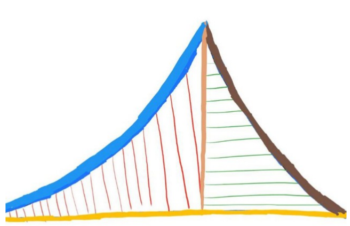
CBSE MATH

Made Simple

G. V. V. Sharma



Contents

Introduction

This book links high school coordinate geometry to linear algebra and matrix analysis through solved problems.

Chapter 1

Vectors

1.1. 2024

1.1.1. 12

1. For any two vectors $\overrightarrow{a}$ and $\overrightarrow{b}$, which of the following statements is always true?

   (a) $\overrightarrow{a} \cdot \overrightarrow{b} \geq |\overrightarrow{a}||\overrightarrow{b}|$

   (b) $\overrightarrow{a} \cdot \overrightarrow{b} = |\overrightarrow{a}||\overrightarrow{b}|$

   (c) $\overrightarrow{a} \cdot \overrightarrow{b} \leq |\overrightarrow{a}||\overrightarrow{b}|$

   (d) $\overrightarrow{a} \cdot \overrightarrow{b} < |\overrightarrow{a}||\overrightarrow{b}|$

2. The unit vector perpendicular to both vectors $\hat{i} + \hat{k}$ and $\hat{i} - \hat{k}$ is:

   (a) $2\hat{j}$

   (b) $\hat{j}$

   (c) $\frac{\hat{i}-\hat{k}}{\sqrt{2}}$

   (d) $\frac{\hat{i}+\hat{k}}{\sqrt{2}}$

3. Direction ratios of a vector parallel to line $\frac{x-1}{2} = -y = \frac{2z+1}{6}$ are :

   (a) $2, -1, 6$

   (b) $2, 1, 6$

   (c) $2, 1, 3$

   (d) $2, -1, 3$

4. Assertion (A): For two non-zero vectors $\overrightarrow{a}$ and $\overrightarrow{b}$, $\overrightarrow{a} \cdot \overrightarrow{b} = \overrightarrow{b} \cdot \overrightarrow{a}$

   Reason (R): For two non-zero vectors $\overrightarrow{a}$ and $\overrightarrow{b}$, $\overrightarrow{a} \times \overrightarrow{b} = \overrightarrow{b} \times \overrightarrow{a}$

   (a) Both Assertion (A) and Reason (R) are true, and Reason (R) is the correct explaination of Assertion (A).

   (b) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explaination of Assertion (A)

   (c) Assertion (A) is true, but Reason (R) is false

   (d) Assertion (A) is false, but Reason (R) is true

5. The position vectors of vertices of $\triangle\ ABC$ are $A(2\hat{i} - \hat{j} + \hat{k})$, $B(\hat{i} - 3\hat{j} - 5\hat{k})$ and $C(3\hat{i} - 4\hat{j} - 4\hat{k})$. Find all the angles of $\triangle\ ABC$.

1.2. 2009

1.2.1. 12

1. Find the value of $\mathbf{p}$ if

$$(2\hat{i} + 6\hat{j} + 27\hat{k}) \times (\hat{i} + 3\hat{j} + p\hat{k}) = \mathbf{0}.$$

2. If $\mathbf{p}$ is a unit vector and $(\mathbf{x}-\mathbf{p})\cdot(\mathbf{x}+\mathbf{p})=80$, then find $|\mathbf{x}|$.

3. Find the shortest distance between the following two lines:

$$\mathbf{r}=(1+\lambda)\hat{\imath}+(2-\lambda)\hat{\jmath}+(\lambda+1)\hat{k};$$

$$\mathbf{r}=(2\hat{\imath}-\hat{\jmath}-\hat{k})+\mu(2\hat{\imath}+\hat{\jmath}+2\hat{k}).$$

4. The scalar product of the vector $\hat{\imath}+\hat{\jmath}+\hat{k}$ with the unit vector along the sum of vectors $2\hat{\imath}+4\hat{\jmath}-5\hat{k}$ and $\lambda\hat{\imath}+2\hat{\jmath}+3\hat{k}$ is equal to one. Find the value of λ.

Chapter 2

linear regression

2.1. 2009

2.1.1. 12

1. A dealer wishes to purchase a number of fans and sewing machines. He has only | $5,760$ to invest and has a space for at most 20 items.A fan costs him | 360 and a sewing machine | 240. His expectation is that he can sell a fan at a profit of | 22 and a sewing machine at a profit of | 18. Assuming that he can sell all the items that he can buy, how should he invest his money in order to maximise the profit ? Formulate this as a linear programming problem and solve it graphically.

Chapter 3

Linear Forms

Chapter 4

Circles

Chapter 5

Intersection of Conics

Chapter 6

Probability

6.1. 2024

6.1.1. 12

1. Let E be an event of a sample space S of an experiment, then $P(S|E) =$

   (a) $P(S \cap E)$

   (b) $P(E)$

   (c) 1

   (d) 0

2. A pair of dice is thrown simultaneously. If X denotes the absolute difference of the numbers appearing on top of the dice, then find the probability distribution of X.

3. Airplanes are by far the safest mode of transportation when the number of transported passengers are measured against personal injuries and fatality tools. Previous records state that the probability of an airplane crash is 0.00001%. Further, there are 95% chances that there

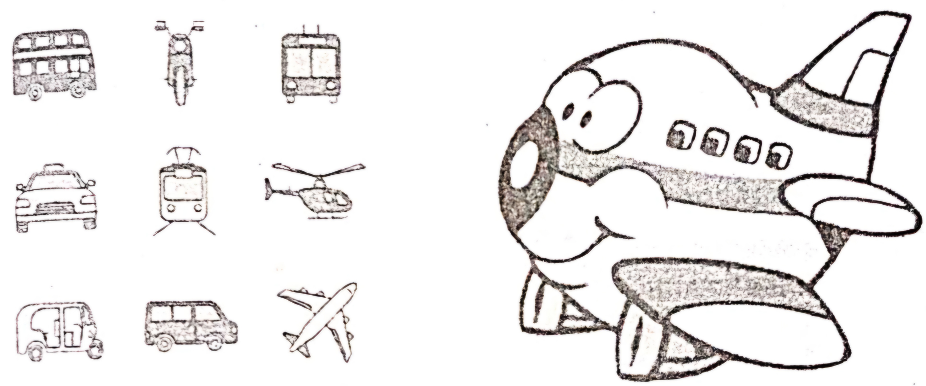


Figure 6.1: 1

will be survivors after a plane crash. Assume that in case of no crash, all travellers survive.

Let E_1 be the event that there is a plane crash and E_2 be the event that there is no crash. Let A be the event that passengers survive after the journey.

On the basis of the above information, answer the following questions:

(i) Find the probability that the airplane will not crash.

(ii) Find $P(A|E_1) + P(A|E_2)$.

(iii) Find $P(A)$

(iv) Find $P(E_2|A)$.

6.2. 2023

6.2.1. 10

1. A student noted the number of cars passing through a spot on a road for 100 periods each of 3 minutes and summarised it in the table given below. Find the mean and median of the following data.

Number of cars	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Frequency (Periods)	7	14	13	12	20	11	15	8

2. Computer-based learning (CBL) refers to any teaching methodology that makes use of computers for information transmission. At an elementary school level, computer applications can be used to display multimedia lesson plans. A survey was done on 1000 elementary and secondary schools of Assam and they were classified by the number of computers they had.

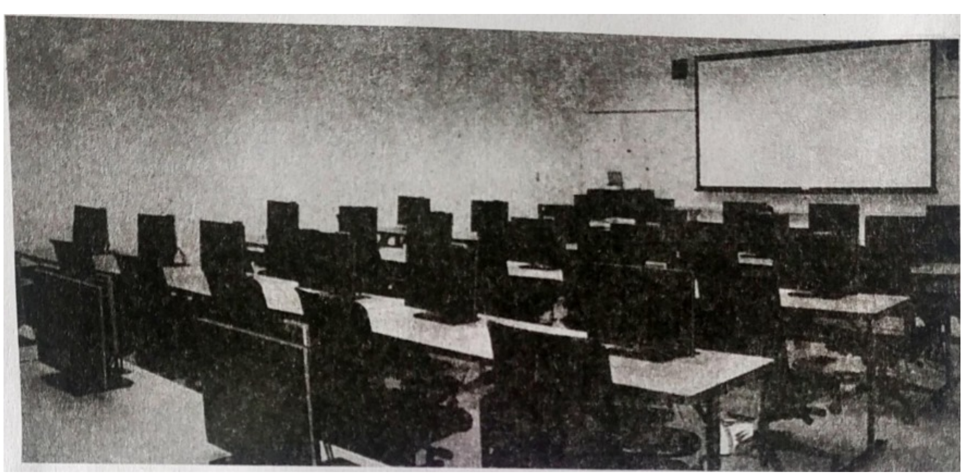


Figure 6.2:

Number of computers	1-10	11-20	21-50	51-100	101 and more
Number of Schools	250	200	290	180	80

One school is chosen at random.Then:

(a) Find the probability that the school chosen at random has more than 100 computers.

(b) i. Find the probability that the school chosen at random has 50 or fewer computers.

ii. Find the probability that the school chosen at random has no more than 20 computers.

(c) Find the probability that the school chosen at random has 10 or less than 10 computers.

6.3. 2006

6.3.1. 10

1. A card is drawn at random from a well-shuffled deck of playing cards. Find the probability that the card drawn is

   (a) a card of spades or an ace

   (b) a red king

   (c) neither a king nor a queen

   (d) either a king or a queen.

6.4. 2009

6.4.1. 12

1. On a multiple choice examination with three possible answers (out of which only one is correct) for each of the five questions, what is the probability that a candidate would get four or more correct answers just by guessing?

2. Colored balls are distributed in three bags as shown in the following table:

Bag	Colour of the Ball		
	Black	White	Red
I	1	2	3
II	2	4	1
III	4	5	3

   A bag is selected at random, and then two balls are randomly drawn from the selected bag. They happen to be black and red. What is the probability that they came from Bag I?

Chapter 7

Construction

Chapter 8

Optimization

Chapter 9

Algebra

9.1. 2024

9.1.1. 12

1. let $f : R_+ \to [-5, \infty)$ be defined as $f(x) = 9x^2 + 6x - 5$ where R_+ is the set of all non-negative real numbers, then f is:

   (a) one-one

   (b) onto

   (c) bijective

   (d) neither one-one nor onto

2. The number of points of discontinuity of $f(x) = \begin{cases} |x| + 3, & if x \leq -3 \\ -2x, & if -3 < x < 3 \\ 6x + 2, & if x \geq 3 \end{cases}$ is:

   (a) 0

   (b) 1

(c) 2

(d) infinite

3. The function $f(x) = x^3 - 3x^2 + 12x - 18$ is:

   (a) strictly decreasing on R

   (b) strictly increasing on R

   (c) neither strictly increasing nor strictly decreasing on R

   (d) strictly decreasing on $(-\infty, 0)$

4. Find the domain of the function $f(x) = \sin^{-1}(x^2 - 4$. Also, find its range.

5. If $f(x) = |\tan 2x|$, then find the value of $f'(x)$ at $x = \frac{\pi}{3}$.

6. If M and m denote the local maximum and local minimum values of the function $f(x) = x + \frac{1}{x}(x \neq 0)$ respectively, find the value of $(M - m)$.

7. Show that $f(x) = e^x - e^{-x} + x - \tan^{-1} x$ is strictly increasing in its domain.

8. Show that a function $f : R \to R$ defined bt $f(x) = \frac{2x}{1+x^2}$ is neither one-one nor onto. Further, find set A so that the given function $f : R \to A$ becomes an onto function.

9. A relation R is defined on $N \times N$ (where N is the set f natural numbers) as:

$$(a, b)R(c, d) \leftrightarrow a - c = b - d$$

   Show that R is an equivalence relation.

10. The month of September is celebrtaed as the Rashtriya Poshan Maah across the country. Following a healthy and well-balanced diet is crucial in order to supply the body with the proper nutrients it needs. A balanced diet also keeps us mentally fit and promotes improved level of energy.

A dietician wishes to minimize the cost of a diet involving two types

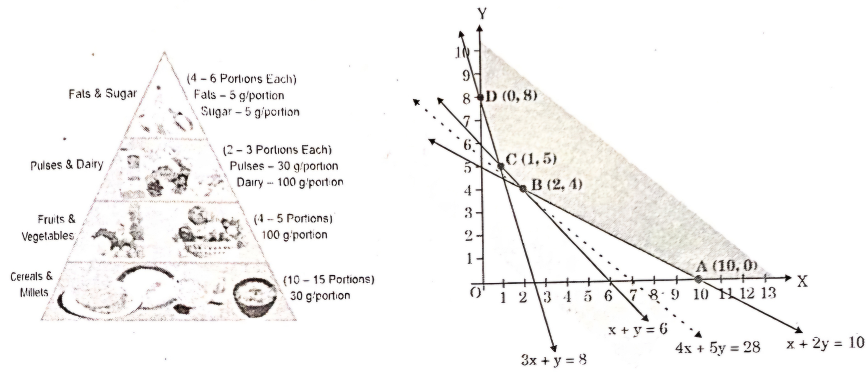


Figure 9.1: 1

of foods,food $X(xkg)$ and fodd $Y(ykg)$ which are available at the rate of $|16/kg$ and $|20/kg$ respectively. The feasible region satisfying the constraints is shown in the graph.

On the basis of the above information, answer the following questions:

(i) Identify and write all the constraints which determine the given feasible region in the above graph.

(ii) If the objective is to minimize cost $Z = 16x + 20y$, find the values of x and y at which cost is minimum. Also, find minimum cost assuming that minimum cost is possiblr for the given unbounded region.

9.2. 2023

9.2.1. 10

1. **Assertion (A):**The polynomial p(x)=$x^2 + 3x + 3$ has two real zeroes.

   Reason (R) : A quadratic polynomial can have at most two zeroes.

   (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

   (b) Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of Assertion (A).

   (c) Assertion (A) is true but Reason (R) is false.

   (d) Assertion (A) is false but Reason (R) is true.

2. Three bells ring at intervals of $6, 12 and 18 minutes$. If all the three bells rang at $6 a.m.$, when will they ring together again ?

3. If the system of linear equations

$$2x + 3y = 7and \tag{9.1}$$

$$2ax + (a + b)\,y = 28 \tag{9.2}$$

   have infinite number of solutions, then find the values of $'a'$and$'b'$.

4. If

$$217x + 131y = 913 and \tag{9.3}$$

$$131x + 217y = 827, \tag{9.4}$$

then solve the equations for the values of x and y.

5. How many terms of the arithmetic progression $45, 39, 33, ...$ must be taken so that their sum is 180? Explain the double answer.

9.3. 2006

9.3.1. 10

1. Solve the system of equations:

   $\frac{bx}{a} - \frac{ay}{b} + a + b = 0$ and $bx - ay + 2ab = 0$.

2. Given that:

   $P = \frac{x+2y}{x+y} + \frac{x}{y}$,Q= $\frac{x+y}{x-y} - \frac{x-y}{x+y}$ and R=$\frac{x+2y}{x+y}$ - $\frac{x}{x+y}$

3. If $(x+2)(x-3)$ is the HCF of the polynomials $p(x) = (x^2+x-2)(3x^2-8x+c)$ and $q(x) = (x^2+x-12)(2x^2+x+b)$,

   find the values of c and b.

4. Using the quadratic formula, solve the equation: $A^2b^2x^2-(4b^4-3a^4)x-$

$12a^2b^2 = 0$.

5. A household article is available for ₹970 cash or ₹210 cash down payment followed by three equal monthly installments. If the rate of interest charged under the installment plan is 16% per annum, find the amount of each installment

6. A man borrows ₹25,200 from a finance company and has to repay it in two equal annual installments. If the interest is charged at the rate of 10% per annum compounded annually, calculate the amount of each installment.

7. The speed of a boat in still water is 11 km/hr. It can go 12 km upstream and return downstream to the original point in 2 hours 45 minutes. Find the speed of the stream.

8. A bucket made up of a metal sheet is in the form of a frustum of a cone. Its depth is 24 cm and the diameters of the top and bottom are 30 cm and 10 cm respectively. Find the cost of milk which can completely fill the bucket at the rate of ₹20 per liter and the cost of the metal sheet used, if it costs ₹10 per 100 cm^2. $(Use \pi = 3.14)$

9. Mrs. Ruchi's salary is Rs.$32,250$ per month exclusive of HRA. She donates ₹12,000 to Prime Minister's Relief Fund (100% exemption). She also donates ₹6,000 to a school and gets a relief of 50% on this donation. She contributes ₹5,000 per month towards her Provident Fund. She pays a quarterly premium of Rs.$2,500$ towards her LIC

policy and invests ₹25,000 in NSCs. If ₹2,700 is the tax deducted each month from her salary for 11 months, find the tax deducted from her salary in the last month of the year.

Income Tax Rates:

Slab	Tax Rate
Up to $Rs.1,35,000$	No tax
From $Rs.1,35,001$ to $Rs.1,50,000$	10% of the taxable income above $Rs.1,35,000$
From $Rs.1,50,001$ to $Rs.2,50,000$	$Rs.1,500 + 20\%$ of the income exceeding $Rs.1,50,000$
$Rs.2,50,001$ and above	$Rs.21,500 + 30\%$ of the amount exceeding $Rs.2,50,000$

Education Cess: 2% of the income tax

9.4. 2009

9.4.1. 12

1. Let $*$ be a binary operation on N given by $a * b = HCF(a, b)$, where $a, b \in N$. Write the value of $22 * 4$.

2. Let $f : N \to N$ be a function defined by:

$$f(n) = \begin{cases} \frac{n+1}{2}, & ifnisodd \\ \frac{n}{2}, & \text{if } n \text{ is even} \end{cases}$$

for all $n \in N$. Find whether the function f is bijective.

3. A manufacturer can sell x items at a price of Rs. $\left(5 - \frac{x}{100}\right)$ each. The

cost price of x is Rs. $\left(\frac{x}{5} + 500\right)$. Find the number of items he should sell to earn maximum profit.

4. Find the intervals in which the function f given by:

$$f(x) = \sin x + \cos x, \quad 0 \leq x \leq 2\pi, \tag{9.5}$$

is strictly increasing or strictly decreasing.

Chapter 10

Geometry

10.1. 2024

10.1.1. 12

1. The coordinates of the foot of the perpendicular drawn from the point $(0, 1, 2)$ on th x-axis are given by:

   (a) $(1, 0, 0)$

   (b) $(2, 0, 0)$

   (c) $(\sqrt{5}, 0, 0)$

   (d) $(0, 0, 0)$

2. If a line makes an angle of 30° with the positive direction of x-axis, 120° with the positive direction of y-axis, then the angle which it makes with the positive direction of z-axis is:

   (a) 90°

   (b) 120°

   (c) 60°

(d) $0°$

3. Find the equation of the line which bisects the line segment joining points $A(2,3,4)$ and $B(4,5,8)$ and is perpendicular to the lines $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$

4. If A_1 denotes the area of region bounded by $y^2 = 4x$ and x-axis in the first quadrant and A_2 denotes the area of region bounded by $y^2 = 4x$, $x = 4$, find $A_1 : A_2$.

10.2. 2023

10.2.1. 10

1. A car has two wipers which do not overlap. Each wiper has a blade of length 21cm sweeping through an angle of $120°$. Find the total area cleaned at each sweep of the two blades.

2. Sides AB and BC and median AD of a triangle ABC are respectively proportional to sides PQ and QR and median PM of $\triangle PQR$.Show that $\triangle ABC \sim \triangle PQR$.

3. Through the mid-point M of the side CD of a parallelogram $ABCD$,the line BM is drawn intersecting AC in L and AD (produced) in E.Prove that $EL = 2BL$.

4. In the given figure, O is the center of the circle .AB and AC are tangents drawn to the circle from point A. If $\angle BAC = 65°$, then find

the measure of $\angle BOC$.

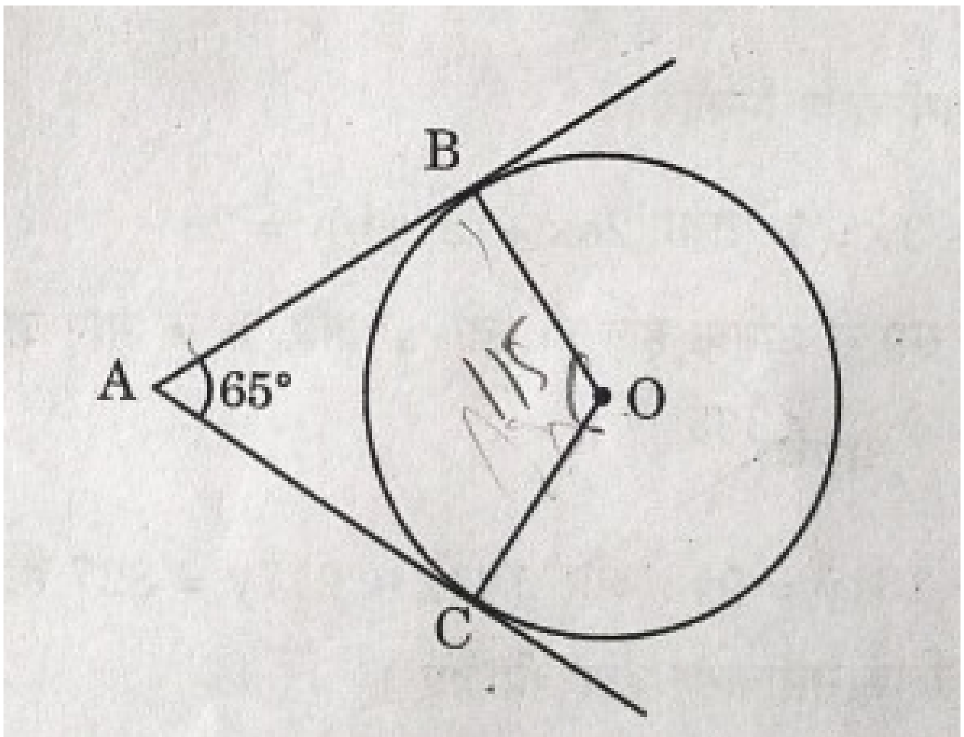


Figure 10.1:

5. In the given figure, O is the centre of the circle and QPR is a tangent to it at P.Prove that $\angle QAP + \angle APR = 90°$.

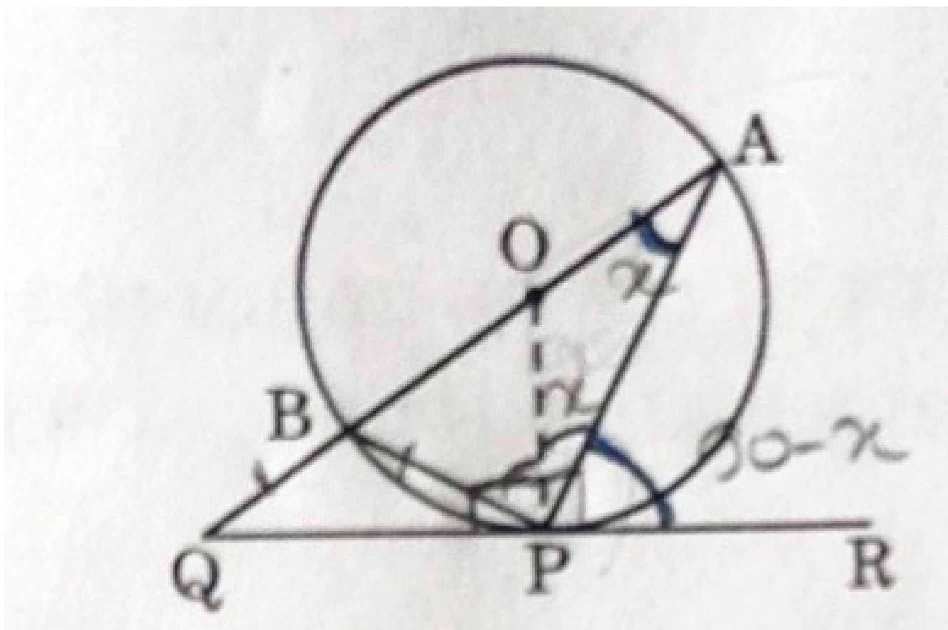


Figure 10.2:

6. In an annual day function of a school, the organizers wanted to give a cash prize along with a memento to their best students.Each memento is made as shown in the figure and its base $ABCD$ is shown from the front side.The rate of silver plating ₹ 20 $per \text{cm}^2$.

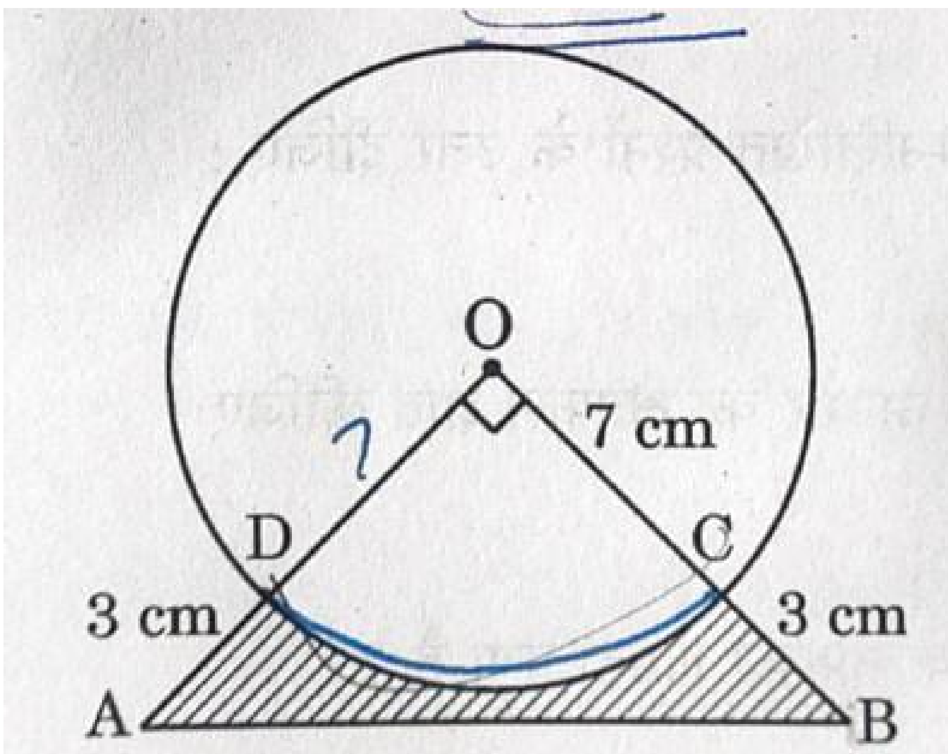


Figure 10.3:

Based on the above, answer the following question:

(a) What is the area of the quadrant $ODOC$?

(b) Find the area of $\triangle AOB$.

(c) i. What is the total cost of silver plating the shaded part $ABCD$?

ii. What is the length of arc CD ?

7. In a coffee shop, coffee is served in two types of cups.One is cylindrical in shape with diameter 7cm and height 14cm and the other is hemispherical with diameter 21cm.

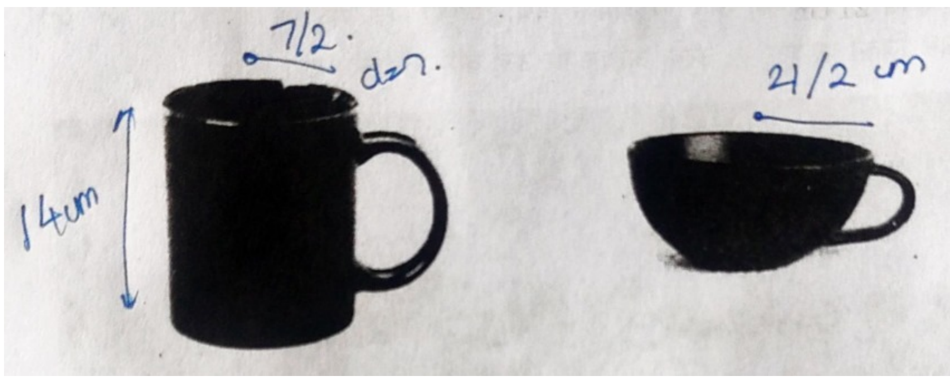


Figure 10.4:

Based on the above, answer the following question:

(a) Find the area of the cylindrical cup.

(b) i. What is the capacity of the hemispherical cup?

ii. Find the capacity of the cylindrical cup.

(c) What is the curved surface area of the cylindrical cup?

8. Show that the points $(-2,3),(8,3)$ and $(6,7)$ are the vertices of a right-angled triangle.

9. If $Q(0,1)$ is equidistant from $P(5,-3)$ and $R(x,6)$,find the values of x.

10.3. 2006

10.3.1. 10

1. In Figure 1, $\angle BAC = 90°$. $AD \parallel BC$. Prove that $AB^2 + CD^2 = BD^2 + AC^2$.

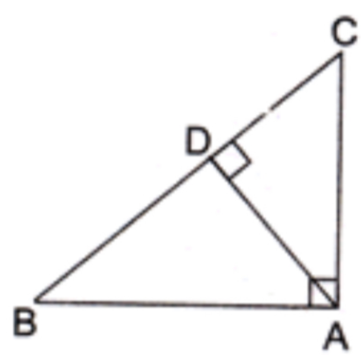


Figure 10.5: 1

2. In Figure 2, $PT = 6$ cm, $AR = 5$ cm. Find the length of PA.

3. Draw the graphs of the following equations: $3x-4y+6 = 0, 3x+y-9 = 0$ Also, determine the co-ordinates of the vertices of the triangle formed by these lines and the x axis.

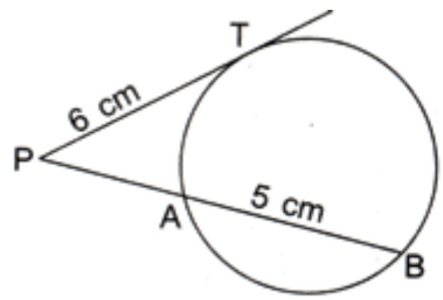


Figure 10.6: 2

4. A solid is in the form of a right circular cylinder with hemispherical ends. The total height of the solid is 58 cm and the diameter of the cylinder is 28cm. Find the total surface area of the solid $\pi \approx \frac{22}{7}$

5. . Construct a triangle ABC in which $BC = 7$ cm, and median $AD =$ 5 cm, $\angle A = 60°$ Write the steps of construction also.

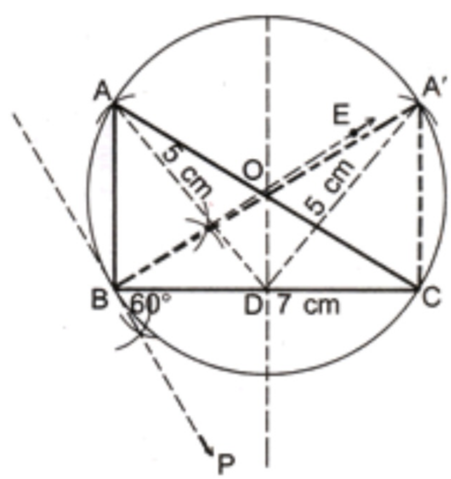


Figure 10.7: 3

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6. Show that the points A(6, 2), B(2, 1), C(1, 5) and D(5, 6) are the vertices of a square

7. Find the value of p for which the points (- 5, 1), (1, p) and $4, -2$ are collinear.

8. . Prove that in a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

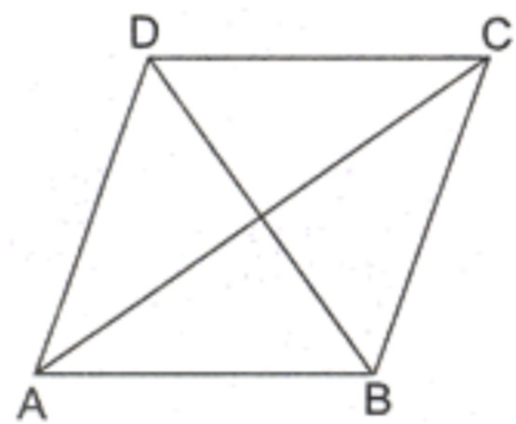

Figure 10.8: 4

Makeing ue of the above, prove the following:

in fig:4, $ABCD$ is a fig:4 rhombus. prove that $4AB^2 = AC^2 + BD^2$.

9. Prove that I a line touch a circle and from the point of contact a chord is drawn, the angles which this chord makes with the given line are equal respectively to the angles formed in the corresponding alternate segments. Using the above, do the following:

AB is a diameter and AC is a chord of a circle such that $\angle BAC = 30°$. The tangent at C intersects AR produced in a point I Prove that

$BC = RD$.

10. A man standing on the deck of a ship, which is 10 m above the water level, observes the angle of elevation of the top of a hill as $60°$ and the angle of depression of the base of the hill as 300. Calculate the distance of the hill from the ship and the height of the hill.

11. From a window x meters high above the ground in a street, the angles of elevation and depression of the top and foot of the other house on the opposite side of the street are α and β respectively. Show that the height of the opposite house is $x(1 + \tan\alpha \cot\beta)$ meters.

10.4. 2009

10.4.1. 12

1. Write the direction cosines of a line equally inclined to the three coordinates axes.

2. Find the equation of the plane determined by the points $A(3, -1, 2)$, $B(5, 2, 4)$ and $C(-1, -1, 6)$. also find the distance of the point $P(6, 5, 9)$ from the plane.

3. Find the area of the region included between the parabola $y^2 = x$ and the line $x + y = 2$

4. The length x of a rectangle is decreasing at the rate of $5cm/$minute and the width y is increasing at the rate of $4cm/$minute. When $x =$

$8cm$and$y = 6cm$, find the rate of change of (a) the perimeter,(b) the area of the rectangle.

Chapter 11

sequences

11.1. 2006

11.1.1. 10

1. The 5^{th} term of an Arithmetic Progression (A.P.) is 26 and the 10th term is 51. Determine the 15^{th} term of the A.P.

2. Find the sum of all the natural numbers less than 100 which are divisible by 6.

Chapter 12

Datahandling

12.1. 2006

12.1.1. 10

1. The following table shows the monthly expenditure of company. Draw apie chart for the data.

	Amount (in Rs.)
Wages	4800
Materials	3200
Taxation	2400
Adm. Expenditure	3000
Miscellaneous	1000

2. The Arithmetic Mean of the following frequency distribution is 47.

Determine the value of p.

Classes	Frequency
0 - 20	8
20 - 40	15
40 - 60	20
60 - 80	p
80 - 100	5

Chapter 13

Discrete

Chapter 14

Number Systems

14.1. 2023

14.1.1. 10

1. Prove that $2+\sqrt{3}$ is an irrational number,given that $\sqrt{3}$ is an irrational number.

2. Find by prime factorisation the LCM of the number 18180 and 7575. Also,find the HCF of the two numbers.

Chapter 15

Differentiation

15.1. 2024

15.1.1. 12

1. The derivative of $\tan^{-1}(x^2)$ w.r.t. x is:

   (a) $\frac{x}{1+x^4}$

   (b) $\frac{2x}{1+x^4}$

   (c) $-\frac{2x}{1+x^4}$

   (d) $\frac{1}{1+x^4}$

2. Overspeeding increases fuel consumption and decreases fuel economy as a result of tyre rolling friction and air resistance. While vehicles reach optimal fuel economy at different speeds, fuel mileage usualy decreases rapidly at speeds above 80km/h.

   The relation btween fuel consumption F(l/100 km) and speed V(km/h) under some constraints is given as $F = \frac{V^2}{500} - \frac{V}{4} + 14$.

   On the basis of the above information, answer the following questions:

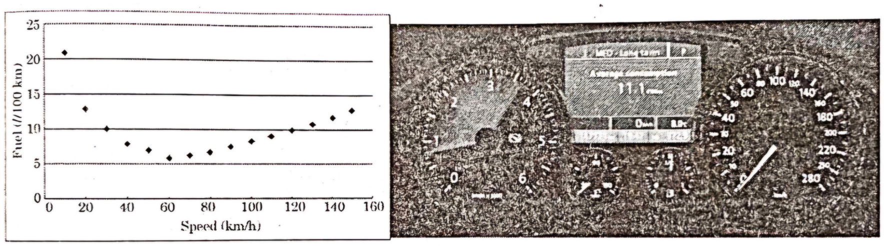

Figure 15.1: 1

(i) Find F, when $V = 40km/h$.

(ii) Find $\frac{dF}{dV}$.

(iii) Find he speed V for which fuel consumption F is minimum.

(iv) Find the quantity of fuel required to travel 600 km at the speed V at which $\frac{dF}{dV} = -0.01$.

1. The differential equation $\frac{dy}{dx} = F(x, y)$ will not be a homogeneous differential equation, if $F(x, y)$ is:

   (a) $\cos x - \sin(\frac{y}{x})$

   (b) $\frac{y}{x}$

   (c) $\frac{x^2+y^2}{xy}$

   (d) $\cos^2(\frac{x}{y})$

2. The degree of the differential equation $(y'')^2 + (y')^3 = x \sin(y')^3$ is:

   (a) 1

   (b) 2

   (c) 3

(d) not defined

3. If $y =$ cosec $(\cot^{-1} x)$, then prove that $\sqrt{1+x^2}\frac{dy}{dx} - x = 0$.

4. If $x = e^{\cos 3t}$ and $y = e^{\sin 3t}$, prove that $\frac{dy}{dx} = -\frac{ylogx}{xlogy}$.

5. Show that: $\frac{d}{dx}(|x|) = \frac{x}{|x|}, x \neq 0$.

6. Find the particular solution of the differetial equation given by $2xy + y^2 - 2x^2\frac{dy}{dx} = 0; y = 2$, when x=1.

7. Find the general solution of the differential equation:

$$ydx = (x + 2y^2)\text{dy}$$

15.2. 2009

15.2.1. 12

1. if $\sin y = x\sin(a+y)$, prove that $\frac{dy}{dx} = \frac{sin^2(a+y)}{\sin a}$.

2. if $(\cos x)^y = (siny)^x$, find $\frac{dy}{dx}$.

3. if $y = \frac{sin^{-1}x}{\sqrt{1-x^2}}$, show that

4. $(1-x^2)\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} - y = 0$

5. Solve the following differential equation :

   $x\frac{dy}{dx} = y - x\tan\frac{y}{x}$

6. solve the following differential equation :

   $cos^2x\frac{dy}{dx} + y = \tan x$

Chapter 16

Integration

16.1. 2024

16.1.1. 12

1. $\int\limits_0^{\frac{\pi}{2}} \frac{\sin x - \cos x}{1 + \sin x \cos x} dx$ is equal to:

   (a) π

   (b) $Zero(0)$

   (c) $\int\limits_0^{\frac{\pi}{2}} \frac{2 \sin x}{1 + \sin x \cos x} dx$

   (d) $\frac{\pi^2}{4}$

2. Find : $\int \frac{e^{4x}-1}{e^{4x}+1} dx$

3. Evaluate:

   $\int\limits_2 -2\sqrt{\frac{2-x}{z+x}} dx$

4. Find:

   $\int \frac{1}{x[(logx)^2 - 3logx - 4]} dx$

5. Find: $\int x^2 \cdot \sin^{-1}(x^{\frac{3}{2}}) dx$

16.2. 2009

16.2.1. 12

1. Evaluate:

$$\int_{0}^{\frac{1}{\sqrt{2}}} \frac{1}{\sqrt{1-x^2}} \quad \mathrm{dx}$$

2. Evaluate:

$$\int \frac{cos\sqrt{x}}{\sqrt{x}} \; \mathrm{dx}$$

3. Evaluate:

$$\int \frac{dx}{\sqrt{5-4x-2x^2}}$$

4. Evaluate:

$$\int xsin^{-1}x\,dx$$

5. Evaluate:

$$\int_{0}^{\pi} \frac{xdx}{a^2\cos^2 x+b^2\sin^2 x}$$

$$\int_0^{\pi} \frac{xdx}{a^2\cos^2 x + b^2\sin^2 x}$$

Chapter 17

Functions

Chapter 18

Matrices

18.1. 2024

18.1.1. 12

1. If the sum of all the elements of 3×3 scalar matrix is 9, then the product of all elements is:

   (a) 0

   (b) 9

   (c) 27

   (d) 729

2. If $\begin{vmatrix} -a & b & c \\ a & -b & c \\ a & b & -c \end{vmatrix} = kabc$, then the value of k is:

   (a) 0

   (b) 1

   (c) 2

(d) 4

3. If $A = [a_{ij}]$ be a 3×3 where $a_{ij} = i - 3j$,then which of the following is false?

   (a) $a_{11} < 0$

   (b) $a_{12} + a_{21} = -6$

   (c) $a_{13} > a_{31}$

   (d) $a_{31} = 0$

4. If $F(x) = \begin{pmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 0 \end{pmatrix}$ and $[F(x)]^2 {=} F(kx)$, then the value of k is:

   (a) 1

   (b) 2

   (c) 0

   (d) -2

5. Assertion (A): For any symmetric matrix A, $B'AB$ is a skew-symmetric matrix.

   Reason (R): A square matrix P is kew-symmetric if $P' = -P$

   (a) Both Assertion and Reason are true, and Reason is the correct explaination of Assertion.

   (b) Both Assertion and Reason are true, but Reason is not the correct explaination of Assertion.

(c) Assertion is true, but Reason is false.

(d) Assertion is false, but Reason is true.

6. Solve the following system of equations,using matrices:

   $\frac{2}{x}+\frac{3}{y}+\frac{10}{z}=4$, $\frac{4}{x}-\frac{6}{y}+\frac{5}{z}=1$, $\frac{6}{x}+\frac{9}{y}-\frac{20}{z}=2$ where $x, y, z \neq 0$

7. If $A=\begin{pmatrix} 1 & \cot x \\ -\cot x & 1 \end{pmatrix}$,then show that $A'A^{-}1=\begin{pmatrix} -\cos 2x & -\sin 2x \\ \sin 2x & -\cos 2x \end{pmatrix}$

18.2. 2009

18.2.1. 12

1. Find the value of x, if
   $\begin{pmatrix} 3x+y & -y \\ 2y-x & 3 \end{pmatrix}=\begin{pmatrix} 1 & 2 \\ -5 & 3 \end{pmatrix}$.

2. Write the value of the following determinant:
   $$\begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix}$$

3. Find the value of x from the following:
   $$\begin{vmatrix} x & 4 \\ 2 & 2x \end{vmatrix}=0$$

4. Using properties of determinants, prove the following:

$$\begin{vmatrix} 1 & 1+p & 1+p+q \\ 2 & 3+2p & 1+3p+2q \\ 3 & 6+3p & 1+6p+3q \end{vmatrix} = 1$$

5. Using matrices, solve the following system of equations:

   $x + y + z = 6$

   $x + 2z = 7$

   $3x + y + z = 12$

6. Obtain the inverse of the following matrix using elementary opertions:

$$A = \begin{pmatrix} 3 & 0 & -1 \\ 2 & 3 & 0 \\ 0 & 4 & 1 \end{pmatrix}$$

Chapter 19

Trignometry

19.1. 2024

19.1.1. 12

1. Find the value of $\tan^{-1}(-\frac{1}{\sqrt{3}}) + \cot^{-1}(\frac{1}{\sqrt{3}}) + \tan^{-1}[\sin(-\frac{\pi}{2})]$

19.2. 2023

19.2.1. 10

1. If $4\cot^2 45° - \sec^2 60° + \sin^2 60° + p = \frac{3}{4}$, then find the value of p.

2. If $\cos A + \cos^2 A = 1$,then find the value of $\sin^2 A + \sin^4 A$

3. The length of the shadow of a tower on the plane ground is $\sqrt{3}$ times the height of the tower. Find the angle of elevation of the sun.

4. The angle of elevation of the top of a tower from a point on the ground which is 30m away from the foot of the tower,is 30°.Find the height of

the tower.

5. Prove that :

$$\left(\frac{1}{\cos\theta} - \cos\theta\right)\left(\frac{1}{\sin\theta} - \sin\theta\right) = \frac{1}{\tan\theta + \cot\theta} \tag{19.1}$$

6. As observed from the top of a 75m high lighthouse from the sea-level, the angles of depression of two ships are 30° and 60°. If one ship is exactly behind the other on the same side of the lighthouse, find the distance between the two ships.
$\left(Use\sqrt{3} = 1.73\right)$

7. From a point on the ground, the angle of elevation of the bottom and top of a transmission tower fixed at the top of 30m high building are 30° and 60°, respectively. Find the height of the transmission tower.
$\left(Use\sqrt{3} = 1.73\right)$.

19.3. 2009

19.3.1. 12

1. write the principle vale of $\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$

2. Prove the following:

$$\cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right) = \frac{x}{2}, x \in \left(0, \frac{\pi}{4}\right)$$

3. If the sum of the lengths of the hypotenuse and a side of a right-angled triangle is given, show that the area of the triangle is maximum when the angle between them is $\frac{\pi}{3}$.