



## 1. Shortest path: Intro Chain

**sam** wants an intro to **aparna** through friends and friends of friends. There are two possible chains of intros:

**sam--jane--kaitlin--nick--ricardo--aparna**

or

**sam--jane--bob--samir--aparna**

The second chain is preferable since it is shorter.

If **sam** wants to be introduced to **michele** through a chain of friends, he is out of luck since there is no chain that leads from **sam** to **michele** in the graph.

Note that this algorithm does NOT have any restriction on the composition of the vertices: a vertex along the shortest chain need NOT be a student at a particular school, or even a student. So, for instance, you may have to find the shortest intro chain from **nick** to **samir**, which has the following solution:

**nick--ricardo--aparana--samir**

which consists of two penn state students, one rutgers student, and one non-student.

## 2. Cliques

Students tend to form cliques with their friends, which creates islands that do not connect with each other. If these cliques could be identified, particularly in the student population at a particular school, introductions could be made between people in different cliques to build larger networks of friendships at that school.

In the sample graph, there are two island cliques for students at rutgers:

```
(sam,rutgers)---(jane,rutgers)----- (bob,rutgers)      (sergei,rutgers)
                |                               |
                (kaitlin,rutgers)              (aparna,rutgers)
```

If we were to look at students at penn state, instead, there is a single clique:

```
(ming,penn state)----(nick,penn state)----(ricardo,penn state)
                |
                (heather,penn state)
```

And again, a single clique for students at ucla:

```
(rich,ucla)---(tom,ucla)
```

And one for students at cornell:

```
(michele,cornell)
```

From these examples, it should be clear that if there is at least one student in the graph that goes to a particular school, then there must be at least one island clique in the graph for students at that school.

## 3. Connectors: Friends who keep friends together

If **jane** were to leave rutgers, **sam** would no longer be able to connect with anyone else--**jane** was the "connector" who could pull **sam** in to hang out with her other friends. Similarly, **aparna** is a connector, since without her, **sergei** would not be able to "reach" anyone else. (And there are more connectors in the graph...)

On the other hand, **samir** is not a connector. Even if he were to leave, everyone else could still "reach" whoever they could when **samir** was there, even though they may have to go through a longer chain.

**Defintion:** In an undirected graph, vertex **v** is a connector if there are at least two other vertices **x** and **w** for which every path between **x** and **w** goes through **v**.

For example,  $v=jane$ ,  $x=sam$ , and  $w=bob$ .

Finding all connectors in an undirected graph can be done using DFS (depth-first search), by keeping track of two additional quantities for every vertex  $v$ . These are:

- $dfsnum(v)$ : This is the dfs number, assigned when a vertex is visited, dealt out in increasing order.
- $back(v)$ : This is a number that is initially assigned when a vertex is visited, and is equal to  $dfsnum$ , but can be changed later as follows:
  - When the DFS backs up from a neighbor,  $w$ , to  $v$ , if  $dfsnum(v) > back(w)$ , then  $back(v)$  is set to  $\min(back(v), back(w))$
  - If a neighbor,  $w$ , is already visited then  $back(v)$  is set to  $\min(back(v), dfsnum(w))$

When the DFS backs up from a neighbor,  $w$ , to  $v$ , if  $dfsnum(v) \leq back(w)$ , then  $v$  is identified as a connector, IF  $v$  is NOT the starting point for the DFS.

If  $v$  is a starting point for DFS, it can be a connector, but another check must be made - see the examples below. The examples don't tell you how to identify such cases--you have to figure it out.

Here are some examples that show how this works.

- Example 1: (B is a connector)

A--B--C

The DFS starts at A. Neighbors for a vertex are stored in REVERSE alphabetical order:

A: B  
B: C,A  
C: B

```
dfs @ A 1/1 (dfsnum/back)
  dfs @ B 2/2
    dfs @ C 3/3
      neighbor B is already visited => C 3/2
      dfsnum(B) <= back(C) B is a CONNECTOR
      nbr A is already visited => B 2/1
      dfsnum(A) <= back(B) A is starting point of DFS, NOT connector in this case
```

- Example 2: (B is a connector)

A--B--C

The same example as the first, except DFS starts at B. This shows how even though B is the starting point, it is still identified (correctly) as a connector. The code is not complete because it does not show HOW B is determined to be a connector in the last line - that's for you to figure out. Neighbors are stored in reverse alphabetical order as before.

```
dfs @ B 1/1
  dfs @ C 2/2
    nbr B is already visited => C 2/1
    dfsnum(B) <= back(C) B is starting point, NOT connector
  dfs @ A 3/3
    nbr B is already visited => A 3/1
    dfsnum(B) <= back(A) B is starting point, but is a CONNECTOR in this case
```

- Example 3: (B and D are connectors)

```
A---B---C
 |       |
 |       |
E---D---F
```

DFS starts at A. Neighbors stored in reverse alphabetical order again:

A: B  
 B: E,C,A  
 C: D,B  
 D: F,E,C  
 E: D,B  
 F: D

```
dfs @ A 1/1
  dfs @ B 2/2
    dfs @ E 3/3
      dfs @ D 4/4
        dfs @ F 5/5
          nbr D is already visited => F 5/4
          dfsnum(D) <= back(F) => D is a CONNECTOR
          nbr E already visited => D 4/3
          dfs @ C 6/6
            nbr D already visited => C 6/4
            nbr B already visited => C 6/2
            dfsnum(D) > back(C) => D 4/2
            dfsnum(E) > back(D) => E 3/2
            nbr B is already visited => E 3/2
            dfsnum(B) <= back(E) => B is a CONNECTOR
            nbr C is already visited => B 2/2
            nbr A is already visited => B 2/1
            dfsnum(A) <= back(B) A is starting point, NOT a connector in this case
```

- Example 4: (B and D are connectors)

```
A---B---C
 |   |
E---D---F
```

Same example as the previous, except DFS starts at D, and neighbors stored in alphabetical order. Connectors are still correctly identified as B and D.

A: B  
 B: A,C,E  
 C: B,D  
 D: C,E,F  
 E: B,D  
 F: D

```
dfs @ D 1/1
  dfs @ C 2/2
    dfs @ B 3/3
      dfs @ A 4/4
        nbr B is already visited => A 4/3
        dfsnum(B) <= back(A) => B is a CONNECTOR
        nbr C is already visited => B 3/2
        dfs @ E 5/5
          nbr B is already visited => E 5/3
          nbr D is already visited => E 5/1
          dfsnum(B) > back(E) => B 3/1
          dfsnum(C) > back(B) => C 2/1
          nbr D is already visited => C 2/1
          dfsnum(D) <= back(C) D is starting point, NOT connector
          dfs @ F 6/6
            nbr D is already visited => F 6/1
            dfsnum(D) <= back(F) D is starting point, is a CONNECTOR
```

## Implementation

You will write a program called [Friends](#) that will read a graph file, build a graph (using the adjacency linked lists

representation), and implement the ~~four~~ three algorithms described above. Details follow, including a user interface.

### 1. (4 pts) User interface

- Your user interface should ask for the input graph file, build the graph, then spin on ~~five~~ four choices-- shortest intro chain, cliques at school, connectors, and quit--until the user quits.

### 2. (6 pts) Graph build

- Input: file that lists names of all people and edges between them. Here is the format of the input file for the sample friendship graph given in the [Background](#) section.

```
15
sam|y|rutgers
jane|y|rutgers
michele|y|cornell
sergei|y|rutgers
ricardo|y|penn state
kaitlin|y|rutgers
samir|n
aparna|y|rutgers
ming|y|penn state
nick|y|penn state
bob|y|rutgers
heather|y|penn state
rachel|n
rich|y|ucla
tom|y|ucla
sam|jane
jane|bob
jane|kaitlin
kaitlin|nick
bob|samir
sergei|aparna
samir|aparna
aparna|ricardo
nick|ricardo
ming|nick
heather|nick
michele|rachel
michele|tom
tom|rich
```

The first line has the number of people in the graph (15 in this case).

The next set of lines has information about the people in the graph, one line per person (15 lines in this example), with the '|' used to separate the fields.

In each line, the first field is the name of the person. Names of people can have any character except '|', and are case *insensitive*. Also, **names are unique over the entire graph**. The second field is 'y' if the person is a student, and 'n' if not. The third field is only present for students, and is the name of the school the student attends. The name of a school can have any character except '|', and is case *insensitive*.

The last set of lines, following the people information, lists the friendships between people, one friendship per line. Since friendship works both ways, any friendship is only listed once, and the order in which the names of the friends is listed does not matter.

- Result: The adjacency linked list representation, along with a data structure to be able to quickly translate from a person's name to vertex number. There should also be a quick way to translate from vertex number to the person's name.
- Output: Graph adjacency linked lists representation in your program.  
**You don't have to print the representation, we will check your code to see if you are storing the graph correctly in this representation.**

### 3. (12 pts) Shortest path (Intro chain)

- Input: Name of person who wants the intro, and the name of the other person, e.g. "sam" and "aparna" for the graph in the Background section. (Neither of these, nor any of the intermediate people are required to be students, in the same school or otherwise.)
- Result: The shortest chain of people in the graph starting at the first and ending at the second.
- Output: Print the chain of people in the shortest path, for example:

sam--jane-bob--samir--aparna

If there is no way to get from the first person to the second person, then the output should be a message to this effect.

### 4. (8 pts) Cliques

- Input: Name of school for which cliques are to be found, e.g. "rutgers"
- Result: The subgraphs for each of the cliques.
- Output: Print the subgraph for each clique, in the same format as the input described in the [Graph build](#) section (a clique is a graph in its own right).

For example:

Clique 1:

<subgraph output>

Clique 2:

<subgraph output>

etc...

Note:

- If there is an edge `x--y` in the graph, then your output must have either `x--y` or `y--x`, but NOT both (so your output can be used as input for any other application that might use your subgraph.)
- If there is even one student at the named school in the graph, then there must be at least one clique in the output. If the graph has no students at all at that school, then the output will be empty. (You can write out an appropriate message as you see fit.)
- In your code, each clique must be created as a separate [Graph](#) object: every subgraph of some graph, is a graph in its own right.

### 5. (20 pts) Connectors (Friends who keep friends together)

- Input: Nothing
- Result: Names of all people who are connectors in the graph
- Output: Print names of all people who are connectors in the graph, comma separated, in any order.

### Special Implementation Notes

- Your program MUST be called [Friends](#) - in other words, you must have a file named [Friends.java](#) with a [main](#) method.
- You may implement as many classes as you want, and separate them into packages as needed.
- You may import any of the classes from [java.lang](#), [java.io](#) and [java.util](#), but you may NOT import classes from any of the other packages in the standard Java API, and you may NOT import classes from any external java APIs. (Of course, if you have more than one package in your application, you can cross-import classes among them.)

## Grading

Since there is no pre-defined structure for the classes you will implement, grading will be done manually. Graders will run your program, and check the printed output for correctness. For [Graph build](#) they will look at the code to make sure you are storing the graph correctly.

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## Submission

ONLY ONE PERSON per team should submit the assignment.

In the [Friends.java](#) file, **make sure you write the names of both members of your team in comment lines at the top.**

Export your entire Eclipse project as a zip archive into a file called [friends.zip](#), and upload this file to Sakai. See the Eclipse page, under [Zipping up a Project](#), for how to do this.

**If you submit an incorrect/incomplete project, you will lose credit.**

So before you submit, you should check that all is well by importing your friends.zip file as a project into Eclipse (in a different workspace, so it doesn't conflict with your existing project), and running it.

The assignments you have been doing all along have been given as project files that you have imported in Eclipse, so this should be old hat by now. If you are still having trouble and cannot figure this out for some reason, ask your TA, or one of the instructors.