
CBSE MATH

Made Simple

G. V. V. Sharma



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Introduction

This book links high school coordinate geometry to linear algebra and matrix analysis through solved problems.

Chapter 1

Vectors

1.1. 2024

1.1.1. 12

1. **Assertion (A):** The vectors

$$\mathbf{a} = 6\hat{i} + 2\hat{j} - 8\hat{k}$$

$$\mathbf{b} = 10\hat{i} - 2\hat{j} - 6\hat{k}$$

$$\mathbf{c} = 4\hat{i} - 4\hat{j} + 2\hat{k}$$

represent the sides of a right angled triangle.

Reason (R): Three non-zero vectors of which none of two are collinear form a triangle if their resultant is zero vector or sum of any two vectors is equal to the third.

2. Find the vector equation of the line passing through the point (2, 3, -5) and making equal angles with the coordinate axes.
3. Find the projection of vector $(\mathbf{b} + \mathbf{c})$ on vector \mathbf{a} , where $\mathbf{a} = 2\hat{i} + 2\hat{j} +$

$$\hat{k}, \mathbf{b} = \hat{i} + 3\hat{j} + \hat{k}, \text{ and } \mathbf{c} = \hat{i} + \hat{k}.$$

4. Find the coordinates of the foot of the perpendicular drawn from the point $(2, 3, -8)$ to the line $\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$.

Also, find the perpendicular distance of the given point from the line.

5. Find the shortest distance between the lines L_1 & L_2 given below:

L_1 : The line passing through $(2, -1, 1)$ and parallel to $\frac{x}{1} = \frac{y}{1} = \frac{z}{3}$

$$L_2: \mathbf{r} = \hat{i} + (2\mu + 1)\hat{j} - (\mu + 2)\hat{k}.$$

6. Given $\mathbf{a} = 2\hat{i} - \hat{j} + \hat{k}$, $\mathbf{b} = 3\hat{i} - \hat{k}$ and $\mathbf{a} = 2\hat{i} + \hat{j} - 2\hat{k}$. Find a vector \mathbf{d} which is perpendicular to both \mathbf{a} and \mathbf{b} and $\mathbf{c} \cdot \mathbf{d} = 3$.

7. The position vectors of points P and Q are \mathbf{p} and \mathbf{q} respectively. The point R divides the line segment PQ in the ratio $3 : 1$ and \mathbf{S} is the midpoint of the line segment PR . The position vector of \mathbf{S} is:

(a) $\frac{\mathbf{p}+3\mathbf{q}}{4}$

(b) $\frac{\mathbf{p}+3\mathbf{q}}{8}$

(c) $\frac{5\mathbf{p}+3\mathbf{q}}{4}$

(d) $\frac{5\mathbf{p}+3\mathbf{q}}{8}$

8. The Cartesian equation of the line passing through the point $(1, -3, 2)$ and parallel to the line:

$$\mathbf{r} = (2 + \lambda)\hat{i} + \lambda\hat{j} + (-1 + \lambda)\hat{k} \quad (1.1)$$

is:

(a) $\frac{x-1}{2} = \frac{y+3}{0} = \frac{z-2}{-1}$

(b) $\frac{x+1}{1} = \frac{y-3}{1} = \frac{z+2}{2}$

(c) $\frac{x+1}{2} = \frac{y-3}{0} = \frac{z+2}{-1}$

(d) $\frac{x-1}{1} = \frac{y+3}{1} = \frac{z-2}{2}$

9. The Cartesian equation of the line passing through the point $(1, -3, 2)$ and parallel to the line:

$\vec{r} = (2 + \lambda)\hat{i} + \lambda\hat{j} + (2\lambda - 1)\hat{k}$ is

(a) $\frac{x-1}{2} = \frac{y+3}{0} = \frac{z-2}{-1}$

(b) $\frac{x+1}{1} = \frac{y-3}{1} = \frac{z+2}{2}$

(c) $\frac{x+1}{2} = \frac{y-3}{0} = \frac{z+2}{-1}$

(d) $\frac{x-1}{1} = \frac{y+3}{1} = \frac{z-2}{2}$

10. The position vectors of points P and Q are \vec{p} and \vec{q} respectively.

The point R divides the line segment PQ in the ratio $3 : 1$ and S is the mid-point of line segment PR . The position vector of S is:

(a) $\frac{\vec{p}+3\vec{q}}{4}$

(b) $\frac{\vec{p}+3\vec{q}}{8}$

(c) $\frac{5\vec{p}+3\vec{q}}{4}$

(d) $\frac{5\vec{p}+3\vec{q}}{8}$

11. **Assertion (A) :** The vectors

$$\vec{a} = 6\hat{i} + 2\hat{j} - 8\hat{k}$$

$$\vec{b} = 10\hat{i} - 2\hat{j} - 6\hat{k}$$

$$\vec{c} = 4\hat{i} - 4\hat{j} + 2\hat{k}$$

represent the sides of a right angled triangle.

Reason (R) : Three non-zero vectors of which none of two are collinear forms a triangle if their resultant is zero vector or sum of any two vectors is equal to the third.

12. The position vectors of points P and Q are \mathbf{p} and \mathbf{q} respectively. The point R divides line segment PQ in the ratio 3: 1 and S is the mid-point of line segment PR . The position vector of S is :

(a) $\frac{\mathbf{p}+3\mathbf{q}}{4}$

(b) $\frac{\mathbf{p}+3\mathbf{q}}{8}$

(c) $\frac{5\mathbf{p}+3\mathbf{q}}{4}$

(d) $\frac{5\mathbf{p}+3\mathbf{q}}{8}$

13. The angle which the line $\frac{x}{1} = \frac{y}{-1} = \frac{z}{0}$ makes with the positive direction of Y-axis is :

(a) $\frac{5\pi}{6}$

(b) $\frac{3\pi}{4}$

(c) $\frac{5\pi}{4}$

(d) $\frac{7\pi}{4}$

14. The Cartesian equation of the line passing through the point (1,-3,2) and parallel to the line

$$\mathbf{r} = (2 + \lambda)\hat{i} + \lambda\hat{j} + (2\lambda - 1)\hat{k} \text{ is}$$

(a) $\frac{x-1}{2} = \frac{y+3}{0} = \frac{z-2}{-1}$

(b) $\frac{x+1}{2} = \frac{y-3}{1} = \frac{z+2}{2}$

(c) $\frac{x+1}{2} = \frac{y-3}{0} = \frac{z+2}{-1}$

(d) $\frac{x-1}{1} = \frac{y+3}{1} = \frac{z-2}{2}$

Chapter 2

Linear Forms

2.1. 2024

2.1.1. 12

1. Students of a school are taken to a railway museum to learn about railways heritage and its history

An exhibit in the museum depicted many rail lines on the track near



the railway station. let L be the set of all rail lines on the railway track and R be the relation on L defined by

$$R = \{(l_1, l_2) : l_1 \text{ is parallel to } l_2\}$$

On the basis of the above information, answer the following questions:

- (a) Find whether the relation R is symmetric or not.
 - (b) Find whether the relation R is transitive or not.
 - (c) If one of the rail lines on the railway track is represented by the equation $y = 3x + 2$ then find the set of rail lines in R related to it.
2. Let S be the relation defined by $S = \{(l_1, l_2) : l_1 \text{ is perpendicular to } l_2\}$ check whether the relation S is symmetric and transitive.

Chapter 3

Circles

3.1. 2024

3.1.1. 12

1. The area of the circle is increasing at a uniform rate of $2\text{ cm}^2/\text{sec}$.

How fast is the circumference of the circle increasing when the radius
 $r = 5\text{ cm}$?

3.2. 2023

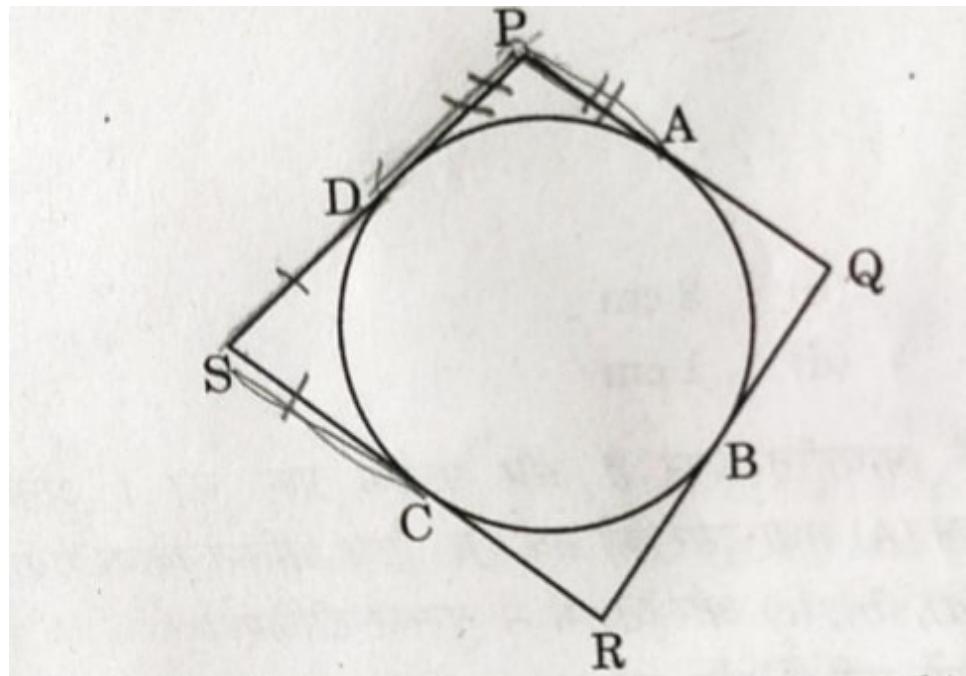
3.2.1. 10

1. The hour-hand of a clock is 6 cm long . The angle swept by it between $7 : 20\text{ a.m.}$ and $7 : 55\text{ a.m.}$ is:

- (a) $(\frac{35}{4})^\circ$
- (b) $(\frac{35}{2})^\circ$
- (c) 35°

(d) 70°

2. In the given figure, the quadrilateral PQRS circumscribes a circleHere
PA + CS is equal to:



- (a) QR
 - (b) PR
 - (c) PS
 - (d) PQ

Chapter 4

Intersection of Conics

Chapter 5

Probability

5.1. 2024

5.1.1. 12

1. Let E be an event of a sample space S of an experiment, then $P(S|E) =$
 - (a) $P(S \cap E)$
 - (b) $P(E)$
 - (c) 1
 - (d) 0
2. A pair of dice is thrown simultaneously. If X denotes the absolute difference of the numbers appearing on top of the dice, then find the probability distribution of X .
3. Airplanes are by far the safest mode of transportation when the number of transported passengers are measured against personal injuries and fatality tools. Previous records state that the probability of an airplane crash is 0.00001%. Further, there are 95% chances that there

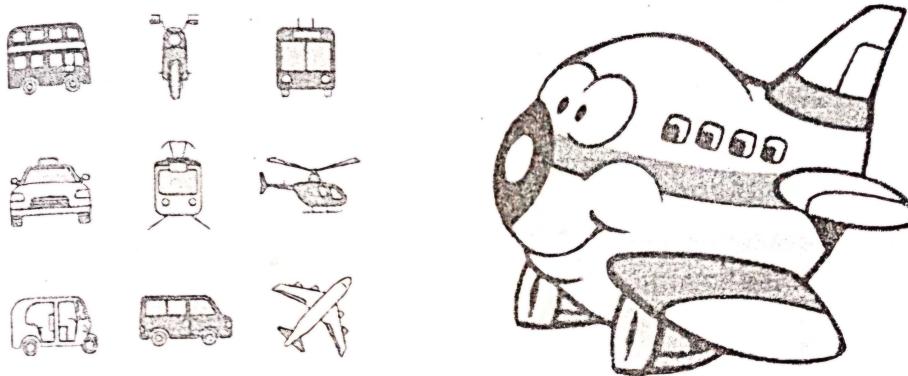


Figure 5.1: 1

will be survivors after a plane crash. Assume that in case of no crash, all travellers survive.

Let E_1 be the event that there is a plane crash and E_2 be the event that there is no crash. Let A be the event that passengers survive after the journey.

On the basis of the above information, answer the following questions:

- (i) Find the probability that the airplane will not crash.
 - (ii) Find $P(A|E_1) + P(A|E_2)$.
 - (iii) Find $P(A)$
 - (iv) Find $P(E_2|A)$.
4. An urn contains 3 red and 2 white marbles. Two marbles are drawn one by one with replacement from the urn. Find the probability distribution of the number of white balls. Also, find the mean of the number of white balls drawn.

5. A departmental store sends bills to charge its customers once a month. Past experience shows that 70% of its customers pay their first month bill in time. The store also found that the customer who pays the bill in time has the probability of 0.8 of paying in time next month and the customer who doesn't pay in time has the probability of 0.4 of paying in time the next month.

Based on the above information, answer the following question:

- (a) Let E_1 and E_2 respectively denote the event of customer paying or not paying the first month bill in time
Find $P(E_1), P(E_2)$.
- (b) Let A denotes the event of customer paying some month's bill in time, then find $P(A|E_1)$ and $P(A|E_2)$.
- (c) Find the probability of customer paying second month's bill in time.
6. Find the probability of customer paying first month's bill in time if it is found that customer has paid the second month's bill in time
7. Bag I contains 3 red and black balls, Bag II contains 5 red and 2 black balls. Two balls are transferred at random from Bag I to Bag II and then a ball is drawn at random from Bag II. Find the probability that the drawn ball is red in colour.
8. If A and B are events such that $P(A | B) = P(B | A) \neq 0$, then:
- (a) $A \subset B$, but $A \neq B$

- (b) $A = B$
- (c) $A \cap B = \emptyset$
- (d) $P(A) = P(B)$

9. The random variable X has the following probability distribution where a and b area some constants:

X	1	2	3	4	5
$P(X)$	0.2	a	a	0.2	b

10. If A and B are events such that $P(A/B) = P(B/A) \neq 0$, then :
- (a) $A \subset B, \text{but} A \neq B$
 - (b) $A = B$
 - (c) $A \cap B = \phi$
 - (d) $P(A) = P(B)$

5.1.2. 10

1. If the probability of a player winning a game is 0.79, then the probability of his losing the same game is:
- (a) 1.79
 - (b) 0.31
 - (c) 0.21
 - (d) 0.21

2. From the data $1, 4, 7, 9, 16, 21, 25$, if all the even numbers are removed, then the probability of getting at random a prime number from the remaining is:
- (a) $\frac{2}{5}$
 - (b) $\frac{1}{5}$
 - (c) $\frac{1}{7}$
 - (d) $\frac{2}{7}$
3. For some data x_1, x_2, \dots, x_n with respective frequencies f_1, f_2, \dots, f_n , the value of $\sum_i^n f_i (x_i - \bar{x})$ is equal to:
- (a) $n\bar{x}$
 - (b) 1
 - (c) Σf_i
 - (d) 0
4. The middle-most observation of every data arranged in order is called:
- (a) mode
 - (b) median
 - (c) mean
 - (d) deviation

5. Two dice are rolled together. The probability of getting a sum of numbers on the two dice as 2, 3, or 5 is:

(a) $\frac{7}{36}$

(b) $\frac{11}{36}$

(c) $\frac{5}{36}$

(d) $\frac{4}{9}$

item At in a pack of 52 playing cards one card is lost. From the remain cards, a card is drawn at random. Find the probability that the drawn card is queen of heart, if the lost card is a black card.

6. BINGO is game of chance. The host has 75 balls numbered 1 through 75. Each player has a BINGO ca rd with some numbers written on it. The participant cancels the number on the card when called out a number written on the ball selected at rand om. Whosoever cancels all the numbers on his/her card, says BINGO and wins the game.



Figure 5.2: image 6

The table given below, shows the data of one game where 48 ball were used before Tara said "BINGO".

Numbers announced	Number of times
0-15	8
15-30	9
30-45	9
45-60	10
60-75	12

Based on the above information, answer the following:

- (a) Write the median class.
- (b) When first ball was picked up, what was the probability of calling out an even number?
- (c) Find median and mode of the given data.

5.2. 2023

5.2.1. 10

1. A student noted the number of cars passing through a spot on a road for 100 periods each of 3 minutes and summarised it in the table given below. Find the mean and median of the following data.

Number of cars	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Frequency (Periods)	7	14	13	12	20	11	15	8

2. Computer-based learning (*CBL*) refers to any teaching methodology that makes use of computers for information transmission. At an elementary school level, computer applications can be used to display multimedia lesson plans. A survey was done on 1000 elementary and secondary schools of Assam and they were classified by the number of computers they had.

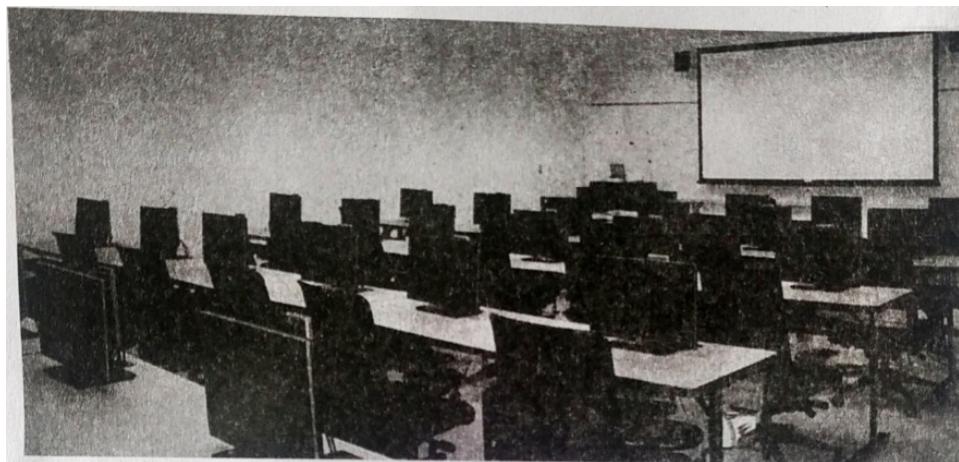


Figure 5.3:

Number of computers	1-10	11-20	21-50	51-100	101 and more
Number of Schools	250	200	290	180	80

One school is chosen at random. Then:

- (a) Find the probability that the school chosen at random has more than 100 computers.
- (b) i. Find the probability that the school chosen at random has 50 or fewer computers.

- ii. Find the probability that the school chosen at random has no more than 20 computers.
- (c) Find the probability that the school chosen at random has 10 or less than 10 computers.
3. For the following distribution:
- | Marks Below | 10 | 20 | 30 | 40 | 50 | 60 |
|---------------------------|----|----|----|----|----|----|
| Number of Students | 3 | 12 | 27 | 57 | 75 | 80 |
- The modal class is:
- (a) $10 - 20$
 - (b) $20 - 30$
 - (c) $30 - 40$
 - (d) $50 - 60$
4. Two dice are thrown together. The probability of getting the difference of numbers on their upper faces equal to 3 is:
- (a) $\frac{1}{9}$
 - (b) $\frac{2}{9}$
 - (c) $\frac{1}{6}$
 - (d) $\frac{1}{12}$
5. A Card is drawn at random from a well-shuffled pack of 52 cards. The probability that the card drawn is not an ace is:

(a) $\frac{1}{13}$

(b) $\frac{9}{13}$

(c) $\frac{4}{13}$

(d) $\frac{12}{13}$

6. DIRECTIONS: In questions number 19 and 20, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct option out of the following: Assertion (A): The probability that a leap year has 53 Sundays is $\frac{2}{7}$.

Reason (R): The probability that a non-leap year has 53 Sundays is $\frac{5}{7}$.

- (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
- (b) Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of Assertion (A).
- (c) Assertion (A) is true but Reason (R) is false.
- (d) Assertion (A) is false but Reason (R) is true.

7. A Bag Contains 100 Cards Numbered 1 to 100 . A Card Is Drawn At Random From The Bag. What Is The Probability That The Number On The Card Is A Perfect Cube?:

(a) $\frac{1}{20}$

(b) $\frac{3}{50}$

(c) $\frac{1}{25}$

(d) $\frac{7}{100}$

8. If three coins are tossed simultaneously, what is the probability of getting at most one tail?:
- (a) $\frac{3}{8}$
 - (b) $\frac{4}{8}$
 - (c) $\frac{5}{8}$
 - (d) $\frac{7}{8}$
9. If a fair coin is tossed twice, find the probability of getting atmost one head.
10. The monthly expenditure on milk in 200 families of a Housing Society is given below:

Monthly Expenditure (in)	1000-1500	1500-2000	2000-2500	2500-3000	3000-3500	3500-4000	4000-4500	4500-5000
Number of Families	24	40	33	x	30	22	16	7

Find the value of x and also, find the median and mean expenditure on milk.

5.3. 2006

5.3.1. 10

1. A card is drawn at random from a well-shuffled deck of playing cards.

Find the probability that the card drawn is

- (a) a card of spades or an ace
- (b) a red king

- (c) neither a king nor a queen
- (d) either a king or a queen.

Chapter 6

Construction

Chapter 7

Optimization

7.1. 2024

7.1.1. 12

1. Solve the following Linear Programming problem graphically:

$$\text{Maximise } Z = 300x + 600y$$

Subject to

$$x + 2y \leq 12 \quad (7.1)$$

$$2x + y \leq 12 \quad (7.2)$$

$$x + \frac{5}{4}y \geq 5 \quad (7.3)$$

$$x \geq 0, y \geq 0. \quad (7.4)$$

2. The surface area of a cube increases at the rate of $72\text{cm}^2/\text{sec}$. find the rate of change of its volume, when the edge of the cube measures 3cm.
3. Solve the following L.P.P. graphically:

Minimise $Z = 6x + 3y$

Subject to constraints

$$4x + y \geq 80$$

$$x + 5y \geq 115$$

$$3x + 2y \leq 150$$

$$x, y \geq 0$$

Chapter 8

Algebra

8.1. 2024

8.1.1. 10

1. If the sum of zeroes of the polynomial $p(x) = 2x^2 - k\sqrt{2}x + 1$ is $\sqrt{2}$, then value of k is:

(a) $\sqrt{2}$

(b) 2

(c) $2\sqrt{2}$

(d) $\frac{1}{2}$

2. If the roots of the equation $ax^2 + bx + c = 0, a \neq 0$ are real and equal, then which of the following relations is true?

(a) $a = \frac{b^2}{c}$

(b) $b^2 = ac$

(c) $ac = \frac{b^2}{4}$

(d) $c = \frac{b^2}{a}$

3. In an A.P., if the first term $a = 7$, n th term $a_n = 84$, and the sum of the first n terms $s_n = \frac{2093}{2}$, then n is equal to:

(a) 22

(b) 24

(c) 23

(d) 26

4. The zeroes of a polynomial $x^2 + px + q$ are twice the zeroes of the polynomial $4x^2 - 5x - 6$. The value of p is:

(a) $-\frac{5}{2}$

(b) $\frac{5}{2}$

(c) -5

(d) 10

5. In the given figure, graphs of two linear equations are shown. The pair of these linear equations is:

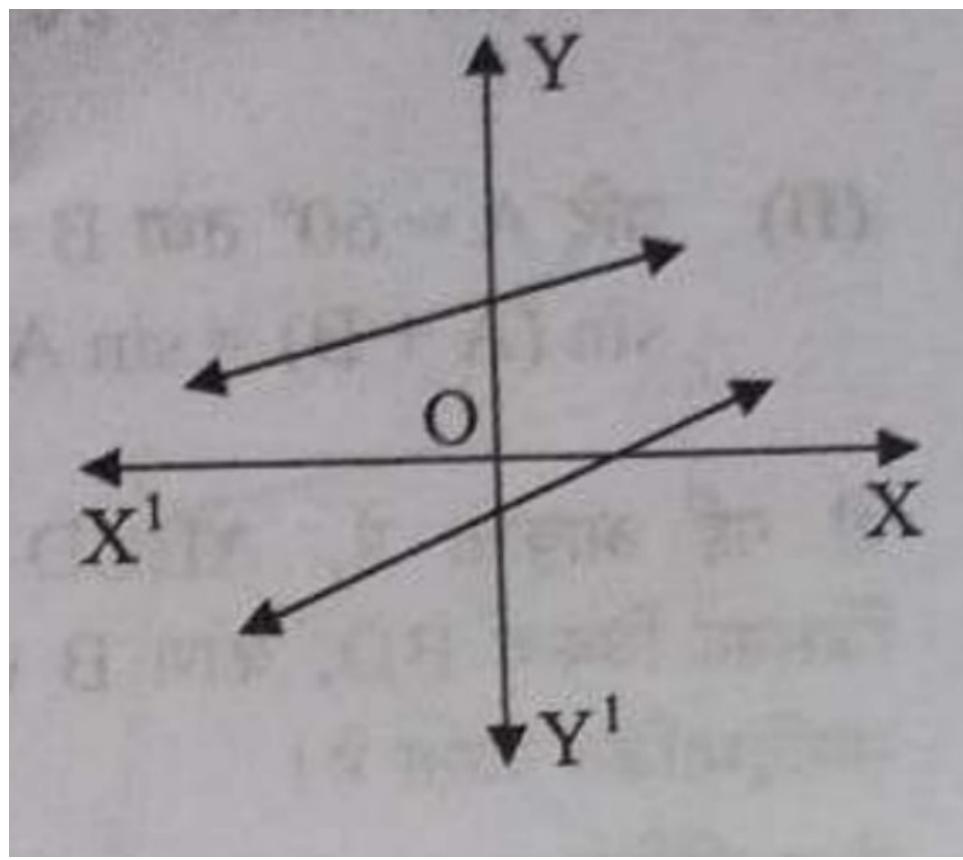


Figure 8.1:

- (a) consistent with a unique solution.
 - (b) consistent with infinitely many solutions.
 - (c) inconsistent.
 - (d) inconsistent but can be made consistent.
6. solve the following system of linear equations $7x - 2y = 5$ and verify

your answer.

7. A rectangular floor area can be completely tiled with 200 square tiles.

If the side length of each tile is increased by 1 unit, it would take only 128 tiles to cover the floor.



Figure 8.2: image5

- (a) Assuming the original length of each side of a tile be x units,
make a quadratic equation from the above information.
 - (b) Write the corresponding quadratic equation in standard form.
 - (c) Find the value of x , the length of side of a tile by factorisation.
8. solve the quadratic equation for x , using quadratic formula.

9. **Assertion (A):** If the graph of a polynomial touches x-axis at only one point, then the polynomial cannot be a quadratic polynomial.

Reason (R): A polynomial of degree n ($n > 1$) can have at most n zeroes.

10. The sum of first and eighth of an A.P. is 32 and their product is 60. Find the first term and common difference of the A.P. Hence, also find the sum of its first 20 terms.

11. In an A.P. of 40 terms, the sum of first 9 terms is 153 and the sum of last 6 terms is 687. Determine the first term and common difference of A.P. Also, find the sum of all the terms of the A.P.

8.1.2. 12

1. let $f : R_+ \rightarrow [-5, \infty)$ be defined as $f(x) = 9x^2 + 6x - 5$ where R_+ is the set of all non-negative real numbers, then f is:

- (a) one-one
- (b) onto
- (c) bijective
- (d) neither one-one nor onto

2. The number of points of discontinuity of $f(x) = \begin{cases} |x| + 3, & if x \leq -3 \\ -2x, & if -3 < x < 3 \\ 6x + 2, & if x \geq 3 \end{cases}$ is:

- (a) 0
- (b) 1
- (c) 2
- (d) infinite
3. The function $f(x) = x^3 - 3x^2 + 12x - 18$ is:
- (a) strictly decreasing on R
- (b) strictly increasing on R
- (c) neither strictly increasing nor strictly decreasing on R
- (d) strictly decreasing on $(-\infty, 0)$
4. Find the domain of the function $f(x) = \sin^{-1}(x^2 - 4)$. Also, find its range.
5. If $f(x) = |\tan 2x|$, then find the value of $f'(x)$ at $x = \frac{\pi}{3}$.
6. If M and m denote the local maximum and local minimum values of the function $f(x) = x + \frac{1}{x}$ ($x \neq 0$) respectively, find the value of $(M - m)$.
7. Show that $f(x) = e^x - e^{-x} + x - \tan^{-1} x$ is strictly increasing in its domain.
8. Show that a function $f : R \rightarrow R$ defined by $f(x) = \frac{2x}{1+x^2}$ is neither one-one nor onto. Further, find set A so that the given function $f : R \rightarrow A$ becomes an onto function.

9. A relation R is defined on $N \times N$ (where N is the set of natural numbers) as:

$$(a, b)R(c, d) \leftrightarrow a - c = b - d$$

Show that R is an equivalence relation.

10. The month of September is celebrated as the Rashtriya Poshan Maah across the country. Following a healthy and well-balanced diet is crucial in order to supply the body with the proper nutrients it needs. A balanced diet also keeps us mentally fit and promotes improved level of energy.

A dietician wishes to minimize the cost of a diet involving two types

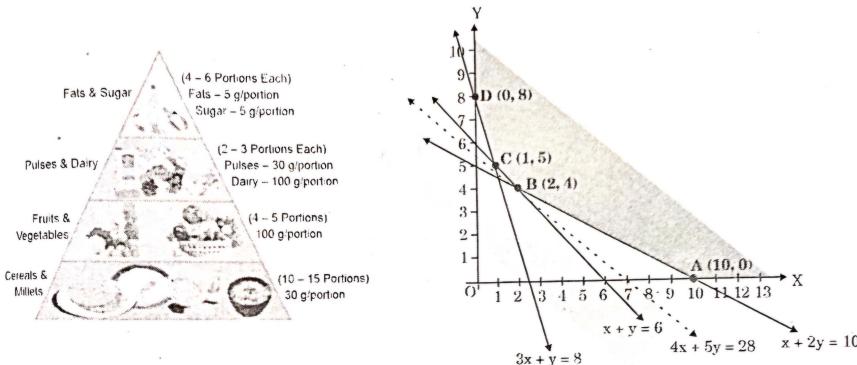


Figure 8.3: 1

of foods, food $X(x\text{kg})$ and food $Y(y\text{kg})$ which are available at the rate of $|16/\text{kg}$ and $|20/\text{kg}$ respectively. The feasible region satisfying the constraints is shown in the graph.

On the basis of the above information, answer the following questions:

- (i) Identify and write all the constraints which determine the given feasible region in the above graph.

- (ii) If the objective is to minimize cost $Z = 16x + 20y$, find the values of x and y at which cost is minimum. Also, find minimum cost assuming that minimum cost is possiblr for the given unbounded region.

8.2. 2023

8.2.1. 10

1. **Assertion (A):** The polynomial $p(x)=x^2 + 3x + 3$ has two real zeroes.

Reason (R) : A quadratic polynomial can have at most two zeroes.

- (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

- (b) Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of Assertion (A).

- (c) Assertion (A) is true but Reason (R) is false.

- (d) Assertion (A) is false but Reason (R) is true.

2. Three bells ring at intervals of 6, 12 and 18 minutes. If all the three bells rang at 6 a.m., when will they ring together again ?

3. If the system of linear equations

$$2x + 3y = 7 \text{ and} \quad (8.1)$$

$$2ax + (a+b)y = 28 \quad (8.2)$$

have infinite number of solutions, then find the values of ' a ' and ' b '.

4. If

$$217x + 131y = 913 \text{ and} \quad (8.3)$$

$$131x + 217y = 827, \quad (8.4)$$

then solve the equations for the values of x and y .

5. How many terms of the arithmetic progression 45, 39, 33, must be taken so that their sum is 180? Explain the double answer.

6. The pair of linear equations $2x = 5y + 6$ and $15y = 6x - 18$ represents two lines which are:

(a) intersecting

(b) parallel

(c) coincident

(d) either intersecting or parallel

7. The next term of the A.P.: $\sqrt{70}, \sqrt{28}, \sqrt{63}$ is:

(a) $\sqrt{70}$

(b) $\sqrt{80}$

(c) $\sqrt{97}$

(d) $\sqrt{112}$

8. The roots of the equation $x^2 + 3x - 10 = 0$ are:

(a) 2, -5

(b) -2, 5

(c) 2, 5

(d) -2, -5

9. If α, β are zeroes of the polynomial $x^2 - 1$, then the value of (($\alpha + \beta$) is:

(a) 2

(b) 1

(c) -1

(d) 0

10. If α, β are the zeroes of the polynomial $p(x) = 4x^2 - 3x - 7$, then $\frac{1}{\alpha} + \frac{1}{\beta}$ is equal to:

(a) $\frac{7}{3}$

(b) $-\frac{7}{3}$

(c) $\frac{3}{7}$

(d) $-\frac{3}{7}$

11. Which of the following quadratic equations has a sum of its roots as 4 ?:

- (a) $2x^2 - 4x + 8 = 0$
- (b) $-x^2 - 4x + 4 = 0$
- (c) $\sqrt{2}x^2 - 4 \div \sqrt{2}x + 1 = 0$
- (d) $4x^2 - 4x + 4 = 0$

12. If the pair of equations $3x - y + 8 = 0$ and $6x - ry + 16 = 0$ represent coincident lines, then the value of 'r' is?:

- (a) $\frac{-1}{2}$
- (b) $\frac{1}{2}$
- (c) -2
- (d) 2

13. The Pair Of Equations $x = a$ and $y = b$ graphically represents Lines Which Are?:

- (a) *parallel*
- (b) *intersecting at (b, a)*
- (c) *coincident*
- (d) *intersecting at (a, b)*

14. If one zero of the polynomial $6x^2 + 37x - (k - 2)$ is the reciprocal of the other, then what is the value of k?:

- (a) -4

(b) -6

(c) 6

(d) 4

15. The zeroes of the polynomial $p(x) = x^2 + 4x + 3$ are given by:

(a) (1, 3)

(b) (-1, 3)

(c) (1, -3)

(d) (-1, -3)

16. If a and b are the zeroes of the quadratic polynomial $p(x) = x^2 - ax - b$ then the value of $\alpha^2 + \beta^2$ is:

(a) $a^2 - 2b$

(b) $a^2 + 2b$

(c) $b^2 - 2a$

(d) $b^2 + 2a$

17. Find the sum and product of the roots of the quadratic equation $2x^2 - 9x + 4 = 0$.

18. Find the discriminant of the quadratic equation $4x^2 - 5 = 0$ and hence comment on the nature of roots of the equation.

19. If one zero of polynomial $p(x) = 6x^2 + 37x - (k - 2)$ is reciprocal of the other, then find the value of k .

20. Find the value of ' p ' for which one root of the quadratic equation $px^2 - 14x + 8 = 0$ is 6 times the other.

8.3. 2006

8.3.1. 10

1. Solve the system of equations:

$$\frac{bx}{a} - \frac{ay}{b} + a + b = 0 \quad \text{and} \quad bx - ay + 2ab = 0.$$

2. Given that:

$$P = \frac{x+2y}{x+y} + \frac{x}{y}, Q = \frac{x+y}{x-y} - \frac{x-y}{x+y} \quad \text{and} \quad R = \frac{x+2y}{x+y} - \frac{x}{x+y}$$

3. If $(x+2)(x-3)$ is the HCF of the polynomials $p(x) = (x^2+x-2)(3x^2-8x+c)$ and $q(x) = (x^2+x-12)(2x^2+x+b)$, find the values of c and b .

4. Using the quadratic formula, solve the equation: $A^2b^2x^2 - (4b^4 - 3a^4)x - 12a^2b^2 = 0$.

5. A household article is available for ₹970 cash or ₹210 cash down payment followed by three equal monthly installments. If the rate of interest charged under the installment plan is 16% per annum, find the amount of each installment

6. A man borrows ₹25,200 from a finance company and has to repay it in two equal annual installments. If the interest is charged at the rate of 10% per annum compounded annually, calculate the amount of each installment.

7. The speed of a boat in still water is 11 km/hr. It can go 12 km upstream and return downstream to the original point in 2 hours 45 minutes. Find the speed of the stream.

8. A bucket made up of a metal sheet is in the form of a frustum of a cone. Its depth is 24 cm and the diameters of the top and bottom are 30 cm and 10 cm respectively. Find the cost of milk which can completely fill the bucket at the rate of ₹20 per liter and the cost of the metal sheet used, if it costs ₹10 per 100 cm². (*Use $\pi = 3.14$*)

9. Mrs. Ruchi's salary is Rs.32,250 per month exclusive of HRA. She donates ₹12,000 to Prime Minister's Relief Fund (100% exemption). She also donates ₹6,000 to a school and gets a relief of 50% on this donation. She contributes ₹5,000 per month towards her Provident Fund. She pays a quarterly premium of Rs.2,500 towards her LIC policy and invests ₹25,000 in NSCs. If ₹2,700 is the tax deducted each month from her salary for 11 months, find the tax deducted from her salary in the last month of the year.

Income Tax Rates:

Slab	Tax Rate
Up to Rs.1, 35, 000	No tax
From Rs.1, 35, 001 to Rs.1, 50, 000	10% of the taxable income above Rs.1, 35, 000
From Rs.1, 50, 001 to Rs.2, 50, 000	Rs.1, 500 + 20% of the income exceeding Rs.1, 50, 000
Rs.2, 50, 001 and above	Rs.21, 500 + 30% of the amount exceeding Rs.2, 50, 000

Education Cess: 2% of the income tax

Chapter 9

Geometry

9.1. 2024

9.1.1. 10

1. A solid sphere is cut into two hemispheres. The ratio of the surface areas of the sphere to that of the two hemispheres taken together is:

(a) 1 : 1

(b) 1 : 4

(c) 2 : 3

(d) 3 : 2

2. The volume of the largest right circular cone that can be carved out from a solid cube of edge 2 cm is:

(a) $\frac{4\pi}{3}$ cu cm

(b) $\frac{5\pi}{3}$ cu cm

(c) $\frac{8\pi}{3}$ cu cm

(d) $\frac{2\pi}{3}$ cu cm

3. **Assertion (A):** The tangents drawn at the end points of a diameter of a circle are parallel.

Reason (R): The diameter of a circle is the longest chord.

- (a) Both Assertion (A) and Reason (R) are true, and Reason (R) is the correct explanation of Assertion (A).
- (b) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation for Assertion (A).
- (c) Assertion (A) is true, but Reason (R) is false.
- (d) Assertion (A) is false, but Reason (R) is true.

4. AD is a median of $\triangle ABC$ with vertices $A(5, -6)$, $B(6, 4)$, and $C(0, 0)$.

The length of AD is equal to:

- (a) $\sqrt{68}$ units
- (b) $2\sqrt{15}$ units
- (c) $\sqrt{101}$ units
- (d) 10 units

5. If the distance between the points $(3, -5)$ and $(x, -5)$ is 15 units, then the values of x are:

- (a) 12, -18
- (b) -12, 18
- (c) 18, 5

(d) $-9, -12$

6. The center of a circle is at $(2, 0)$. If one end of a diameter is at $(6, 0)$, then the other end is at:

(a) $(0, 0)$

(b) $(4, 0)$

(c) $(-2, 0)$

(d) $(-6, 0)$

7. In the given figure, $ABCD$ is a quadrilateral. diagonal BD bisects $\angle B$ and $\angle D$ both.

Prove that :

(a) $\triangle ABD \sim \triangle CBD$

(b) $AB = BC$

8. Find the ratio in which the point $(8, y)$ divides the line segment joining the points $(1, 2)$ and $(2, 3)$. Also, find the value of y .

9. $ABCD$ is a rectangle formed by the points $A(-1, -1), B(-1, 6), C(3, 6)$ and $D(3, 1)$, P, Q, R and S are mid-points of sides AB, BC, CD and DA respectively. Show that diagonals of the quadrilateral $PQRS$ bisect each other.

10. In the given figure, AB is a diameter of the circle O . AQ, BP and PQ are tangents to the circle. Prove that $\angle POQ = 90^\circ$.

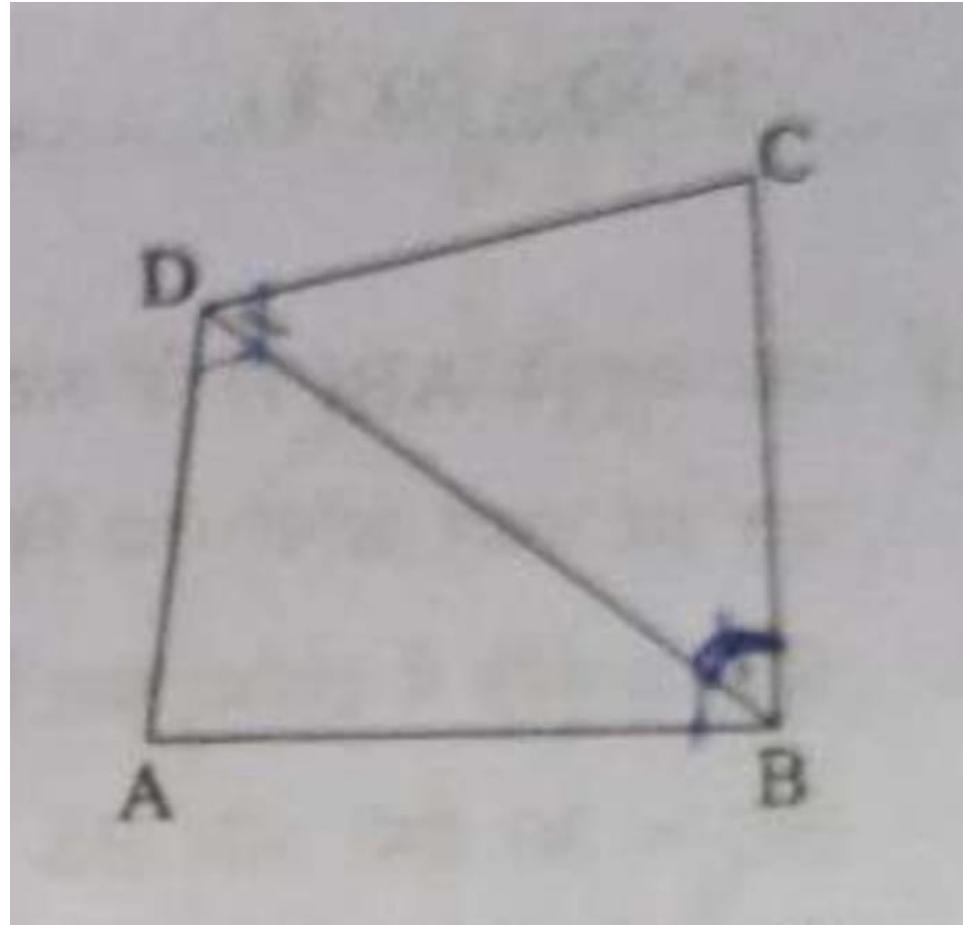


Figure 9.1: image 1

11. A circle with centre O and radius 8cm is inscribed in a quadrilateral $ABCD$ in which P, Q, R, S are the points of contact as shown. If AD is perpendicular to DC , $BC = 30\text{cm}$ and $BS = 24\text{cm}$, then find the length DC .

12. The difference between the outer and inner radii of a hollow right circular cylinder of length 14cm is 1cm. If the volume of the metal

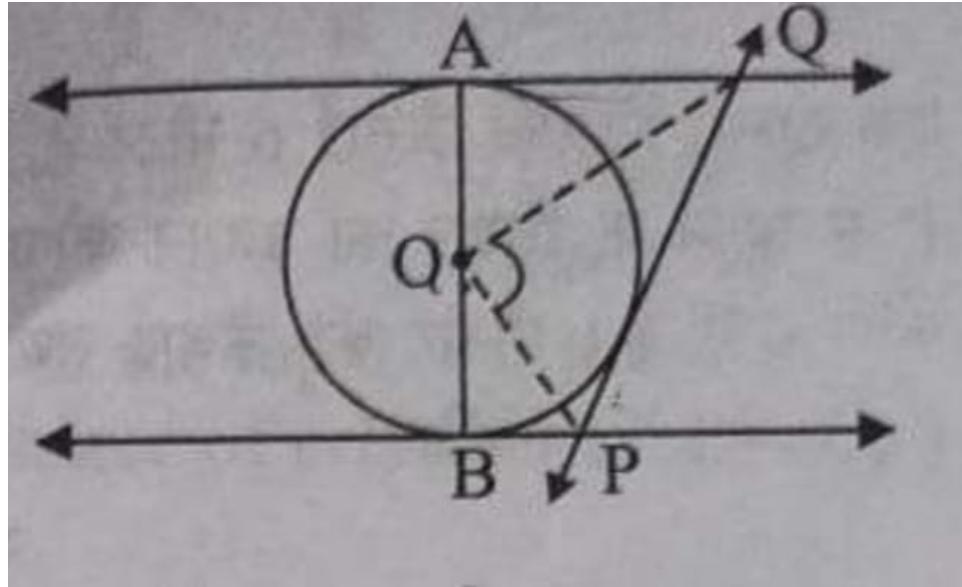


Figure 9.2: image 2

used in making the cylinder is 176cm^3 , find the outer and inner radii of the cylinder.

13. An arc of a circle of radius 21cm subtends an angle of 60° at the centre.

Find:

(a) the length of the arc.

(b) the area of the minor segment of the circle made by the corresponding chord.

14. If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then prove that the other two sides are divided in the same ratio.

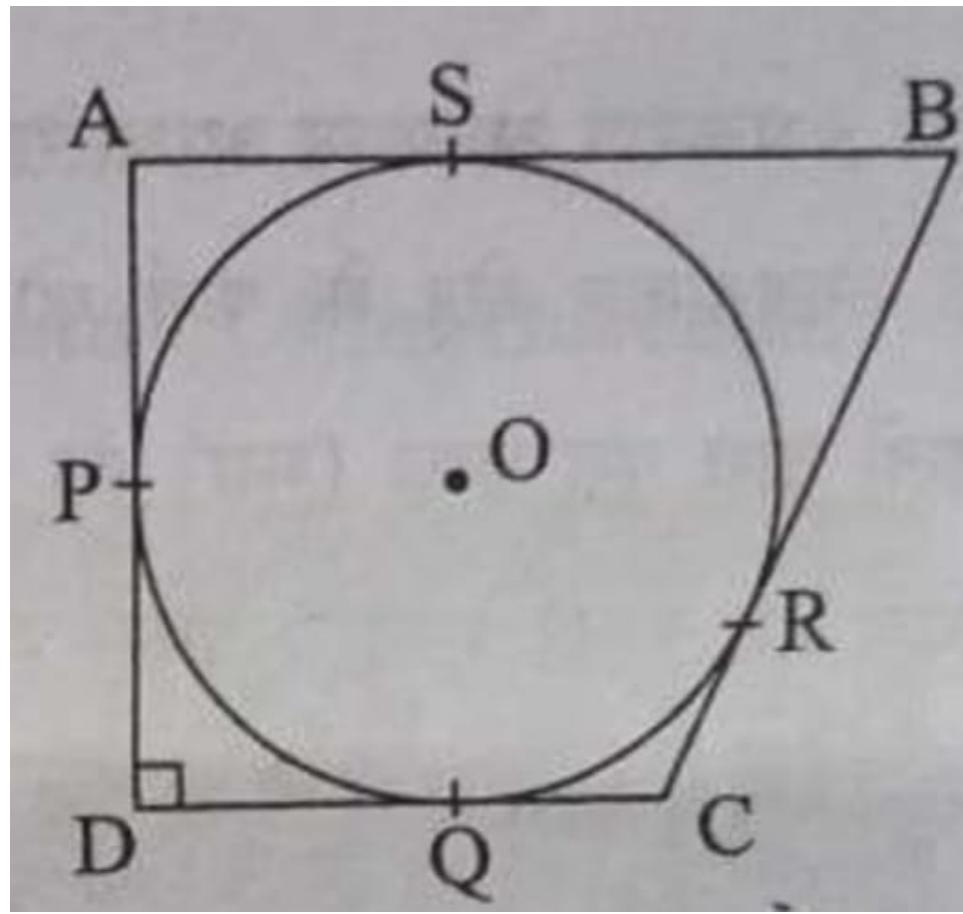


Figure 9.3: image 3

15. In the given figure PA, QB and RC are each perpendicular to AC . if

$$AP = x, bq = y \text{ and } cr = z, \text{ then prove that } \frac{1}{x} + \frac{1}{z} = \frac{1}{y}$$

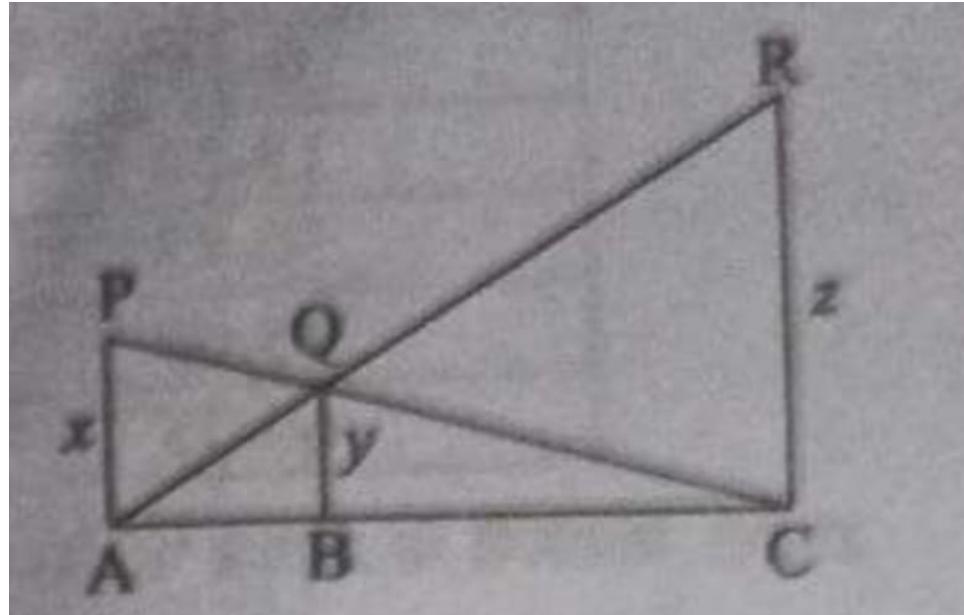


Figure 9.4: image 4

16. A backyard is in the shape of a triangle ABC with right angle at $\angle B = 90^\circ$. The side $BC = 15m$ and $AB = 7m$. A circular pit was dug inside it such that it touches the walls AC , BC and AB at P , Q and R respectively such that $AP = xm$. Based on the above information, answer the following question:

- find the length of AR in terms of x .
- write the type of quadrilateral of $BQOR$.
- Find the length C in terms of x and hence find the value of x .
- Find x and hence find the radius r of circle.

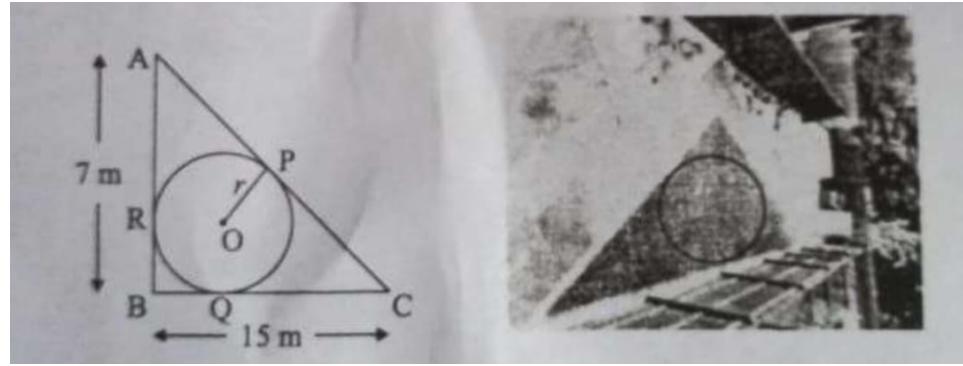


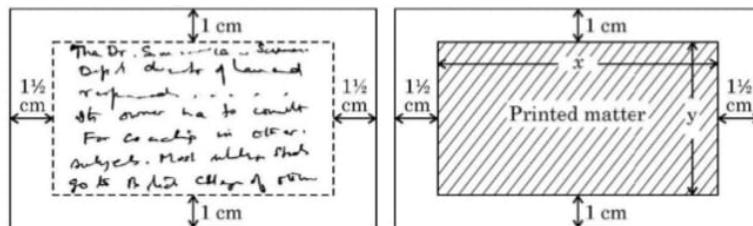
Figure 9.5: image 7

9.1.2. 12

1. The coordinates of the foot of the perpendicular drawn from the point $(0, 1, 2)$ on the x -axis are given by:
 - (a) $(1, 0, 0)$
 - (b) $(2, 0, 0)$
 - (c) $(\sqrt{5}, 0, 0)$
 - (d) $(0, 0, 0)$

2. If a line makes an angle of 30° with the positive direction of x -axis, 120° with the positive direction of y -axis, then the angle which it makes with the positive direction of z -axis is:
 - (a) 90°
 - (b) 120°
 - (c) 60°
 - (d) 0°

3. Find the equation of the line which bisects the line segment joining points $A(2, 3, 4)$ and $B(4, 5, 8)$ and is perpendicular to the lines $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$
4. If A_1 denotes the area of region bounded by $y^2 = 4x$ and x -axis in the first quadrant and A_2 denotes the area of region bounded by $y^2 = 4x$, $x = 4$, find $A_1 : A_2$.
5. Sand is pouring from a pipe at the rate of $15\text{cm}^3/\text{minute}$. The falling sand forms a cone on the ground such that the height of the cone is always one- third of the radius of the base. How fast is the height of the sand cone increasing at the instant when the height is 4cm
6. A rectangular visiting card is to contain 24sq.cm. of printed matter. The margins at the top and bottom of the card are to be 1cm and the margins on the left and right are to be 1cm as shown below:



On the basis of the above information, answer the following questions:

- (i) Write the expression for the area of the visiting card in terms of x .
- (ii) Obtain the dimensions of the card of minimum area.

7. The angle which the line $\frac{x}{1} = \frac{y}{-1} = \frac{z}{2}$ makes with the positive direction of the y -axis is:
- (a) $\frac{5\pi}{6}$
 - (b) $\frac{3\pi}{4}$
 - (c) $\frac{5\pi}{4}$
 - (d) $\frac{7\pi}{4}$
8. Given a curve $y = 7x - x^3$ and x increases at the rate of 2 units per second, the rate at which the slope of the curve is changing when $x = 5$ is:
- (a) -60 units/sec
 - (b) 60 units/sec
 - (c) -70 units/sec
 - (d) -140 units/sec
9. The area bounded by the curve $y = \sqrt{x}$, the y -axis, and between the lines $y = 0$ and $y = 3$ is:
- (a) $2\sqrt{3}$
 - (b) 27
 - (c) 9
 - (d) 3
10. Given a curve $y = 7x - x^3$ and x increases at the rate of 2 units per second. The rate at which the slope of the curve is changing, when $x = 5$ is:

(a) -60 units/sec

(b) 60 units/sec

(c) -70 units/sec

(d) -140 units/sec

11. The area of the region bounded by the curve $y^2 = 4x$ and $x = 1$ is:

(a) $\frac{4}{3}$

(b) $\frac{8}{3}$

(c) $\frac{64}{3}$

(d) $\frac{32}{3}$

12. The angle which the line $\frac{x}{1} = \frac{y}{-1} = \frac{z}{0}$ makes with the positive direction of Y-axis is:

(a) $\frac{5\pi}{6}$

(b) $\frac{3\pi}{4}$

(c) $\frac{5\pi}{4}$

(d) $\frac{7\pi}{4}$

13. Area of the region bounded by curve $y^2 = 4x$ and the $X-axis$ between $x = 0$ and $x = 1$ is :

(A) $\frac{2}{3}$

(B) $\frac{8}{3}$

(C) 3

(D) $\frac{4}{3}$

14. Given a curve $y = 7x - x^3$ and x increases at the rate of 2 units per second.

The rate at which the slope of the curve is changing, when $x = 5$ is :

- (A) -60 units/sec
(B) 60 units/sec
(C) -70 units/sec
(D) -140 units/sec

9.2. 2023

9.2.1. 10

1. A car has two wipers which do not overlap. Each wiper has a blade of length 21cm sweeping through an angle of 120° . Find the total area cleaned at each sweep of the two blades.
2. Sides AB and BC and median AD of a triangle ABC are respectively proportional to sides PQ and QR and median PM of $\triangle PQR$. Show that $\triangle ABC \sim \triangle PQR$.
3. Through the mid-point M of the side CD of a parallelogram $ABCD$, the line BM is drawn intersecting AC in L and AD (produced) in E . Prove that $EL = 2BL$.
4. In the given figure, O is the center of the circle. AB and AC are

tangents drawn to the circle from point A . If $\angle BAC = 65^\circ$, then find the measure of $\angle BOC$.

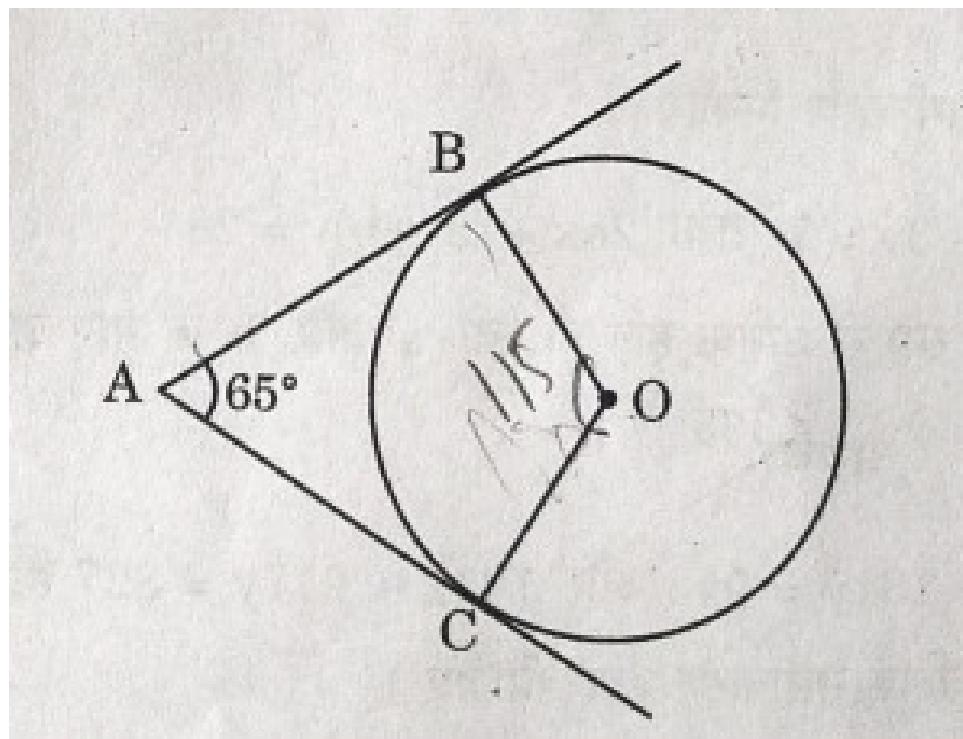


Figure 9.6:

5. In the given figure, O is the centre of the circle and QPR is a tangent to it at P . Prove that $\angle QAP + \angle APR = 90^\circ$.

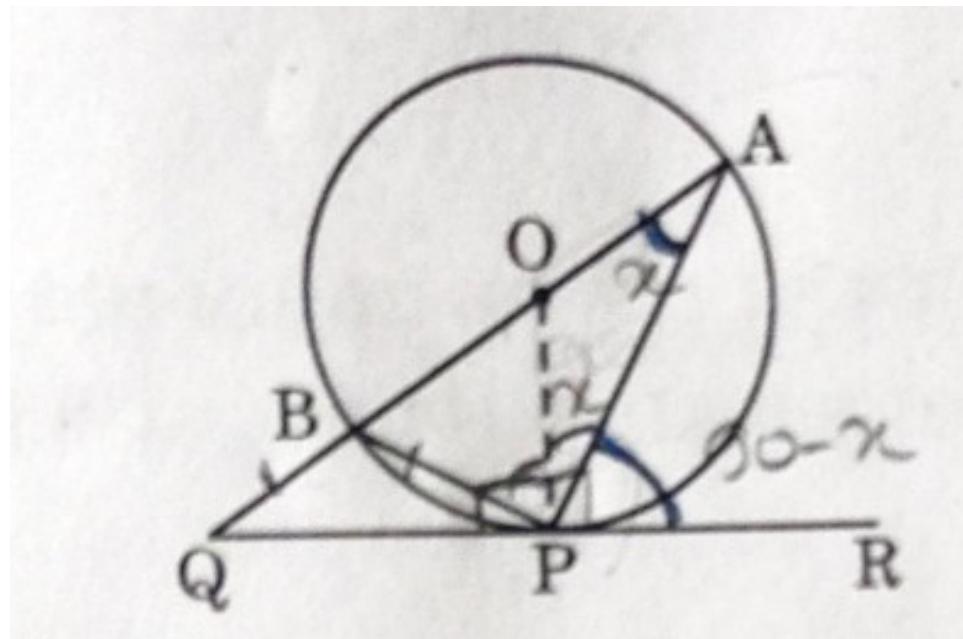


Figure 9.7:

6. In an annual day function of a school, the organizers wanted to give a cash prize along with a memento to their best students. Each memento is made as shown in the figure and its base $ABCD$ is shown from the front side. The rate of silver plating $\text{₹ } 20 \text{ per cm}^2$.

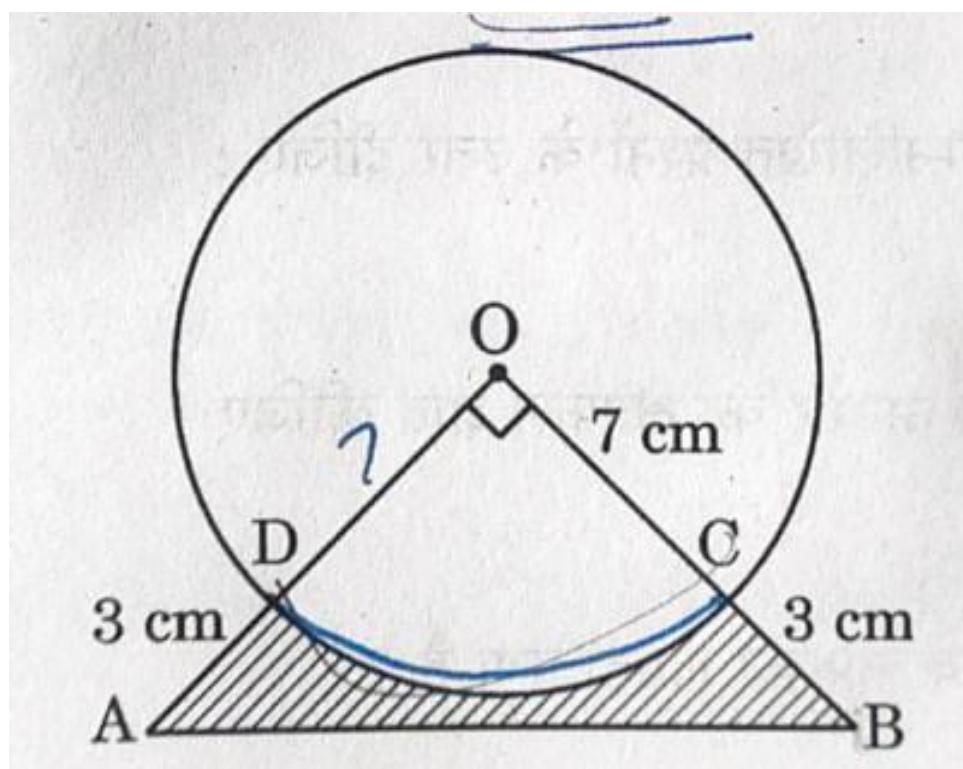


Figure 9.8:

Based on the above, answer the following question:

- What is the area of the quadrant $ODOC$?
- Find the area of $\triangle AOB$.
- i. What is the total cost of silver plating the shaded part $ABCD$?
ii. What is the length of arc CD ?

7. In a coffee shop, coffee is served in two types of cups. One is cylindrical in shape with diameter 7cm and height 14cm and the other is hemispherical with diameter 21cm.

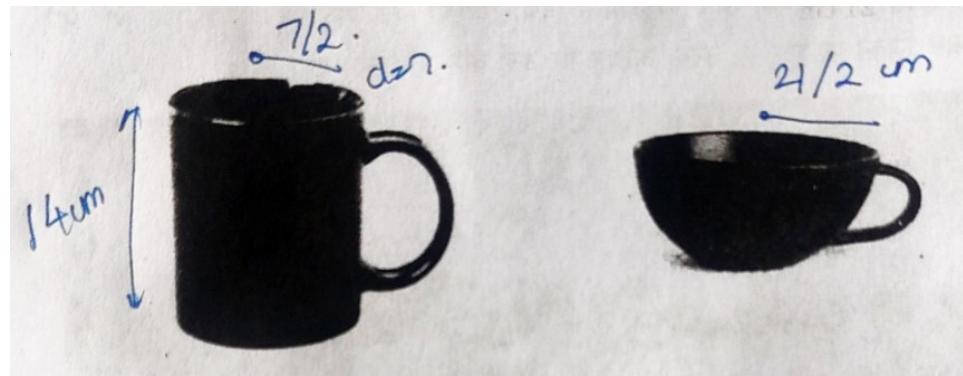


Figure 9.9:

Based on the above, answer the following question:

- (a) Find the area of the cylindrical cup.
- (b)
 - i. What is the capacity of the hemispherical cup?
 - ii. Find the capacity of the cylindrical cup.
- (c) What is the curved surface area of the cylindrical cup?

8. Show that the points $(-2, 3)$, $(8, 3)$ and $(6, 7)$ are the vertices of a right-angled triangle.
9. If $Q(0, 1)$ is equidistant from $P(5, -3)$ and $R(x, 6)$, find the values of x .
10. In the given figure, TA is a tangent to the circle with center O such that $OT = 4\text{cm}$, $\angle OTA = 30^\circ$, then the length of TA is:
- (a) $2 \times \sqrt{3}\text{cm}$
 - (b) 2cm
 - (c) $2 \times \sqrt{2}\text{cm}$
 - (d) $\sqrt{3}\text{cm}$
11. In the given figure, $\triangle ABC \sim \triangle QPR$. If $AC = 6\text{cm}$, $BC = 5\text{cm}$, $QR = 3\text{cm}$ and $PR = x$; then the value of x is:
- (a) 3.6cm
 - (b) 2.5cm
 - (c) 10cm
 - (d) 3.2cm
12. The distance of the point $(-6, 8)$ from origin is:
- (a) 6
 - (b) -6
 - (c) 8

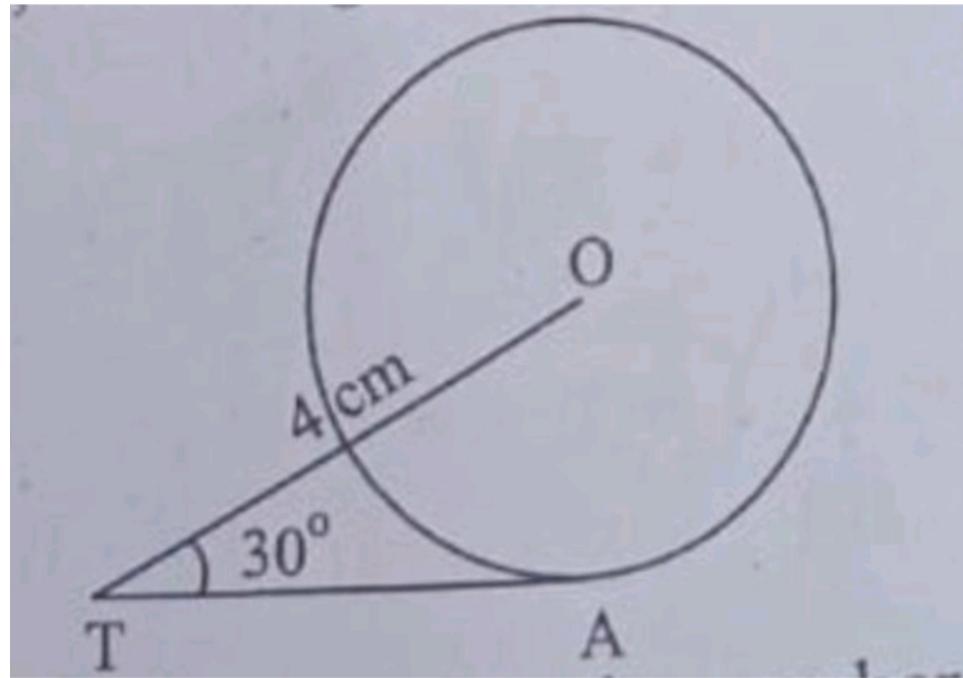


Figure 9.10: image1

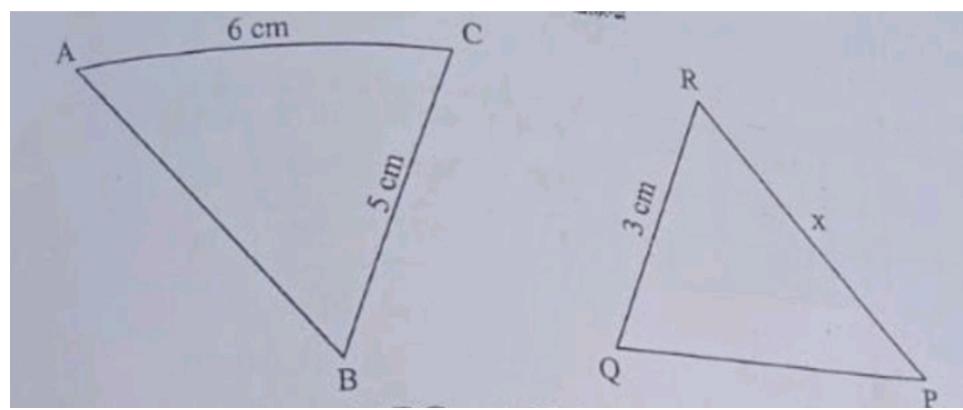


Figure 9.11: image2

(d) 10

13. What is the area of a semi-circle of diameter (d)?

(a) $\frac{1}{16} \times \pi \times d^2$

(b) $\frac{1}{4} \times \pi \times d^2$

(c) $\frac{1}{8} \times \pi \times d^2$

(d) $\frac{1}{2} \times \pi \times d^2$

14. In the given figure, PT is a tangent at T to the circle with centre (O).

If $\angle TPO = 25^\circ$, then x is equal to:

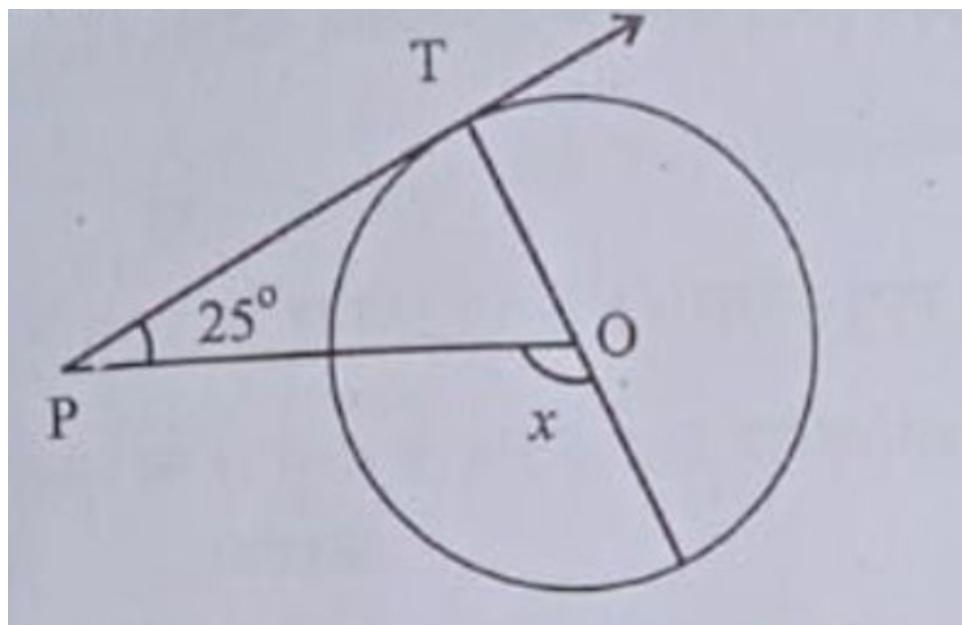


Figure 9.12: image3

(a) 25°

(b) 65°

(c) 90°

(d) 115°

15. In the given figure, $PQ \parallel AC$. If $BP = 4\text{cm}$, $AP = 2.4\text{cm}$, and $BQ = 5\text{ cm}$, then the length of BC is:

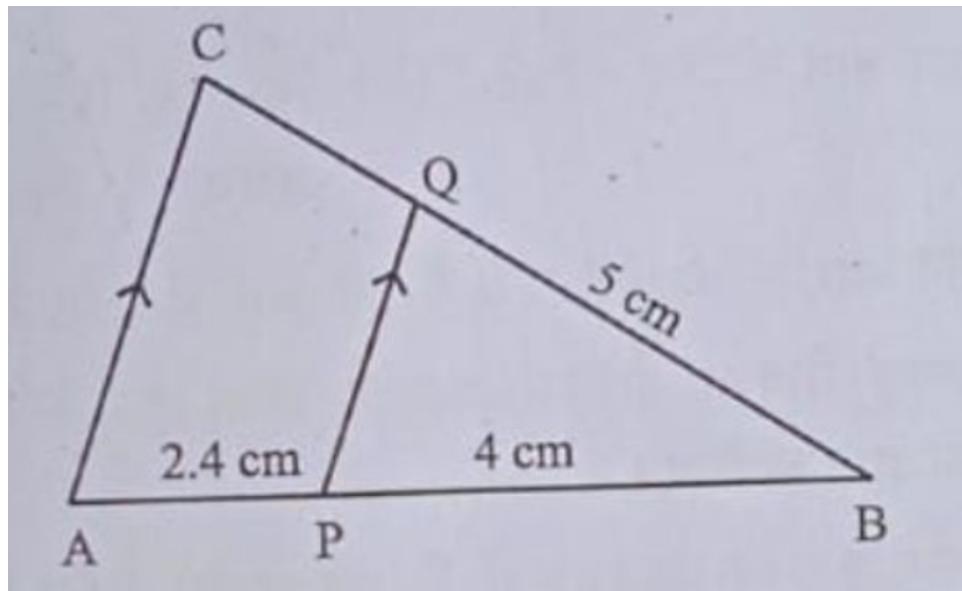


Figure 9.13: image4

- (a) 8cm
- (b) 3cm
- (c) 0.3cm
- (d) $\frac{25}{3}\text{cm}$

16. The points $(-4, 0)$, $(4, 0)$, and $(0, 3)$ are the vertices of a:

- (a) right triangle
- (b) isosceles triangle
- (c) equilateral triangle

(d) scalene triangle

17. What is the length of the arc of the sector of a circle with radius 14cm and a central angle of 90° :

(a) 22cm

(b) 44cm

(c) 88cm

(d) 11cm

18. If $\triangle ABC \sim \triangle PQR$ with $\angle A = 32^\circ$ and $\angle R = 65^\circ$, then the measure of $\angle B$ is?:

(a) 32°

(b) 65°

(c) 83°

(d) 97°

19. The coordinates of vertex A of a triangle ABCD whose three vertices are given as $B(0,0)$, $C(3,0)$, and $D(0,4)$ are?:

(a) $(4,0)$

(b) $(0,3)$

(c) $(3,4)$

(d) $(5,3)$

20. The area of the triangle formed by the line axes is: $\frac{x}{a} + \frac{y}{b} = 1$ with the coordinate axes is:

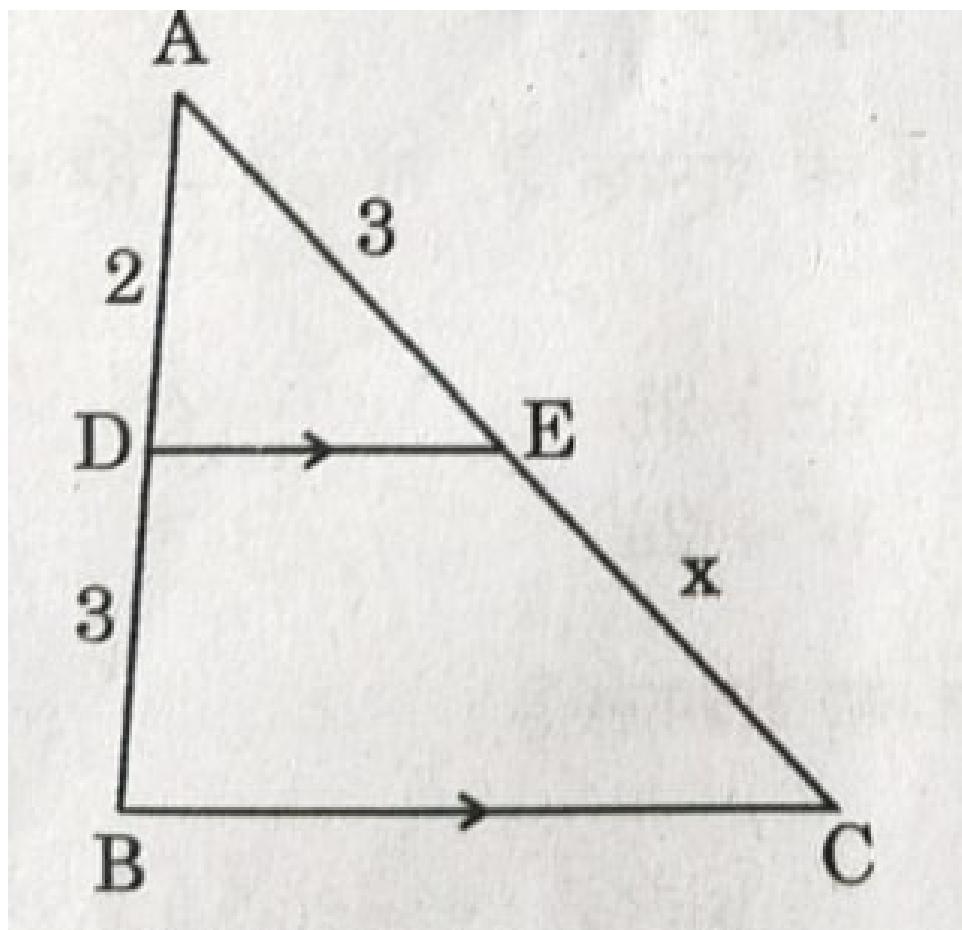
(a) ab

(b) $\frac{1}{2}ab$

(c) $\frac{1}{4}ab$

(d) $2ab$

item In the given figure, $DE \parallel BC$. If $AD = 2\text{units}$, $DB = AE = 3\text{units}$ and $EC = x\text{units}$, then the value of x is:



(a) 2

(b) 3

(c) 5

(d) $\frac{9}{2}$

21. In the given figure, $AB \parallel PQ$. If $AB = 6\text{cm}$, $PQ = 2\text{cm}$ and $OB = 3\text{cm}$, then the length of OP is:
- (a) 9cm
(b) 3cm
(c) 4cm
(d) 1cm
22. What Is The Total Surface Area Of a Solid Hemisphere Of Diameter ' d '?:
- (a) $3\pi d^2$
(b) $2\pi d^2$
(c) $\frac{1}{2}\pi d^2$
(d) $\frac{3}{4}\pi d^2$
23. From an external point, two tangents are drawn to a circle. Prove that the line joining the external point to the center of the circle bisects the angle between the two tangents.
24. Two concentric circles are of radii 5cm and 3cm. Find the length of the chord of the larger circle which touches the smaller circle.
25. In a $\triangle PQR$, N is a point on PR , such that $QN \perp PR$. If $PN \times NR = QN^2$, prove that $\angle PQR = 90^\circ$.

26. In the given figure, $\triangle ABC$ and $\triangle DBC$ are on the same base BC .

$$AD \text{ intersects } BC \text{ at } O. \text{ prove that } \frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle DBC)} = \frac{AO}{DO}$$

27. A wooden article was made by scooping out a hemisphere from each end of a solid cylinder, as shown in the figure. If the height of the cylinder is 10cm and its base is of radius 3.5cm, find the total surface area of the article.

28. Governing council of a local public development authority of Dehradun decided to build an adventurous playground on the top of a hill, which will have adequate space for parking. After survey, it was decided to build rectangular playground, with a semi-circular area allotted for parking at one end of the playground. The length and breadth of the rectangular playground are 14 units and 7 units, respectively. There are two quadrants of radius 2 units on one side for special seats.
- Based on the above information, answer the following questions:
- (a) What is the total perimeter of the parking area?
 - (b)
 - i. What is the total area of parking and the two quadrants?
 - ii. What is the ratio of area of playground to the area of parking area?
 - (c) Find the cost of fencing the playground and parking area at the rate of ₹2 per unit.

- (d) Two schools P and Q decided to award prizes to their students for two games of Hockey ₹ x per student and Cricket ₹ y per student. School P decided to award a total of ₹9,500 for the two games to 5 and 4 students respectively; while school Q decided to award ₹7,370 for the two games to 4 and 3 students respectively. Based on the given information, answer the following questions:
- i. Represent the following information in algebraically (in terms of x and y).
 - ii. A. What is the prize amount for hockey?
B. Prize amount on which game is more and by how much?
 - iii. What will be the total prize amount if there are 2 students each from two games?

29. Jagadish has a field which is in the shape of a right angled triangle AQC . He wants to leave a space in the form of a square $PQRS$ inside the field for growing wheat and the remaining for growing vegetables (*as shown in the figure*). In the field, there is a pole marked as O . Based on the above information, answer the following questions:
- (a) Taking O as origin, coordinates of P are $(-200, 0)$ and of Q are $(200, 0)$. $PQRS$ being a square, what are the coordinates of R and S ?
- (b) i. What is the area of square $PQRS$?
ii. What is the length of diagonal PR in square $PQRS$?
- (c) If S divides CA in the ratio $K : 1$, what is the value of K , where point A is $(200, 800)$?

9.3. 2006

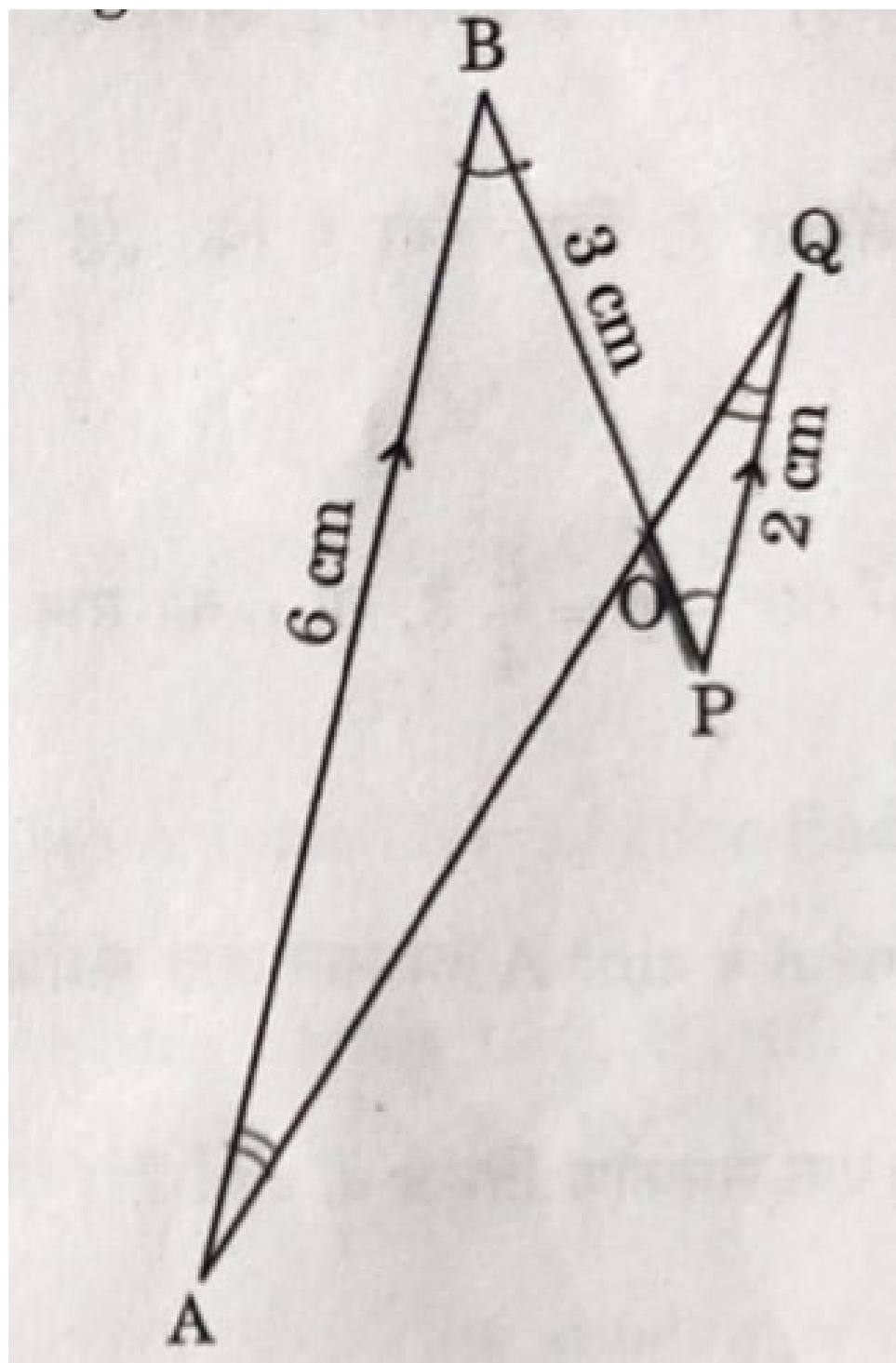
9.3.1. 10

1. In Figure 1, $\angle BAC = 90^\circ$. $AD \parallel BC$. Prove that $AB^2 + CD^2 = BD^2 + AC^2$.
2. In Figure 2, $PT = 6$ cm, $AR = 5$ cm. Find the length of PA .
3. Draw the graphs of the following equations: $3x - 4y + 6 = 0$, $3x + y - 9 = 0$ Also, determine the co-ordinates of the vertices of the triangle formed by these lines and the x axis.

4. A solid is in the form of a right circular cylinder with hemispherical ends. The total height of the solid is 58 cm and the diameter of the cylinder is 28cm. Find the total surface area of the solid $\pi \approx \frac{22}{7}$
5. . Construct a triangle ABC in which $BC = 7$ cm, and median $AD = 5$ cm, $\angle A = 60^\circ$ Write the steps of construction also.
- .
6. Show that the points $A(6, 2)$, $B(2, 1)$, $C(1, 5)$ and $D(5, 6)$ are the vertices of a square
7. Find the value of p for which the points $(- 5, 1)$, $(1, p)$ and $4, -2$ are collinear.
8. . Prove that in a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.
Makeing ue of the above, prove the following:
in fig:4, $ABCD$ is a fig:4 rhombus. prove that $4AB^2 = AC^2 + BD^2$.
9. Prove that I a line touch a circle and from the point of contact a chord is drawn, the angles which this chord makes with the given line are equal respectively to the angles formed in the corresponding alternate segments. Using the above, do the following:
 AB is a diameter and AC is a chord of a circle such that $\angle BAC = 30^\circ$.
The tangent at C intersects AR produced in a point I Prove that $BC = RD$.
10. A man standing on the deck of a ship, which is 10 m above the water level, observes the angle of elevation of the top of a hill as 60° and

the angle of depression of the base of the hill as 30° . Calculate the distance of the hill from the ship and the height of the hill.

11. From a window x meters high above the ground in a street, the angles of elevation and depression of the top and foot of the other house on the opposite side of the street are α and β respectively. Show that the height of the opposite house is $x(1 + \tan \alpha \cot \beta)$ meters.



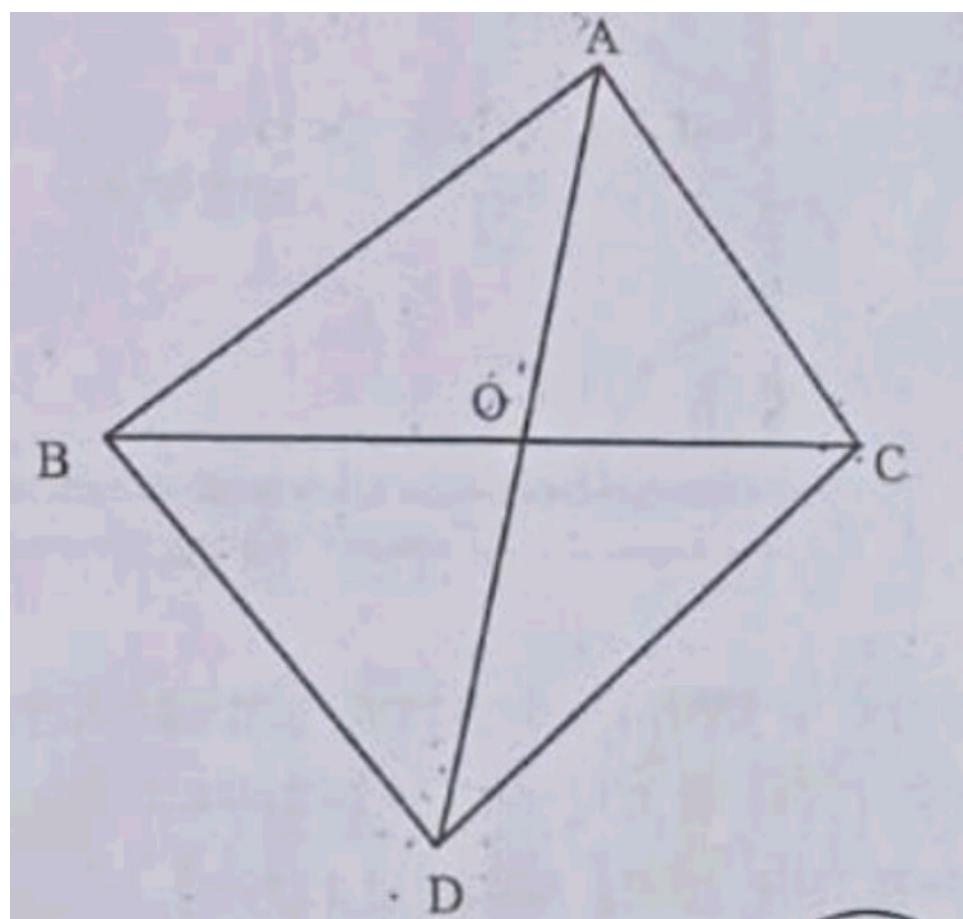


Figure 9.14: Image 1

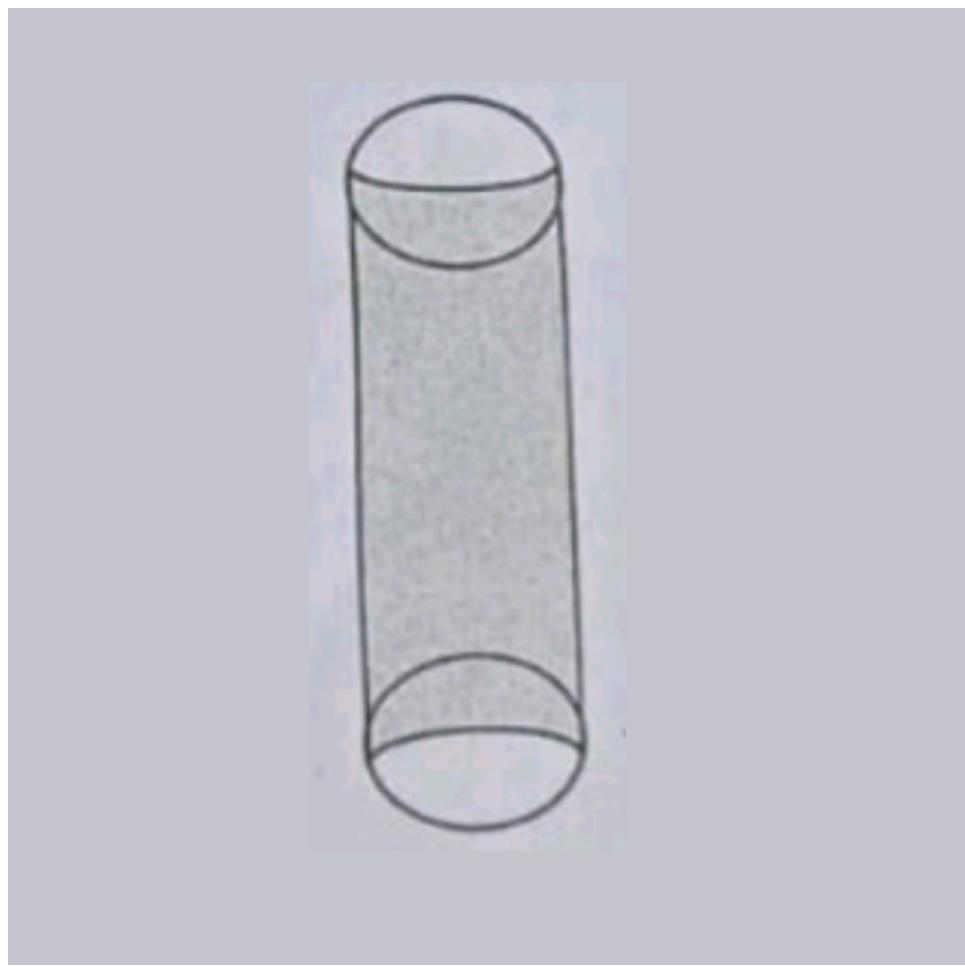


Figure 9.15: Image 2

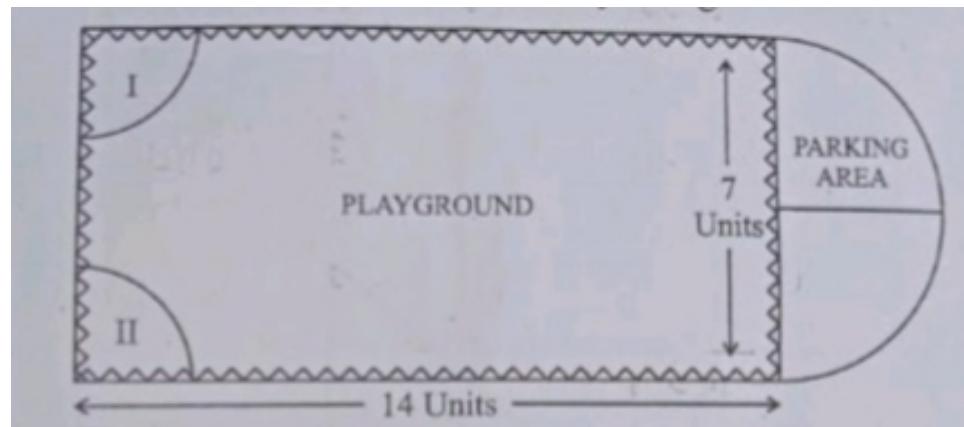


Figure 9.16: Image 3



Figure 9.17: Image 4

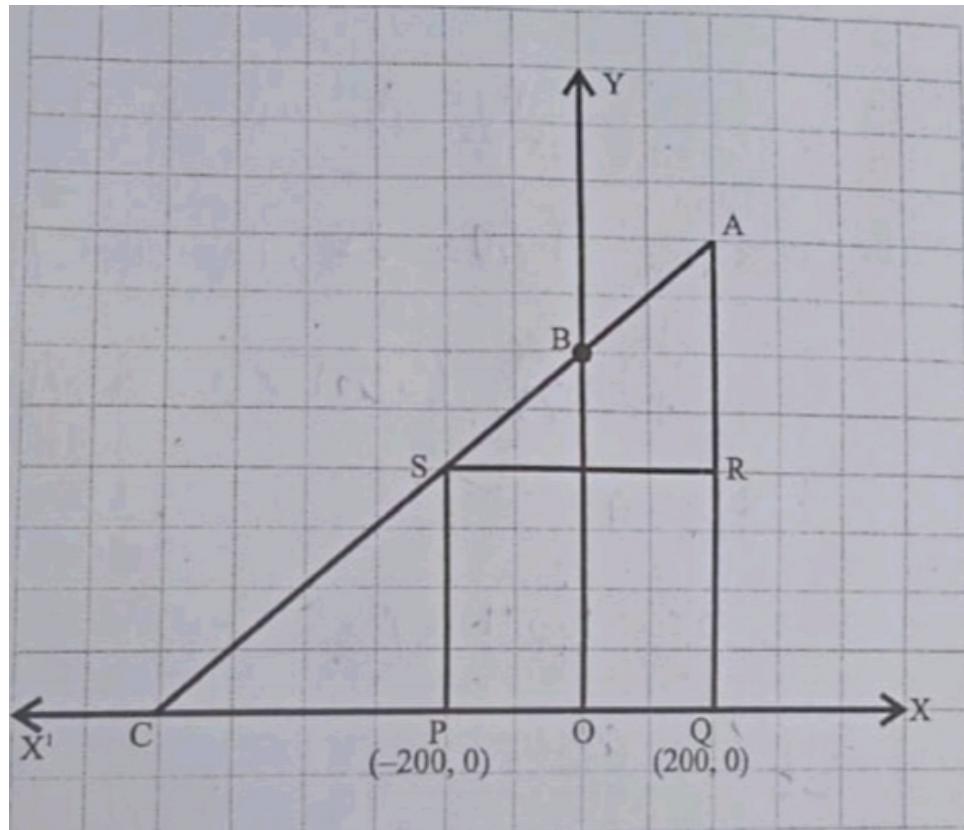


Figure 9.18: Image 5

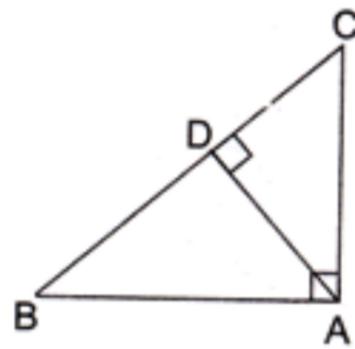


Figure 9.19: 1

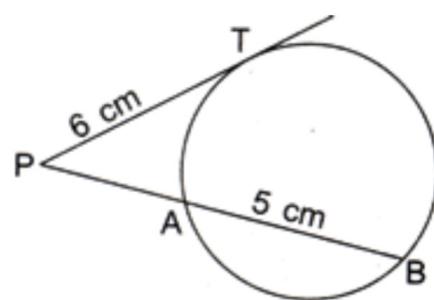


Figure 9.20: 2

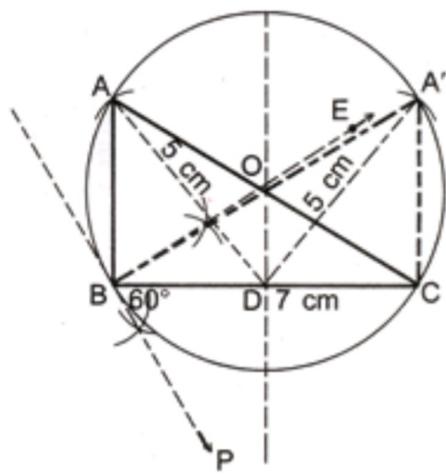


Figure 9.21: 3

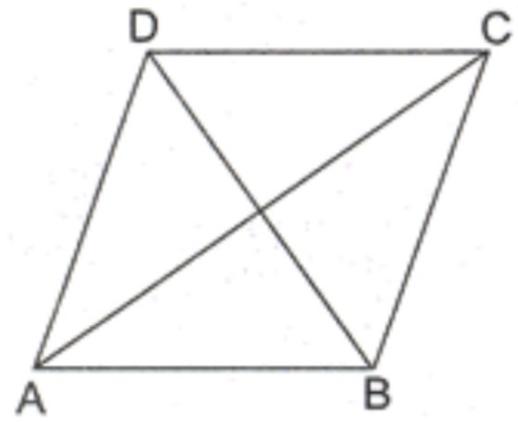


Figure 9.22: 4

Chapter 10

sequences

10.1. 2006

10.1.1. 10

1. The 5th term of an Arithmetic Progression (A.P.) is 26 and the 10th term is 51. Determine the 15th term of the A.P.

2. Find the sum of all the natural numbers less than 100 which are divisible by 6.

Chapter 11

Datahandling

11.1. 2006

11.1.1. 10

1. The following table shows the monthly expenditure of company. Draw pie chart for the data.

	Amount (in Rs.)
Wages	4800
Materials	3200
Taxation	2400
Adm. Expenditure	3000
Miscellaneous	1000

2. The Arithmetic Mean of the following frequency distribution is 47.

Determine the value of p .

Classes	Frequency
0 - 20	8
20 - 40	15
40 - 60	20
60 - 80	p
80 - 100	5

Chapter 12

Discrete

12.1. 2023

12.1.1. 10

1. **Assertion(A):** a, b, c are in *A.P.* if and if only if $2b = a + c$.

Reason(R): The sum of first n natural numbers is n^2 .

- (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
 - (b) Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of Assertion (A).
 - (c) Assertion (A) is true but Reason (R) is false.
 - (d) Assertion (A) is false but Reason (R) is true.
2. How many terms are there in *A.P.* whsoe first and fifth term are -14 and 2 , respectively and the last term is 62 .
 3. Which term of the *A.P.* : $65, 61, 57, 53, \dots$ is the first negative term?

Chapter 13

Number Systems

13.1. 2024

13.1.1. 10

1. If two positive integers p and q can be expressed as $p = 18a^2b^4$ and $q = 20a^3b^2$ where a and b are prime numbers, then $\text{LCM}(p, q)$ is:
 - (a) $2a^2b^2$
 - (b) $180a^2b^2$
 - (c) $12a^2b^2$
 - (d) $180a^3b^4$
2. Prove that $5 - 2\sqrt{3}$ is an irrational number. It is given that $\sqrt{3}$ is an irrational number.
3. show that the number $5x11x17 + 3x11$ is a composite number.
4. In a teachers workshop, the number of teachers teaching French, Hindi and English are 48, 80 and 144 respectively. Find the minimum number

of root required if in each room the same number of teachers are seated and all of them are of the same subject.

5. Three years ago, Rashmi was thrice as old as Nazma. Ten years later, Rashmi will be twice as old as Nazma. How old are Rashmi and Nazma

13.2. 2023

13.2.1. 10

1. Prove that $2+\sqrt{3}$ is an irrational number, given that $\sqrt{3}$ is an irrational number.
2. Find by prime factorisation the *LCM* of the number 18180 and 7575. Also, find the *HCF* of the two numbers.
3. The ratio of HCF to LCM of the least composite number and the least prime number is:
 - (a) 1 : 2
 - (b) 2 : 1
 - (c) 1 : 1
 - (d) 1 : 3
4. If p and q are natural numbers and p is a multiple of q , then what is the HCF of p and q ?
 - (a) pq

(b) p

(c) q

(d) $p + q$

5. Two numbers are in the ratio $2 : 3$ and their LCM is 180. What is the HCF of these numbers?
6. Prove that $\sqrt{5}$ is an irrational number.

Chapter 14

Differentiation

14.1. 2024

14.1.1. 12

1. The derivative of $\tan^{-1}(x^2)$ w.r.t. x is:

(a) $\frac{x}{1+x^4}$

(b) $\frac{2x}{1+x^4}$

(c) $-\frac{2x}{1+x^4}$

(d) $\frac{1}{1+x^4}$

2. Overspeeding increases fuel consumption and decreases fuel economy as a result of tyre rolling friction and air resistance. While vehicles reach optimal fuel economy at different speeds, fuel mileage usually decreases rapidly at speeds above 80km/h.

The relation between fuel consumption $F(1/100 \text{ km})$ and speed $V(\text{km/h})$ under some constraints is given as $F = \frac{V^2}{500} - \frac{V}{4} + 14$.

On the basis of the above information, answer the following questions:

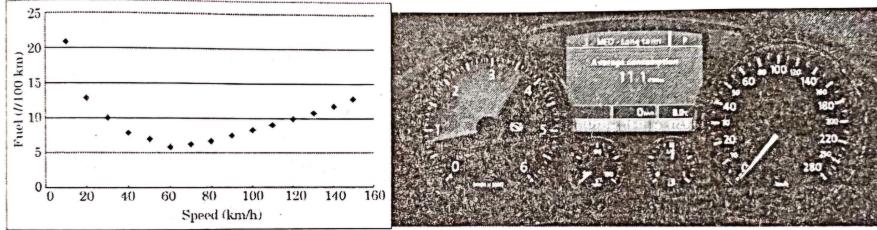


Figure 14.1: 1

- (i) Find F, when $V = 40 \text{ km/h}$.
- (ii) Find $\frac{dF}{dV}$.
- (iii) Find the speed V for which fuel consumption F is minimum.
- (iv) Find the quantity of fuel required to travel 600 km at the speed V at which $\frac{dF}{dV} = -0.01$.

1. The differential equation $\frac{dy}{dx} = F(x, y)$ will not be a homogeneous differential equation, if $F(x, y)$ is:

(a) $\cos x - \sin(\frac{y}{x})$

(b) $\frac{y}{x}$

(c) $\frac{x^2+y^2}{xy}$

(d) $\cos^2(\frac{x}{y})$

2. The degree of the differential equation $(y'')^2 + (y')^3 = x \sin(y')^3$ is:

(a) 1

(b) 2

(c) 3

(d) not defined

3. If $y = \operatorname{cosec}(\cot^{-1} x)$, then prove that $\sqrt{1+x^2} \frac{dy}{dx} - x = 0$.
4. If $x = e^{\cos 3t}$ and $y = e^{\sin 3t}$, prove that $\frac{dy}{dx} = -\frac{y \log x}{x \log y}$.
5. Show that: $\frac{d}{dx}(|x|) = \frac{x}{|x|}, x \neq 0$.
6. Find the particular solution of the differential equation given by $2xy + y^2 - 2x^2 \frac{dy}{dx} = 0; y = 2$, when $x=1$.
7. Find the general solution of the differential equation:

$$ydx = (x + 2y^2)dy$$

8. Verify whether the function f defined by $f(x) = \begin{cases} x \sin\left(\frac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$ is continuous at $x = 0$ or not.

9. Check for differentiability of the function f defined by $f(x) = |x - 5|$, at the point $x = 5$.

10. Find the particular solution of the differential equation :

$$\frac{dy}{dx} - 2xy = 3x^2 e^{x^2}; y(0) = 5$$

11. Solve the following differential equation:

$$x^2 dy + y(x + y)dx = 0$$

12. Find the values of a and b so that the following function is differentiable

$$\text{for all values of } x \quad f(x) = \begin{cases} ax + b, & x > -1 \\ bx^2 - 3, & x \leq -1 \end{cases}$$

13. Find $\frac{dy}{dx}$, if $(\cos x)^y = (\cos y)^x$.

14. If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, prove that $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$.

15. If $y = (\tan^{-1} x)^2$, show that $(x^2+1)^2 \frac{d^2y}{dx^2} + 2x(x^2+1) \frac{dy}{dx} = 2$.

16. If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, prove that $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$.

17. The order of the differential equation:

$$\frac{d^3y}{dx^3} + x \left(\frac{dy}{dx} \right)^5 = 4 \log \left(\frac{d^4y}{dx^4} \right)$$

is:

(a) not defined

(b) 3

(c) 4

(d) 5

18. Derivative of $e^{\sin^2 x}$ with respect to $\cos x$ is:

(a) $\sin x e^{a \sin^2 x}$

(b) $\cos x e^{\sin^2 x}$

(c) $2\cos x e^{\sin^2 x}$

(d) $-2 \sin^2 x \cos x e^{\sin^2 x}$

19. Derivative of $e^{\sin^2 x}$ with respect to $\cos x$ is:

(a) $\sin x e^{\sin^2 x}$

(b) $\cos x e^{\sin^2 x}$

(c) $-2\cos x e^{\sin^2 x}$

(d) $-2\sin^2 x \cos x e^{\sin^2 x}$

20. If $\sin(xy) = 1$, then $\frac{dy}{dx}$ is equal to:

(a) $\frac{x}{y}$

(b) $-\frac{x}{y}$

(c) $\frac{y}{x}$

(d) $-\frac{y}{x}$

21. The general solution of the differential equation

$\frac{dy}{dx} = e^{x+y}$ is:

(a) $e^x + e^{-y} = c$

(b) $e^{-x} + e^{-y} = c$

(c) $e^{x+y} = c$

(d) $2e^{x+y} = c$

22. If $x = a \sin^3 \theta$, $y = b \cos^3 \theta$, then find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{4}$.

23. Find the particular solution of the differential equation

$\frac{dy}{dx} - 2xy = 3x^2 e^{x^2}$; $y(0) = 5$.

24. Solve the following differential equation :

$$x^2 dy + y(x+y) dx = 0$$

25. If $xe^y = 1$, then the value of $\frac{dy}{dx}$ at $x = 1$ is :

(a) -1

(b) 1

(c) $-e$

(d) $-\frac{1}{e}$

26. Derivative of $e^{\sin^2 x}$ with respect to $\cos x$ is :

(A) $\sin x e^{\sin^2 x}$

(B) $\cos x e^{\sin^2 x}$

(C) $-2 \cos x e^{\sin^2 x}$

(D) $-2 \sin^2 x \cos x e^{\sin^2 x}$

27. The order of the differential equation $\frac{d^4 y}{dx^4} - \sin[\frac{d^2 y}{dx^2}] = 5$ is

(a) 4

(b) 3

(c) 2

(d) notdefined

Chapter 15

Integration

15.1. 2024

15.1.1. 12

1. $\int_0^{\frac{\pi}{2}} \frac{\sin x - \cos x}{1 + \sin x \cos x} dx$ is equal to:

(a) π

(b) $Zero(0)$

(c) $\int_0^{\frac{\pi}{2}} \frac{2 \sin x}{1 + \sin x \cos x} dx$

(d) $\frac{\pi^2}{4}$

2. Find : $\int \frac{e^{4x} - 1}{e^{4x} + 1} dx$

3. Evaluate:

$$\int_2^z -2\sqrt{\frac{2-x}{z+x}} dx$$

4. Find:

$$\int \frac{1}{x[(\log x)^2 - 3\log x - 4]} dx$$

5. Find: $\int x^2 \cdot \sin^{-1}(x^{\frac{3}{2}}) dx$

6. Find: $\int \cos^3 x \cdot e^{\log(\sin x)} dx$
7. Find: $\int \frac{1}{5+4x-x^2} dx$
8. Evaluate: $\int_0^\pi \frac{e^{\cos x}}{e^{\cos x} + e^{-\cos x}} dx = 0$
9. Find the area of the region bounded by the curve $4x^2 + y^2 = 36$. using integration.
10. If $\int_{-2}^3 x^2 dx = k \int_0^3 x^2 dx + \int_2^3 x^2 dx$, then the value of k is:
- (a) 2
 - (b) 1
 - (c) 0
 - (d) $\frac{1}{2}$
11. The value of $\int_1^0 \log x dx$ is:
- (a) 0
 - (b) 1
 - (c) e
 - (d) $e \log e$
12. The value of $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cot \theta \operatorname{cosec}^2 \theta d\theta$ is:
- (a) $\frac{1}{2}$
 - (b) $-\frac{1}{2}$
 - (c) 0
 - (d) $-\frac{\pi}{8}$

13. The integral $\int \frac{dx}{\sqrt{9-4x^2}}$ is equal to:

- (a) $\frac{1}{6} \sin^{-1}\left(\frac{2x}{3}\right) + c$
- (b) $\frac{1}{2} \sin^{-1}\left(\frac{2x}{3}\right) + c$
- (c) $\sin^{-1}\left(\frac{2x}{3}\right) + c$
- (d) $\frac{3}{2} \sin^{-1}\left(\frac{2x}{3}\right) + c$

14. $\int \frac{1}{x(\log x)^2} dx$ is equal to :

- (A) $2\log(\log x) + c$
- (B) $-\frac{1}{\log x} + c$
- (C) $\frac{(\log x)^3}{3} + c$
- (D) $\frac{3}{(\log x)^3} + c$

15. The value of $\int_{-1}^1 x|x|dx$ is :

- (A) $\frac{1}{6}$
- (B) $\frac{1}{3}$
- (C) $\frac{1}{6}$
- (D) 0

Chapter 16

Functions

16.1. 2024

16.1.1. 12

1. **Assertion (A):** Domain of $y = \cos^{-1}(x)$ is $[-1, 1]$.

Reason (R): The range of the principal value branch of $y = \cos^{-1}(x)$ is $[0, \pi] - \{\frac{\pi}{2}\}$.

2. A function $f : \mathbb{R} \rightarrow \mathbb{R}$ defined as $f(x) = x^2 - 4x + 5$ is:

(a) injective but not surjective

(b) surjective but not injective

(c) both injective and surjective

(d) neither injective nor surjective

3. The function $f(x) = \frac{x}{2} + \frac{2}{x}$ has a local minimum at x equal to:

(a) 2

(b) 1

(c) 0

(d) -2

4. **Assertion (A):** Domain of $y = \cos^{-1}(x)$ is $[-1, 1]$.

Reason (R): The range of the principal value branch of $y = \cos^{-1}(x)$ is $[0, \pi] - \{\frac{\pi}{2}\}$. **Direction:** one labeled Assertion (A) and the other labeled Reason (R). Select the correct answer from the following options

- (A) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of the Assertion (A).
- (B) Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of the Assertion (A).
- (C) Assertion (A) is true, but Reason (R) is false.
- (D) Assertion (A) is false, but Reason (R) is true.

5. The function $f(x) = \frac{x}{2} + \frac{2}{x}$ has a local minima at x is equal to:

(a) 2

(b) 1

(c) 0

(d) -2

6. A function $f : \mathbb{R} \rightarrow \mathbb{R}$ defined as $f(x) = x^2 - 4x + 5$ is :

(A) injective but not surjective

(B) surjective but not injective

(C) both injective and surjective

(D) neither injective nor surjective

7. The function $f(x)=\frac{x}{2}+\frac{2}{x}$ has a local minima at x is equal to:

(A) 2

(B) 1

(C) 0

(D) -2

Chapter 17

Matrices

17.1. 2024

17.1.1. 12

1. If the sum of all the elements of 3×3 scalar matrix is 9, then the product of all elements is:

(a) 0

(b) 9

(c) 27

(d) 729

2. If $\begin{vmatrix} -a & b & c \\ a & -b & c \\ a & b & -c \end{vmatrix} = kabc$, then the value of k is:

(a) 0

(b) 1

(c) 2

(d) 4

3. If $A = [a_{ij}]$ be a 3×3 where $a_{ij} = i - 3j$, then which of the following is false?

(a) $a_{11} < 0$

(b) $a_{12} + a_{21} = -6$

(c) $a_{13} > a_{31}$

(d) $a_{31} = 0$

4. If $F(x) = \begin{pmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 0 \end{pmatrix}$ and $[F(x)]^2 = F(kx)$, then the value of k is:

(a) 1

(b) 2

(c) 0

(d) -2

5. Assertion (A): For any symmetric matrix A , $B'AB$ is a skew-symmetric matrix.

Reason (R): A square matrix P is kew-symmetric if $P' = -P$

(a) Both Assertion and Reason are true, and Reason is the correct explanation of Assertion.

(b) Both Assertion and Reason are true, but Reason is not the correct explanation of Assertion.

- (c) Assertion is true, but Reason is false.
- (d) Assertion is false, but Reason is true.
6. Solve the following system of equations, using matrices:

$$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4, \frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1, \frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2 \text{ where } x, y, z \neq 0$$

7. If $A = \begin{pmatrix} 1 & \cot x \\ -\cot x & 1 \end{pmatrix}$, then show that $A'A^{-1} = \begin{pmatrix} -\cos 2x & -\sin 2x \\ \sin 2x & -\cos 2x \end{pmatrix}$
8. If $A = \begin{pmatrix} 1 & 2 & -3 \\ 2 & 0 & -3 \\ 1 & 2 & 0 \end{pmatrix}$ then find A^{-1} and use it to solve the following system of equations :

$$x + 2y - 3z = 1 \quad (17.1)$$

$$2x - 3z = 2 \quad (17.2)$$

$$x + 2y = 3 \quad (17.3)$$

9. Find the product of the matrices $\begin{pmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & 4 \end{pmatrix} \begin{pmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{pmatrix}$, then find AB and use it to solve the system of linear equations :

$$x - 2y = 3 \quad (17.4)$$

$$2x - y - z = 2 \quad (17.5)$$

$$-2y + z = 3 \quad (17.6)$$

10. Find the product of the matrices $\begin{pmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{pmatrix}$ and $\begin{pmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{pmatrix}$ and hence solve the system of linear equations:

$$x + 2y - 3z = -4$$

$$2x + 3y + 2z = 2$$

$$3x - 3y - 4z = 11$$

11. For the matrix $A = \begin{pmatrix} 2 & -1 & 1 \\ \lambda & 2 & 0 \\ 1 & -2 & 3 \end{pmatrix}$ to be invertible, the value of λ is:

(a) 0

(b) 10

(c) $\mathbb{R} - \{10\}$

(d) $\mathbb{R} - \{-10\}$

12. If $A = \begin{pmatrix} x & 0 \\ 1 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 4 & 0 \\ -1 & 1 \end{pmatrix}$ then the value of x for which $A^2 = B$ is:

(a) -2

(b) 2

(c)

(d) 2 or -2

(e) 1

13. Let $f(x) = \begin{pmatrix} x^2 & \sin x \\ p & -1 \end{pmatrix}$ where p is a constant. The value of p for which $f'(0) = 1$ is:

(a) R

(b) 1

(c) 0

(d) -1

14. If A is a square matrix of order 3 such that the value of $|\text{adj}A| = 8$, then the value of $|A^T|$ is:

(a) $\sqrt{2}$

(b) $-\sqrt{2}$

(c) 8

(d) $2\sqrt{2}$

15. If the inverse of the matrix $\begin{pmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix}$ is the matrix $\begin{pmatrix} 1 & 3 & 3 \\ 1 & \lambda & 3 \\ 1 & 3 & 4 \end{pmatrix}$, then the value of λ is:

(a) -4

(b) 1

(c) 3

(d) 4

16. Find the matrix A^2 , where $A = \begin{pmatrix} a_{ij} \end{pmatrix}$ is a 2×2 matrix whose elements are given by $a_{ij} = \max(i, j) - \min(i, j)$:

(a) $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

(b) $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

(c) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

(d) $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$

17. If A is a square matrix of order 2 and $|A| = -2$, then value of $|5A'|$ is:

(a) -50

(b) -10

(c) 10

(d) 50

18. The product of matrix P and Q is equal to a diagonal matrix. If the order of matrix Q is 3×2 , then the order of matrix P is:

(a) 2×2

(b) 3×3

(c) 2×3

(d) 3×2

19. If $A = \begin{pmatrix} 9 & c & -1 \\ b & 0 & 5 \\ 1 & -5 & 5 \end{pmatrix}$ is a skew symmetric matrix, then the value of $2a - (b + c)$ is :

- (a) 0
- (b) 1
- (c) -10
- (d) 10

20. If A is a square matrix of order 3 such that the value of $|adj.A| = 8$, then the value of $|A^T|$ is :

- (a) $\sqrt{2}$
- (b) $-\sqrt{2}$
- (c) 8
- (d) $2\sqrt{2}$

21. If inverse of matrix $\begin{pmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix}$ is the matrix $\begin{pmatrix} 1 & 3 & 3 \\ 1 & \lambda & 3 \\ 1 & 3 & 4 \end{pmatrix}$, then value of λ is :

- (a) -4
- (b) 1
- (c) 3
- (d) 4

22. If $\begin{pmatrix} x & 2 & 0 \end{pmatrix} \begin{pmatrix} 5 \\ -1 \\ x \end{pmatrix} = \begin{pmatrix} 3 & 1 \end{pmatrix} \begin{pmatrix} -2 \\ x \end{pmatrix}$, then value of x is :

(a) -1

(b) 0

(c) 1

(d) 2

23. Find the matrix $(A)^2$, where $A = [a_{ij}]$ is a 2×2 matrix whose elements are given by $a_{ij} = \text{maximum } (i, j) - \text{minimum } (i, j)$:

(a) $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

(b) $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

(c) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

(d) $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$

Chapter 18

Trigonometry

18.1. 2024

18.1.1. 10

1. If $\sec \theta - \tan \theta = m$, then the value of $\sec \theta + \tan \theta$ is:

(a) $1 - \frac{1}{m}$

(b) $m^2 - 1$

(c) $\frac{1}{m}$

(d) $-m$

2. If $\cos(\alpha + \beta) = 0$ then the value of $\cos\left(\frac{\alpha+\beta}{2}\right)$ is equal to:

(a) $\frac{1}{\sqrt{2}}$

(b) $\frac{1}{2}$

(c) 0

(d) $\sqrt{2}$

1. Simplify: $\cos^{-1} x + \cos^{-1} \left[\frac{x \sqrt{3-3x^2}}{2} \right]; -\frac{1}{2} \leq x \leq 1$

2. Evaluate: $2\sqrt{2} \cos 45^\circ \sin 10^\circ + 2\sqrt{3} \cos 30^\circ$
3. If $A = 60^\circ$ and $B = 30^\circ$, verify that : $\sin(A+B) = \sin A \cos B + \cos A \sin B$
4. Prove that : $\frac{\tan \theta}{1-\cot \theta} + \frac{\cot \theta}{1-\tan \theta} = 1 + \sec \theta \csc \theta$
5. A pole $6m$ high is fixed on the top of a tower. The angle of elevation of the top of the pole observed from a point P on the ground is 60° and the angle of depression of the point P from the top of the tower is 45° . Find the height of the tower and the distance of point P from the foot of the tower

18.1.2. 12

- Find the value of $\tan^{-1}(-\frac{1}{\sqrt{3}}) + \cot^{-1}(\frac{1}{\sqrt{3}}) + \tan^{-1}[\sin(-\frac{\pi}{2})]$
- If $a = \sin^{-1}\left(\frac{\sqrt{2}}{2}\right) + \cos^{-1}\left(\frac{-1}{2}\right)$ and $b = \tan^{-1}(\sqrt{3}) + \cot^{-1}\left(\frac{-1}{\sqrt{3}}\right)$, then find the value of $a + b$.
- Find the value k if

$$\sin^{-1} \left[k \tan \left(2 \cos^{-1} \frac{\sqrt{3}}{2} \right) \right] = \frac{\pi}{3}. \quad (18.1)$$

18.2. 2023

18.2.1. 10

1. If $4 \cot^2 45^\circ - \sec^2 60^\circ + \sin^2 60^\circ + p = \frac{3}{4}$, then find the value of p.
2. If $\cos A + \cos^2 A = 1$, then find the value of $\sin^2 A + \sin^4 A$
3. The length of the shadow of a tower on the plane ground is $\sqrt{3}$ times the height of the tower. Find the angle of elevation of the sun.
4. The angle of elevation of the top of a tower from a point on the ground which is 30m away from the foot of the tower, is 30° . Find the height of the tower.
5. Prove that :

$$\left(\frac{1}{\cos \theta} - \cos \theta \right) \left(\frac{1}{\sin \theta} - \sin \theta \right) = \frac{1}{\tan \theta + \cot \theta} \quad (18.2)$$

6. As observed from the top of a 75m high lighthouse from the sea-level, the angles of depression of two ships are 30° and 60° . If one ship is exactly behind the other on the same side of the lighthouse, find the distance between the two ships.

$$(Use \sqrt{3} = 1.73)$$

7. From a point on the ground, the angle of elevation of the bottom and top of a transmission tower fixed at the top of 30m high building are 30° and 60° , respectively. Find the height of the transmission tower. $(Use \sqrt{3} = 1.73)$.

8. If a pole 6 m high casts a shadow $2 \times \sqrt{3}$ m long on the ground, then

sun's elevation is:

(a) 60°

(b) 45°

(c) 30°

(d) 90°

9. $(\sec^2 \theta - 1)(\operatorname{cosec}^2 \theta - 1)$ is equal to:

(a) -1

(b) 1

(c) 0

(d) 2

10. Evaluate

$$2 \sec^2 \theta + 3 \csc^2 \theta - 2 \sin \theta \cos \theta \text{ if } \theta = 45^\circ. \quad (18.3)$$

11. If

$$\sin \theta - \cos \theta = 0, \text{ then find the value of } \sin^4 \theta + \cos^4 \theta. \quad (18.4)$$

12. Prove that

$$\frac{\sin A - 2 \sin^3 A}{2 \cos^3 A - \cos A} = \tan A \quad (18.5)$$

13. Prove that

$$\sec A (1 - \sin A) (\sec A + \tan A) = 1. \quad (18.6)$$

14. A straight highway leads to the foot of a tower. A man standing on the top of the 75m high tower observes two cars at angles of depression of 30° and 60° , which are approaching the foot of the tower. If one car is exactly behind the other on the same side of the tower, find the distance between the two cars. use ($\sqrt{3} = 1.73$).
15. From the top of a 7m building, the angle of elevation of the top of a cable tower is 60° and the angle of depression of its foot is 30° . Determine the height of the tower.

