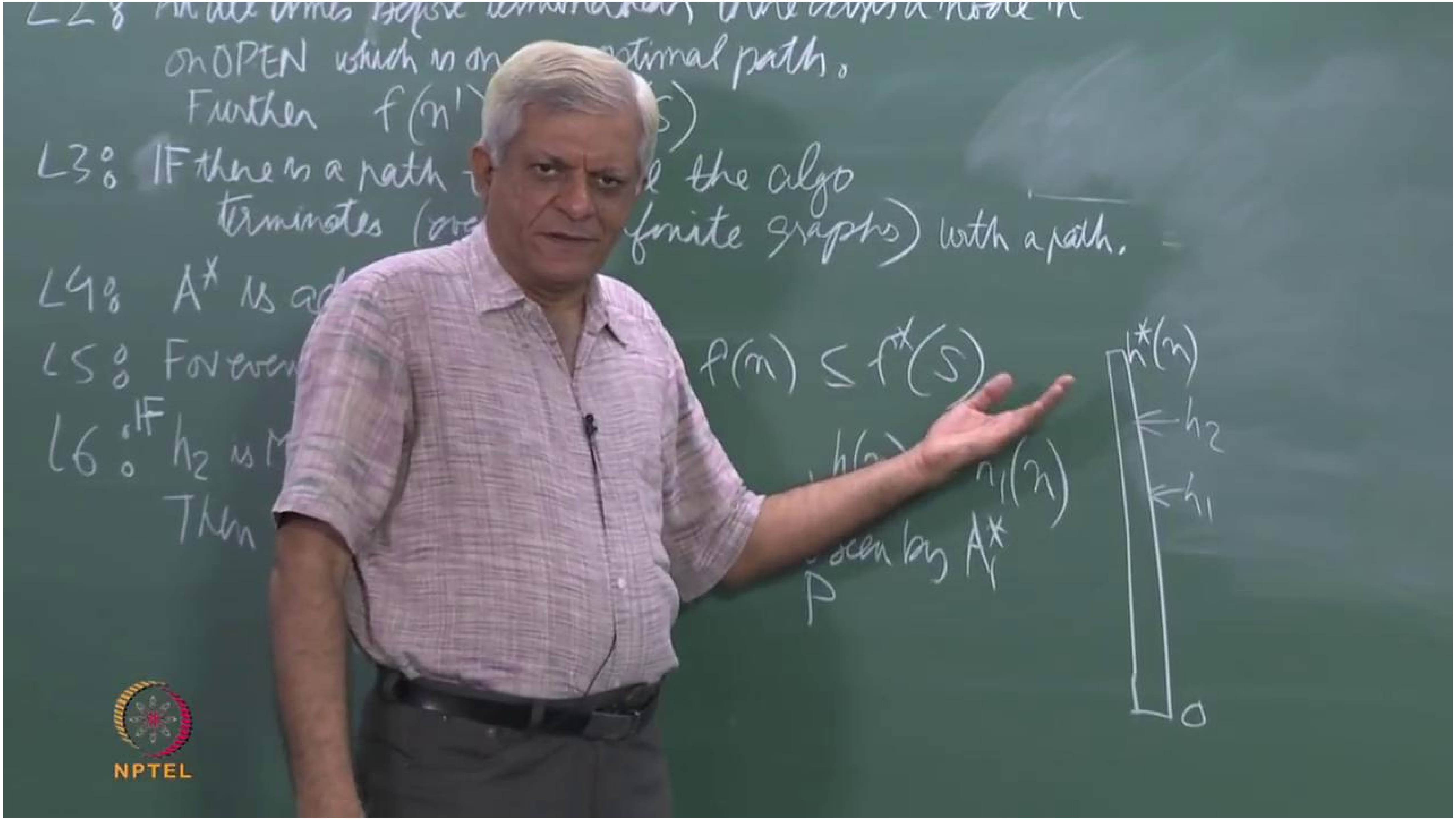
L6: h_2 is MORE INFORMED than h_1, $h_2(n) > h_1(n)$

Then every node seen by A_2^* is also seen by A_1^*

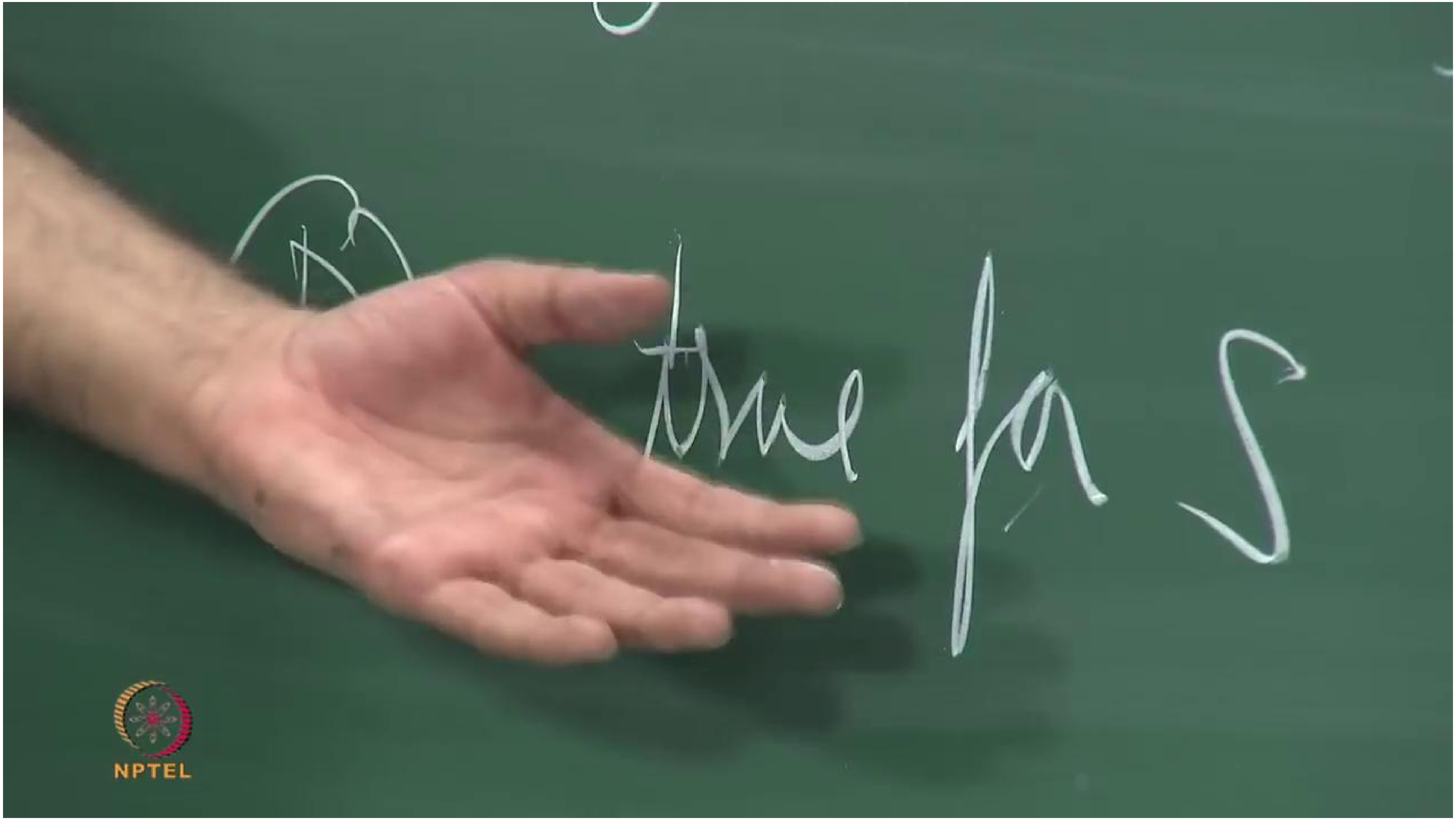
Let

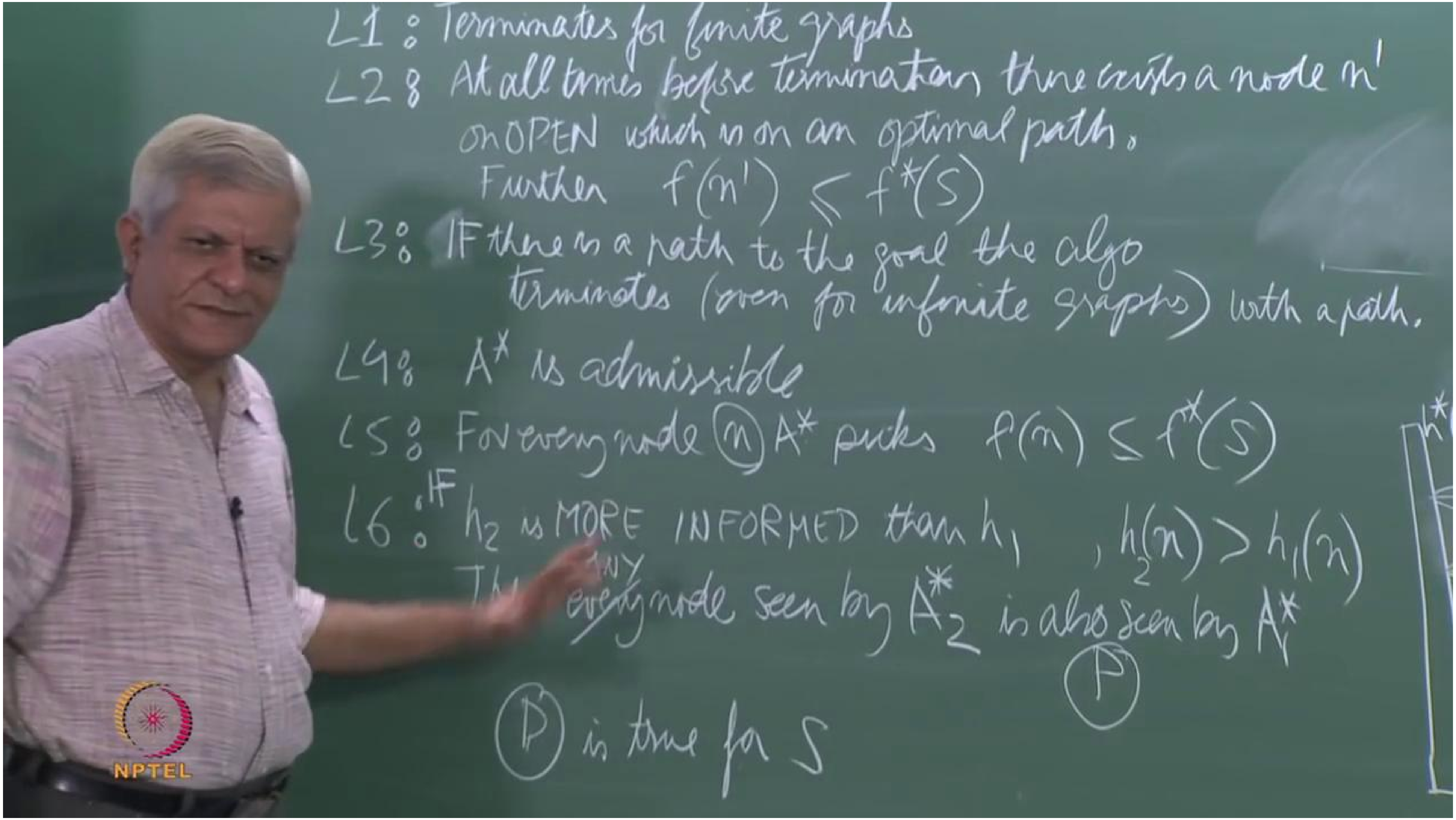

L2: At all times before termination there exists a node n'
on OPEN which is on an optimal path.
Further $f(n') \leq f^*(S)$
L3: IF there is a path to the goal the algo
terminates (even for infinite graphs) with a path.
L4: A^* is admissible
L5: For every node n A^* picks $f(n) \leq f^*(S)$
L6: h_2 is MORE INFORMED than h_1, $h_2(n) > h_1(n)$
Then every node seen by A^*_2 is also seen by A^*_1
$\boxed{f(n')} =$
NPTEL


L1 : Terminates for finite graphs
L2 : At all times before termination there exists a node n'
on OPEN which is on an optimal path.
Further f(n') ≤ f*(S)
IF there is a path to the goal the algo
terminates (even for infinite graphs) with a path.
A* is admissible
For every node n A* picks f(n) ≤ f*(S)
IF
h2 is MORE INFORMED than h1 , h2(n) > h1(n)
seen by A*2 is also seen by A*1
P
NPTEL



onOPEN which is on optimal path.
Further f(n'
L3: IF there is a path the algo
terminates (over finite graphs) with a path.
L4: A* is ad
L5: For every f(n) ≤ f*(S)
L6: IF h2 is M
Then
h*(n)
h2
h1
h1(n)
seen by A1*
NPTEL


time
NPTEL


L1 : Terminates for finite graphs
L2 : At all times before termination there exists a node n'
on OPEN which is on an optimal path.
Further $f(n') \leq f^*(S)$
L3 : IF there is a path to the goal the algo
terminates (even for infinite graphs) with a path.
L4 : A^* is admissible
L5 : For every node n A^* picks $f(n) \leq f^*(S)$
L6 : IF h_2 is MORE INFORMED than h_1 , $h_2(n) > h_1(n)$
Then ANY ~~every~~ node seen by A^*_2 is also seen by A^*_1
P
P is true for S
NPTEL

ADMISSIBILITY of A^*

picks node

there exists a node n'

l pat

(2) Let (P) be true for depth K

(3) Let N be a node at depth (K+1)
picked by A_2^*

Let A_1^* terminate without picking N

the c

te with a path.

$h^*(n)$

h_2

h_1

ADMISSIBILITY of A^*

picks node w

(2) Let (P) be true for depth K

(3) Let N be a node at depth (K+1) picked by A_2^*

Let A_1^* terminate without picking N

...an there exists a node n'
...timal path.
...(S)
...l the algo
...finite graphs) ...n.

...s f(n) ... *(n)
... h2
...an h1
...A*
...A2 i...

ADMISSIBILITY of A*

picks node w...

(2) Let (P) be true for depth K

(3) Let N be a node at depth (K+1)
picked by A_2^*
Let A_1^* terminate without picking N

$$f_2(N) = g_2(N) + h_2(N)$$

ADMISSIBILITY of A^*

(2) Let (P) be true for depth K

(3) Let N be a node at depth (K+1) picked by A_2^*

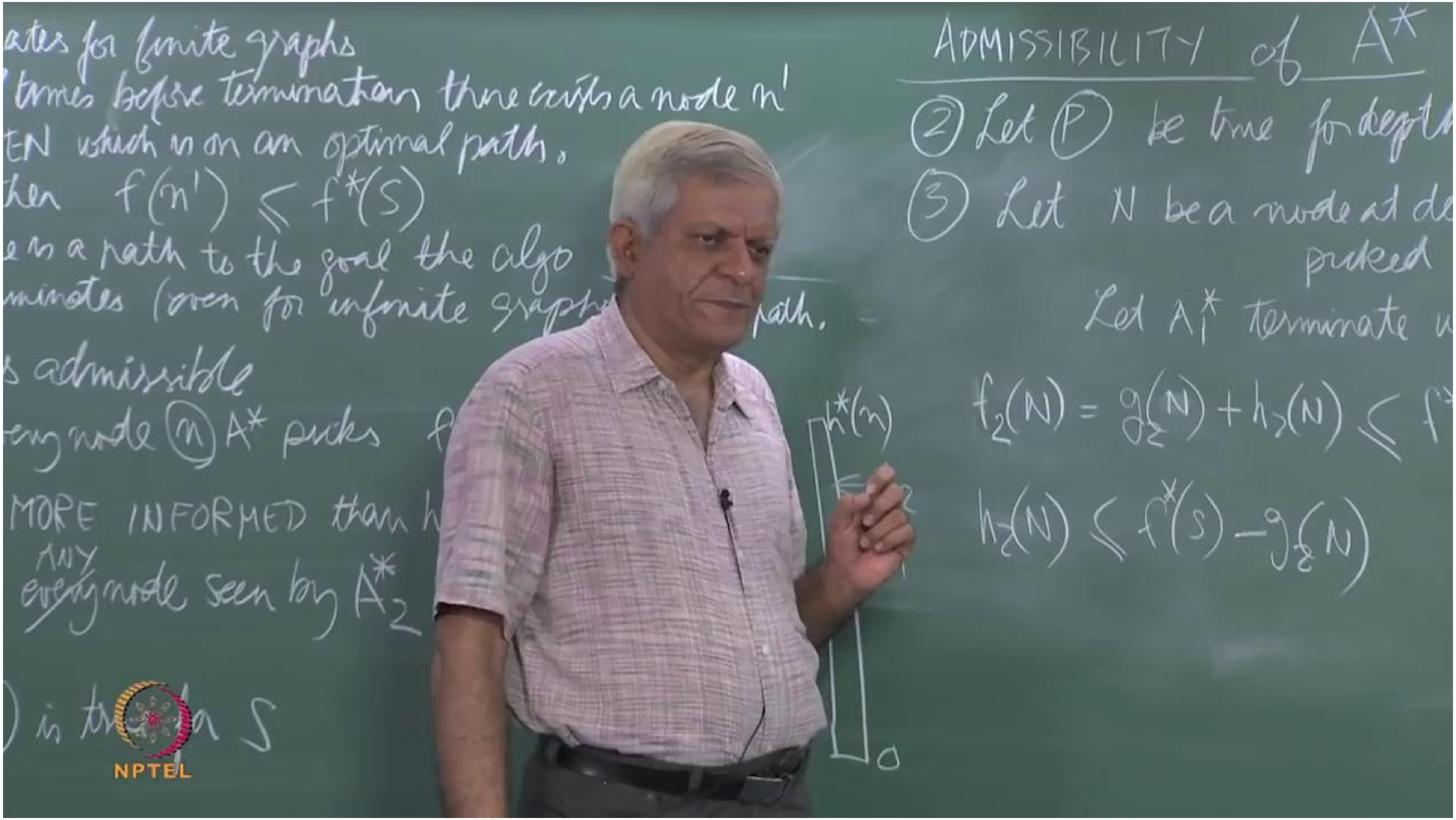
Let A_1^* terminate without picking N

$$f_2(N) = g_2(N) + h_2(N) < f^*(S)$$

$$h_2(N)$$

$$f_2(N) = g_2(N) + h_2(N) \leq f^*(S)$$

$$h_2(N) \leq f^*(S) - g_2(N)$$


ates for finite graphs
times before termination there exists a node n'
EN which is on an optimal path.
then f(n') ≤ f*(S)
e is a path to the goal the algo
minates (even for infinite graph
path.
s admissible
ery node (n) A* picks f
MORE INFORMED than h
ANY
every node seen by A*2
in th
S
h*(n)
ADMISSIBILITY of A*
(2) Let (P) be true for depth
(3) Let N be a node at de
picked
Let A1* terminate w
f2(N) = g2(N) + h2(N) ≤ f*
h2(N) ≤ f*(S) − g2(N)

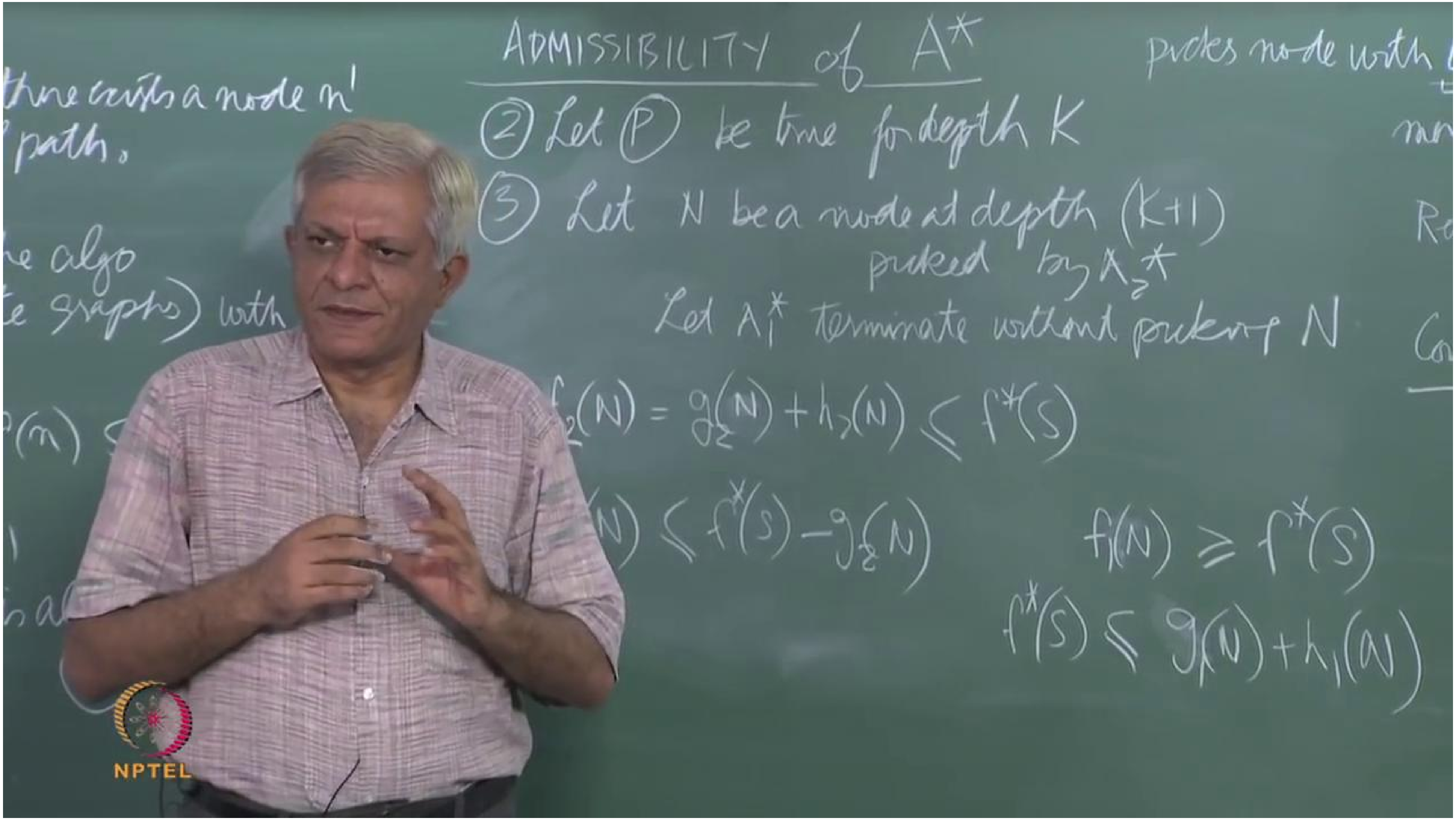
$$f_1(N) \geq f^*(S)$$

NPTEL

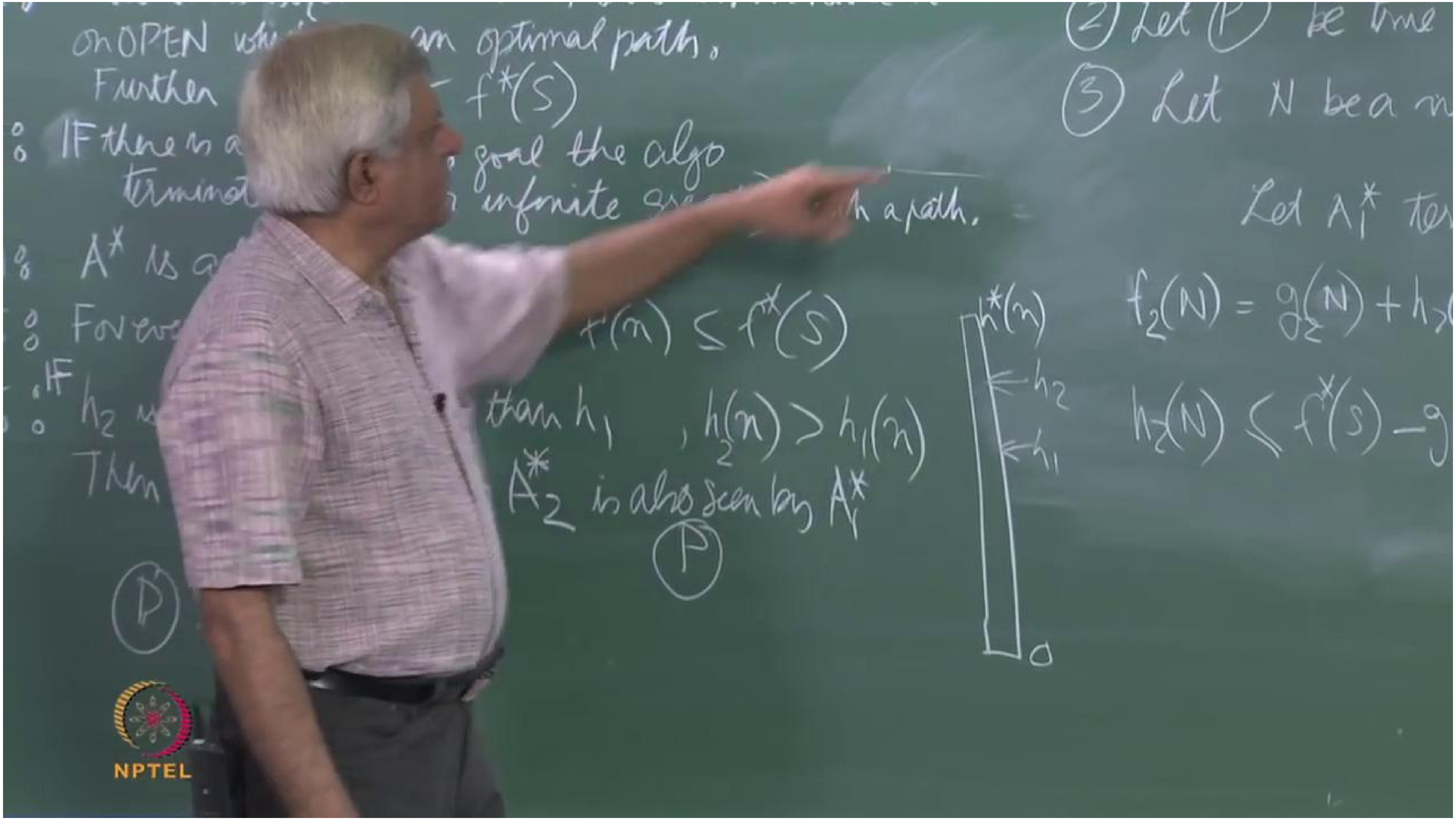
$$f_1(N) \geq f^*(S)$$

$$f^*(S) \leq g_1(N) + h_1(N)$$

NPTEL


ADMISSIBILITY of A*
picks node with
there exists a node n'
path.
(2) Let (P) be true for depth K
(3) Let N be a node at depth (K+1)
picked by A₂*
Let A₁* terminate without picking N
$f_2(N) = g_2(N) + h_2(N) \leq f^*(S)$
$(N) \leq f^*(S) - g_2(N)$
$f_1(N) \geq f^*(S)$
$f^*(S) \leq g_1(N) + h_1(N)$
he algo
e graphs) with
NPTEL

$$\boxed{g_2(N) \qquad g_1(N')}$$

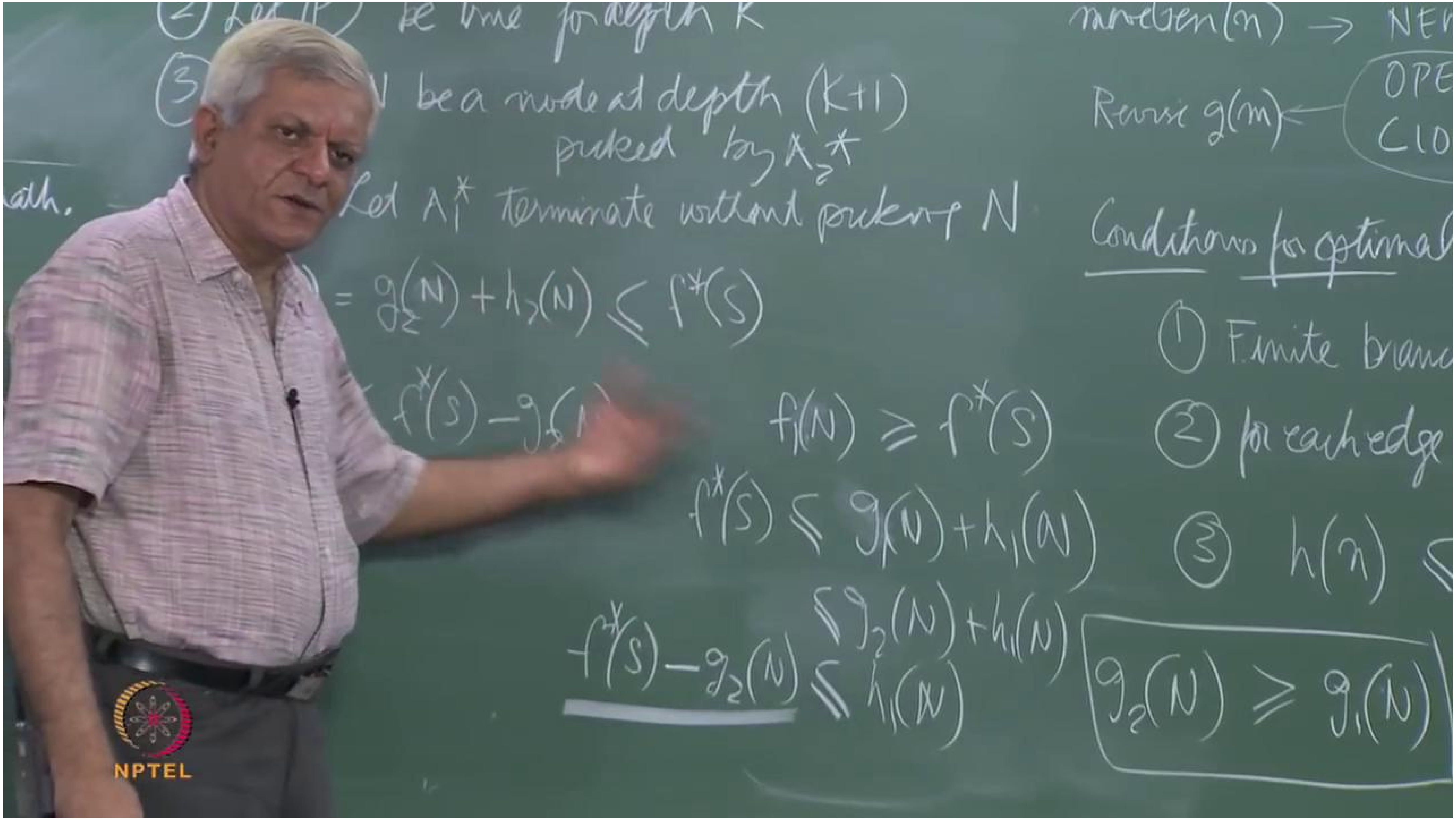


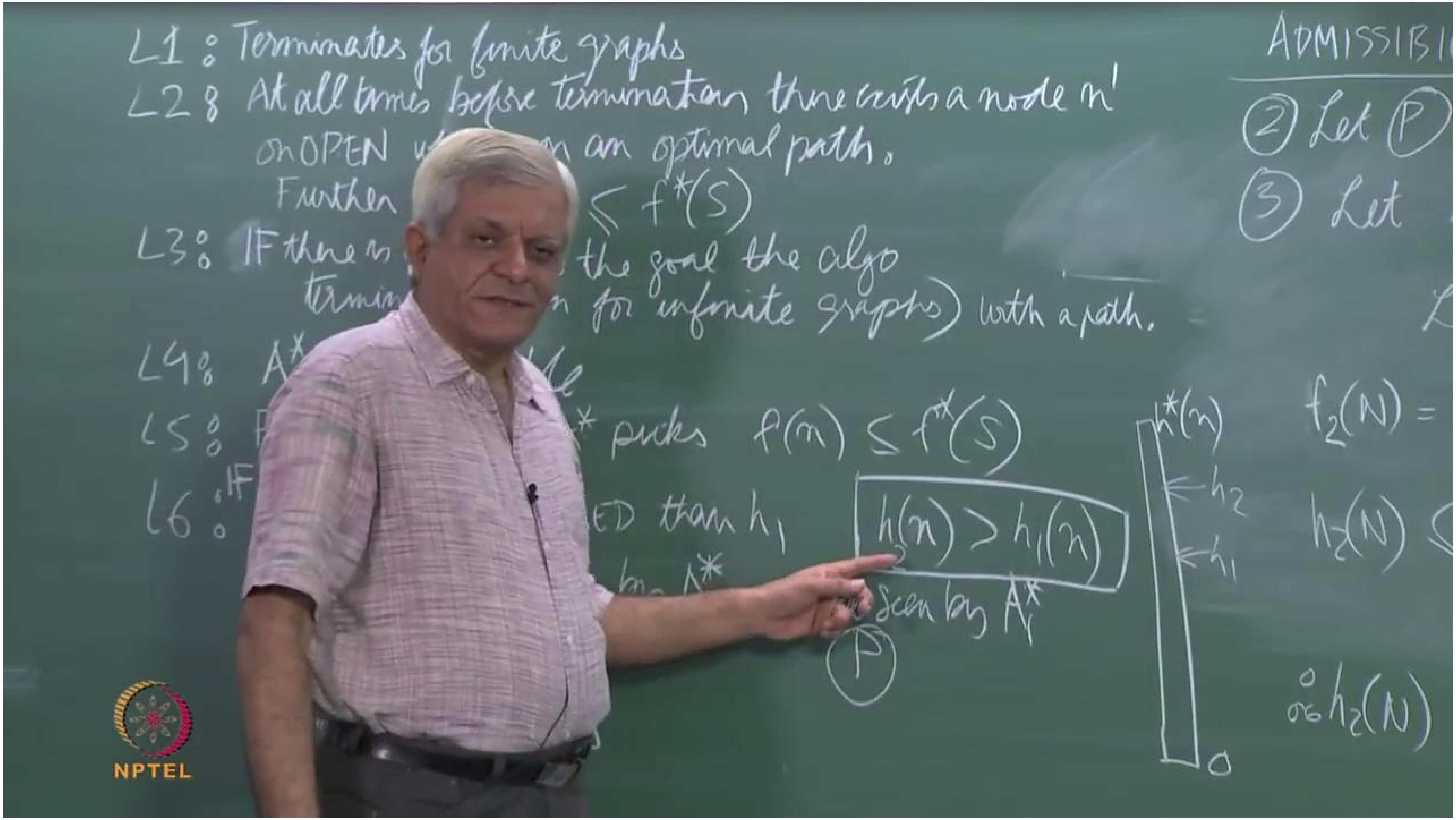
on OPEN wh... an optimal path.
Further ... f*(S)
IF there is a ... goal the algo
terminat... infinite ... a path.
A* is a
For eve... f(n) ≤ f*(S)
IF
h2 ... than h1 , h2(n) > h1(n)
Then ... A2* is also seen by A1*
P
P
h*(n)
h2
h1
(2) Let P be the
(3) Let N be a n
Let A1* te
f2(N) = g2(N) + h2(
h2(N) ≤ f*(S) − g
NPTEL

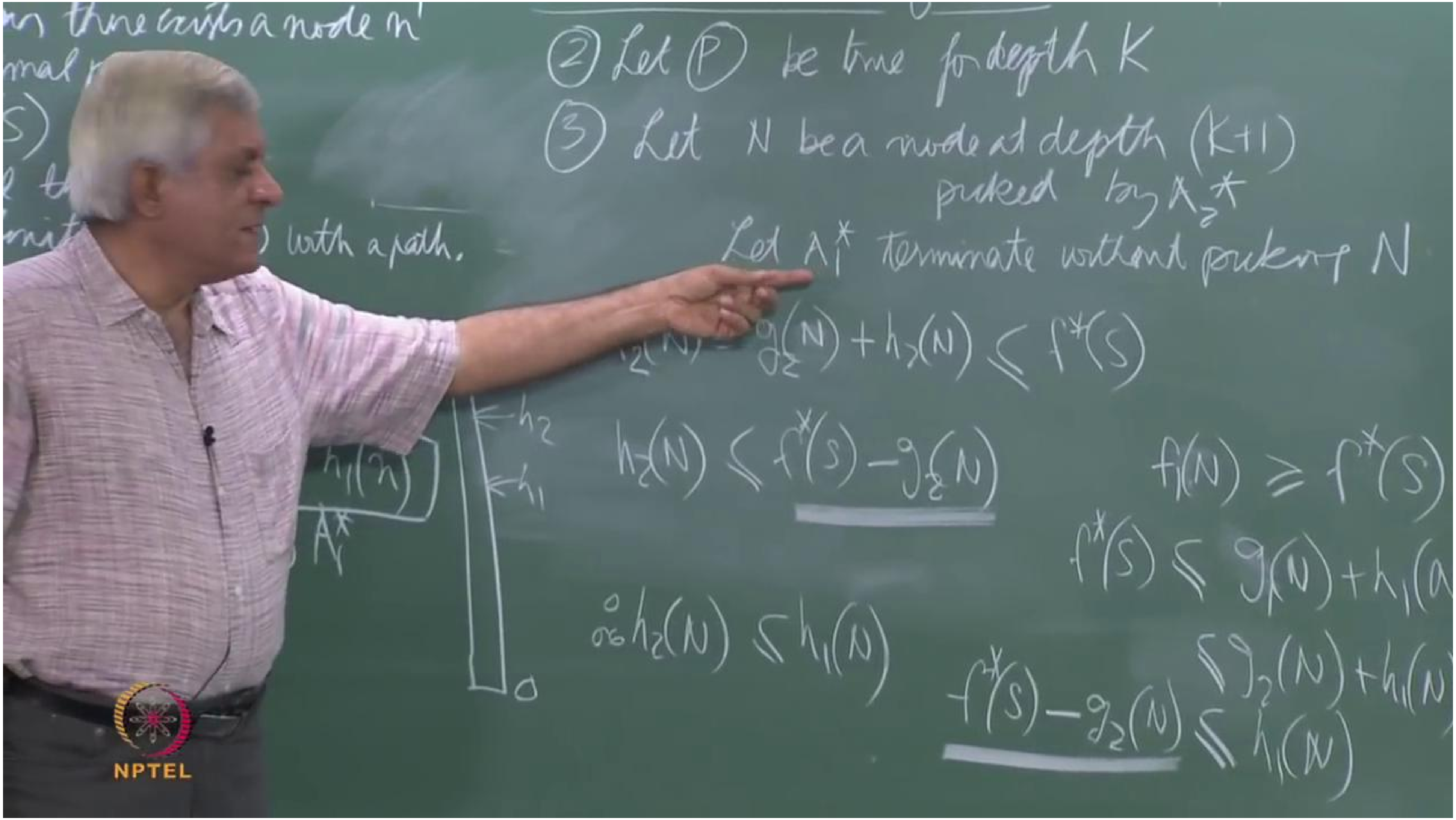
$$\boxed{g_2(N) \geq g_1(N)}$$

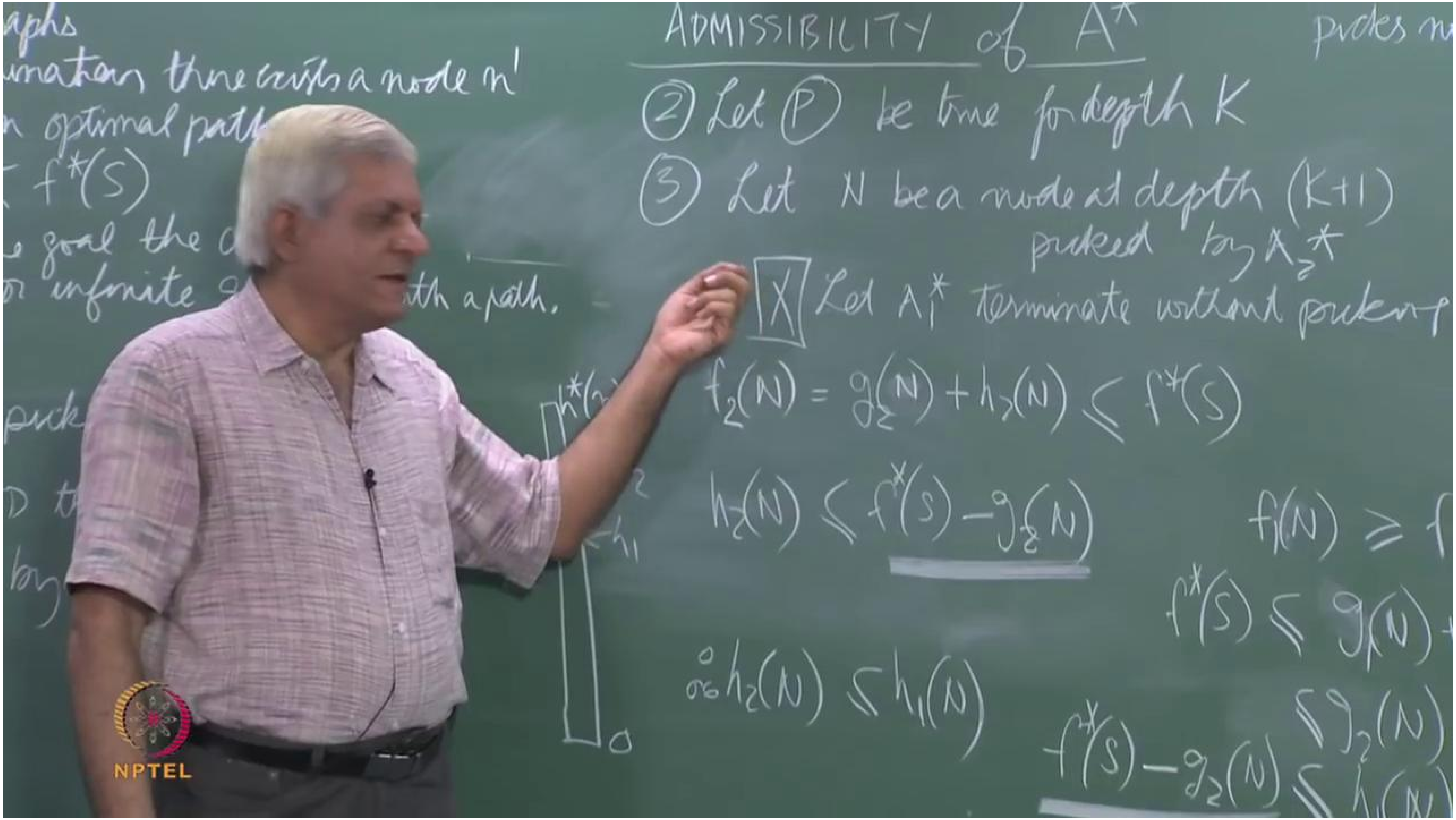
$$f^{-1}(s) - g_2(N) \le g_2(N) + h_1(N)$$

$$\ge h_1(N)$$


be true for depth K
moveGen(n) → NEW
be a node at depth (K+1)
picked by A_2^*
Revise g(m)
Let A_1^* terminate without picking N
Conditions for
$= g_2(N) + h_2(N) \leq f^*(S)$
1 Finite
$f_1(N) \geq f^*(S)$
2 for each edge
$f^*(S) \leq g_1(N) + h_1(N)$
$\leq g_2(N) + h_1(N)$
3 h(n)
$f^*(S) - g_2(N) \leq h_1(N)$
$g_2(N) \geq g_1(N)$
NPTEL


L1 : Terminates for finite graphs
L2 : At all times before termination there exists a node n'
on OPEN wh... on an optimal path.
Further ... ≤ f*(S)
L3 : IF there is ... the goal the algo
termin... for infinite graphs) with a path.
L4 : A* ...le
L5 : ...* picks f(n) ≤ f*(S)
L6 : IF ... ED than h1
h2(n) > h1(n)
... by A*
seen by A1*
P
h*(n)
h2
h1
0
ADMISSIBI
2 Let P
3 Let
f2(N) =
h2(N)
∴ h2(N)
NPTEL


there exists a node n'
mal p
(S)
with a path.
② Let (P) be true for depth K
③ Let N be a node at depth (K+1)
picked by A2*
Let A1* terminate without picking N
= g2(N) + h2(N) ≤ f*(S)
h2
h1
h1(n)
A1*
h2(N) ≤ f*(S) − g2(N)
f1(N) ≥ f*(S)
f*(S) ≤ g1(N) + h1(
∴ h2(N) ≤ h1(N)
≤ g2(N) + h1(N)
f*(S) − g2(N) ≤ h1(N)
NPTEL


ADMISSIBILITY of A*
② Let P be true for depth K
③ Let N be a node at depth (K+1)
picked by A2*
Let A1* terminate without picking
f2(N) = g2(N) + h2(N) ≤ f*(S)
h2(N) ≤ f*(S) − g2(N)
∴ h2(N) ≤ h1(N)
f1(N) ≥ f
f*(S) ≤ g1(N) +
≤ g2(N)
f*(S) − g2(N) ≤ h1(N)
mation there exists a node n'
optimal path
f*(S)
goal the
infinite
th a path.
NPTEL