Triangle Singularities in a Hilbert's House

CD24, Bochum

Ajay S. Sakthivasan [1]

M. Döring [2,3] M. Mai [2,4] A. Rusetsky [1,5]

[1]HISKP & BCTP, University of Bonn

[2]INS & Department of Physics, GWU

[3]Thomas Jefferson National Accelerator Facility

[4]AEC, University of Bern

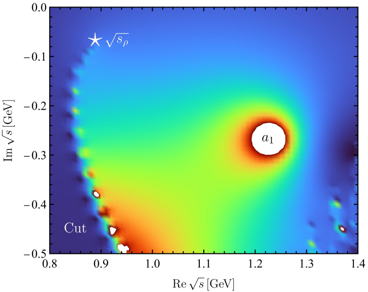
[5]Tbilisi State University

August 26, 2024

Presentation Outline

Introduction

- Hadronic resonances correspond to poles of the S-matrix.

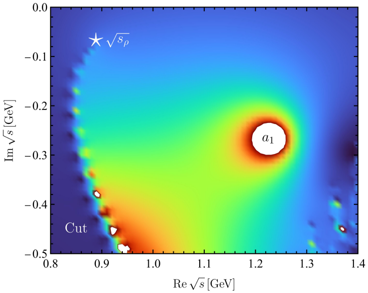


Pole position of the $a_1(1260)$.[1]

[1] Figure from: 10.1103/PhysRevD.105.054020

Introduction

- Hadronic resonances correspond to poles of the S-matrix.
- Experimentally — peaks in invariant mass distributions.

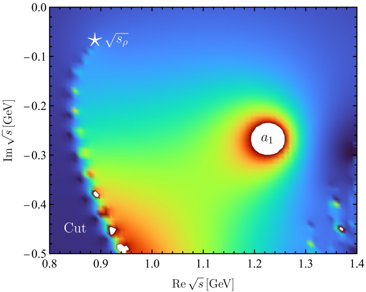


Pole position of the $a_1(1260)$.[1]

[1] Figure from: 10.1103/PhysRevD.105.054020

Introduction

- Hadronic resonances correspond to poles of the S-matrix.
- Experimentally — peaks in invariant mass distributions.
- All observed peaks correspond to hadronic states? No.



Pole position of the $a_1(1260)$.[1]

[1] Figure from: 10.1103/PhysRevD.105.054020

- Landau Singularities may not correspond to poles of the S-matrix, but can *mimic* a resonance.

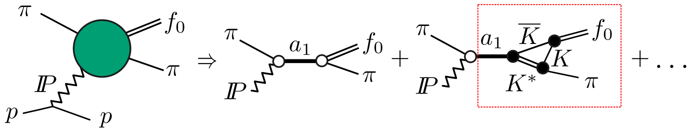
- Landau Singularities may not correspond to poles of the S-matrix, but can *mimic* a resonance.
- Originally proposed by Landau. Recently used to explain a lot of observed peaks (*one-loop level, rescattering effects not taken into account*).

- Landau Singularities may not correspond to poles of the S-matrix, but can *mimic* a resonance.
- Originally proposed by Landau. Recently used to explain a lot of observed peaks (*one-loop level, rescattering effects not taken into account*).
- We try to achieve the following:

- Landau Singularities may not correspond to poles of the S-matrix, but can *mimic* a resonance.
- Originally proposed by Landau. Recently used to explain a lot of observed peaks (*one-loop level, rescattering effects not taken into account*).
- We try to achieve the following:
  1. To systematically study the effect of final-state rescattering.

- Landau Singularities may not correspond to poles of the S-matrix, but can *mimic* a resonance.
- Originally proposed by Landau. Recently used to explain a lot of observed peaks (*one-loop level, rescattering effects not taken into account*).
- We try to achieve the following:
  1. To systematically study the effect of final-state rescattering.
  2. To consistently describe the unstable particles.

- Landau Singularities may not correspond to poles of the S-matrix, but can *mimic* a resonance.
- Originally proposed by Landau. Recently used to explain a lot of observed peaks (*one-loop level, rescattering effects not taken into account*).
- We try to achieve the following:
  1. To systematically study the effect of final-state rescattering.
  2. To consistently describe the unstable particles.
  3. Apply this to an $a_1(1420)$ (no spin-)system, and eventually the $a_1(1420)$ system with spin.[2]

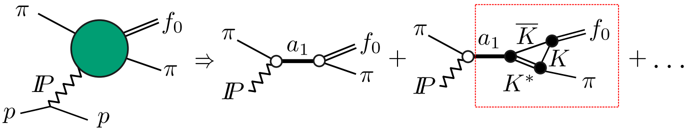


[2] Figure from: 10.1103/physrevlett.127.082501

- Landau Singularities may not correspond to poles of the S-matrix, but can *mimic* a resonance.
- Originally proposed by Landau. Recently used to explain a lot of observed peaks (*one-loop level, rescattering effects not taken into account*).
- We try to achieve the following:
  1. To systematically study the effect of final-state rescattering.
  2. To consistently describe the unstable particles.

  } **IVU**

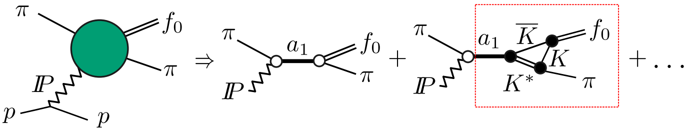
  3. **Apply this to an $\mathbf{a_1}$(1420) (no spin-)system**, and eventually the $a_1(1420)$ system with spin.[2]



[2]Figure from: 10.1103/physrevlett.127.082501

- Landau Singularities may not correspond to poles of the S-matrix, but can *mimic* a resonance.
- Originally proposed by Landau. Recently used to explain a lot of observed peaks (*one-loop level, rescattering effects not taken into account*).
- We try to achieve the following:
  1. To systematically study the effect of final-state rescattering.
  2. To consistently describe the unstable particles.

     IVU
  3. **Apply this to an $\mathbf{a_1}$(1420) (no spin-)system**, and eventually the $a_1(1420)$ system with spin.[2]



[2] Figure from: 10.1103/physrevlett.127.082501

- Effects of Final State Interactions on Landau singularities (arXiv:2407.17969 [hep-ph]) — under review.

- Effects of Final State Interactions on Landau singularities (arXiv:2407.17969 [hep-ph]) — under review.
- a1(1420): resonance like structure in COMPASS experiment. (10.1103/PhysRevLett.115.082001)
- Interpretation as a kinematical singularity — Mikhasenko et al. (10.1103/PhysRevD.91.094015), Bayar et al. (10.1103/PhysRevD.94.074039)
- Relevance to lattice QCD and ChPT — Korpa et al. (10.1103/PhysRevD.107.L031505), Isken et al. (10.1103/PhysRevD.109.034032), Mai et al. (10.1103/PhysRevLett.127.222001)
- Review — Guo et al. (10.1016/j.ppnp.2020.103757)

The Landau Equations

- The Landau Equations: singularities corresponding to Feynman integrals (*Landau singularities*).

The Landau Equations

- The Landau Equations: singularities corresponding to Feynman integrals (*Landau singularities*).
- For the Feynman integral with N internal propagators and l loop momenta,

$$I_1 = \int \prod_{i=1}^{l} d^4 k_i \; \frac{1}{\prod_{r=1}^{N} (q_r(k_i)^2 - m_r^2)}.$$

The Landau Equations

- The Landau Equations: singularities corresponding to Feynman integrals (*Landau singularities*).
- For the Feynman integral with N internal propagators and l loop momenta,

$$I_1 = \int \prod_{i=1}^{l} d^4 k_i \; \frac{1}{\prod_{r=1}^{N} (q_r(k_i)^2 - m_r^2)}.$$

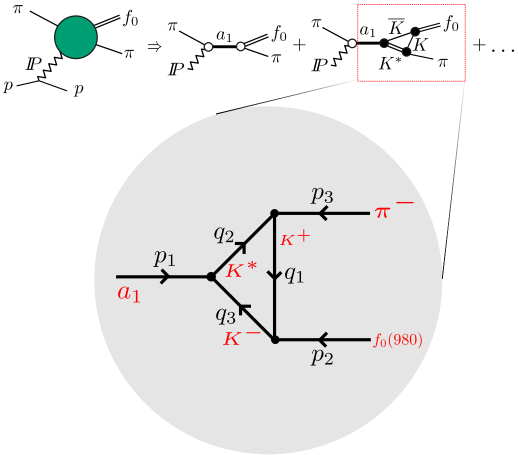
- For some real parameters α_i, the Landau equations read,

$$\alpha_i (q_i^2 - m_i^2) = 0 \implies \alpha_i = 0 \text{ or } q_i^2 = m_i^2,$$
$$\sum_{i \in \text{loop}} \alpha_i \; q_i^\mu (k_j) = 0.$$

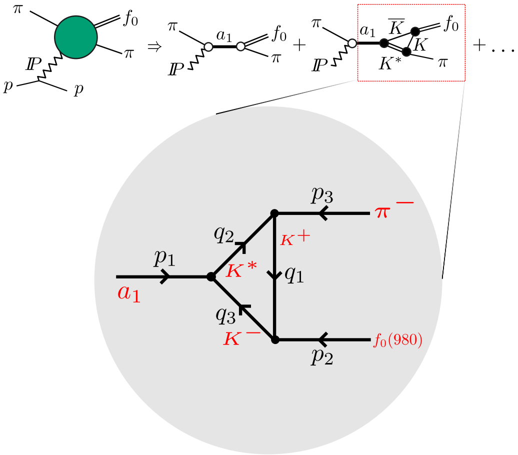
- $\alpha_i \neq 0 \implies$ all propagators go on-shell for the leading singularity. One also needs $\alpha_i > 0$.

- $\alpha_i \neq 0 \implies$ all propagators go on-shell for the leading singularity. One also needs $\alpha_i > 0$.
- $\alpha_i = 0$ case is called the subleading singularity — equivalent to contracting the propagator.

- The leading Landau singularity associated with the graph:



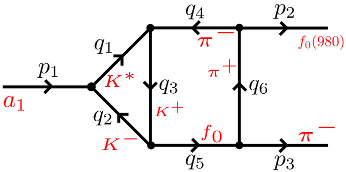
- The leading Landau singularity associated with the graph:



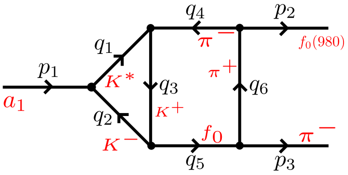
- Landau singularity for $p_3 = m_\pi$: $p_2/\mathrm{GeV} \equiv \sqrt{\sigma}/\mathrm{GeV} \in [0.986, 1.024]$ and $p_1/\mathrm{GeV} \equiv \sqrt{s}/\mathrm{GeV} \in [1.385, 1.436]$.

- What happens to singularities after rescattering? Ladder diagrams.

- What happens to singularities after rescattering? Ladder diagrams.
- Triangle +1 box diagram:



- What happens to singularities after rescattering? Ladder diagrams.
- Triangle $+1$ box diagram:

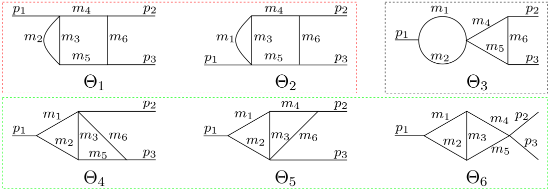


- Check for leading and subleading singularities. . .

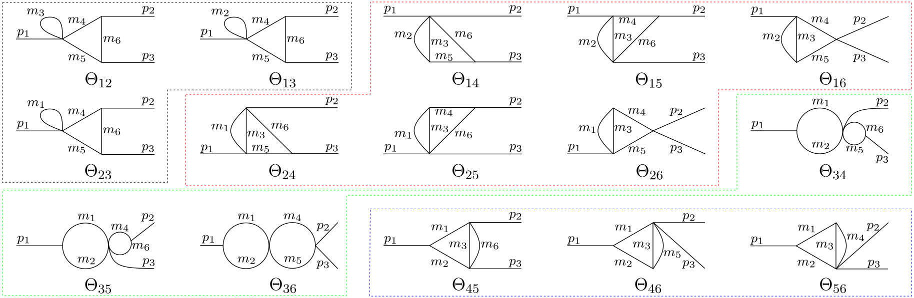
- Leading singularity is not present.

- Leading singularity is not present.
- For subleading singularities go through all contractions and check for singular graphs. . .

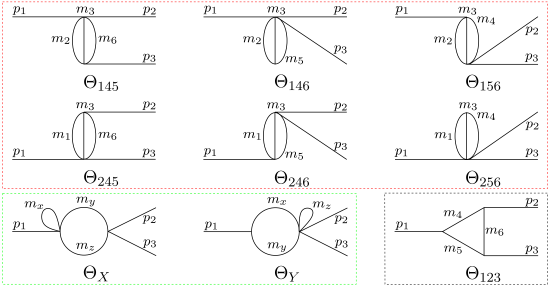
- Leading singularity is not present.
- For subleading singularities go through all contractions and check for singular graphs. . .



- Leading singularity is not present.
- For subleading singularities go through all contractions and check for singular graphs...

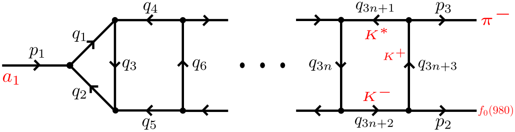


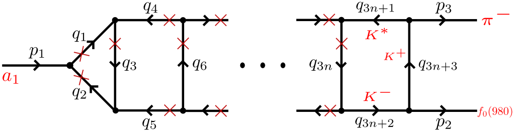
- Leading singularity is not present.
- For subleading singularities go through all contractions and check for singular graphs. . .



- Accommodating infinite rescatterings.

- Accommodating infinite rescatterings.
- The triangle singularity is present and no other singularities.





- Landau equations give the conditions for the singularity to be present.

- Landau equations give the conditions for the singularity to be present.
- One still needs to evaulate the corresponding Feynman integrals.

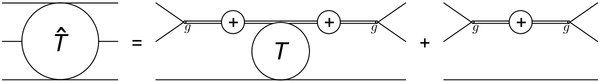
- Study the system using Infinite Volume 3-Body Unitary Formalism, due to Mai et al.[3]

[3] 10.1140/epja/i2017-12368-4

- Study the system using Infinite Volume 3-Body Unitary Formalism, due to Mai et al.[3]
- **In brief**: the system is split into a two-body subsystem — the isobar, and the spectator, which is on-shell.

[3] 10.1140/epja/i2017-12368-4

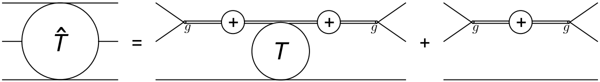
- Study the system using Infinite Volume 3-Body Unitary Formalism, due to Mai et al.[3]
- **In brief**: the system is split into a two-body subsystem — the isobar, and the spectator, which is on-shell.
- The full three-body amplitude is split into a connected and a disconnected amplitude.[4]



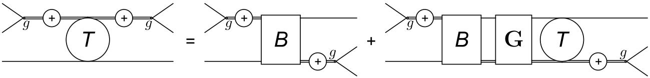
[3] 10.1140/epja/i2017-12368-4

[4] Figure from: 10.1140/epja/i2017-12368-4

- Study the system using Infinite Volume 3-Body Unitary Formalism, due to Mai et al.[3]
- **In brief**: the system is split into a two-body subsystem — the isobar, and the spectator, which is on-shell.
- The full three-body amplitude is split into a connected and a disconnected amplitude.[4]



- One considers the Bethe-Salpeter ansatz, $T = B + BGT$, to determine T.[4]



[3] 10.1140/epja/i2017-12368-4

[4] Figure from: 10.1140/epja/i2017-12368-4

- The equation to be solved,

$$T(p,q) = B(p,q) + \int \frac{d^3\mathbf{l}}{(2\pi)^3}\frac{1}{2E_l}B(p,l)\tau(\sigma(l))T(l,q).$$

- The equation to be solved,

$$T(p,q) = B(p,q) + \int \frac{d^3\mathbf{l}}{(2\pi)^3}\frac{1}{2E_l}B(p,l)\tau(\sigma(l))T(l,q).$$

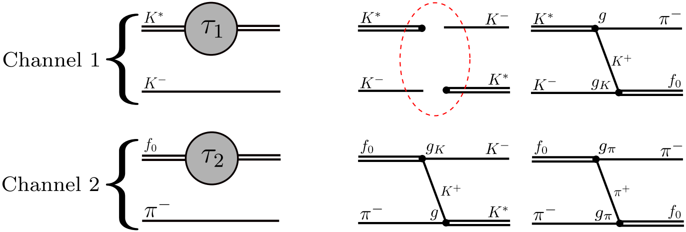
- We have the explicit forms of B and τ.

- The equation to be solved,

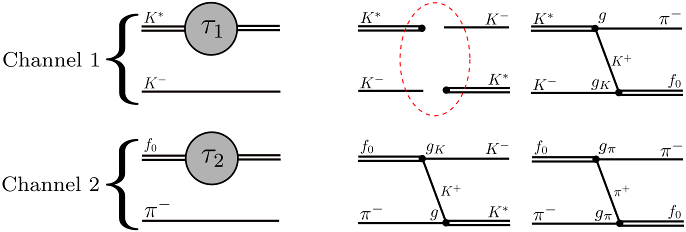
$$T(p,q) = B(p,q) + \int \frac{d^3\mathbf{l}}{(2\pi)^3}\frac{1}{2E_l}B(p,l)\tau(\sigma(l))T(l,q).$$

- We have the explicit forms of B and τ.
- In this work, we consider scalar propagators, in relative s-wave.

- Need to carry out a coupled channel analysis.

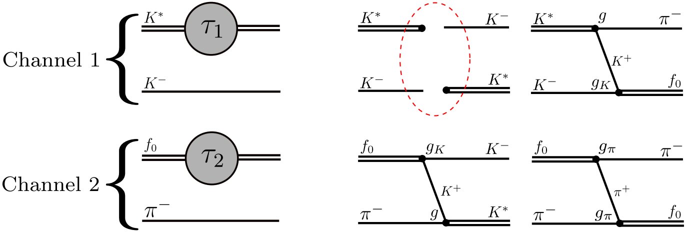


- Need to carry out a coupled channel analysis.



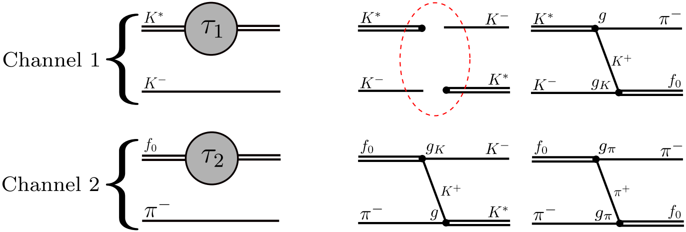
- B is parametrised through the spectator masses.

- Need to carry out a coupled channel analysis.



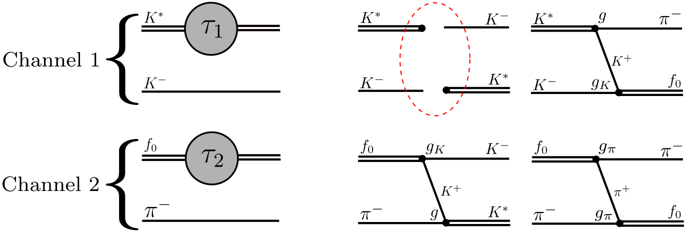
- B is parametrised through the spectator masses.
- There are two unknowns corresponding to each isobar — coupling and bare mass.

- Need to carry out a coupled channel analysis.



- B is parametrised through the spectator masses.
- There are two unknowns corresponding to each isobar — coupling and bare mass.
- These are fixed through PDG values of physical mass and width.

- Need to carry out a coupled channel analysis.

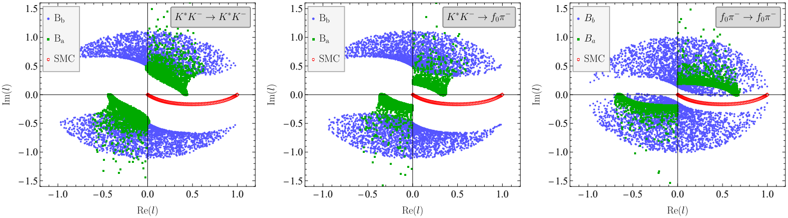


- B is parametrised through the spectator masses.
- There are two unknowns corresponding to each isobar — coupling and bare mass.
- These are fixed through PDG values of physical mass and width.
- For f_0 we also make use of g_K/g_p ratio from BaBaR collaboration.

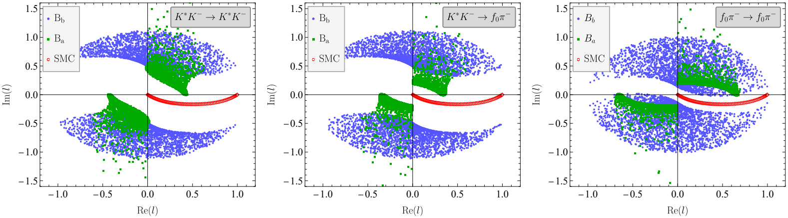
- The integral is solved numerically, using Gaussian quadrature.

- The integral is solved numerically, using Gaussian quadrature.
- But, singularities on the integration contour may prevent numerical implementation.

- The integral is solved numerically, using Gaussian quadrature.
- But, singularities on the integration contour may prevent numerical implementation.
- Contour deformation: Evaluate the integral along a complex contour, that is far from the singularities → obtain $T(\sqrt{s}, q \in \mathbb{C})$.

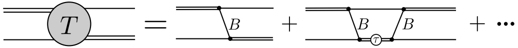


- The integral is solved numerically, using Gaussian quadrature.
- But, singularities on the integration contour may prevent numerical implementation.
- Contour deformation: Evaluate the integral along a complex contour, that is far from the singularities $\rightarrow$ obtain $T(\sqrt{s}, q \in \mathbb{C})$.

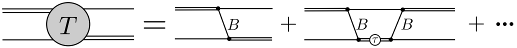


- But, we need the amplitudes for $q \in \mathbb{R} \rightarrow$ can be done in different ways!

- The solution of the Bethe-Salpeter equation can be given in form of the Born series.



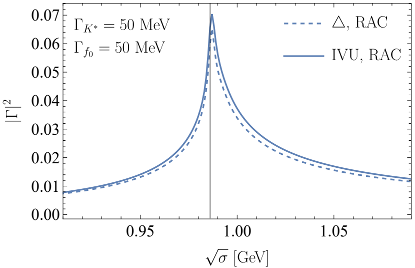
- The solution of the Bethe-Salpeter equation can be given in form of the Born series.



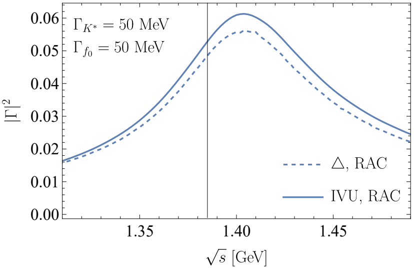
- This defines the loop expansion for the invariant mass distributions.

- **Rational Analytic Continuation**: We use a continued fraction to obtain the isobar-spectator amplitude for $q \in \mathbb{R}$.

- **Rational Analytic Continuation**: We use a continued fraction to obtain the isobar-spectator amplitude for $q \in \mathbb{R}$.
- At $1 + \infty$-loop level,

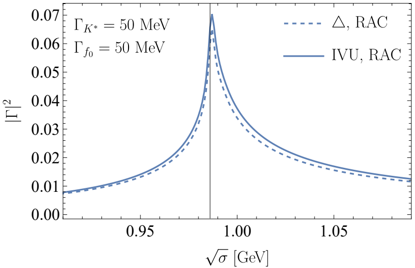


Amplitude squared vs. $\sqrt{\sigma}$, for $\sqrt{s} = 1.42$ GeV.

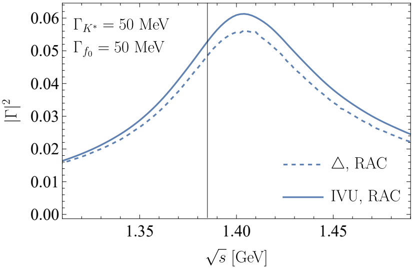


Amplitude squared vs. $\sqrt{s}$, for $\sqrt{\sigma} = 0.99$ GeV.

- **Rational Analytic Continuation**: We use a continued fraction to obtain the isobar-spectator amplitude for $q \in \mathbb{R}$.
- At $1 + \infty$-loop level,



Amplitude squared vs. $\sqrt{\sigma}$, for $\sqrt{s} = 1.42$ GeV.

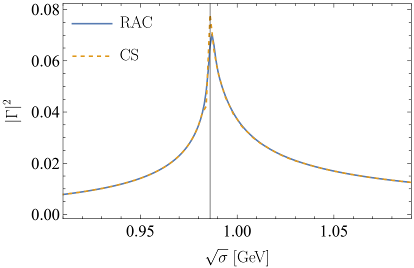


Amplitude squared vs. $\sqrt{s}$, for $\sqrt{\sigma} = 0.99$ GeV.

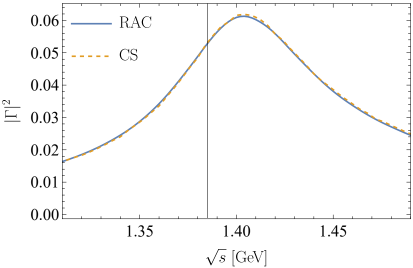
- We see the Landau singularity in both the plots.

- **Cahill & Sloan Method**: We make use Cauchy's integral theorem instead.

- **Cahill & Sloan Method**: We make use Cauchy's integral theorem instead.
- Matches with the other method.



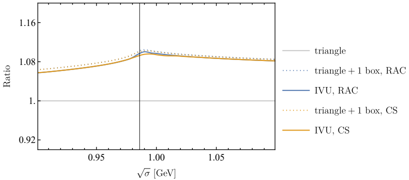
Amplitude squared vs. $\sqrt{\sigma}$, for $\sqrt{s} = 1.42$ GeV.



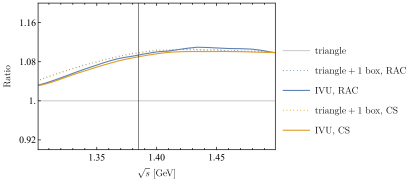
Amplitude squared vs. $\sqrt{s}$, for $\sqrt{\sigma} = 0.99$ GeV.

- Triangle diagram contributes the most to the amplitude.

- Triangle diagram contributes the most to the amplitude.
- With decreasing contributions from subsequent rescatterings.

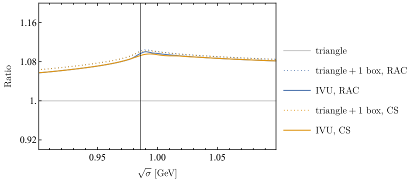


Ratios of amplitudes to triangle amplitudes vs. $\sqrt{\sigma}$, for $\sqrt{s} = 1.42$ GeV.

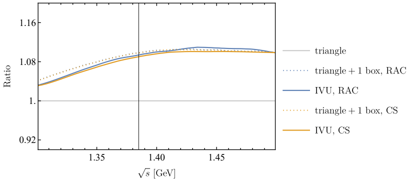


Ratios of amplitudes to triangle amplitudes vs. $\sqrt{s}$, for $\sqrt{\sigma} = 0.99$ GeV.

- Triangle diagram contributes the most to the amplitude.
- With decreasing contributions from subsequent rescatterings.



Ratios of amplitudes to triangle amplitudes vs. $\sqrt{\sigma}$, for $\sqrt{s} = 1.42$ GeV.



Ratios of amplitudes to triangle amplitudes vs. $\sqrt{s}$, for $\sqrt{\sigma} = 0.99$ GeV.

- The amplitudes at triangle $+1$ box are about 1.1x the triangle diagram amplitudes.

Conclusions

- Triangle singularity at 1-loop level and at higher loops as subleading singularity.

Conclusions

- Triangle singularity at 1-loop level and at higher loops as subleading singularity.
- The isobar-spectator model numerically implemented and the triangle singularity observed at $1 + \infty$-loop level.

Conclusions

- Triangle singularity at 1-loop level and at higher loops as subleading singularity.
- The isobar-spectator model numerically implemented and the triangle singularity observed at $1 + \infty$-loop level.
- The singularity is of logarithmic nature at all levels.

Conclusions

- Triangle singularity at 1-loop level and at higher loops as subleading singularity.
- The isobar-spectator model numerically implemented and the triangle singularity observed at $1 + \infty$-loop level.
- The singularity is of logarithmic nature at all levels.
- The contributions of the higher order diagrams depend on the system considered.

Conclusions

- Triangle singularity at 1-loop level and at higher loops as subleading singularity.
- The isobar-spectator model numerically implemented and the triangle singularity observed at $1 + \infty$-loop level.
- The singularity is of logarithmic nature at all levels.
- The contributions of the higher order diagrams depend on the system considered.
- Outlook:
  1. Realistic model, i.e., spin with pseudovector source — in progress.

Conclusions

- Triangle singularity at 1-loop level and at higher loops as subleading singularity.
- The isobar-spectator model numerically implemented and the triangle singularity observed at $1 + \infty$-loop level.
- The singularity is of logarithmic nature at all levels.
- The contributions of the higher order diagrams depend on the system considered.
- Outlook:
  1. Realistic model, i.e., spin with pseudovector source — in progress.
  2. $a_1(1420)$ on the lattice? Implement finite volume unitarity[5]— in progress.

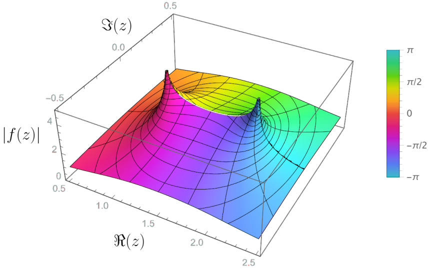
[5] 10.1140/epja/i2017-12440-1

Thanks for listening!

Landau singularities

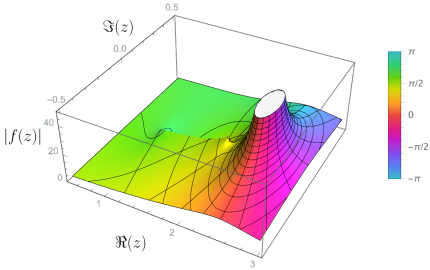
Endpoint singularities: singularities in g, hitting the contour C. E.g.,

$$f(z) = \int_1^2 dw \; \frac{1}{w - z}. \qquad (1)$$

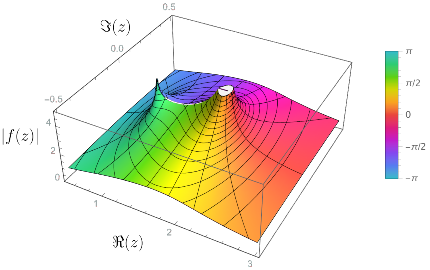


Pinch singularities: Contour C, gets trapped between two singularities in g. E.g.,

$$f(z) = \int_1^2 dw \, \frac{1}{(w-z)(w-5/2)}. \tag{2}$$



Pinch singularity avoided when the singularities approach from the same side of the contour.



The Other Method

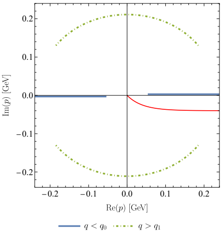
- **Cahill & Sloan Method**[6]: Instead of a continued fraction, use the knowledge of the location of the branch cuts to analytically continue the amplitude for $q \in \mathbb{R}$.[7]

[6] 10.1016/0375-9474(71)90156-4

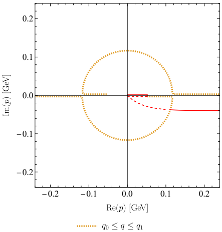
[7] For a comprehensive review: *The Quantum Mechanical Three-Body Problem*, Schmid & Ziegelmann.

The Other Method

- **Cahill & Sloan Method**[6]: Instead of a continued fraction, use the knowledge of the location of the branch cuts to analytically continue the amplitude for $q \in \mathbb{R}$.[7]



Branch cuts of B for q outside the triangle singularity region.



Branch cuts of B for q in the triangle singularity region.

[6] 10.1016/0375-9474(71)90156-4

[7] For a comprehensive review: *The Quantum Mechanical Three-Body Problem*, Schmid & Ziegelmann.