AE706

Assignment 3

Computation of Streamline Patterns for Steady Inviscid Incompressible Fluid Flows

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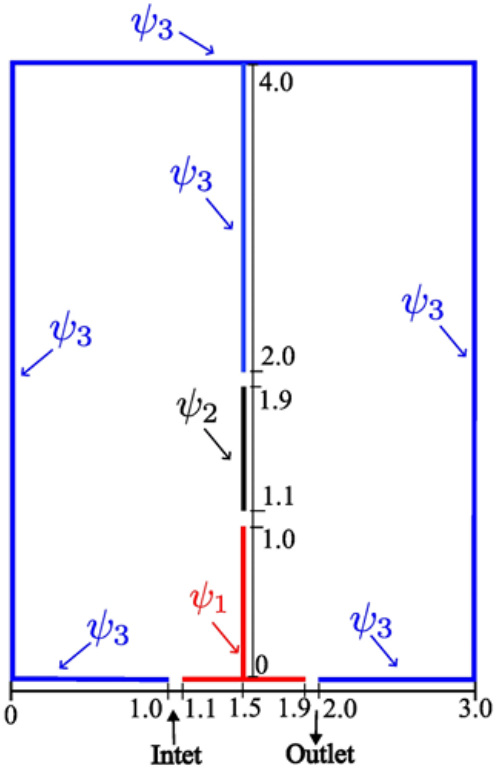
1 Problem Statement

This study examines the numerical simulation of the stream function ψ within a two-dimensional rectangular chamber of dimensions $3m \times 4m$ (width $\times$ height). The chamber features a central divider located at $x = 1.5$, which spans the entire height, and contains two openings (slots) positioned at $y = 1.0$ to 1.1 and $y = 1.9$ to 2.0.

To determine the streamline distribution, we solve the stream-function equation, given by:

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = 0, \tag{1}$$

which governs the flow within the chamber. The boundary conditions are applied along the chamber walls, inlet, outlet, and central divider, with three test cases incorporating different values of ψ at designated locations. The chamber configuration, including the inlet, outlet, and slot positions, is depicted in Figure **??**.



Schematic of the 2D chamber (3m $\times$ 4m) with a centre wall at $x = 1.5$ and slots at $y = 1.0$ to 1.1 and $y = 1.9$ to 2.0. The inlet and outlet are located at the bottom wall.

2 Governing Equation and discretization

The given two-dimensional inviscid incompressible equation is:

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = 0 \tag{2}$$

Using the second derivative approximation:

$$\frac{\partial^2 \psi}{\partial x^2} \approx \frac{\psi_{i+1,j} - 2\psi_{i,j} + \psi_{i-1,j}}{(\Delta x)^2} \tag{3}$$

$$\frac{\partial^2 \psi}{\partial y^2} \approx \frac{\psi_{i,j+1} - 2\psi_{i,j} + \psi_{i,j-1}}{(\Delta y)^2} \tag{4}$$

Substituting these into equation (1):

$$\frac{\psi_{i+1,j} - 2\psi_{i,j} + \psi_{i-1,j}}{(\Delta x)^2} + \frac{\psi_{i,j+1} - 2\psi_{i,j} + \psi_{i,j-1}}{(\Delta y)^2} = 0 \tag{5}$$

For $\Delta x = \Delta y = 0.1$, simplifying gives:

$$\psi_{i+1,j} + \psi_{i-1,j} + \psi_{i,j+1} + \psi_{i,j-1} - 4\psi_{i,j} = 0 \tag{6}$$

Rearranging for $\psi_{i,j}$, the Point Jacobi formula is:

$$\psi_{i,j}^{n+1} = \frac{1}{4}\left(\psi_{i+1,j}^{n} + \psi_{i-1,j}^{n} + \psi_{i,j+1}^{n} + \psi_{i,j-1}^{n}\right) \tag{7}$$

where $\psi_{i,j}^{n+1}$ is the updated value of $\psi_{i,j}$ at the $(n+1)$th iteration level.

3 Boundary Conditions

The chamber walls are the streamlines, so ψ is constant along the walls.

Right boundary ($x = 3$): ψ_3
Top boundary ($y = 4$): ψ_3
Bottom boundary:

- Left section ($x = 0$ to $x = 1.1$): ψ_3
- Inlet ($x = 1.1$ to $x = 2.0$): ψ_1
- Right section ($x = 2$ to $x = 3$): ψ_3

Internal boundary:

- Lower part ($x = 1.5, y = 1.1$): ψ_1
- Middle part with slot ($x = 1.5, y = 1.1$ to $y = 2.0$): ψ_2
- Upper part ($x = 1.5, y = 2.0$ to $y = 4.0$): ψ_3

	ψ_1	ψ_2	ψ_3
Test 1	100	150	300
Test 2	100	200	300
Test 3	100	250	300

4 Convergence and Error Calculation

The iteration stops when the solution converges. The convergence condition is based on the L_2 norm of the error:

$$\text{ERROR} = \frac{\|\psi_{i,j}^{n+1} - \psi_{i,j}^{n}\|_2}{\|\psi_{i,j}^{n+1}\|_2} < 10^{-4} \tag{8}$$

where the L_2 norm is defined as:

$$\|\psi_{i,j}\|_2 = \sqrt{\sum_{i=2}^{IM-1} \sum_{j=2}^{JM-1} |\psi_{i,j}|^2} \tag{9}$$

Here, IM and JM are the maximum indices for i and j.

The L_2 norm for the difference in the solution at two consecutive iterations is given by:

$$\|\psi_{i,j}^{n+1} - \psi_{i,j}^{n}\|_2 = \sqrt{\sum_{i=2}^{IM-1} \sum_{j=2}^{JM-1} |\psi_{i,j}^{n+1} - \psi_{i,j}^{n}|^2} \tag{10}$$

5 Results

5.1 Test 1 ($\psi 1 = 100, \psi 2 = 150, \psi 3 = 300$)

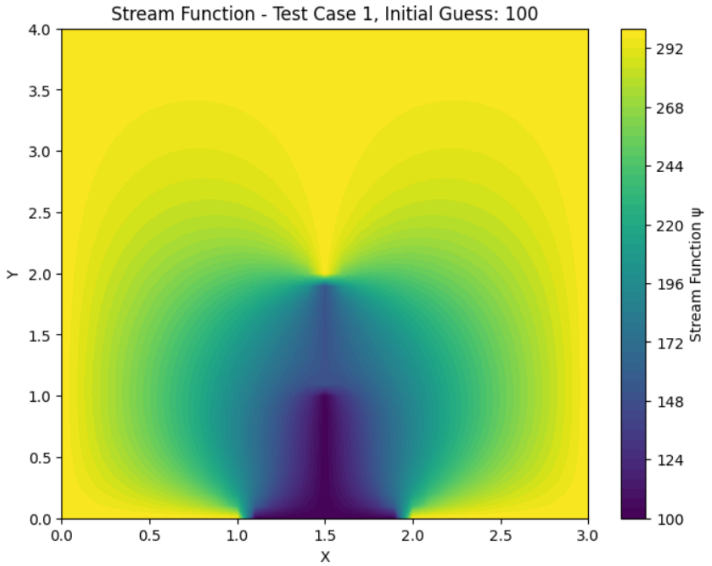


Figure 1: Caption

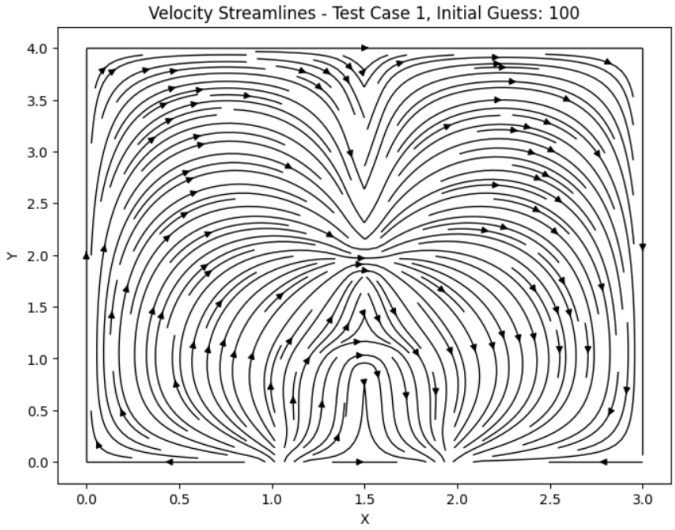
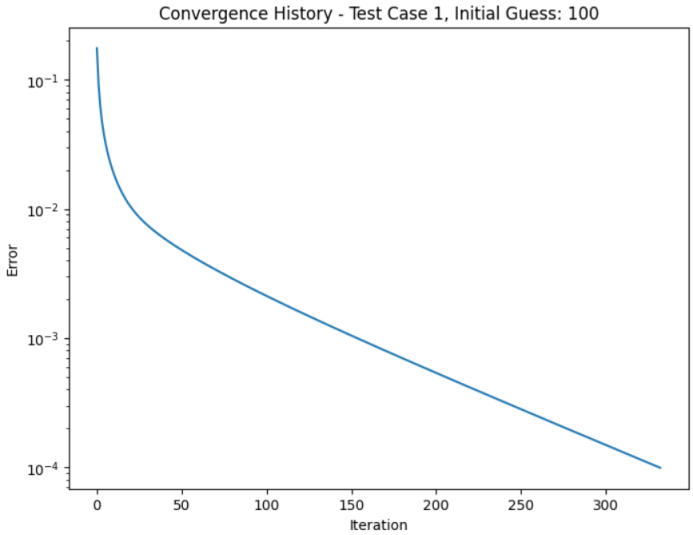


Figure 2: Caption



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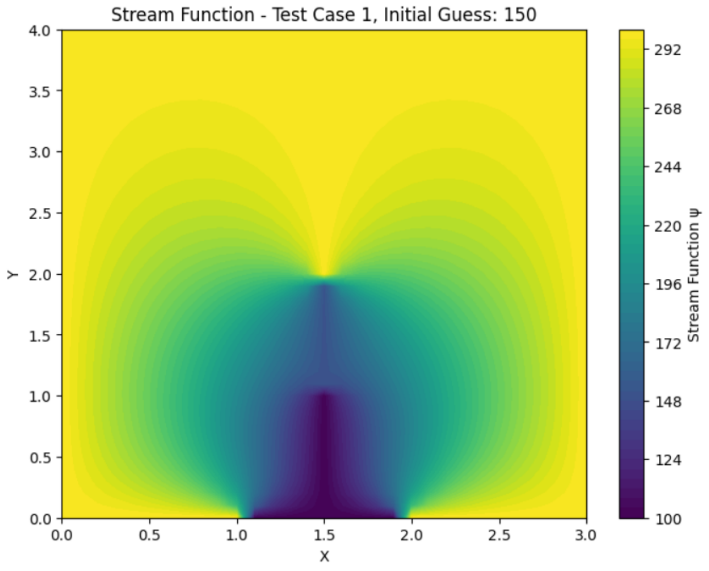

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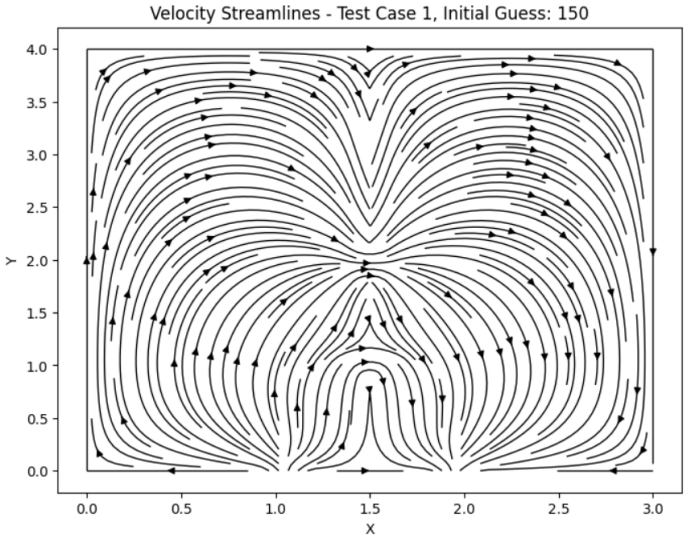

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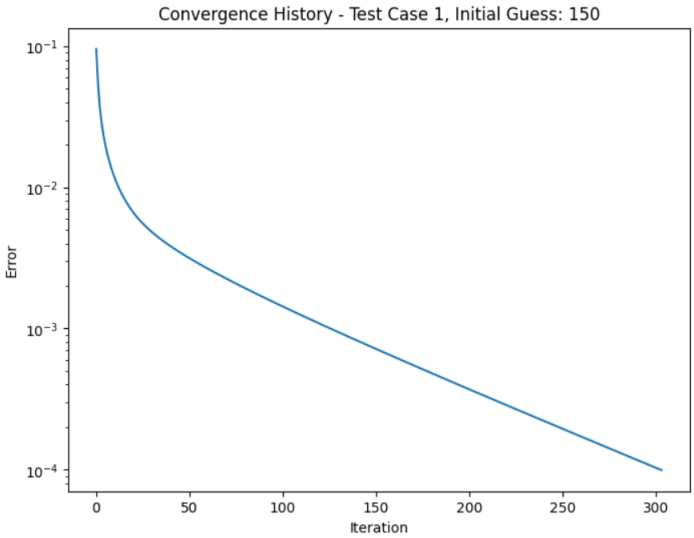

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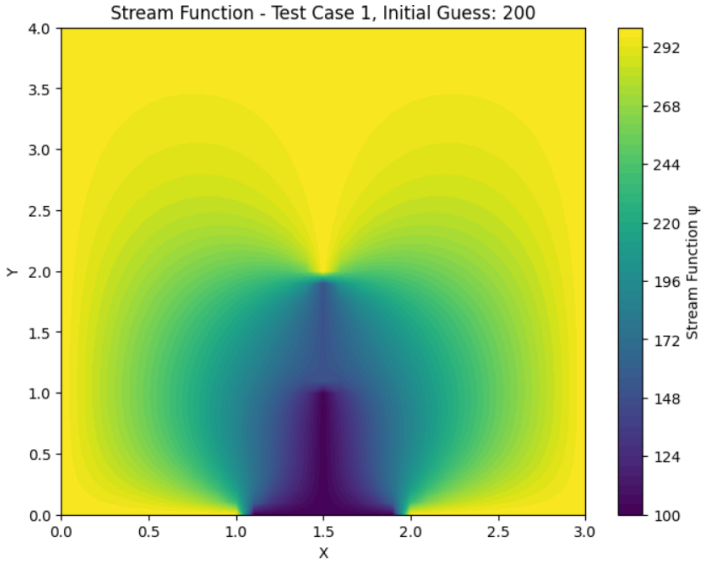

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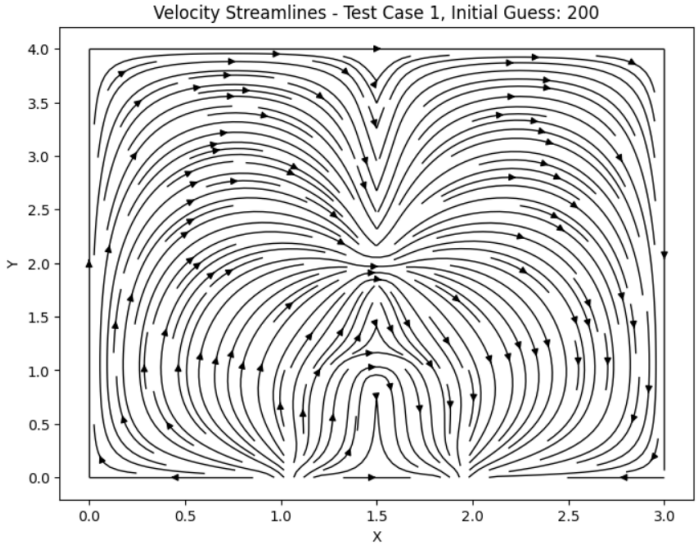

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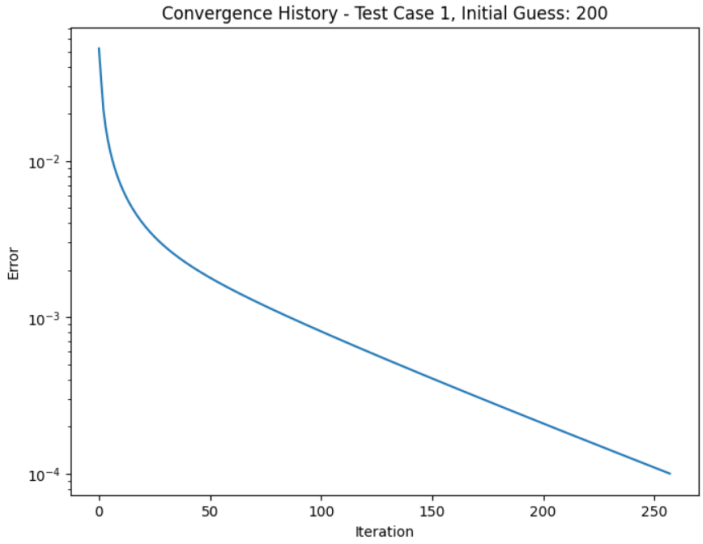
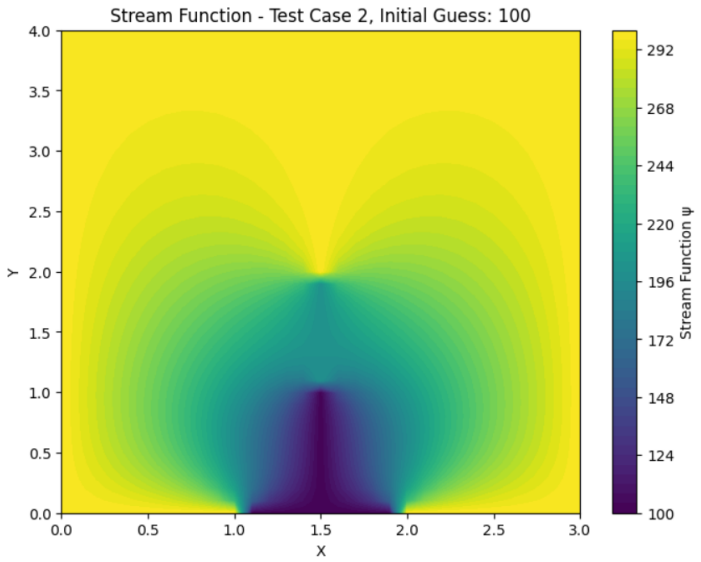


Figure 9: Caption

5.2 Test 2 ($\psi 1 = 100, \psi 2 = 150, \psi 3 = 300$)



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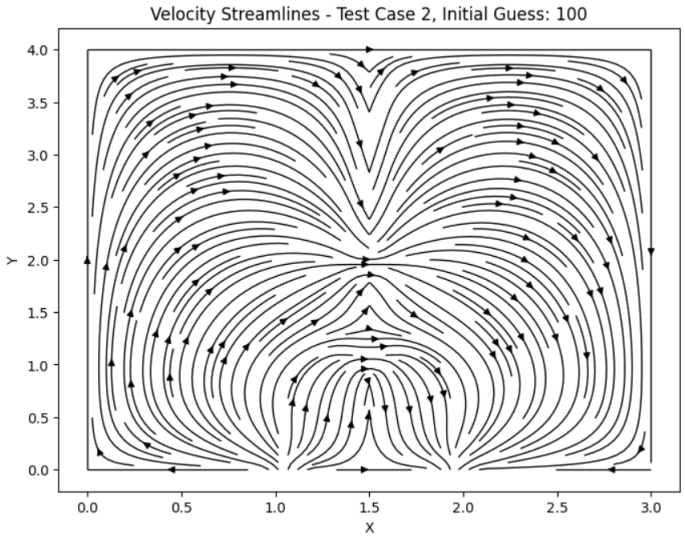

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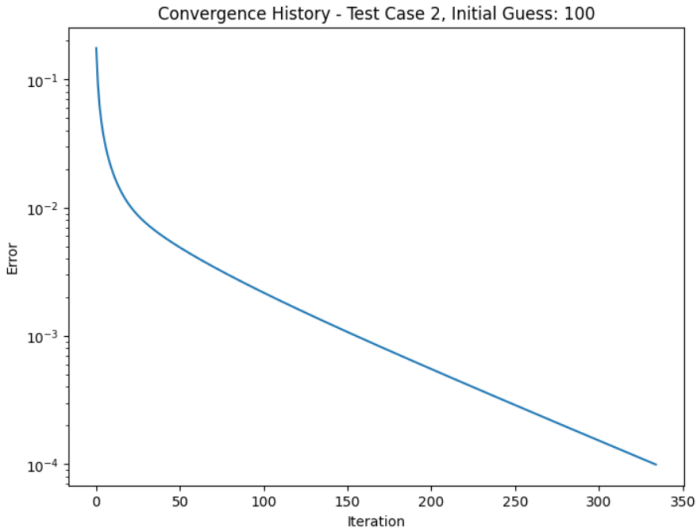

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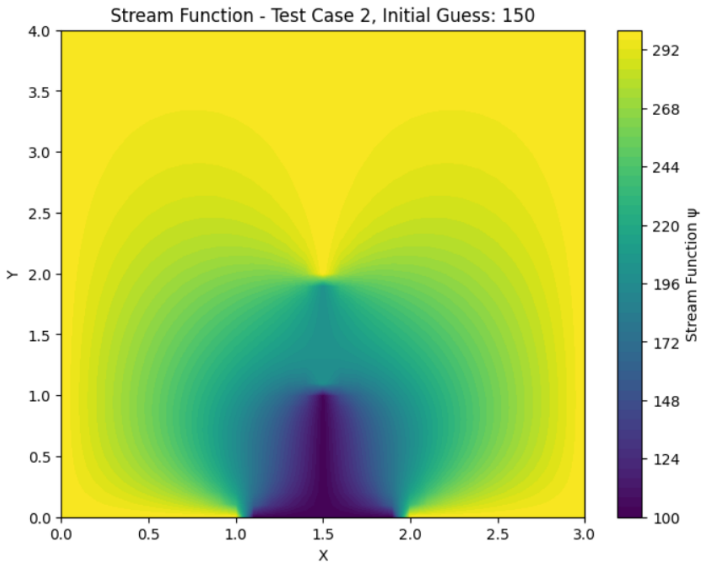

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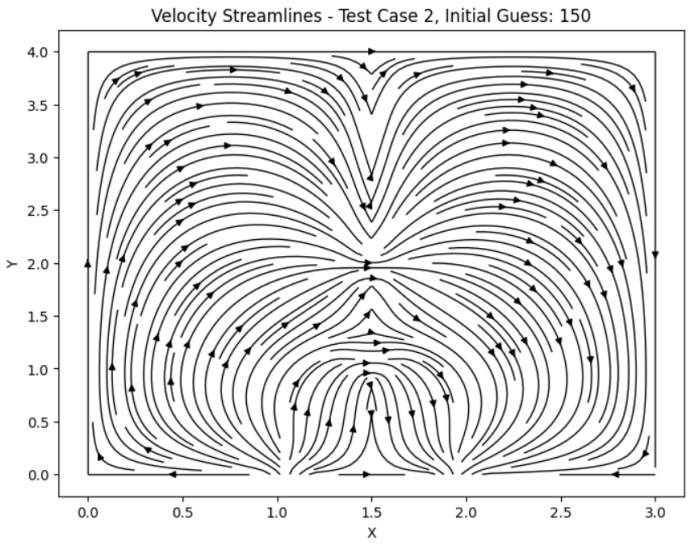

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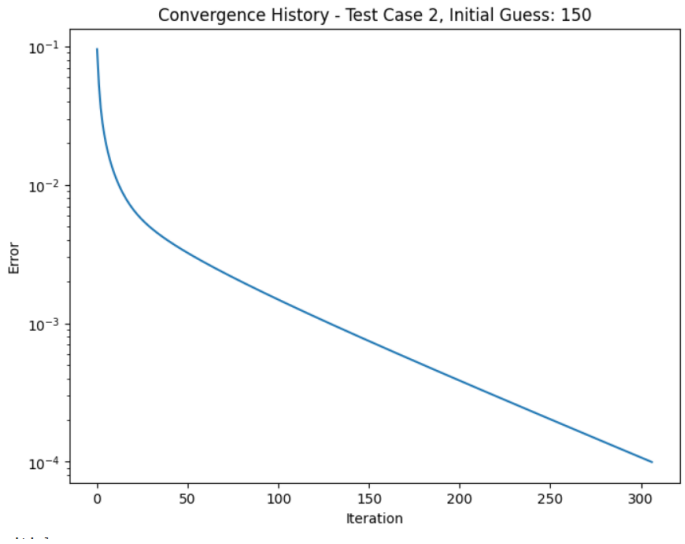

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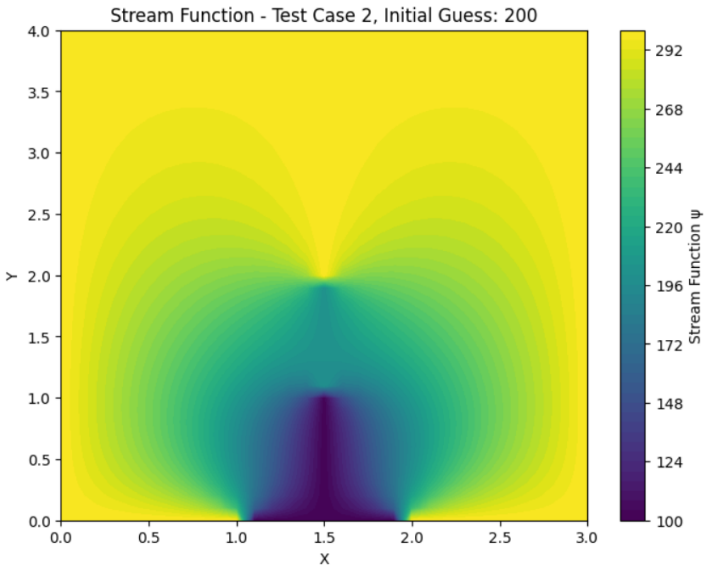

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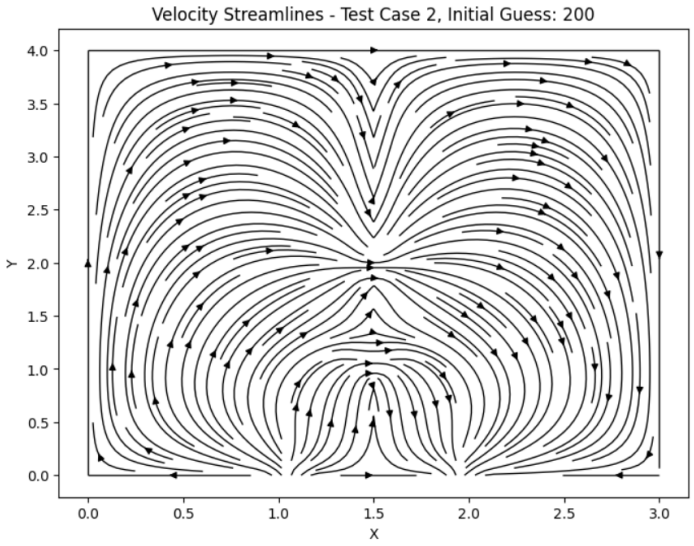

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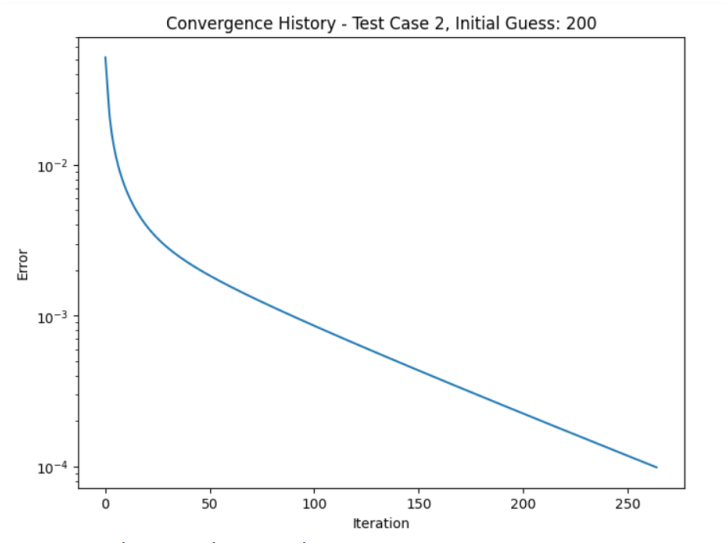
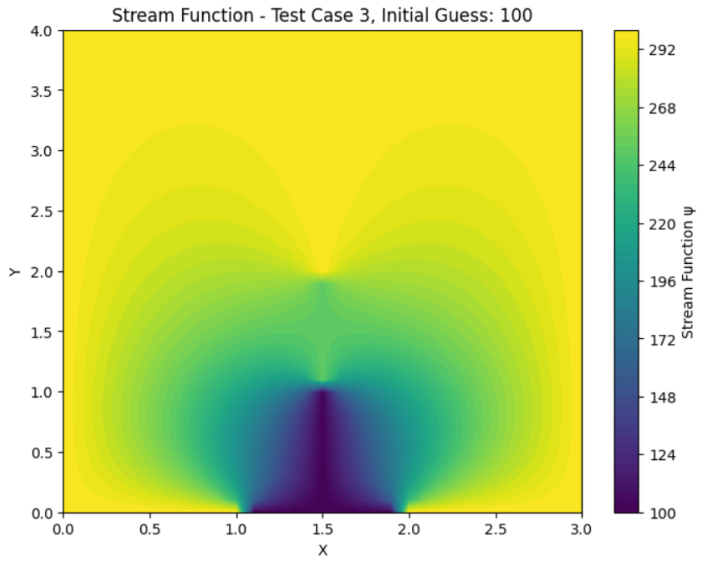


Figure 18: Caption

5.3 Test 2 ($\psi 1 = 100, \psi 2 = 150, \psi 3 = 300$)



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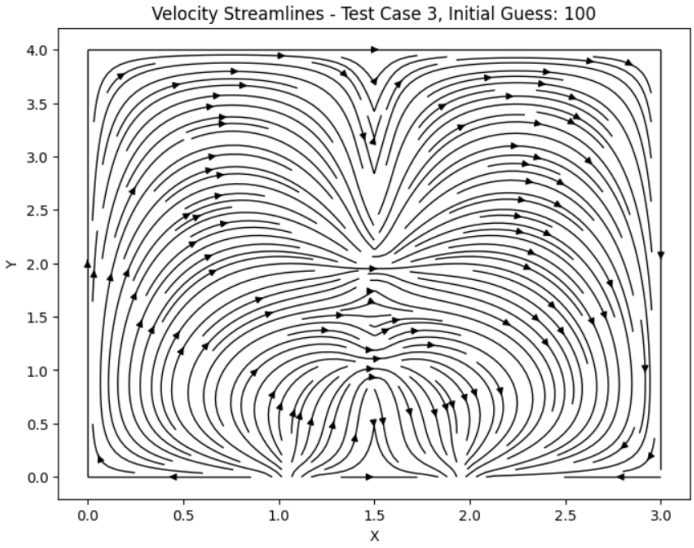

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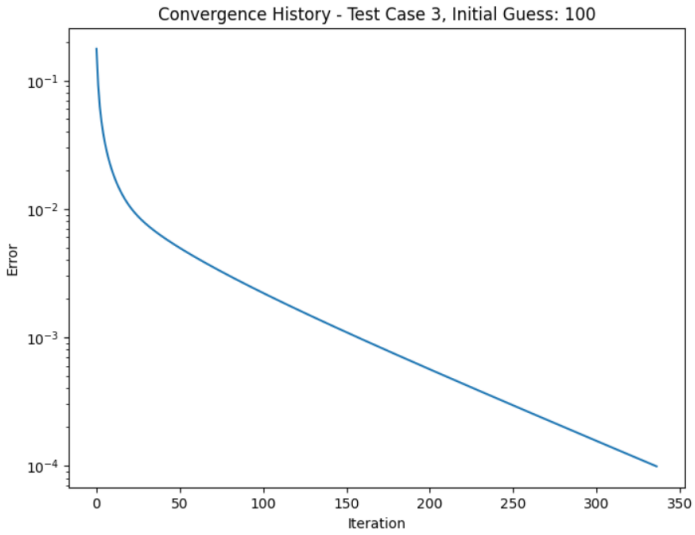

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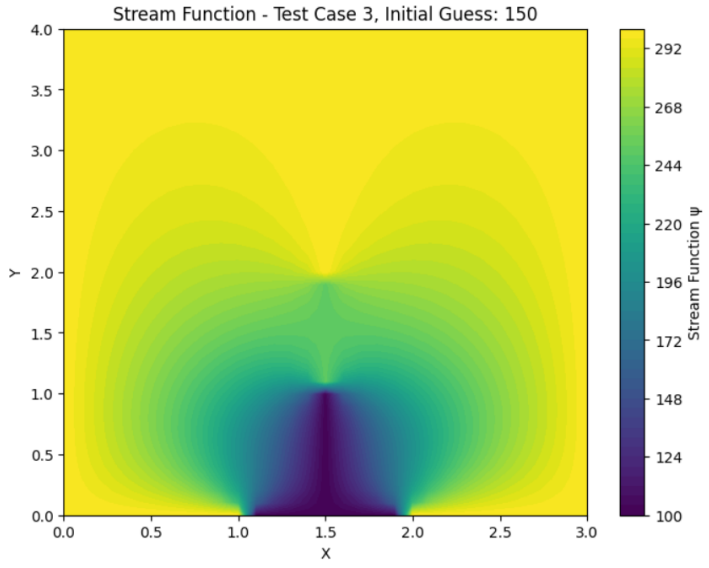

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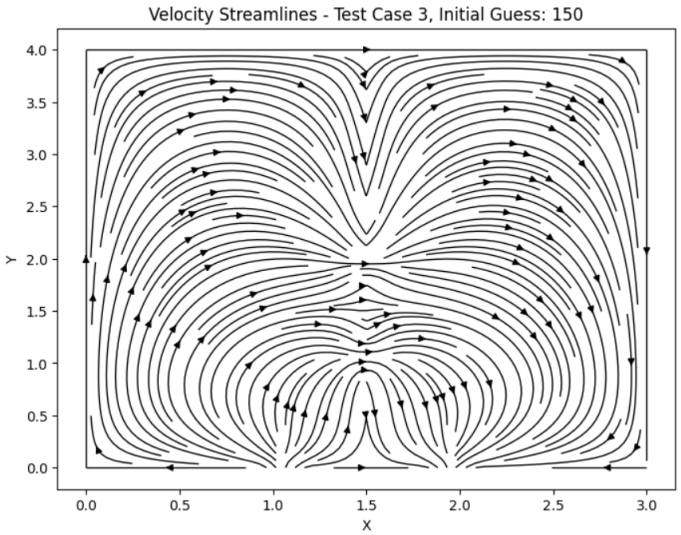

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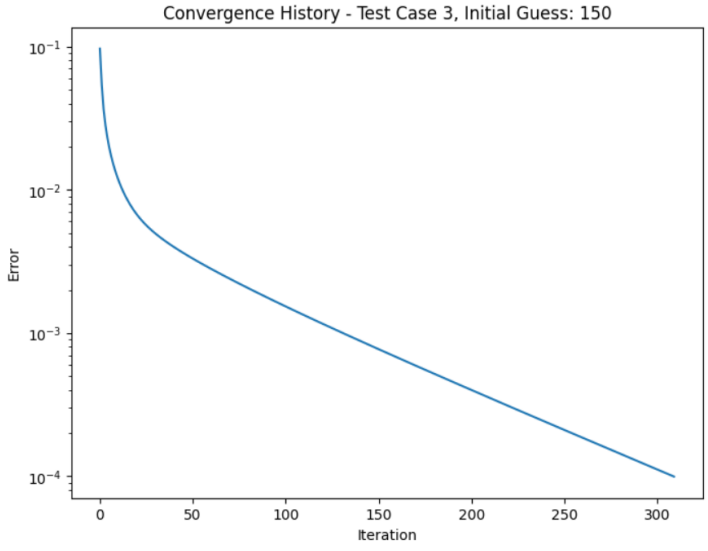

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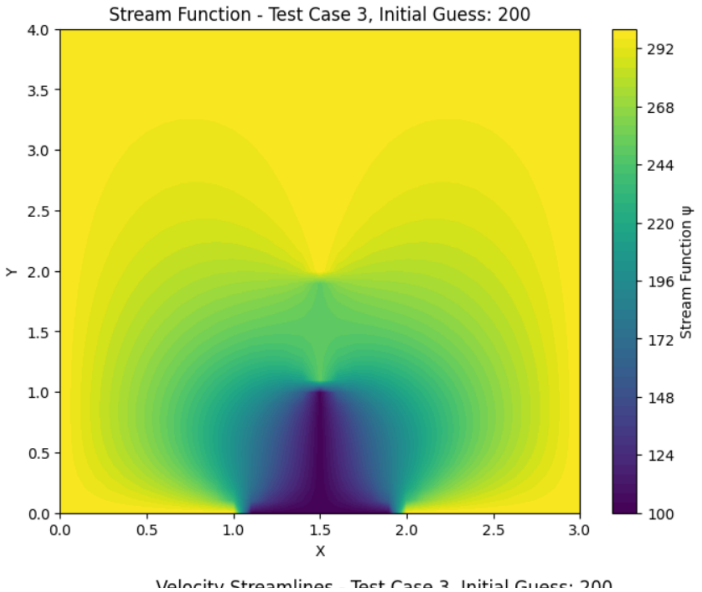

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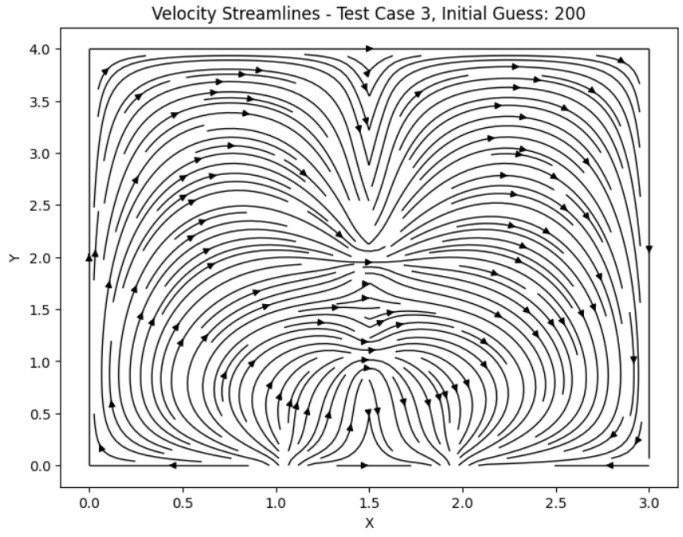

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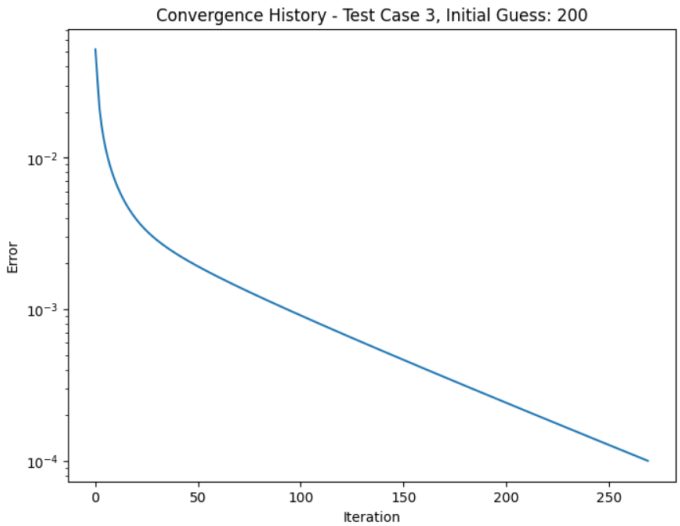

Figure 27: Caption

6 Discussion

1. With the increase of initial guess, the number of iterations decreases, leading to faster convergence. The pattern is visible in all three test cases. For the first test condition, the number of iterations decreases from 334 to 260 as the initial guess increases from 100 to 200. The same pattern is observed in other test cases as well.

2. The order of error for each initial guess and test case is found to be the same.

3. The reason behind faster convergence for an initial guess of 200 is that the stream function inside the chamber is closer to the guess value of 200, as observed in the tabulated results. This leads to a smaller number of iterations, whereas initial guesses of 100 and 150 take more iterations as they are further from the actual stream function values inside the chamber. However, there is no effect on the final solution regardless of the initial guess used.

4. The streamline pattern is consistent with the boundary conditions provided in the problem statement. The walls maintain a constant stream function.

5. The velocity field stream plots appear to originate from the inlet and terminate at the outlet.