CBSE MATH

Made Simple

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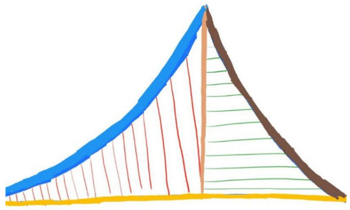


Contents

Introduction

This book links high school coordinate geometry to linear algebra and matrix analysis through solved problems.

Chapter 1

Vectors

1.1. 2024

1.1.1. 12

1. **Assertion (A):** The vectors

$$\mathbf{a} = 6\hat{\imath} + 2\hat{\jmath} - 8\hat{k}$$
$$\mathbf{b} = 10\hat{\imath} - 2\hat{\jmath} - 6\hat{k}$$
$$\mathbf{c} = 4\hat{\imath} - 4\hat{\jmath} + 2\hat{k}$$

represent the sides of a right angled triangle.

Reason (R): Three non-zero vectors of which none of two are collinear form a triangle if their resultant is zero vector or sum of any two vectors is equal to the third.

2. Find the vector equation of the line passing through the point (2, 3, -5) and making equal angles with the coordinate axes.

3. Find the projection of vector $(\mathbf{b}+\mathbf{c})$ on vector $\mathbf{a}$, where $\mathbf{a} = 2\hat{\imath}+2\hat{\jmath}+$

$\hat{k}, \mathbf{b} = \hat{\imath} + 3\hat{\jmath} + \hat{k},$ and $\mathbf{c} = \hat{\imath} + \hat{k}$.

4. Find the coordinates of the foot of the perpendicular drawn from the point $(2, 3, -8)$ to the line $\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$.

   Also, find the perpendicular distance of the given point from the line.

5. Find the shortest distance between the lines L_1 & L_2 given below:

   L_1: The line passing through (2,-1, 1) and parallel to $\frac{x}{1} = \frac{y}{1} = \frac{z}{3}$

   L_2: $\mathbf{r} = \hat{\imath} + (2\mu + 1)\hat{\jmath} - (\mu + 2)\hat{k}$.

6. Given $\mathbf{a} = 2\hat{\imath} - \hat{\jmath} + \hat{k}$,$\mathbf{b} = 3\hat{\imath} - \hat{k}$ and $\mathbf{a} = 2\hat{\imath} + \hat{\jmath} - 2\hat{k}$. Find a vector $\mathbf{d}$ which is perpendicular to both $\mathbf{a}$ and $\mathbf{b}$ and $\mathbf{c} \cdot \mathbf{d} = 3$.

Chapter 2

Linear Forms

2.1. 2024

2.1.1. 12

1. Students of a school are taken to a railway museum to learn about railways heritage and its history

   An exhibt in the museum depicted many rail lines on the tack near

   the railway station. let L be the set of all ral lines on the railwa track and R be the relation on L defined by

R=(l_1, l_2) : $l_1$is parallel to l_2

On the basis of the above information, answer the following questions:

(a) Find whether the relation R is symmetric or not.

(b) Find whether the relation R is transitive or not.

(c) If one of the rail lines on the railway track is represented by the equation $y = 3x + 2$ then find the set of rail lines in R related to it.

2. Let S be the relation defined by $S = (l_1, l_2) : l_1$ is perpendicular to l_2 check whether the relation S is symmetric and transitive.

Chapter 3

Circles

Chapter 4

Intersection of Conics

Chapter 5

Probability

5.1. 2024

5.1.1. 12

1. Let E be an event of a sample space S of an experiment, then $P(S|E) =$

   (a) $P(S \cap E)$

   (b) $P(E)$

   (c) 1

   (d) 0

2. A pair of dice is thrown simultaneously. If X denotes the absolute difference of the numbers appearing on top of the dice, then find the probability distribution of X.

3. Airplanes are by far the safest mode of transportation when the number of transported passengers are measured against personal injuries and fatality tools. Previous records state that the probability of an airplane crash is 0.00001%. Further, there are 95% chances that there

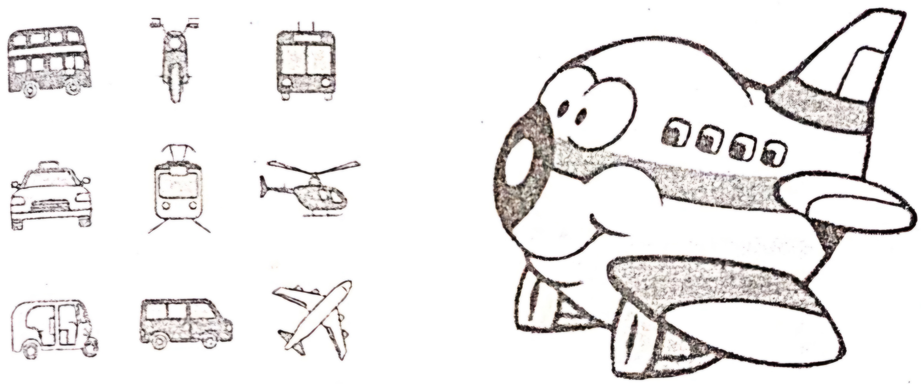


Figure 5.1: 1

will be survivors after a plane crash. Assume that in case of no crash, all travellers survive.

Let E_1 be the event that there is a plane crash and E_2 be the event that there is no crash. Let A be the event that passengers survive after the journey.

On the basis of the above information, answer the following questions:

(i) Find the probability that the airplane will not crash.

(ii) Find $P(A|E_1) + P(A|E_2)$.

(iii) Find $P(A)$

(iv) Find $P(E_2|A)$.

1. An urn contains 3 red and 2 white marbles. Two marbles are drawn one by one with replacement from the urn. Find the probability distribution of the number of white balls. Also, find the mean of the number of white balls drawn.

2. A departmental store sends bills to charge its customers once a month Past experience shows that 70% of its customers pay their first month bill in time. The store also found that the customer who pays the bill in time has the probability of 0.8 of paying in time next month and the customer who doesn't pay in time has the probability of 0.4 of paying in time the next month.

   Based on the above information, answer the following question:

   (a) Let E_1 and E_2 respectively denote the event of customer paying or not paying the first month bill in time
   Find $P(E_1), P(E_2)$.

   (b) Let A denotes the event of customer paving some month's bill in time, then find $P(A|E_1)$ and $P(A|E_2)$.

   (c) Find the probability of customer paying second month's bill in time.

3. Find the probability of customer paying first month's bill in time if it is found that customer has paid the second month's bill in time

4. Bag I contains 3 red and black balls, Bag II contains 5 red and 2 black balls. Two balls are transferred at random from Bag I to Bag II and then a ball is drawn at random from Bag II. Find the probability that the drawn ball is red in colour.

5.1.2. 10

1. If the probability of a player winning a game is 0.79, then the probability of his losing the same game is:

   (a) 1.79

   (b) 0.31

   (c) 0.21

   (d) 0.21

2. From the data $1, 4, 7, 9, 16, 21, 25$, if all the even numbers are removed, then the probability of getting at random a prime number from the remaining is:

   (a) $\frac{2}{5}$

   (b) $\frac{1}{5}$

   (c) $\frac{1}{7}$

   (d) $\frac{2}{7}$

3. For some data $x_1, x_2, \ldots, x_n$ with respective frequencies $f_1, f_2, \ldots, f_n$, the value of $\sum_i^n f_i\,(x_i - \overline{x})$ is equal to:

   (a) $n\overline{x}$

   (b) 1

   (c) Σf_i

   (d) 0

4. The middle-most observation of every data arranged in order is called:

(a) mode

(b) median

(c) mean

(d) deviation

5. Two dice are rolled together. The probability of getting a sum of numbers on the two dice as 2, 3, or 5 is:

   (a) $\frac{7}{36}$

   (b) $\frac{11}{36}$

   (c) $\frac{5}{36}$

   (d) $\frac{4}{9}$

5.2. 2023

5.2.1. 10

1. A student noted the number of cars passing through a spot on a road for 100 periods each of 3 minutes and summarised it in the table given below. Find the mean and median of the following data.

Number of cars	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Frequency (Periods)	7	14	13	12	20	11	15	8

2. Computer-based learning (CBL) refers to any teaching methodology that makes use of computers for information transmission. At an elementary school level, computer applications can be used to display multimedia lesson plans. A survey was done on 1000 elementary and secondary schools of Assam and they were classified by the number of computers they had.

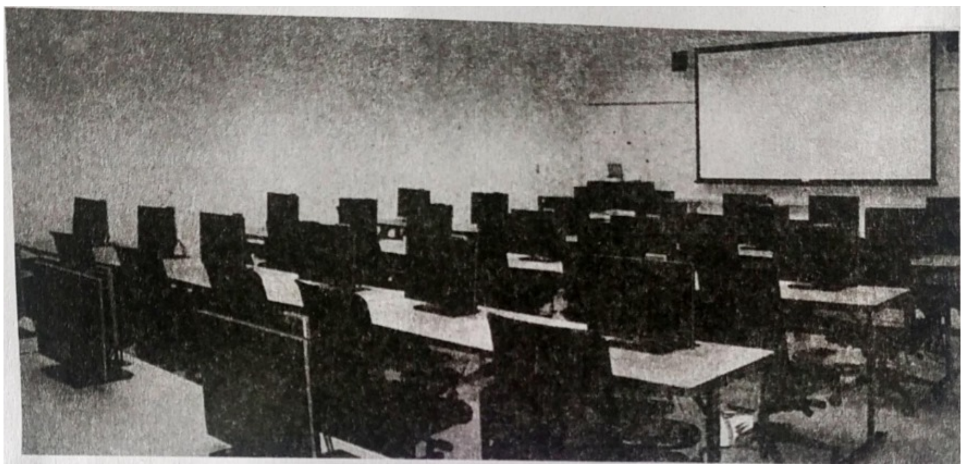


Figure 5.2:

Number of computers	1-10	11-20	21-50	51-100	101 and more
Number of Schools	250	200	290	180	80

One school is chosen at random.Then:

(a) Find the probability that the school chosen at random has more than 100 computers.

(b) i. Find the probability that the school chosen at random has 50 or fewer computers.

ii. Find the probability that the school chosen at random has no more than 20 computers.

(c) Find the probability that the school chosen at random has 10 or less than 10 computers.

5.3. 2006

5.3.1. 10

1. A card is drawn at random from a well-shuffled deck of playing cards.

   Find the probability that the card drawn is

   (a) a card of spades or an ace

   (b) a red king

   (c) neither a king nor a queen

   (d) either a king or a queen.

Chapter 6

Construction

Chapter 7

Optimization

7.1. 2024

7.1.1. 12

1. Solve the following Linear Programming problem graphically:

   Maximise $Z = 300x + 600y$

   Subject to

$$x + 2y \le 12 \tag{7.1}$$

$$2x + y \le 12 \tag{7.2}$$

$$x + \frac{5}{4}y \ge 5 \tag{7.3}$$

$$x \ge 0, y \ge 0. \tag{7.4}$$

Chapter 8

Algebra

8.1. 2024

8.1.1. 10

1. If the sum of zeroes of the polynomial $p(x) = 2x^2 - k\sqrt{2}x + 1$ is $\sqrt{2}$,then value of k is:

   (a) $\sqrt{2}$

   (b) 2

   (c) $2\sqrt{2}$

   (d) $\frac{1}{2}$

2. If the roots of the equation $ax^2 + bx + c = 0$,$a \neq 0$ are real and equal, then which of the following relations is true?

   (a) $a = \frac{b^2}{c}$

   (b) $b^2 = ac$

   (c) $ac = \frac{b^2}{4}$

(d) $c = \frac{b^2}{a}$

3. In an A.P., if the first term $a = 7$, nth term $a_n = 84$, and the sum of the first n terms $s_n = \frac{2093}{2}$, then n is equal to:

   (a) 22

   (b) 24

   (c) 23

   (d) 26

4. The zeroes of a polynomial $x^2 + px + q$ are twice the zeroes of the polynomial $4x^2 - 5x - 6$. The value of p is:

   (a) $-\frac{5}{2}$

   (b) $\frac{5}{2}$

   (c) -5

   (d) 10

5. In the given figure, graphs of two linear equations are shown. The pair of these linear equations is:

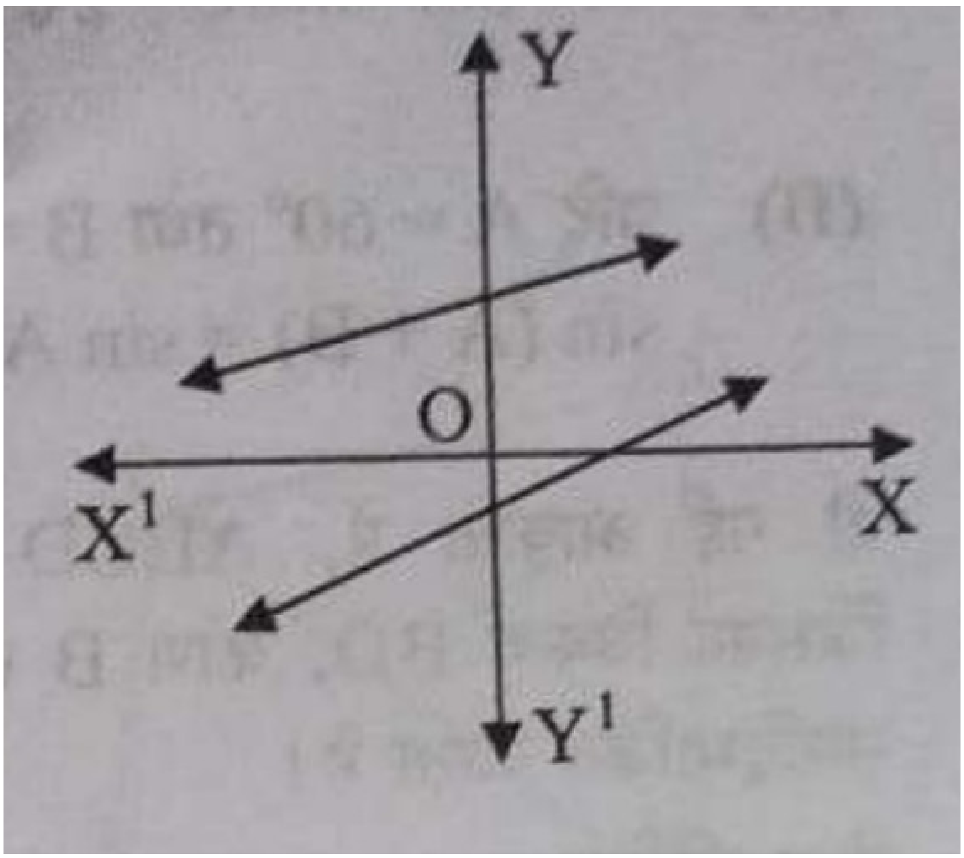


Figure 8.1:

(a) consistent with a unique solution.

(b) consistent with infinitely many solutions.

(c) inconsistent.

(d) inconsistent but can be made consistent.

8.1.2. 12

1. let $f : R_+ \to [-5, \infty)$ be defined as $f(x) = 9x^2 + 6x - 5$ where R_+ is the set of all non-negative real numbers, then f is:

   (a) one-one

   (b) onto

   (c) bijective

   (d) neither one-one nor onto

2. The number of points of discontinuity of $f(x) = \begin{cases} |x| + 3, & ifx \leq -3 \\ -2x, & if -3 < x < 3 \\ 6x + 2, & ifx \geq 3 \end{cases}$ is:

   (a) 0

   (b) 1

   (c) 2

   (d) infinite

3. The function $f(x) = x^3 - 3x^2 + 12x - 18$ is:

   (a) strictly decreasing on R

   (b) strictly increasing on R

   (c) neither strictly increasing nor strictly decreasing on R

   (d) strictly decreasing on $(-\infty, 0)$

4. Find the domain of the function $f(x) = \sin^{-1}(x^2 - 4$. Also, find its range.

5. If $f(x) = |\tan 2x|$, then find the value of $f'(x)$ at $x = \frac{\pi}{3}$.

6. If M and m denote the local maximum and local minimum values of the function $f(x) = x + \frac{1}{x}(x \neq 0)$ respectively, find the value of $(M - m)$.

7. Show that $f(x) = e^x - e^{-x} + x - \tan^{-1} x$ is strictly increasing in its domain.

8. Show that a function $f : R \to R$ defined bt $f(x) = \frac{2x}{1+x^2}$ is neither one-one nor onto. Further, find set A so that the given function $f : R \to A$ becomes an onto function.

9. A relation R is defined on $N \times N$ (where N is the set f natural numbers) as:

$$(a, b)R(c, d) \leftrightarrow a - c = b - d$$

Show that R is an equivalence relation.

10. The month of September is celebrtaed as the Rashtriya Poshan Maah across the country. Following a healthy and well-balanced diet is crucial in order to supply the body with the proper nutrients it needs. A balanced diet also keeps us mentally fit and promotes improved level of energy.

A dietician wishes to minimize the cost of a diet involving two types of foods,food $X(xkg)$ and fodd $Y(ykg)$ which are available at the rate

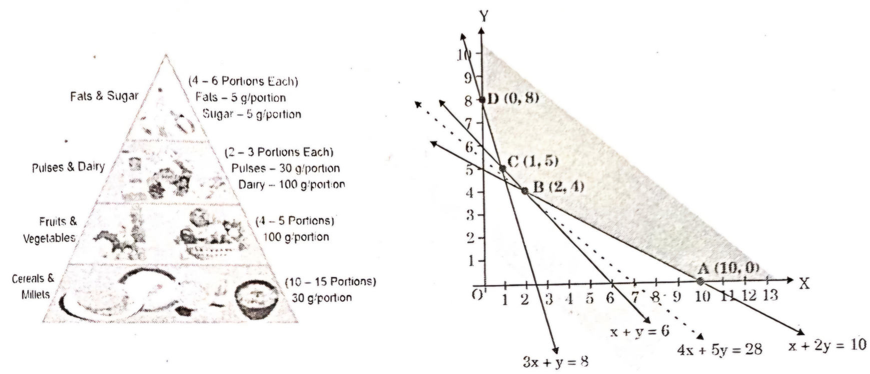


Figure 8.2: 1

of $|16/kg$ and $|20/kg$ respectively. The feasible region satisfying the constraints is shown in the graph.

On the basis of the above information, answer the following questions:

(i) Identify and write all the constraints which determine the given feasible region in the above graph.

(ii) If the objective is to minimize cost $Z = 16x + 20y$, find the values of x and y at which cost is minimum. Also, find minimum cost assuming that minimum cost is possiblr for the given unbounded region.

8.2. 2023

8.2.1. 10

1. **Assertion (A):**The polynomial p(x)=$x^2 + 3x + 3$ has two real zeroes.

   Reason (R) : A quadratic polynomial can have at most two zeroes.

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(b) Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of Assertion (A).

(c) Assertion (A) is true but Reason (R) is false.

(d) Assertion (A) is false but Reason (R) is true.

2. Three bells ring at intervals of $6, 12 and 18 minutes$. If all the three bells rang at $6 a.m.$, when will they ring together again ?

3. If the system of linear equations

$$2x + 3y = 7 and \tag{8.1}$$

$$2ax + (a + b) y = 28 \tag{8.2}$$

have infinite number of solutions, then find the values of $'a'$and$'b'$.

4. If

$$217x + 131y = 913 and \tag{8.3}$$

$$131x + 217y = 827, \tag{8.4}$$

then solve the equations for the values of x and y.

5. How many terms of the arithmetic progression $45, 39, 33, ...$ must be taken so that their sum is 180? Explain the double answer.

8.3. 2006

8.3.1. 10

1. Solve the system of equations:

   $\frac{bx}{a} - \frac{ay}{b} + a + b = 0$ and $bx - ay + 2ab = 0.$

2. Given that:

   $P = \frac{x+2y}{x+y} + \frac{x}{y}$,Q= $\frac{x+y}{x-y} - \frac{x-y}{x+y}$ and R= $\frac{x+2y}{x+y}$ - $\frac{x}{x+y}$

3. If $(x+2)(x-3)$ is the HCF of the polynomials $p(x) = (x^2+x-2)(3x^2-8x+c)$ and $q(x) = (x^2+x-12)(2x^2+x+b)$,
   find the values of c and b.

4. Using the quadratic formula, solve the equation: $A^2b^2x^2-(4b^4-3a^4)x-12a^2b^2 = 0.$

5. A household article is available for ₹970 cash or ₹210 cash down payment followed by three equal monthly installments. If the rate of interest charged under the installment plan is 16% per annum, find the amount of each installment

6. A man borrows ₹25,200 from a finance company and has to repay it in two equal annual installments. If the interest is charged at the rate of 10% per annum compounded annually, calculate the amount of each

installment.

7. The speed of a boat in still water is 11 km/hr. It can go 12 km upstream and return downstream to the original point in 2 hours 45 minutes. Find the speed of the stream.

8. A bucket made up of a metal sheet is in the form of a frustum of a cone. Its depth is 24 cm and the diameters of the top and bottom are 30 cm and 10 cm respectively. Find the cost of milk which can completely fill the bucket at the rate of ₹20 per liter and the cost of the metal sheet used, if it costs ₹10 per 100 cm^2. $(Use \pi = 3.14)$

9. Mrs. Ruchi's salary is Rs.$32,250$ per month exclusive of HRA. She donates ₹12,000 to Prime Minister's Relief Fund (100% exemption). She also donates ₹6,000 to a school and gets a relief of 50% on this donation. She contributes ₹5,000 per month towards her Provident Fund. She pays a quarterly premium of Rs.$2,500$ towards her LIC policy and invests ₹25,000 in NSCs. If ₹2,700 is the tax deducted each month from her salary for 11 months, find the tax deducted from her salary in the last month of the year.

Income Tax Rates:

Slab	Tax Rate
Up to Rs.$1,35,000$	No tax
From Rs.$1,35,001$ to Rs.$1,50,000$	10% of the taxable income above Rs.$1,35,000$
From Rs.$1,50,001$ to Rs.$2,50,000$	Rs.$1,500 + 20\%$ of the income exceeding Rs.$1,50,000$
Rs.$2,50,001$ and above	Rs.$21,500 + 30\%$ of the amount exceeding Rs.$2,50,00$

Education Cess: 2% of the income tax

Chapter 9

Geometry

9.1. 2024

9.1.1. 10

1. A solid sphere is cut into two hemispheres. The ratio of the surface areas of the sphere to that of the two hemispheres taken together is:

   (a) 1 : 1

   (b) 1 : 4

   (c) 2 : 3

   (d) 3 : 2

2. The volume of the largest right circular cone that can be carved out from a solid cube of edge 2 cm is:

   (a) $\frac{4\pi}{3}$ cucm

   (b) $\frac{5\pi}{3}$ cucm

   (c) $\frac{8\pi}{3}$ cucm

(d) $\frac{2\pi}{3}$ cucm

3. **Assertion (A):** The tangents drawn at the end points of a diameter of a circle are parallel.

   Reason (R): The diameter of a circle is the longest chord.

   (a) Both Assertion (A) and Reason (R) are true, and Reason (R) is the correct explanation of Assertion (A).

   (b) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation for Assertion (A).

   (c) Assertion (A) is true, but Reason (R) is false.

   (d) Assertion (A) is false, but Reason (R) is true.

4. AD is a median of $\triangle ABC$ with vertices $A(5, -6)$, $B(6, 4)$, and $C(0, 0)$. The length of AD is equal to:

   (a) $\sqrt{68}$ units

   (b) $2\sqrt{15}$ units

   (c) $\sqrt{101}$ units

   (d) 10 units

5. If the distance between the points $(3, -5)$ and $(x, -5)$ is 15 units, then the values of x are:

   (a) $12, -18$

   (b) $-12, 18$

   (c) $18, 5$

(d) $-9, -12$

6. The center of a circle is at $(2, 0)$. If one end of a diameter is at $(6, 0)$, then the other end is at:

   (a) $(0, 0)$

   (b) $(4, 0)$

   (c) $(-2, 0)$

   (d) $(-6, 0)$

9.1.2. 12

1. The coordinates of the foot of the perpendicular drawn from the point $(0, 1, 2)$ on th x-axis are given by:

   (a) $(1, 0, 0)$

   (b) $(2, 0, 0)$

   (c) $(\sqrt{5}, 0, 0)$

   (d) $(0, 0, 0)$

2. If a line makes an angle of 30° with the positive direction of x-axis, 120° with the positive direction of y-axis, then the angle which it makes with the positive direction of z-axis is:

   (a) 90°

   (b) 120°

   (c) 60°

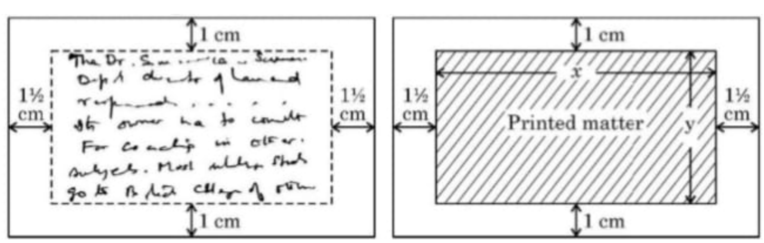
(d) 0°

3. Find the equation of the line which bisects the line segment joining points $A(2,3,4)$ and $B(4,5,8)$ and is perpendicular to the lines $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$

4. If A_1 denotes the area of region bounded by $y^2 = 4x$ and x-axis in the first quadrant and A_2 denotes the area of region bounded by $y^2 = 4x$, $x = 4$, find $A_1 : A_2$.

5. Sand is pouring from a pipe at the rate of $15cm^3/minute$. The falling sand forms a cone on the ground such that the height of the cone is always one- third of the radius of the base. How fast is the height of the sand cone increasing at the instant when the height is $4cm$

6. A rectangular visiting card is to contain $24sq.cm.$ of printed matter. The margins at the top and bottom of the card are to be $1cm$ and the margins on the left and right are to be cm as shown below:



On the basis of the above information, answer the following questions:

(i) Write the expression for the area of the visiting card in terms of x.

(ii) Obtain the dimensions of the card of minimum area.

9.2. 2023

9.2.1. 10

1. A car has two wipers which do not overlap. Each wiper has a blade of length 21cm sweeping through an angle of $120°$. Find the total area cleaned at each sweep of the two blades.

2. Sides AB and BC and median AD of a triangle ABC are respectively proportional to sides PQ and QR and median PM of $\triangle PQR$.Show that $\triangle ABC \sim \triangle PQR$.

3. Through the mid-point M of the side CD of a parallelogram $ABCD$,the line BM is drawn intersecting AC in L and AD (produced) in E.Prove that $EL = 2BL$.

4. In the given figure, O is the center of the circle .AB and AC are tangents drawn to the circle from point A. If $\angle BAC = 65°$, then find the measure of $\angle BOC$.

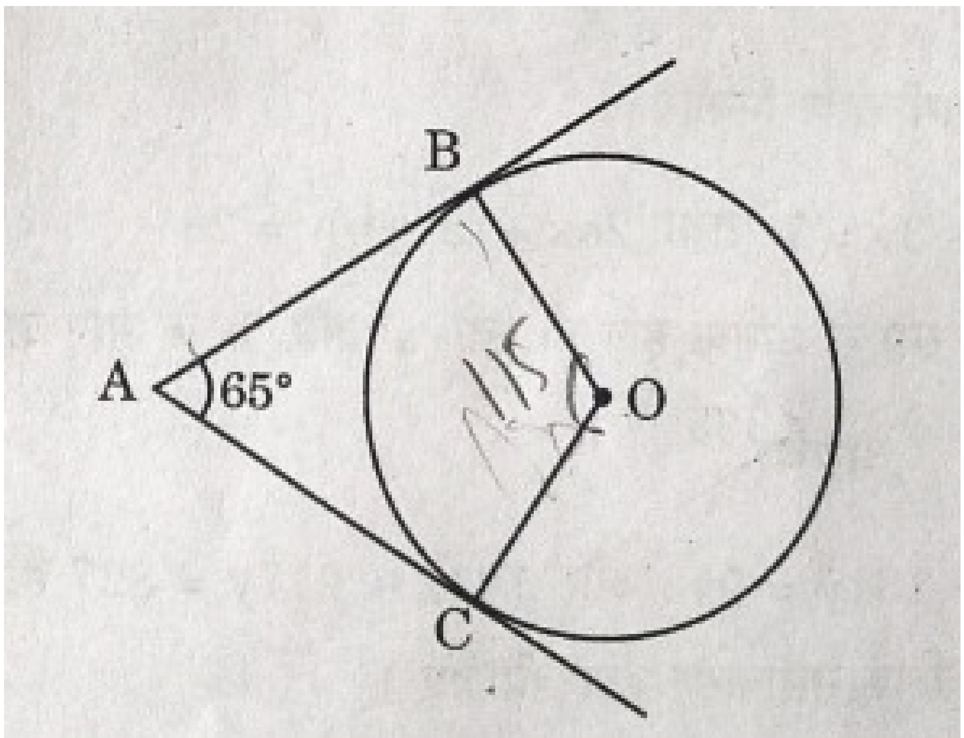


Figure 9.1:

5. In the given figure, O is the centre of the circle and QPR is a tangent to it at P. Prove that $\angle QAP + \angle APR = 90°$.

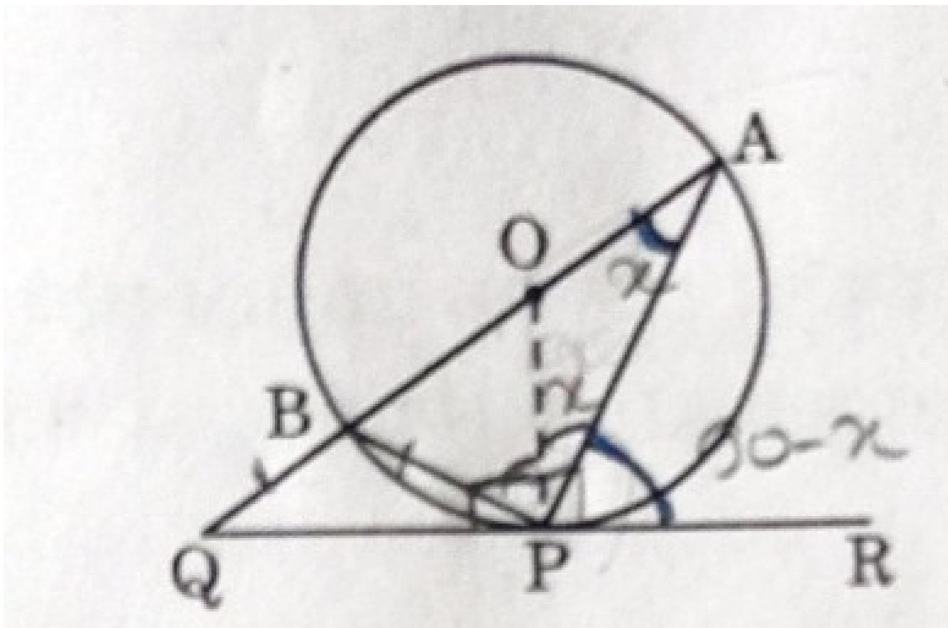


Figure 9.2:

6. In an annual day function of a school, the organizers wanted to give a cash prize along with a memento to their best students.Each memento is made as shown in the figure and its base $ABCD$ is shown from the front side.The rate of silver plating ₹ 20 percm^2.

   Based on the above, answer the following question:

   (a) What is the area of the quadrant $ODOC$?

   (b) Find the area of $\triangle AOB$.

   (c) i. What is the total cost of silver plating the shaded part $ABCD$?

       ii. What is the length of arc CD ?

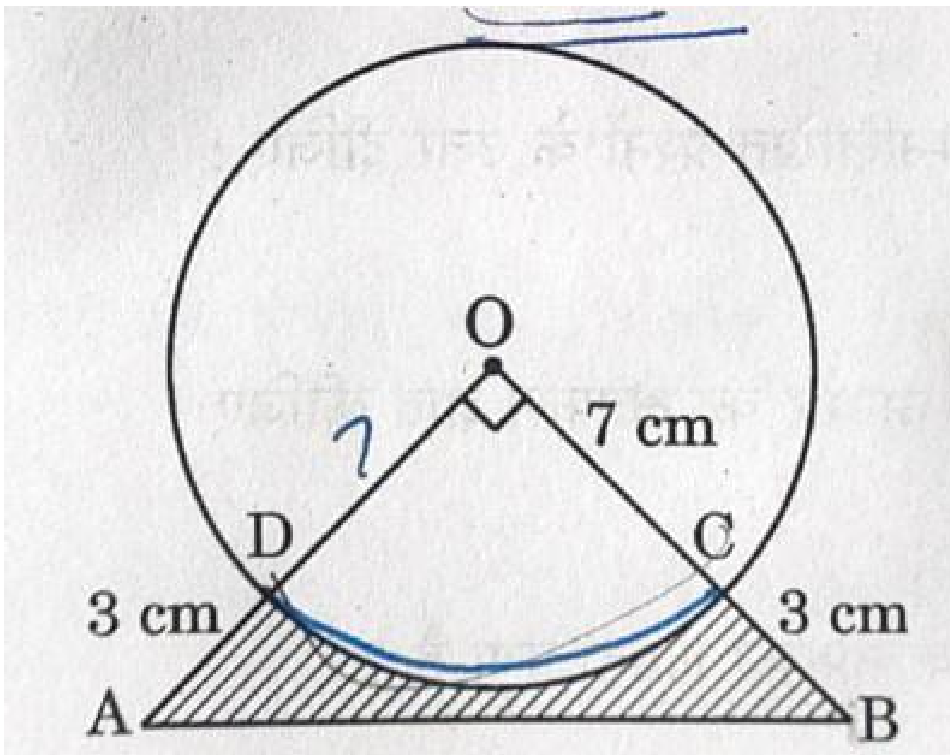


Figure 9.3:

7. In a coffee shop, coffee is served in two types of cups.One is cylindrical in shape with diameter 7cm and height 14cm and the other is hemispherical with diameter 21cm.

   Based on the above, answer the following question:

   (a) Find the area of the cylindrical cup.

   (b) i. What is the capacity of the hemispherical cup?

   ii. Find the capacity of the cylindrical cup.

   (c) What is the curved surface area of the cylindrical cup?

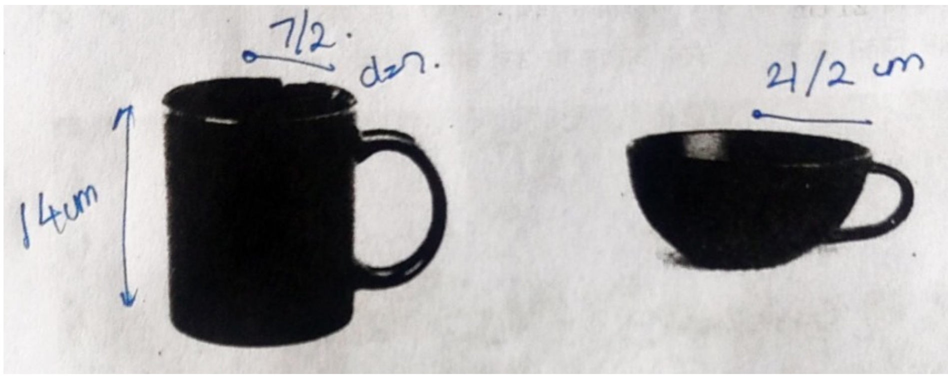


Figure 9.4:

8. Show that the points $(-2, 3), (8, 3)$ and $(6, 7)$ are the vertices of a right-angled triangle.

9. If $Q(0, 1)$ is equidistant from $P(5, -3)$ and $R(x, 6)$,find the values of x.

9.3. 2006

9.3.1. 10

1. In Figure 1, $\angle BAC = 90°$. $AD \parallel BC$. Prove that $AB^2 + CD^2 = BD^2 + AC^2$.

2. In Figure 2, $PT = 6$ cm, $AR = 5$ cm. Find the length of PA.

3. Draw the graphs of the following equations: $3x-4y+6 = 0, 3x+y-9 = 0$ Also, determine the co-ordinates of the vertices of the triangle formed by these lines and the x axis.

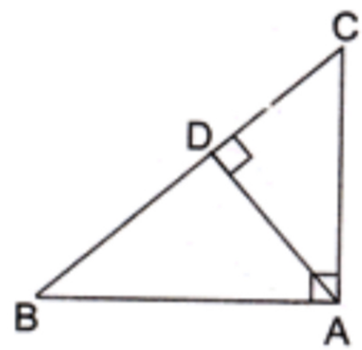


Figure 9.5: 1

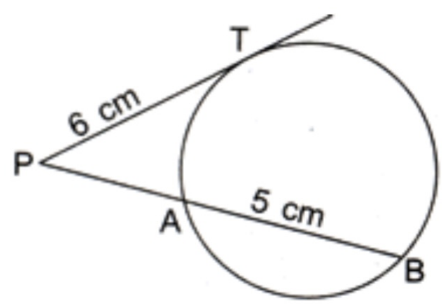


Figure 9.6: 2

4. A solid is in the form of a right circular cylinder with hemispherical ends. The total height of the solid is 58 cm and the diameter of the cylinder is 28cm. Find the total surface area of the solid $\pi \approx \frac{22}{7}$

5. . Construct a triangle ABC in which $BC = 7$ cm, and median $AD =$ 5 cm, $\angle A = 60°$ Write the steps of construction also.

   .

6. Show that the points A(6, 2), B(2, 1), C(1, 5) and D(5, 6) are the vertices of a square

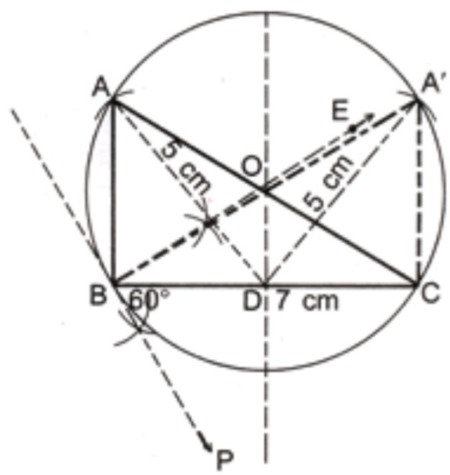


Figure 9.7: 3

7. Find the value of p for which the points (- 5, 1), (1, p) and $4, -2$ are collinear.

8. . Prove that in a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.
   Makeing ue of the above, prove the following:
   in fig:4, $ABCD$ is a fig:4 rhombus. prove that $4AB^2 = AC^2 + BD^2$.

9. Prove that I a line touch a circle and from the point of contact a chord is drawn, the angles which this chord makes with the given line are equal respectively to the angles formed in the corresponding alternate segments. Using the above, do the following:
   AB is a diameter and AC is a chord of a circle such that $\angle BAC = 30°$. The tangent at C intersects AR produced in a point I Prove that $BC = RD$.

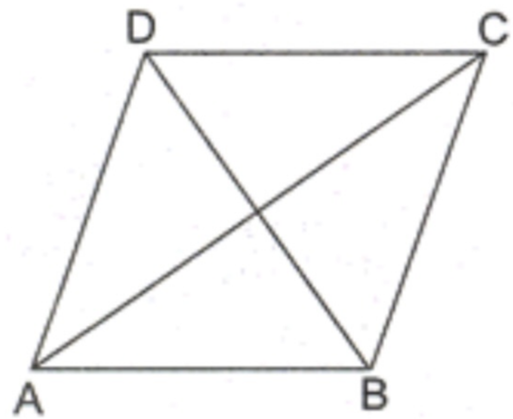


Figure 9.8: 4

10. A man standing on the deck of a ship, which is 10 m above the water level, observes the angle of elevation of the top of a hill as $60°$ and the angle of depression of the base of the hill as 300. Calculate the distance of the hill from the ship and the height of the hill.

11. From a window x meters high above the ground in a street, the angles of elevation and depression of the top and foot of the other house on the opposite side of the street are α and β respectively. Show that the height of the opposite house is $x(1 + \tan\alpha \cot\beta)$ meters.

Chapter 10

sequences

10.1. 2006

10.1.1. 10

1. The 5^{th} term of an Arithmetic Progression (A.P.) is 26 and the 10th term is 51. Determine the 15^{th} term of the A.P.

2. Find the sum of all the natural numbers less than 100 which are divisible by 6.

Chapter 11

Datahandling

11.1. 2006

11.1.1. 10

1. The following table shows the monthly expenditure of company. Draw apie chart for the data.

	Amount (in Rs.)
Wages	4800
Materials	3200
Taxation	2400
Adm. Expenditure	3000
Miscellaneous	1000

2. The Arithmetic Mean of the following frequency distribution is 47.

Determine the value of p.

Classes	Frequency
0 - 20	8
20 - 40	15
40 - 60	20
60 - 80	p
80 - 100	5

Chapter 12

Discrete

Chapter 13

Number Systems

13.1. 2024

13.1.1. 10

1. If two positive integers p and q can be expressed as $p = 18a^2b^4$ and $q = 20a^3b^2$ where a and b are prime numbers, then $\text{LCM}(p, q)$ is:

   (a) $2a^2b^2$

   (b) $180a^2b^2$

   (c) $12a^2b^2$

   (d) $180a^3b^4$

13.2. 2023

13.2.1. 10

1. Prove that $2+\sqrt{3}$ is an irrational number,given that $\sqrt{3}$ is an irrational number.

2. Find by prime factorisation the LCM of the number 18180 and 7575. Also,find the HCF of the two numbers.

Chapter 14

Differentiation

14.1. 2024

14.1.1. 12

1. The derivative of $\tan^{-1}(x^2)$ w.r.t. x is:

    (a) $\frac{x}{1+x^4}$

    (b) $\frac{2x}{1+x^4}$

    (c) $-\frac{2x}{1+x^4}$

    (d) $\frac{1}{1+x^4}$

2. Overspeeding increases fuel consumption and decreases fuel economy as a result of tyre rolling friction and air resistance. While vehicles reach optimal fuel economy at different speeds, fuel mileage usualy decreases rapidly at speeds above 80km/h.

    The relation btween fuel consumption F(l/100 km) and speed V(km/h) under some constraints is given as $F = \frac{V^2}{500} - \frac{V}{4} + 14$.

    On the basis of the above information, answer the following questions:

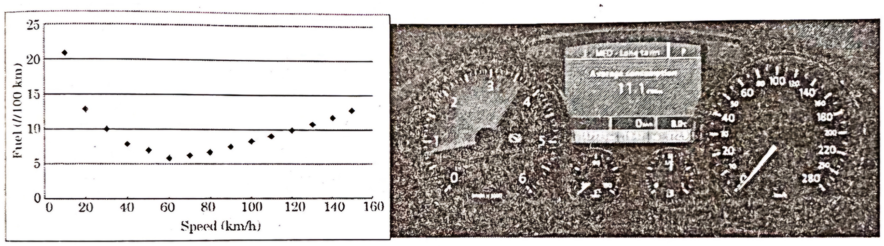


Figure 14.1: 1

(i) Find F, when $V = 40km/h$.

(ii) Find $\frac{dF}{dV}$.

(iii) Find he speed V for which fuel consumption F is minimum.

(iv) Find the quantity of fuel required to travel 600 km at the speed V at which $\frac{dF}{dV} = -0.01$.

1. The differential equation $\frac{dy}{dx} = F(x, y)$ will not be a homogeneous differential equation, if $F(x, y)$ is:

   (a) $\cos x - \sin(\frac{y}{x})$

   (b) $\frac{y}{x}$

   (c) $\frac{x^2+y^2}{xy}$

   (d) $\cos^2(\frac{x}{y})$

2. The degree of the differential equation $(y'')^2 + (y')^3 = x \sin(y')^3$ is:

   (a) 1

   (b) 2

   (c) 3

(d) not defined

3. If $y =$ cosec $(\cot^{-1} x)$, then prove that $\sqrt{1+x^2}\frac{dy}{dx} - x = 0$.

4. If $x = e^{\cos 3t}$ and $y = e^{\sin 3t}$, prove that $\frac{dy}{dx} = -\frac{ylogx}{xlogy}$.

5. Show that: $\frac{d}{dx}(|x|) = \frac{x}{|x|}, x \neq 0$.

6. Find the particular solution of the differetial equation given by $2xy + y^2 - 2x^2\frac{dy}{dx} = 0; y = 2$, when x=1.

7. Find the general solution of the differential equation:

$$ydx = (x + 2y^2)\text{dy}$$

8. Verify whether the function f defined by $f(x) = \begin{cases} x\sin\left(\frac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$

   is continuous at x = 0 or not.

9. Check for differentiability of the function f defined by $f(x) = |x - 5|$, at the point $x = 5$.

10. Find the particular solution of the differential equation :

    $\frac{dy}{dx} - 2xy = 3x^2e^{x^2}; y(0) = 5$

11. Solve the following differential equation:

    $x^2dy + y(x + y)dx = 0$

12. Find the values of a and b so that the following function is differentiable for all values of x $f(x) = \begin{cases} ax + b, & x > -1 \\ bx^2 - 3, & x \leq -1 \end{cases}$

13. Find $\frac{dy}{dx}$, if $(\cos x)^y = (\cos y)^x$.

14. If $\sqrt{1-x^2}+\sqrt{1-y^2}=a(x-y)$, prove that $\frac{dy}{dx}=\sqrt{\frac{1-y^2}{1-x^2}}$.

Chapter 15

Integration

15.1. 2024

15.1.1. 12

1. $\int\limits_{0}^{\frac{\pi}{2}} \frac{\sin x - \cos x}{1 + \sin x \cos x} dx$ is equal to:

    (a) π

    (b) $Zero(0)$

    (c) $\int\limits_{0}^{\frac{\pi}{2}} \frac{2 \sin x}{1 + \sin x \cos x} dx$

    (d) $\frac{\pi^2}{4}$

2. Find : $\int \frac{e^{4x} - 1}{e^{4x} + 1} dx$

3. Evaluate:

    $\int\limits_{2} -2\sqrt{\frac{2-x}{z+x}} dx$

4. Find:

    $\int \frac{1}{x[(logx)^2 - 3logx - 4]} dx$

5. Find: $\int x^2 \cdot \sin^{-1}(x^{\frac{3}{2}}) dx$

6. Find: $\int \cos^3 x \cdot e^{\log(\sin x)} dx$

7. Find: $\int \frac{1}{5+4x-x^2} dx$

8. Evaluate: $\int_0^\pi \frac{e^{\cos x}}{e^{\cos x}+e^{-\cos x}} dx = 0$

9. Find the area of the region bounded by the curve $4x^2 + y^2 = 36$. using integration.

Chapter 16

Functions

16.1. 2024

16.1.1. 12

1. **Assertion (A):** Domain of $y = \cos^{-1}(x)$ is [-1, 1].

   Reason (R): The range of the principal value branch of $y = \cos^{-1}(x)$ is $[0, \pi]$- $\{\frac{\pi}{2}\}$.

Chapter 17

Matrices

17.1. 2024

17.1.1. 12

1. If the sum of all the elements of 3×3 scalar matrix is 9, then the product of all elements is:

   (a) 0

   (b) 9

   (c) 27

   (d) 729

2. If $\begin{vmatrix} -a & b & c \\ a & -b & c \\ a & b & -c \end{vmatrix} = kabc$, then the value of k is:

   (a) 0

   (b) 1

   (c) 2

(d) 4

3. If $A = [a_{ij}]$ be a 3×3 where $a_{ij} = i - 3j$,then which of the following is false?

   (a) $a_{11} < 0$

   (b) $a_{12} + a_{21} = -6$

   (c) $a_{13} > a_{31}$

   (d) $a_{31} = 0$

4. If $F(x) = \begin{pmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 0 \end{pmatrix}$ and $[F(x)]^2 = F(kx)$, then the value of k is:

   (a) 1

   (b) 2

   (c) 0

   (d) -2

5. Assertion (A): For any symmetric matrix A, $B'AB$ is a skew-symmetric matrix.

   Reason (R): A square matrix P is kew-symmetric if $P' = -P$

   (a) Both Assertion and Reason are true, and Reason is the correct explaination of Assertion.

   (b) Both Assertion and Reason are true, but Reason is not the correct explaination of Assertion.

(c) Assertion is true, but Reason is false.

(d) Assertion is false, but Reason is true.

6. Solve the following system of equations,using matrices:

$\frac{2}{x}+\frac{3}{y}+\frac{10}{z}=4,\ \frac{4}{x}-\frac{6}{y}+\frac{5}{z}=1,\ \frac{6}{x}+\frac{9}{y}-\frac{20}{z}=2$ where $x,y,z\neq 0$

7. If $A=\begin{pmatrix}1 & \cot x\\ -\cot x & 1\end{pmatrix}$,then show that $A'A^{-1}=\begin{pmatrix}-\cos 2x & -\sin 2x\\ \sin 2x & -\cos 2x\end{pmatrix}$

8. If $A=\begin{pmatrix}1 & 2 & -3\\ 2 & 0 & -3\\ 1 & 2 & 0\end{pmatrix}$ then find A^{-1} and use it to solve the following system of equations :

$$x+2y-3z=1 \tag{17.1}$$

$$2x-3z=2 \tag{17.2}$$

$$x+2y=3 \tag{17.3}$$

9. Find the product of the matrices $\begin{pmatrix}1 & 2 & -3\\ 2 & 3 & 2\\ 3 & -3 & 4\end{pmatrix}\begin{pmatrix}-6 & 17 & 13\\ 14 & 5 & -8\\ -15 & 9 & -1\end{pmatrix}$,then find AB and use it to solve the system of linear equations :

$$x-2y=3 \tag{17.4}$$

$$2x-y-z=2 \tag{17.5}$$

$$-2y+z=3 \tag{17.6}$$

Chapter 18

Trignometry

18.1. 2024

18.1.1. 10

1. If $\sec\theta - \tan\theta = m$, then the value of $\sec\theta + \tan\theta$ is:

   (a) $1 - \frac{1}{m}$

   (b) $m^2 - 1$

   (c) $\frac{1}{m}$

   (d) $-m$

2. If $\cos(\alpha+\beta) = 0$ then the value of $\cos\left(\frac{\alpha+\beta}{2}\right)$ is equal to:

   (a) $\frac{1}{\sqrt{2}}$

   (b) $\frac{1}{2}$

   (c) 0

   (d) $\sqrt{2}$

1. Simplify: $\cos^{-1} x + \cos^{-1}\left[\frac{x}{2}\frac{\sqrt{3-3x^2}}{2}\right]; -\frac{1}{2} \le x \le 1$

18.1.2. 12

1. Find the value of $\tan^{-1}(-\frac{1}{\sqrt{3}}) + \cot^{-1}(\frac{1}{\sqrt{3}}) + \tan^{-1}[\sin(-\frac{\pi}{2})]$

2. If $a = \sin^{-1}\left(\frac{\sqrt{2}}{2}\right) + \cos^{-1}\left(\frac{-1}{2}\right)$ and $b = \tan^{-1}(\sqrt{3}) + \cot^{-1}\left(\frac{-1}{\sqrt{3}}\right)$, then find the value of $a + b$.

18.2. 2023

18.2.1. 10

1. If $4\cot^2 45° - \sec^2 60° + \sin^2 60° + p = \frac{3}{4}$, then find the value of p.

2. If $\cos A + \cos^2 A = 1$,then find the value of $\sin^2 A + \sin^4 A$

3. The length of the shadow of a tower on the plane ground is $\sqrt{3}$ times the height of the tower. Find the angle of elevation of the sun.

4. The angle of elevation of the top of a tower from a point on the ground which is 30m away from the foot of the tower,is 30°.Find the height of the tower.

5. Prove that :

$$\left(\frac{1}{\cos\theta} - \cos\theta\right)\left(\frac{1}{\sin\theta} - \sin\theta\right) = \frac{1}{\tan\theta + \cot\theta} \tag{18.1}$$

6. As observed from the top of a 75m high lighthouse from the sea-level, the angles of depression of two ships are 30° and 60°. If one ship is exactly behind the other on the same side of the lighthouse, find the

distance between the two ships.

$\left(Use\sqrt{3} = 1.73\right)$

7. From a point on the ground, the angle of elevation of the bottom and top of a transmission tower fixed at the top of 30m high building are 30° and 60°, respectively. Find the height of the transmission tower. $\left(Use\sqrt{3} = 1.73\right)$.