Contents

Chapter 1

Triangle

Consider a triangle with vertices

$$\mathbf{A} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 5 \\ 4 \end{pmatrix}, \mathbf{c} = \begin{pmatrix} -4 \\ 0 \end{pmatrix}, \tag{1.1}$$

1.1. Vectors

1.2. Median

1.3. Altitude

1.3.1. $\mathbf{D}_1$ is a point on BC such that

$$AD_1 \perp BC \tag{1.3.1.1}$$

and AD_1 is defined to be the altitude. Find the normal vector of AD_1.

Solution: Given

$$\mathbf{A} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{1.3.1.2}$$

$$\mathbf{B} = \begin{pmatrix} 5 \\ 4 \end{pmatrix} \tag{1.3.1.3}$$

$$\mathbf{C} = \begin{pmatrix} -4 \\ 0 \end{pmatrix} \tag{1.3.1.4}$$

The normal vector of AD_1 is orthogonal to AD_1 and hence parallel to BC

Direction vector $\mathbf{m_{BC}}$

$$= \mathbf{C} - \mathbf{B} \tag{1.3.1.5}$$

$$= \begin{pmatrix} -4 \\ 0 \end{pmatrix} - \begin{pmatrix} 5 \\ 4 \end{pmatrix} \tag{1.3.1.6}$$

$$m_{BC} = \begin{pmatrix} 9 \\ 4 \end{pmatrix} \tag{1.3.1.7}$$

Normal vector of AD_1 is

$$\mathbf{n} = \begin{pmatrix} 9 \\ 4 \end{pmatrix} \tag{1.3.1.8}$$

1.3.2. Find the equation of AD_1.

Solution: from (1.3.1.8)

$$\mathbf{n} = \begin{pmatrix} 9 \\ 4 \end{pmatrix} \tag{1.3.2.1}$$

The equation of AD_1 is

$$\mathbf{n}^\top (\mathbf{x} - \mathbf{A}) = 0 \tag{1.3.2.2}$$

$$\implies \begin{pmatrix} 9 & 4 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 9 & 4 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{1.3.2.3}$$

$$\begin{pmatrix} 9 & 4 \end{pmatrix} \mathbf{x} = 9 \tag{1.3.2.4}$$

see Fig. 1.1

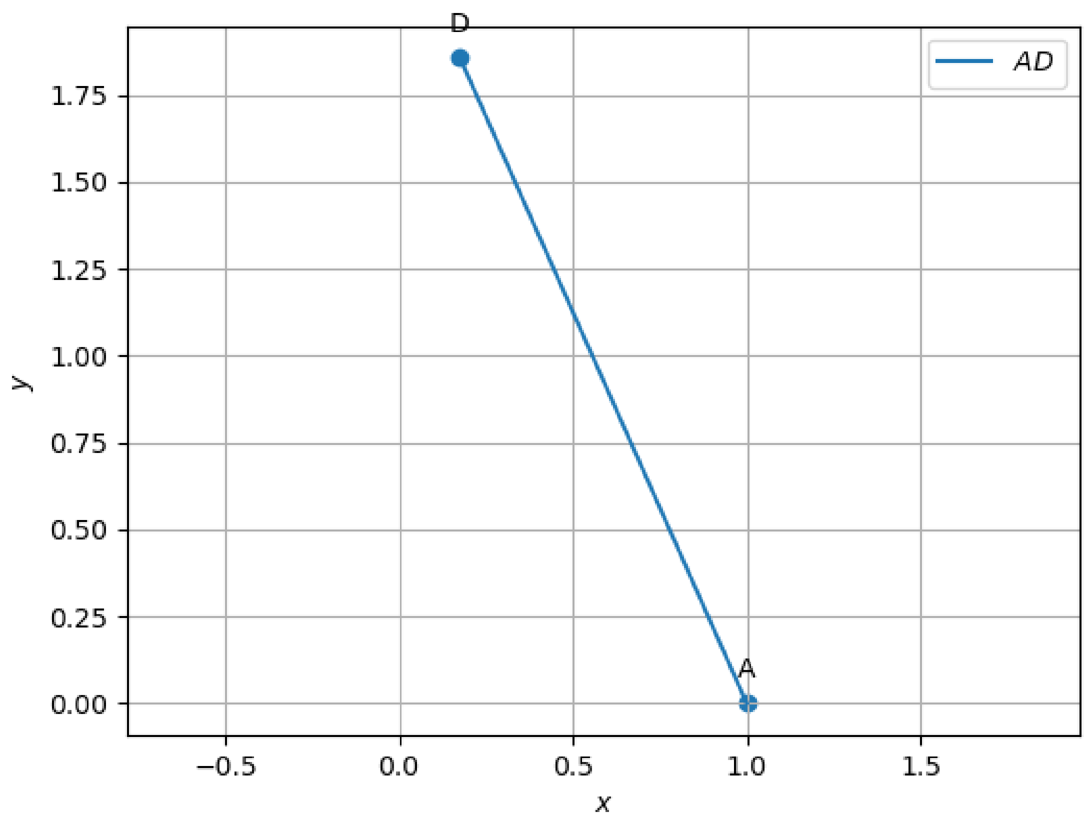

Figure 1.1: Line AD

1.3.3. Find the equations of the altitudes BE_1 and CF_1 to the sides AC and AB respectively.

Solution: Given

$$\mathbf{A} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{1.3.3.1}$$

$$\mathbf{B} = \begin{pmatrix} 5 \\ 4 \end{pmatrix} \tag{1.3.3.2}$$

$$\mathbf{C} = \begin{pmatrix} -4 \\ 0 \end{pmatrix} \tag{1.3.3.3}$$

Direction vector

$$\mathbf{m_{AB}} = \mathbf{B} - \mathbf{A} \tag{1.3.3.4}$$

$$= \begin{pmatrix} 5 \\ 4 \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{1.3.3.5}$$

$$= \begin{pmatrix} -4 \\ -4 \end{pmatrix} \tag{1.3.3.6}$$

$$\mathbf{m_{AC}} = \mathbf{C} - \mathbf{A} \tag{1.3.3.7}$$

$$= \begin{pmatrix} -4 \\ 0 \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{1.3.3.8}$$

$$= \begin{pmatrix} -5 \\ 0 \end{pmatrix} \tag{1.3.3.9}$$

$$\tag{1.3.3.10}$$

Normal vector of BE_1 is orthogonal to BE_1 and hence parallel to AC

and normal vector of $CF_!$ is orthogonal to CF_1 and hence parallel to AB

$$\mathbf{n_{BE_1}} = \mathbf{m_{AC}} \tag{1.3.3.11}$$

$$= \begin{pmatrix} -4 \\ -4 \end{pmatrix} \tag{1.3.3.12}$$

$$\mathbf{n_{CF_1}} = \mathbf{m_{AB}} \tag{1.3.3.13}$$

$$= \begin{pmatrix} -5 \\ 0 \end{pmatrix} \tag{1.3.3.14}$$

$$\tag{1.3.3.15}$$

Equation of line is represented by :

$$\mathbf{n}^\top \left(\mathbf{x} - \mathbf{p} \right) = 0 \tag{1.3.3.16}$$

(a) The equation of line CF_1

$$\mathbf{n}_{CF_1}^\top \left(\mathbf{x} - \mathbf{C} \right) = 0 \tag{1.3.3.17}$$

$$\mathbf{n}^\top CF_1 \mathbf{x} = \mathbf{n}^\top CF_1 \mathbf{C} \tag{1.3.3.18}$$

$$\begin{pmatrix} -5 \\ 0 \end{pmatrix}^\top \mathbf{x} = \begin{pmatrix} -5 \\ 0 \end{pmatrix}^\top \begin{pmatrix} 5 \\ 4 \end{pmatrix} \tag{1.3.3.19}$$

$$\begin{pmatrix} -5 & 0 \end{pmatrix} \mathbf{x} = -25 \tag{1.3.3.20}$$

(b) The equation of line BE_1

$$\mathbf{n}_{BE_1}^{\top}\left(\mathbf{x}-\mathbf{B}\right)=0 \tag{1.3.3.21}$$

$$\mathbf{n}^{\top}CF_1\mathbf{x}=\mathbf{n}^{\top}BE_1\mathbf{B} \tag{1.3.3.22}$$

$$\begin{pmatrix}-4\\-4\end{pmatrix}^{\top}\mathbf{x}=\begin{pmatrix}-4\\-4\end{pmatrix}^{\top}\begin{pmatrix}-4\\0\end{pmatrix} \tag{1.3.3.23}$$

$$\begin{pmatrix}-4 & -4\end{pmatrix}\mathbf{x}=16 \tag{1.3.3.24}$$

see Fig. 1.2

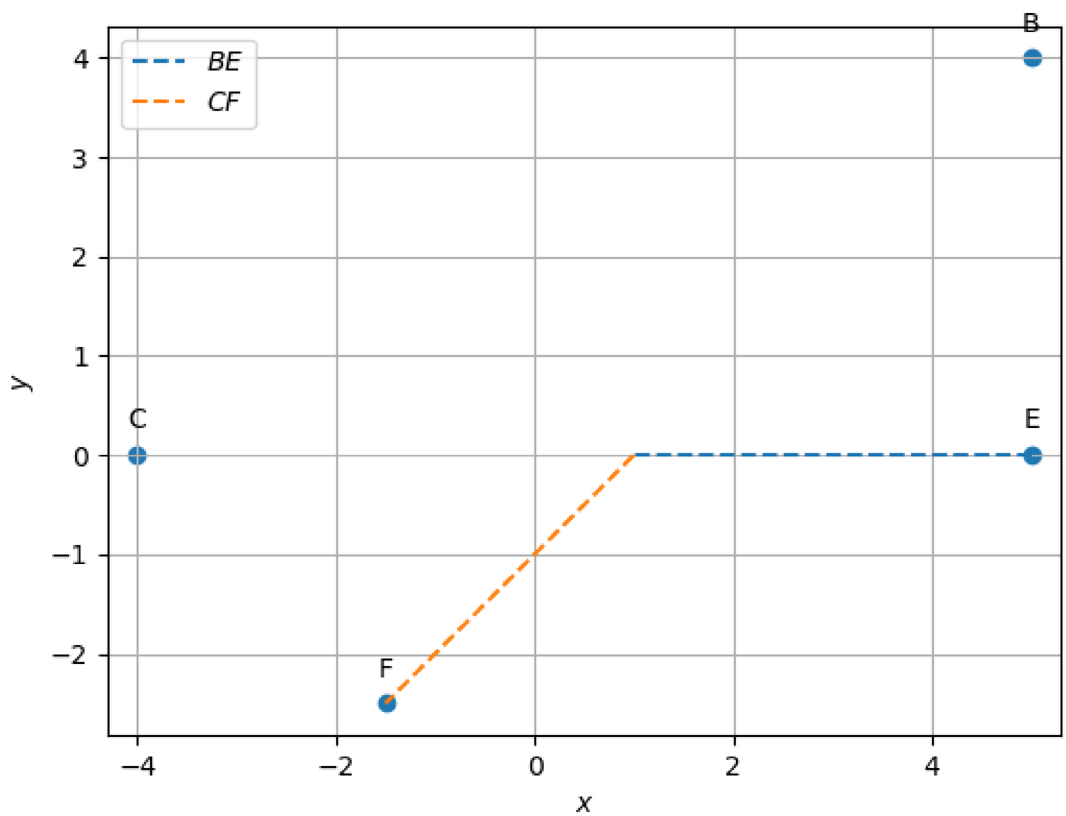

Figure 1.2: Lines $\mathbf{BE_1} and \mathbf{CF_1}$

1.3.4. Find the intersection $\mathbf{H}$ of BE_1 and CF_1.

Solution: Equation of $\mathbf{BE_1}$

$$\begin{pmatrix} -5 & 0 \end{pmatrix} \mathbf{x} = -25 \tag{1.3.4.1}$$

Equation of $\mathbf{CF_1}$

$$\begin{pmatrix} -4 & -4 \end{pmatrix} \mathbf{x} = 16 \tag{1.3.4.2}$$

Therefore ,we need to solve the following equation to get $\mathbf{H}$:

$$\begin{pmatrix} -5 & 0 \\ -4 & -4 \end{pmatrix} \mathbf{x} = \begin{pmatrix} -25 \\ 16 \end{pmatrix} \tag{1.3.4.3}$$

which can be solved as

$$\begin{pmatrix} -5 & 0 & -25 \\ -4 & -4 & 16 \end{pmatrix} \xleftrightarrow{R_1 \leftarrow -R_1/5} \begin{pmatrix} 1 & 0 & 5 \\ -4 & -4 & 16 \end{pmatrix} \tag{1.3.4.4}$$

$$\xleftrightarrow{R_2 \leftarrow R_2 + 4R_1} \begin{pmatrix} 1 & 0 & 5 \\ 0 & -4 & 36 \end{pmatrix} \tag{1.3.4.5}$$

$$\xleftrightarrow{R_2 \leftarrow -R_2/4} \begin{pmatrix} 1 & 0 & 5 \\ 0 & 1 & -9 \end{pmatrix} \tag{1.3.4.6}$$

yielding

$$\mathbf{H} = \begin{pmatrix} 5 \\ -9 \end{pmatrix}, \tag{1.3.4.7}$$

See Fig. 1.3

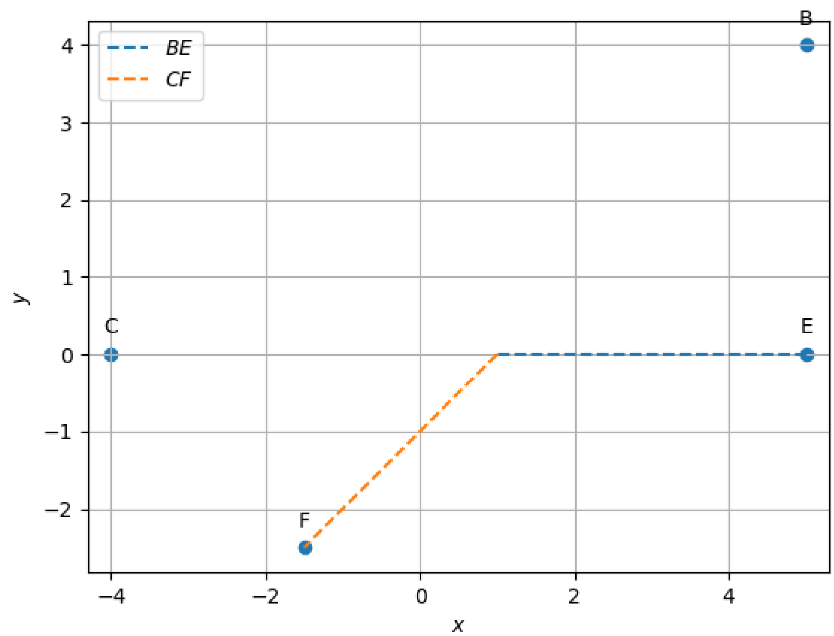

Figure 1.3: Intersection point $\mathbf{H}$ of altitudes BE_1 and CF_1 plotted using python

1.3.5. Verify that

$$\left(\mathbf{A}-\mathbf{H}\right)^{\top}\left(\mathbf{B}-\mathbf{C}\right)=0 \tag{1.3.5.1}$$

Solution:

$$\mathbf{A} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{1.3.5.2}$$

$$\mathbf{B} = \begin{pmatrix} 5 \\ 4 \end{pmatrix} \tag{1.3.5.3}$$

$$\mathbf{C} = \begin{pmatrix} -4 \\ 0 \end{pmatrix} \tag{1.3.5.4}$$

$$\mathbf{H} = \begin{pmatrix} 5 \\ -9 \end{pmatrix} \tag{1.3.5.5}$$

$$\mathbf{A} - \mathbf{H} = \begin{pmatrix} -4 \\ 9 \end{pmatrix}, \mathbf{B} - \mathbf{C} = \begin{pmatrix} 9 \\ 4 \end{pmatrix} \tag{1.3.5.6}$$

$$\implies (\mathbf{A} - \mathbf{H})^\top (\mathbf{B} - \mathbf{C}) = \begin{pmatrix} -4 & 9 \end{pmatrix} \begin{pmatrix} 9 \\ 4 \end{pmatrix} = 0 \tag{1.3.5.7}$$

see Fig. 1.4

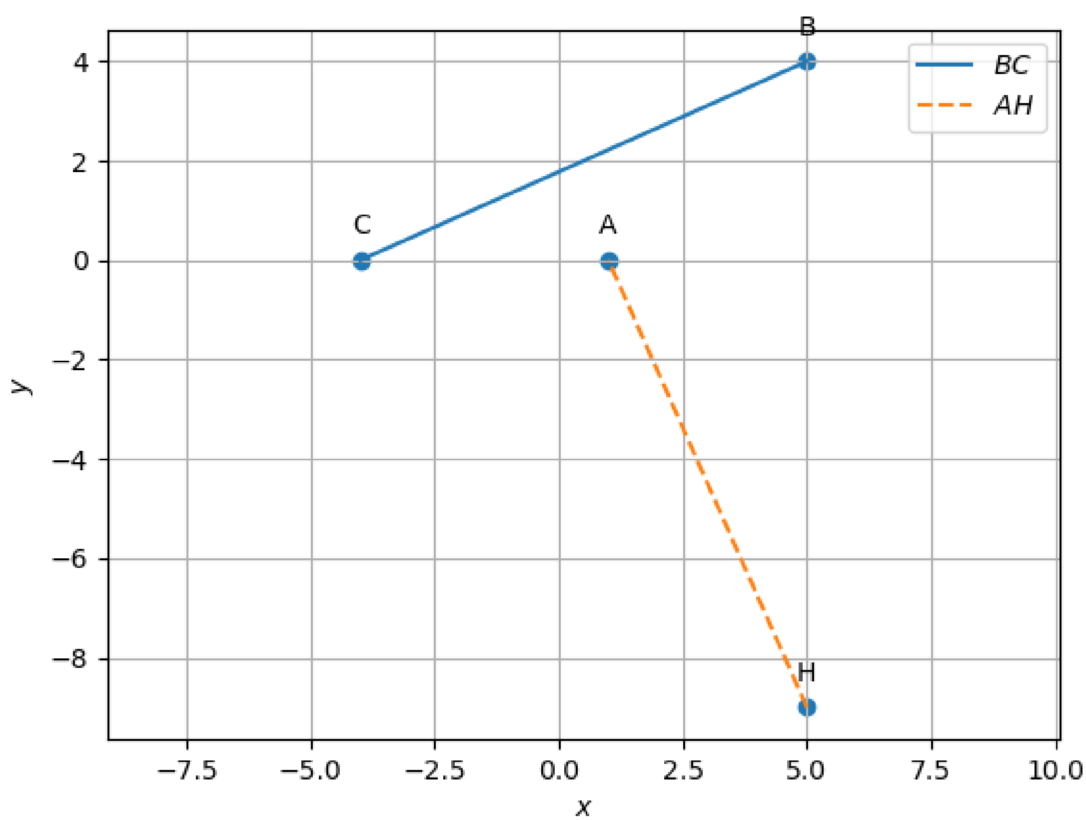

Figure 1.4: Plot of points A,B,C and H